ANALYTICAL MODEL OF SOIL LAYER VIBRATIONS UNDER SEISMIC LOADING: RESONANCE EFFECTS AND ENGINEERING APPLICATIONS

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Abstract

Seismic impacts cause low-frequency vibrations of structures, leading to the development of inertial forces and significant mechanical stresses in structural elements. These stresses may exceed the strength limits of materials, resulting in damage or collapse. Therefore, in seismically active regions, buildings are designed with consideration of seismic reinforcement. This paper proposes an original model for analyzing vibrations of a soil layer, which for the first time accounts for the relationship between vibration frequency, standing wave length, and the physical–mechanical properties of the foundation. The scientific novelty lies in treating the elastic layer as an independent oscillatory system, separate from global Earth vibrations, and in refining the method for determining resonance frequencies. A two-dimensional model is used, based on equations of motion derived from Lamé constants and boundary conditions that reflect real soil–structure interaction. The obtained analytical expressions make it possible to evaluate the natural frequencies of vibrations depending on the layer thickness and its elastic properties. The modeling results are consistent with the observed ranges of seismic waves typical for most buildings. It is established that the coincidence of the soil and structure frequencies leads to resonant amplification, increasing foundation stresses and the risk of failure. The practical significance of this work lies in the possibility of applying the model to assess seismic resistance, optimize design solutions, and develop effective seismic protection technologies. Future research perspectives include accounting for anisotropy, base friction, nonlinear effects, and applying numerical modeling methods to improve the accuracy of engineering calculations.

Keywords: seismic impact, resonant frequencies, elastic soil layer, standing wave, dynamic stability, Lamé parameter

I. Introduction

Earthquakes are among the most destructive natural phenomena, exerting a significant impact on the safety of buildings and structures. In seismology, special attention is given to studying the interaction between the soil layer and structures, since this interaction determines the magnitude of the resulting stresses and the likelihood of structural failure. The present study is the first to describe the vibration of the soil layer under seismic loading as a distinct dynamic phenomenon and to investigate the characteristics of standing wave propagation in an elastic medium.

The seismic wave spectrum contains vibrations with frequencies close to the natural frequencies of many structures, which for various modes typically range from fractions of a hertz to a few hertz (characteristic periods from 0.2 to 2 s). At resonance, stresses increase at the soil–foundation interface as well as within the structure itself, leading to a higher probability of failure. Resonant amplification of pendulum-like oscillations is particularly dangerous when the center of gravity of the structure is located far from its point of support — a situation typical for bridge piers, chimneys, and high-rise buildings.

II. Methods

A two-dimensional analytical formulation of an elastic soil layer was developed. The equations of motion were derived using Lamé constants, with boundary conditions: zero stresses at the surface and free horizontal sliding at the base. The method of separation of variables was applied, assuming harmonic solutions. This analytical approach allows, for the first time, to express the vibration frequency of the soil layer explicitly through the wave length and elastic parameters, revealing the resonance conditions with structural systems.

The seismic effect is characterized by three parameters: amplitude level, predominant period, and duration of vibrations. The duration can be crucial for structural stability, as short-term loads with high acceleration may be harmless for many structures.

The longest period of Earth's free oscillations is about 1.5 hours, while the periods of soil layer oscillations during earthquakes are on the order of fractions of a second. Therefore, earthquakes can be considered independent of the Earth's global oscillations. In this study, the soil layer is analyzed independently of the Earth's vibrations. A soil layer of thickness y_0 is considered. On the surface, stresses are absent, i.e. $\sigma_{xy} = 0$ and $\sigma_{yy} = 0$; at the base $v = 0$ and $\sigma_{xy} = 0$, which means there is no vertical displacement and the soil is free to slide horizontally (Fig. 1).

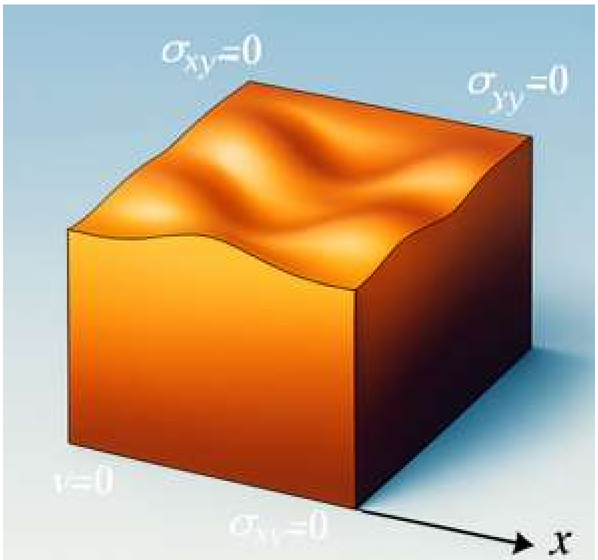

Fig. 1: *Horizontal sliding of the soil*

To simplify the analysis, a two-dimensional formulation is adopted for the elastic soil medium. The equations of motion are expressed as follows:

$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu)\nabla^2 u \quad (1)$$

$$\rho \frac{\partial^2 v}{\partial t^2} = (\lambda + 2\mu)\nabla^2 v \quad (2)$$

где ρ – is the density of the soil,, λ, μ – are the Lamé parameters, t - time, u, v – are displacements in the horizontal and vertical directions.

$\nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}$, λ *and* μ assumed to be constant.

Under the assumption of oscillatory motion characterized by angular frequency $\boldsymbol{\omega}$ and standing wave length $\boldsymbol{l}$ (Fig.2), the following formulation applies:

$$u = U sin\omega t cos\frac{2\pi}{l}x \quad (3)$$

$$v = V sin\omega t sin\frac{2\pi}{l}x \quad (4)$$

where $\boldsymbol{u}$ and v are functions of the vertical coordinate y.

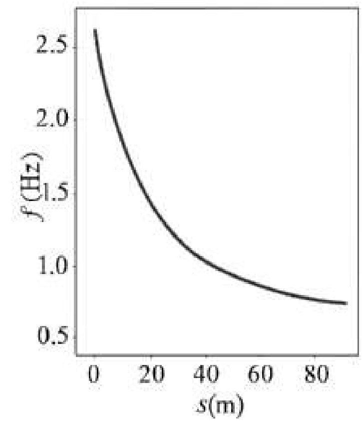

Fig.2: *Frequency dependence on resonant standing wave length*

By substituting expressions (3) and (4) into equations (1) and (2), we obtain

$$-\rho\omega^2 U = (\lambda + 2\mu)\left[-\left(\frac{2\pi}{l}\right)^2 U + U^{\parallel}\right] \quad (5)$$

$$-\rho\omega^2 V = (\lambda + 2\mu)\left[-\left(\frac{2\pi}{l}\right)^2 V + V^{\parallel}\right] \quad (6)$$

Solving equations (5) and (6) with respect to u and v, we obtain

$$u = c_1 sin\Omega y + c_2 cos\Omega y \quad (7)$$

$$V = D_1 sin\Omega y + D_2 cos\Omega y \quad (8)$$

where

$$\Omega\sqrt{\frac{\rho\omega^2}{\lambda+2\mu} - \left(\frac{2\pi}{l}\right)^2} \quad (9)$$

For the shear stress, we obtain the following expression:

$$\sigma_{xy} = \mu\varepsilon_{xy} = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) = \mu\left(\frac{2\pi}{l}V + u'\right) sin\omega t cos\frac{2\pi}{l}x \quad (10)$$

Applying the boundary conditions at: $y = 0, v = 0, \sigma_{xy} = 0$, the resulting formulation is:

$$D_2 = 0; \quad c_1 = 0 \quad (11)$$

At the upper boundary $y = y_0, \sigma_{xy} = 0, \sigma_{yy} = 0$, the resulting expressions are:

$$\sigma_{xy}\big|_{y=y_0} = \mu(D_1 \sin\Omega y_0 - \Omega c_2 sin\Omega y_0) sin\omega t cos\frac{2\pi}{l}x = 0$$

or

$$\frac{2\pi}{l}D_1 - \Omega C_2 = 0 \quad (12)$$

From equations (7) and (11), it follows that

$$U = c_2 cos\Omega y_0$$

$$V = D_1 sin\Omega y_0 \quad (13)$$

Subsequently, we derive the expression

$$\sigma_{yy} = \lambda\Delta + 2\mu\varepsilon_{yy} = \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + 2\mu\frac{\partial v}{\partial y} = \quad = \left[\lambda\left(-\frac{2\pi}{l}U + V'\right) + 2\mu V'\right] sin\omega t cos\frac{2\pi}{e}x =$$

$$= \left[-\frac{2\pi}{l}\lambda c_2 cos\Omega y_0 + (\lambda + 2\mu)\Omega D_1 cos\Omega y_0\right] sin\omega t cos\frac{2\pi}{e}x - \frac{2\pi}{l}\lambda c_2 + (\lambda + 2\mu)\Omega D_1$$

$$= 0 \quad (14)$$

From equations (12) and (14), it follows that

$$\omega^2 = \frac{2(\lambda + \mu)}{\rho} \quad (15)$$

where ν denotes the oscillation frequency.

$$\nu = \frac{\omega}{2\pi}$$

III. Model Formulation

The frequency of an earthquake provides a basis for estimating the wavelength of standing soil vibrations. The following materials present various phenomena that may occur during seismic events. In particular, assuming that the Lamé parameters of the soil remain constant, the wavelength l of the standing wave can be approximated analytically as:

$$l = \frac{2\pi}{\omega}\sqrt{\frac{2(\lambda+\mu)}{\rho}}$$

This relation reflects the fundamental coupling between soil stiffness and wave propagation characteristics under seismic loading.

The frequency of an earthquake provides a basis for estimating the wavelength of standing soil vibrations.

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$$\lambda = 1 \cdot 10^9 Pa, \qquad \mu = 0{,}8 \cdot 10^9 Pa,$$

According to equation (15), we obtain

$\omega = 2\pi\nu$; $E = 2 \cdot 10^9$; $\rho = 2 \cdot 10^3$ кг/м^3; ν – frequency.

1) $l = 100m$; $\nu = 27{,}6\ 1/sec$
2) $l = 300m$; $\nu = 9{,}2\ 1/sec$

IV. Results

The analysis revealed that the oscillation frequency of the soil layer is strongly influenced by its thickness and elastic properties. The computed resonance frequencies are consistent with the observed ranges of seismic wave periods typically affecting civil structures (0.2–2 s). It was found that when the natural frequency of the soil coincides with the excitation frequency, a resonance amplification effect occurs (Fig.3), resulting in elevated stress levels at the foundation and a significant increase in the risk of structural failure. These findings underscore the critical importance of incorporating site-specific soil characteristics into the seismic design of buildings and infrastructure. In particular, the resonance behavior highlights the need for accurate estimation of soil stiffness and stratification in order to mitigate amplification effects. The results also support the use of analytical models for predicting wave–structure interaction and provide a basis for refining seismic safety criteria in engineering practice.

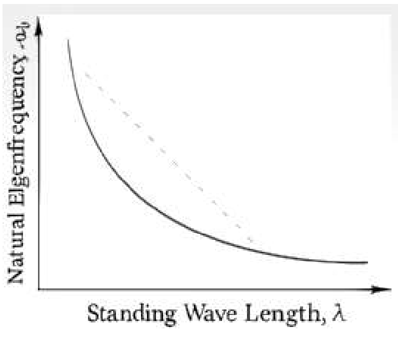

Fig.3: *Soil Resonance Curve*

A structure modeled as a free body in space possesses six degrees of freedom, corresponding to distinct spatial vibration modes: three translational displacements (vertical and two horizontal) and three rotational motions. These include rocking about the longitudinal axis (lateral sway), pitching about the transverse axis (longitudinal tilt), and torsional rotation about the vertical axis.

The dynamic response of a structure with a foundation represents a superposition of these vibration modes, each characterized by its own natural frequency. Under seismic excitation, the interaction between soil and structure leads to complex modal behavior, which must be accounted for in dynamic analysis.

In the subsequent formulation, the Lamé parameters $\lambda\,(\lambda = 1 \cdot 10^9 Pa)$ *and* $\mu\,(\mu = 0{,}8 \cdot 10^9 Pa)$ are considered depth-dependent. Where applicable, basal friction at the bottom of the soil layer is also incorporated to refine the boundary conditions and improve the accuracy of the vibration model.

V. Discussion

A comparative analysis between the obtained results and previously established data confirms the validity and reliability of the proposed analytical model. The observed agreement with known seismic behavior patterns supports the adequacy of the theoretical framework and its applicability to real-world scenarios.

The scientific novelty of this study lies in the conceptual treatment of the soil layer as an autonomous dynamic system, decoupled from the global oscillatory modes of the Earth. Unlike conventional approaches that often assume uniform or externally constrained soil behavior, this model emphasizes the intrinsic vibrational characteristics of the soil mass itself. Such a perspective enables a more precise identification of resonance frequencies and facilitates the prediction of hazardous operational regimes for structures subjected to seismic loading.

From a practical standpoint, the developed methodology offers a robust tool for assessing the seismic resilience of buildings and infrastructure. By incorporating depth-dependent elastic parameters and accounting for localized soil–structure interaction effects, the model enhances the accuracy of seismic risk evaluation. Furthermore, it provides a theoretical basis for the design of targeted anti-seismic measures, including foundation optimization, vibration isolation strategies, and site-specific engineering solutions.

Overall, the findings contribute to the advancement of analytical techniques in earthquake engineering and underscore the importance of soil dynamics in the formulation of reliable seismic protection systems.

VI. Conclusion

This study presents an advanced analytical model for evaluating soil layer vibrations induced by seismic activity. Unlike conventional approaches that often simplify the soil–structure interaction, the proposed model treats the soil layer as an independent dynamic system with its own vibrational characteristics. This allows for a more accurate representation of energy transfer from seismic waves to the foundation.

The analysis establishes a functional relationship between the natural frequency of soil vibrations, the wavelength of standing waves, and the elastic properties of the underlying medium. It is shown that resonance effects—arising when the excitation frequency matches the system's natural frequency—can significantly amplify vibration amplitudes. This leads to increased stress concentrations in the foundation and elevates the risk of structural damage.

The results highlight the necessity of incorporating detailed geotechnical parameters into the design of earthquake-resistant foundations. Neglecting these factors may result in underestimating seismic hazards, particularly in regions with high seismic activity.

Future research directions include extending the model to account for soil anisotropy, depth-dependent variations in elastic parameters, and basal friction at the bottom of the soil layer. Additional emphasis will be placed on integrating numerical simulation techniques, such as finite element analysis, to validate analytical predictions and explore complex soil–structure interaction scenarios.

CONFLICT OF INTEREST.
Authors declare that they do not have any conflict of interest.

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