MATRIX MODEL OF ACCURACY IN MACHINING CONICAL SURFACES ON CNC LATHES

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Abstract

The article presents the development of a matrix model for accuracy in machining conical surfaces on CNC lathes. Spatial computational schemes of elastic displacements of technological subsystems were constructed based on the balance of force interaction between the cutting tool and the workpiece. These schemes cover both single-tool setups with a rotating carriage and single-tool two-coordinate setups. Additionally, models of dimensional distortions were developed, taking into account the characteristics of the setups, and were analyzed based on cutting conditions, cone angle, and the comprehensive compliance characteristics of the technological systems. It is also possible to calculate setup parameters using the proposed models. The developed models can be applied in computer-aided design (CAD) systems for machining conical surfaces on CNC lathes, as well as for studying and predicting the influence of various parameters on the dimensional accuracy achieved during conical surface machining..

Keywords: conical surface machining, single-tool setups with rotating carriage, single-tool two-coordinate setups, matrix model of accuracy, CNC lathes, dimensional distortions, combined compliance matrix of technological system subsystems.

I. Introduction

In machining processes such as turning, milling, and grinding, the dimensional accuracy and form error of the produced surface are determined by the dynamics of the interaction (i.e., the force characteristics of the process) between the tool and the workpiece. The interaction of the main elements of the technological system is significantly influenced by auxiliary elements, such as fixtures, handles, machine carriages, and others [1-10].

From the perspective of the mechanism of error formation in machining, the technological system is considered a feedback system [3-4]. The application of the laws of analytical mechanics allows for the description of the error formation mechanism in machining, but it leads to nonlinear systems of equations.

Let us examine how the developed general matrix models of force interaction among the subsystems of the technological system [3-4] during the machining of conical surfaces are transformed into models of machining errors.

II. Single-tool setups with a rotating carriage

On conventional vertical multi-spindle semi-automatic lathes [1], as well as on several modern CNC machines [2-4], a rotating carriage is provided, which allows for the machining of conical surfaces using a contour cutting tool. The guideways of the carriage are inclined at an angle corresponding to the cone's generatrix, enabling the feed along the contour of the workpiece being machined. In such a setup, the cutting tool is oriented normal to the machined surface, meaning that its elastic displacements caused by the force interaction between the elements of the technological system are also measured relative to this normal. At the same time, the resultant dimension (diameter) is measured perpendicular to the axis of the machined surface. The misalignment of the cutting tool's orientation and the resultant dimension leads to a change in the degree of influence of each component of the cutting force on the distortion of the resultant dimension compared to setups using longitudinal or transverse carriages aligned with the axes of the workpiece and the directions of the dimensions being machined. As a result, the model of dimensional distortion [3, 4] developed for machining a workpiece with a carriage aligned along its axes is not applicable in this case. The situation is further complicated by the fact that the formulas for the components of the cutting forces are developed in the tool's coordinate system: the y-axis is directed normal to the feed vector, the x-axis is along the feed vector, and the z-axis is perpendicular to the first two axes. As a result, when determining the resultant force, it is necessary to account for the angle of rotation of the tool's coordinate system in relation to the workpiece's coordinate system. Consequently, the traditional scheme for calculating the coordinates of a single-tool setup [3, 4] is transformed into the form presented in Figure 1 [11].The situation is further complicated by the fact that the formulas for the components of the cutting forces are developed in the tool's coordinate system: the y-axis is directed normal to the feed vector, the x-axis is along the feed vector, and the z-axis is perpendicular to the first two axes. As a result, when determining the resultant force, it is necessary to account for the angle of rotation of the tool's coordinate system in relation to the workpiece's coordinate system. Consequently, the traditional scheme for calculating the coordinates of a single-tool setup [3, 4] is transformed into the form presented in Figure 1 [11].

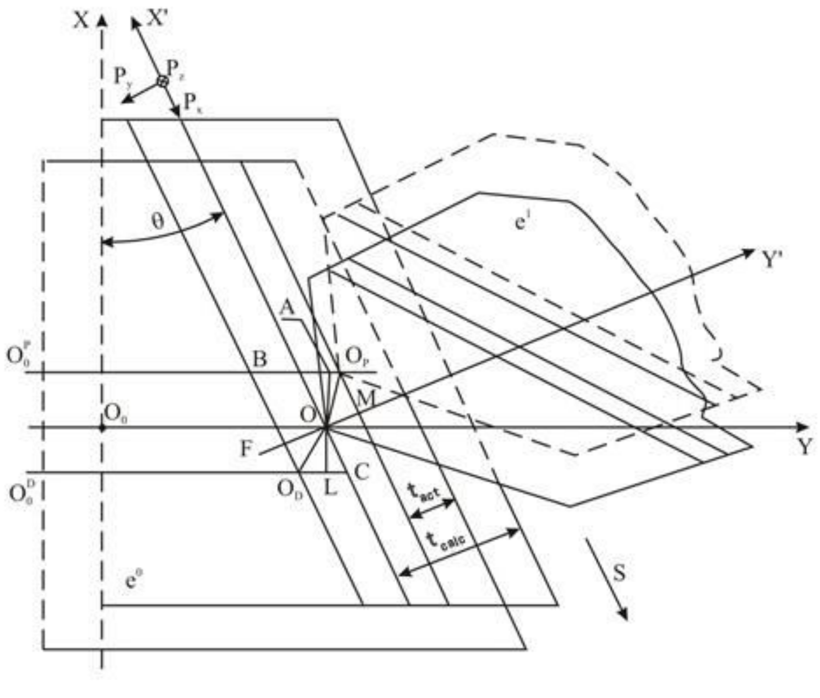

Figure 1: *Spatial computational scheme of elastic displacements in technological subsystems during single-tool machining with a rotating carriage [11]*

In Figure 1, the following designations are presented: ODL - movement of the "spindle-workpiece" subsystem along the OY axis, OL - movement of the "spindle-workpiece" subsystem along the OX axis, movement of the "carriage-tool" subsystem along the OY' axis, OpM - movement of the "carriage-tool" subsystem along the OX' axis, BO_P – dimensional distortion in the diametral direction, t_{calc} – calculated depth of cut, t_{act} – actual depth of cut after processing, S- feed rate along the contour, e^0, e^1 - compliance matrices of the "spindle-workpiece" and "carriage-tool" subsystems, respectively, θ – inclination angle of the cone's generatrix relative to the axis of the processed workpiece.

As can be seen, the considered setup allows the decomposition of the technological system into two subsystems: O_0XYZ– the coordinate system of the "spindle-workpiece" subsystem, and $OX'Y'Z'$- the coordinate system of the "carriage-tool" subsystem [3, 4]. As a result, the problem of elastic displacements in the technological system with this setup is reduced to studying a two-body system, for which a matrix model of the contact point's displacements has been developed [3]. Under the influence of cutting forces, the workpiece undergoes displacement ($O_0^D O_D$) due to elastic deformations in the "spindle-workpiece" subsystem. In response to the cutting forces, the tool also undergoes displacement ($O_0^P O_P$) due to elastic deformations in the "carriage-tool" subsystem, but in the opposite direction.

The basis for the model of dimensional distortions in the studied setup can be taken as the matrix model [3-4], developed for the setup with the carriage oriented along the coordinate axes of the workpiece:

$$\overline{g^{01}} = e^{01}\,\bar{P} \tag{1}$$

where $\overline{g^{01}}$ is the vector of dimensional distortions, e^{01} is the combined compliance matrix of the subsystems of the technological system, and $\overline{P}$ is the resulting cutting force applied to the workpiece.

Equation (1), written in matrix form, is developed for the workpiece coordinate system. For this reason, to adapt it to the studied setup, the vector $\overline{P}$ must also be expressed in the same coordinate system. Using known coordinate transformation relations, the vector $\overline{P}$ is obtained as:

$$\bar{P} = \begin{pmatrix} P_x\cos\theta + P_y\sin\theta \\ P_y\cos\theta - P_x\sin\theta \\ P_z \end{pmatrix} \tag{2}$$

In formula (2), P_x, P_y, and P_z are the components of the cutting force in the tool's coordinate system. The following well-known formulas from cutting theory apply to them: $p_i = c_i t^{x_i} s^{y_i} v^{z_i}$ $i = x; y; z$ [12], where c_i– are coefficients depending on the type of material being processed and other cutting conditions; t, s, v – represent the depth of cut, feed rate, and cutting speed, respectively; x y z – are exponents corresponding to the respective variables.

Considering these formulas and applying relation (2), the matrix model for the case of single-tool processing (1) can be represented in expanded form as follows [11]:

$$\begin{pmatrix} g_x \\ g_y \\ g_z \end{pmatrix} = \begin{pmatrix} e_{xx}^{01} & e_{xy}^{01} & e_{xz}^{01} \\ e_{yx}^{01} & e_{yy}^{01} & e_{yz}^{01} \\ e_{zx}^{01} & e_{zy}^{01} & e_{zz}^{01} \end{pmatrix} = \begin{pmatrix} c_x t^{x_x} s^{y_x} v^{z_x}\cos\theta + c_y t^{x_y} s^{y_y} v^{z_y}\sin\theta \\ c_y t^{x_y} s^{y_y} v^{z_y}\cos\theta - c_x t^{x_x} s^{y_y} v^{z_x}\sin\theta \\ c_z t^{x_z} s^{y_z} v^{z_z} \end{pmatrix} \tag{3}$$

Considering the projections of the displacement vector $\overline{g}$ and the dimensional distortions in the directions of interest: g_y - diametral dimension (along the Y axis); g_x - linear dimension (along the X axis). The magnitude of the distortion of the diametral dimension g_y will be determined by the element-wise multiplication of the second row of the compliance matrix of the technological system e^{01} and the force vector.

The obtained model describes the distortions of the performed dimensions and the distortion of the shape of the processed surface. It can be used to calculate the setup dimensions and serves as the basis for determining other indicators of processing accuracy.

III. Single-tool two-coordinate setups

CNC lathes allow for the machining of conical and even complex contoured surfaces without the need to rotate the carriage [13-16]. The formation of the contoured surface is achieved by superimposing two coordinate movements of the tool that are performed simultaneously: the controlled movement along the Z coordinate, referred to as feed Sz, and the controlled movement along the X coordinate, referred to as feed Sx. The main difference between such setups and those discussed in [13-19] lies in the control over two coordinates. Therefore, it is reasonable to refer to these setups as two-coordinate setups.

Since the tool's trajectory in a single-tool two-coordinate setup is formed by two simultaneous feeds, the scheme for forming the processing error becomes significantly more complicated. In the single-tool single-coordinate setup [16] and in the single-tool setup with a rotating carriage [5], the tool was oriented relative to the processed surface (perpendicular to it); however, in the current case, the tool is oriented relative to the axis of the processed surface. Therefore, when determining the distortion of the performed dimension, it is necessary to account for the transition from the tool's coordinate movements to movements relative to the processed surface.

It is evident that for single-tool two-coordinate setups, the decomposition of the technological system into two subsystems is quite acceptable [13-19]. Therefore, as a methodological basis for the model of dimensional distortions in such setups, one can use the analytical model of movements in a two-body system with elastically deformable connections [4,11].

To identify the fundamental features of the influence of two-coordinate control on the distortion of the performed dimension, let us first consider the simplest case of machining—a conical surface. Taking into account the specifics of error formation during two-coordinate control of the machining process, the overall calculation scheme of the deformational interaction of the two-body system is transformed for the technological subsystems to the form presented in Figure 2.

Here, matrices e^0 and e^1 characterize the compliances of the technological subsystems—subsystem 0 (spindle – chuck – workpiece) and subsystem 1 (carriage – tool holder – tool), respectively. The line G describes the contour of the workpiece in its initial state (without any force acting on the technological system). The line D describes the contour of the part, i.e., the contour after the allowance has been removed (its calculated position). Point M is the calculated position of the tool tip (which determines the setup dimension).

After applying cutting forces $\bar{P}$, the technological subsystems undergo elastic displacements g, and accordingly, the previously described elements of the calculation model also shift. As a result of the displacement of subsystem 0, the contour of the workpiece will move to line G0, and line D (the calculated contour of the part) will take the position of line D0. Point M is the contact point between the workpiece and the tool; therefore, we can consider that there are two overlapping points M here: one belongs to the workpiece, and the other belongs to the tool tip. As a result of the force applied, both technological subsystems experience elastic displacements, causing point M on the workpiece (subsystem 0) to move to position M0, and point M on the tool tip (subsystem 1) to move to position M1. Thus, the actual contour of the part formed in the force interaction moves to line D1.

Thus, the tool tip, instead of the setup radius Rs, will form the actual radius Ract, thereby determining the distortion of the performed dimension.

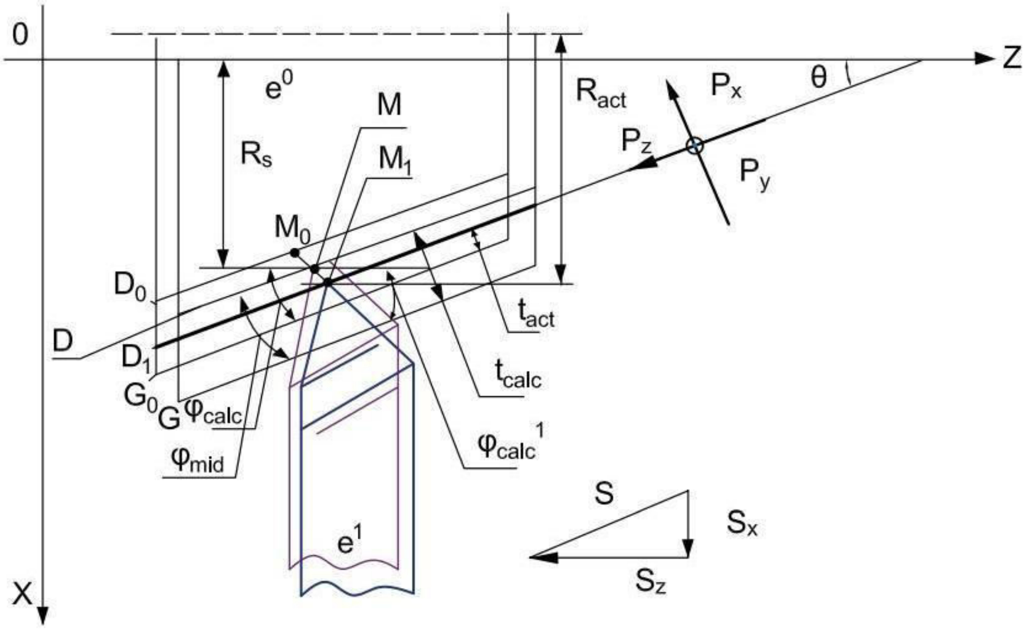

Figure 2: *The calculation scheme of elastic displacements of technological subsystems during single-tool two-coordinate machining of an external cone*

The calculation scheme in Figure 2 illustrates the force interaction between two contacting subsystems. In analytical mechanics, such a scheme is reduced to the interaction of a two-body system with elastic connections and, as shown in [4,11], is described by the matrix model (1), which in this case takes the following form:

$$\overline{MM_0} = g_0 = e^0\bar{P} \tag{4}$$

$$\overline{MM_1} = g_1 = -e^1\bar{P} \tag{5}$$

$$G = g_0 - g_1 = (e_0 + e_1)\bar{P} = e_{01}\bar{P} \tag{6}$$

The resulting model is presented in a compact matrix form. To identify distortions in the measured dimensions, it is necessary to expand it and derive expressions for all coordinate projections of the vectors $\overline{P}$ and g.

Naturally, for the desired vector g, only the notations of its coordinate projections can be written: gz, gx, gy.

The situation is more complex for the given vector $\overline{P}$, which defines the force loading of the technological system. As mentioned earlier, the control of the tool's movement is two-parameter: the coordinate feeds Sz and Sx are set so that, when executed simultaneously, the tool tip moves along the vector $\overline{S}$, i.e., along the formed contour (line D1). All known formulas for cutting forces (for example $p_i = c_i t^{x_i} s^{y_i} v^{z_i}$ $i = x; y; z$ [12] or formulas in [20-21]) define the components of cutting forces in a coordinate system associated with the formed surface: the X-axis is directed normal to the formed surface, the Z-axis follows the feed vector of the tool, i.e., along the formed surface, and the Y-axis is normal to the ZX plane. It should be noted that, according to ISO 841–74 and GOST 23597–79 standards, the information about the Z, X, and Y axes provided in the article should be interpreted for CNC machines as follows: Z ⇒X, X ⇒Y, and Y ⇒ Z. The point is that on CNC machines, the Z-axis runs along the spindle axis, while the transverse movement of the tool is along the X-axis. Accordingly, the feeds are labeled in the same way. When the tool is oriented normal to the formed surface, in single-tool, single-coordinate setups, the tool's coordinate system aligns with this kinematic coordinate system, and applying the formulas of cutting theory presents no difficulties [6, 20-21]. In our case, the misalignment of these systems results in the force vector applied to subsystem 0 taking the following form:

$$\bar{P} = \begin{pmatrix} P_x\, cos\theta - P_z\; sin\theta \\ P_x\; \sin\theta + P_z\; \cos\theta \\ P_y \end{pmatrix} \quad (7)$$

Taking into account the introduced notations and the transformations performed, the general model (1) in its expanded form will be represented as follows:

$$\begin{pmatrix} g_z \\ g_x \\ g_y \end{pmatrix} = \begin{pmatrix} e_{zz}^{01}\; e_{zx}^{01}\; e_{zy}^{01} \\ e_{xz}^{01}\; e_{xx}^{01}\; e_{xy}^{01} \\ e_{yz}^{01}\; e_{yz}^{01}\; e_{yy}^{01} \end{pmatrix} = \begin{pmatrix} c_y t^{x_y} \left(\frac{s_z}{\sin\theta}\right)^{y_y} v^{z_y}\cos\theta - \; c_x t^{x_x} \left(\frac{s_z}{\sin\theta}\right)^{y_x} v^{z_x}\sin\theta \\ c_y t^{x_y} \left(\frac{s_z}{\sin\theta}\right)^{y_y} v^{z_y}\sin\theta + \; c_x t^{x_x} \left(\frac{s_z}{\sin\theta}\right)^{y_x} v^{z_x}\cos\theta \\ c_y t^{x_z} \left(\frac{s_z}{\sin\theta}\right)^{y_z} v^{z_z} \end{pmatrix} \quad (5)$$

Here, the contour feed s is defined through the coordinate feeds s_z and s_x. Since these feeds are functionally related (ensuring the tool follows the specified trajectory), s_z is more often taken as the independent variable. As a result, the contour feed is given as:

$$s = \frac{s_z}{\sin\theta} \quad (6)$$

It should also be noted that the depth of cut t in formula (5) is understood as the actual depth tf, which differs from the calculated depth. The calculated depth of cut tcalc in Figure 2 is the distance between lines G and D. The actual depth of cut tact in this scheme is the distance between lines G1 and D0. The relationship between these quantities is given by the formula:

$$t_{act} = t_{calc} - (g_z sin\theta + g_x cos\theta) \quad (7)$$

As shown in the works of A.A. Koshin [22] and V.I. Guzeyev [23], the stiffness of modern lathe machines is quite high, and the force levels during operation at standard cutting modes are such that the elastic deformations of the components of the technological system are much smaller than the specified depths of cut. Therefore, the terms in parentheses in formula (7) can often be neglected.

The derived formula (5) describes the coordinate components of the vector of the total displacement of the contact point. These components in the previously discussed setups directly determined the distortions in the measured dimensions in the specified directions [3-4, 13-16, 19]. In a two-coordinate setup, the scheme for determining the distortion of the performed dimension becomes more complex. The point is that the actual radius of the part Rf, due to the displacement of the workpiece and the tool along the Z-axis, no longer relates to the calculated cross-section of the part, but rather to a cross-section that is shifted from the calculated one by the amount gz. In this cross-section, the part should have a different radius, and the distortion in dimension must be measured from it. Therefore, an additional formula applies for the distortion of the measured diametrical dimension in a two-coordinate setup:

$$\Delta R = g_x + g_z\,\mathrm{tg}\,\theta \quad (8)$$

Thus, the overall analytical matrix model of the force interaction of a two-body system with elastic connections (1) for two-coordinate setups in cone machining is transformed into the matrix equation (5) and the additional relationship (8).

IV. Conclusion

1. Matrix models of machining errors have been developed for single-tool setups on a rotating carriage and for single-tool two-coordinate setups, taking into account the combined effects of cutting forces and elastic deformations of the technological system in all coordinate directions.
2. The resulting matrix models of machining accuracy for conical surfaces reflect dimensional distortions depending on cutting conditions, cone angle, and the comprehensive compliance characteristics of the technological system. These models can also be used to calculate setup dimensions.
3. The use of the developed accuracy models enhances the potential for automated design of operations on automatic lathes.

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