APPLICATION AREAS OF CURVES AND SURFACES IN ENGINEERING

Ahmed Imanov[1], Zakir Galandarov[1]
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[1]Azerbaijan Technical University, Baku, Azerbaijan
ehmed.imanov@aztu.edu.az, zakir.qelenderov@aztu.edu.az

Abstract

The general classification of curves widely used in constructing surfaces for rapid movements along curved surfaces in engineering is presented. Key parameters ensuring the smoothness of curved lines are analyzed. The potential positive or negative effects of the curvature of these lines on moving objects, profiled surfaces, and similar applications are examined. The study considers the fields of application of curved lines and surfaces in engineering. Surfaces obtained using the formula representing the curved line, resolved as an inverse problem based on curvature, are investigated. The formula derived from curvature and its modified version are applied in analyzing the relationship between smoothness and potential energy, further confirming the accuracy of this connection. Types of surfaces are shown, and examples are provided of surfaces generated based on guiding curves and frames. The efficiency of surface modeling using computer technologies is also examined.

Keywords: curvature, torsion, centrifugal force, speed, main road, transition curve, circle, tangent, transcendental.

I. Introduction

In descriptive geometry, a line can be understood as the trajectory of a moving point's successive positions. If the moving point changes direction during its motion, the resulting trajectory forms a curved line. Curved lines can also be derived by other methods. Curved lines whose points lie on the same plane are called plane curves, while those that do not belong to a single plane are known as space curves. Curves are divided into two groups: algebraic and transcendental. The maximum degree of an algebraic equation defines the order of a curve. For plane curves, the geometric order is determined by its maximum intersections with a straight line, while for space curves, it is defined by the maximum intersections with a plane. The direction at any point on a curve is defined by its tangent. Curved surfaces are primarily obtained from the trajectory traced by a line, known as the generating line, as it moves along another line, called the guiding line.

Reference [1] examined the general application areas of functional curves and surfaces used in engineering, highlighting the importance of smoothness and curvature of curves on profiled surfaces. Information was provided on the application of computer technologies in 2D and 3D modeling of curves, surfaces, aerodynamic surfaces, and technical design. In [2], the research generalized curves with monotonically varying curvature, known as super spirals. In [3], an expert evaluation was conducted on the aesthetic quality of curves with varying smoothness parameters. However, these studies did not explore the application of curvature as an inverse problem in deriving mathematical expressions for curves. Taking this into account, the presented article extensively addresses the application of curvature as an inverse problem in the formation of curves and surfaces commonly used in engineering.

Nikolaos Eliou et al. [4] constructed the transition section of roads using a Symmetrically Projected Transition Curve (SPTC), derived from comparing clothoid and cubic curves. In the formulation of the SPTC, trigonometric expressions were approximated by polynomials. Esveld C. [5] applied cubic curves to railway tracks, while Kasper H. et al. [6] used clothoid curves for designing transition lines in highways. In [7], the application of various types of curves (clothoid, cubic curves, and Bernoulli lemniscate) to transition curves was discussed, with an attempt to justify their use. E.A. Gavrilenko et al. [8] proposed a classification of curves based on the dynamics of changes in their differential-geometric properties.

As is known, the movement of a point changing position along a curved line is related to the variation of two quantities:

- ds represents the displacement, which is the distance traveled from the initial position;
- dφ, the angle of rotation of the tangent relative to the initial position (Figure 1).

One of the key parameters of a curved line is the curvature coefficient (or simply curvature). Curvature is defined as the inverse of the radius of curvature at a given point on the curve and represents the ratio of the angle of rotation of the tangent to the distance traveled by the point. It is expressed by the differential equation as follows:

$$k = \frac{d\varphi}{ds} \tag{1}$$

For example, the curvature of a straight line is zero at all points.

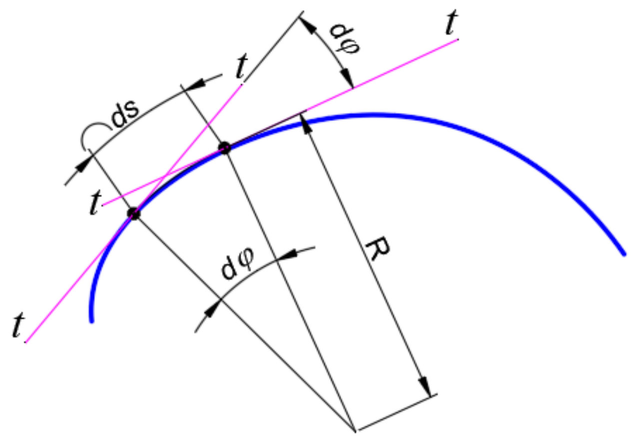

Figure 1: *Key Parameters of a Curved Line*

Application Areas of Curved Lines in Engineering. Curved lines used in engineering, whether in a plane or in space, express the functional characteristics of any object. Curved lines have found extensive applications during the design phase in engineering. Examples include:The profiling of blades' surfaces in turbo engines to enhance quality performance indicators.

- The design of transition curves that ensure comfortable and safe passageways on main roads, with the condition of providing maximum smoothness.
- The profiling of surfaces of pushers in distributors that ensure a gentle and impact-free motion of the valves that open and close timely in internal combustion engines.
- The profiling of surfaces of the stator to ensure that the plates located on the rotor of sliding (shutter) pumps move in contact with and without separation from the stator, among others.

Such curved lines can generally be classified as local convex (with negative curvature), local concave (with positive curvature), and transition points (where the curvature changes). In space curves, torsion can also be added. Without considering the specific characteristics of the designed surfaces, the primary requirement imposed on all engineering curves is their smoothness. Smoothness refers to the differentiability of a function or geometric figure (curved line, surface, etc.) at all points in a given section. Different projects utilize curved lines with varying degrees of smoothness. For example:

- In classical design on main roads, clothoid curves are most commonly used for constructing transition curves, as their smoothness is ensured by a second-order derivative.
- In the profiling of the surfaces of pushers in distributors in internal combustion engines, the smoothness is generally maintained at a level not lower than three, which is why smooth curves that allow for derivatives of the third order are utilized.
- In the design of space curves, the smoothness of the curve must be of third order to ensure the continuity of the function's torsion.
- Regardless of the application area of the curved lines, higher-order smooth curves, such as transcendental curves, are also used for their superior smoothness.

One of the key parameters that ensures the smoothness of curved lines is the minimal number or complete absence of extremum points in the curvature graph over a given interval. For example:

$$y(t) = \frac{C_2}{p} \cdot \ell n\left(sin(p \cdot t) + \sqrt{B^2 - cos^2(p \cdot t)}\right) + C_1 \qquad (2)$$

Here C_1, C_2, B, p, a -is constant and $B = \frac{p}{a}$. Expression (2) is derived from the specific solution of the differential equation of curvature in Cartesian coordinates, with an additional constant included [9]. Expression (2) is a transcendental equation, and the smoothness is considered high. This equation can be used in the construction of all the planar engineering curves mentioned above. When the boundary conditions used to determine the constants in expression (2) are chosen correctly, there are no extremum points in the curvature graph, or at worst, there may be only one. The presence of excessive extremum points in curvature, for instance, in profiled technical surfaces and the design of objects, can lead to the following negative consequences:

- In a cam mechanism, it can cause premature wear of the cam and pusher due to the impact of the pusher.
- In aerodynamically profiled surfaces (e.g., blades), the flow of working fluids can become turbulent due to non-smooth surface irregularities, increasing the drag on the blade and causing undesirable local separations in the working fluid.
- On main roads, it can lead to hazardous movements and unnecessary braking or acceleration, increasing energy consumption during transition curves (10).
- It may cause premature wear of the stators and plates in sliding pumps.
- In computer graphics and CAD systems, it can result in incorrect visual perception of objects.

One of the main conditions imposed on profiled aerodynamic surfaces is that the maximum curvature should be minimized. If the substitution from $C_2 = 1$ [10,11] is applied in expression (2), then the curvature is defined by a sinusoidal curve. This ensures that the curvature is considered as a boundary condition in advance. That is, the minimum of the maximum value of curvature is directly used in determining the constants in the equation. For example, when a transition curve is connected to a circular road with a given radius on main roads, the radius of curvature at the endpoint of the transition curve must equal the radius of the circular road; otherwise, there will be an undesirable jump in the centripetal force.

Figure 2 (dimensions are given in meters), expression of k_2 curvation (2), k_1 is a curvation which is defined for the expression $C_2 = 1$. A seen from curvek_2, to ensure safe movement, the rate of change of curvature is low at the entrance of the road (at the beginning) and increases as it approaches the exit.

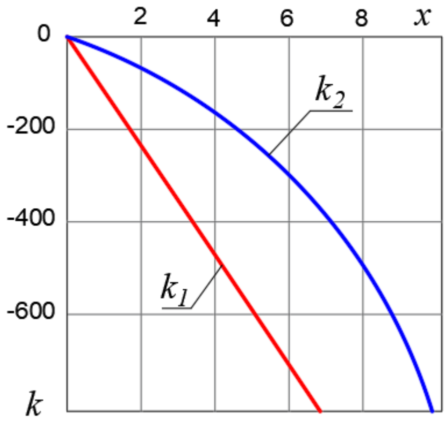

Figure 2: *Dependence of Curvature on the x Coordinate*

As is known, the smoothness of a curved line is directly related to its potential energy. The goal of selecting curves with low potential energy in high-speed curved trajectories is to ensure that the moving object behaves like an elastic body. It is well understood that less work is required to deform an elastic medium along a flow line with lower potential energy. The motion of an object along a convex curved trajectory with low potential energy will require relatively less work when taking friction into account.

The curved line with minimum potential energy from the given lines is referred to as elastic. The elasticity of the curved line defined by expression (2) can be expressed as follows:

$$E_{eyx} = \int_{l_1}^{l_2} k^2(s)\, ds = \int_{x_1}^{x_2} k^2(x) \cdot \sqrt{1 + \frac{C^2{}_2 \cos^2(px)}{\left(\frac{p}{a}\right)^2 - \cos^2(px)}}\, dx \tag{3}$$

where, E_{eyx}-is the potential energy of the curved line., x_1, x_2, ℓ_2, ℓ_2 beginning of the curve and its endpoints (interval), k - the curvature coefficient, defined by (4):

$$k(x) = \frac{-C_2\left(\frac{p}{a}\right)^2 \cdot p \sin(px)}{\sqrt{\left(\left(\frac{p}{a}\right)^2 + \cos^2(px)(C_2{}^2 - 1)\right)^3}} \tag{4}$$

(3) the potential energy of the curved line obtained by the expression.
In expression (3) $C_2 = 1$ (then from (4) $k(x) = -a \sin(px)$) is approximately 10% greater than the energy of the curved line obtained for the condition.

In addition to the properties that govern the quality of the smoothness of the aforementioned curve, there are also parameters that determine its technical aesthetics. When the surfaces present in real life are expressed mathematically, they tend to be more aesthetically pleasing (for example, the contour of a bird's wing). When modeling aesthetically pleasing surfaces, curves known as ℓog - aesthetic curves, which have a constant curvature represented as a straight line on a logarithmic scale, are widely used. As it can be seen from figure (2) $C_2 = 1$variant and ℓog in an expression (2) can be considered a special case of aesthetic curves.

II. Application of Curves in Engineering.

In descriptive geometry, there are various methods for generating surfaces using curved lines. Surfaces can be formed through the continuous motion of a guiding line, referred to as a generating line, or they can be created as a framework formed by multiple lines. Both methods are widely used in computer technologies.

For example, consider the modeling of a cam. In the kinematic analysis of cam mechanisms, it is essential to determine the motion law of the follower, which corresponds to the cam profile. Since the profile curve of the cam is complex, constructing it graphically in several positions can pose challenges, leading to inaccuracies in calculations.

Utilizing computational tools and advanced modeling techniques allows for more precise generation of these curves, enabling engineers to achieve higher accuracy and efficiency in the design and analysis of mechanical systems.

The application of modern computer technologies has made kinematic analyses more accurate and easier to perform. For this purpose, various software solutions in the fields of CAD (Computer-Aided Design) and CAE (Computer-Aided Engineering) are widely used around the world. For example, SolidWorks, produced by Dassault Systèmes, holds a leading position globally.

The profile of the cam is drawn in the SolidWorks program based on the displacement-rotation angle dependency graph obtained from (2) and using circles in the polar coordinate system (Figure 3). For example, the phase angles used as boundary conditions in the construction of the profile are as follows; $\varphi_u = 60°$ - divergence angle, $\varphi_{ud} = 40°$ - distant stopping angle, $\varphi_y = 60°$- approaching angle, $\varphi_{yd} = 200°$- the angle of stopping near has been accepted.[12].

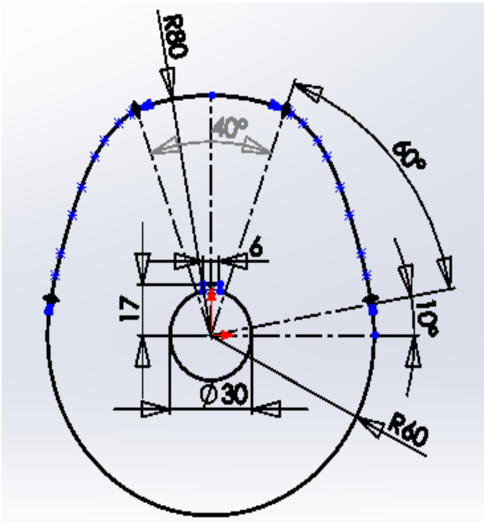

Figure 3: *The profile of the cam construction in SolidWORKS software.*

In Figure 3, other dimensions are taken arbitrarily and do not affect the intended calculations. The diameter of the cam's roller is assumed to be 25 mm. When constructing the contour in SolidWORKS, (2) is used directly. That is, the departure and approach lines are obtained using the mathematical expression (2) within the given interval. After the contour is established, it is extruded to obtain a 3D model.

To demonstrate the practical application of the cam, let's consider the assembly sequence of the designed cam mechanism. Just like the 3D model of the cam, 3D models of all parts related to the cam mechanism are also created. The 3D model of the parts included in the assembly is created and each is saved in a separate file. The 3D modeling is carried out based on the "bottom-up" principle. In Assembly mode, the 3D assembly of the cam mechanism is developed based on the 3D model of the parts (Figure 4).

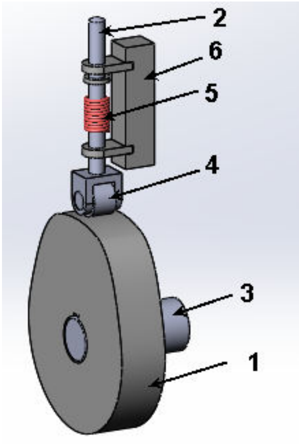

Figure 4: *3D model of the cam mechanism (1 - cam, 2 - pusher, 3 - shaft, 4 - roller, 5 - spring, 6 - support).*

Another example of a framework-type surface is blades. Aerodynamic blades are created in the form of a framework by twisting two-dimensional profiles located at varying distances along the height. The blade shown in Figure 5 is created based on three cross-sections (two-dimensional profiles). Using the expression (2) ($C_2 = 1$) he belly and back parts of the profile in SolidWorks are joined with the entry and exit circles to form a complete closed contour. In the height direction of the frame, the exit lines are assumed to be straight. In SolidWorks, using the Boundary Boss/Base command, the 3D model of the blade is created based on three profiles and the side straight lines (Figure 5).

For practical application, for example, to perform CFD (Computational Fluid Dynamics) analyses based on the 3D model of the blade, either SolidWorks is used, or the 3D model is converted into Parasolid (.x_t) format and transferred to the ANSYS program.

Figure 5: *3D Model of the Blade.*

III. Conclusion.

The conclusions and recommendations derived from the article are as follows:

• The smoothness of curves should be considered, and the appropriate selection should be made based on the application field of the curve;

• Smoothness should be ensured in torsion, which is one of the key properties of space curves;

• It is advisable to keep the minimum of the maximum curvature value and the rate of change of curvature within a certain limit;

• Efforts should be made to minimize the potential energy of curvature;

• The design of aerodynamic surfaces should be based on intersecting lines, and the optimal variant should be selected;

• To ensure the effective use of surfaces in engineering and design, their 3D models should be created using computer technologies;

• Based on the presented data and methodology, it will be possible to select the appropriate applications of curves in engineering in the future.

References

[1] V. G. Muftiev, R.A. Ziatdinov. Functionality and Aesthetic of Curves in Industrial Design: A Multicriteria Approach to Quality Evaluation of Forms in Future CAD Systems. Published in *Vestnik Mashinostroyeniya*, Issue 7 (2018).
http://www.mashin.ru/eshop/journals/vestnik_mashinostroeniya/2039/18/.

[2] Ziatdinov R. Family of superspirals with completely monotonic curvature given in terms of Gauss hypergeometric function// Computer Aided Geometric Design. 2012. Vol. 29. No.7. P. 510-518

[3] Levien R. L. From Spiral to Spline: Optimal Techniques in Interactive Curve Design. PhD.thesis, University of California, Berkeley, 2009

[4] Nikolaos Eliou, Georgios Kaliabetsos, A new, simple and accurate transition curve type, for use in road and railway alignment design, European Transport Research Review An Open Access Journal, 29 September 2013.

[5] Esveld C., Modern railway track, 2nd edn. T.U. Delft Press, The Netherlands, 2001.

[6] Kasper H., Schuerba W., Lorenz H. The clothoid as an element of horizontal alignment. F. Dummlers, Publishing House, Bonn, 1954.

[7] Agata Basak, The Study of Geometry of the Selected Transition Curves in the Design of Circular RoadsAdvances in Science and Technology Research Journal 2022, 16(4), 270–278, https://doi.org/10.12913/22998624/152936, ISSN 2299–8624, License CC-BY 4.0

[8] E.A. Gavrilenko, YU.V. Kholodnyak. Forming of geometric characteristics of monotonous curves. Applied Geometry and Computer Technologies. Bulletin of Kharkiv National Technical University, Issue 3(58), 2016.

[9] A.S. İmanov, I.A. Khalilov 3D modeling and analysis of gas flow in the interblade channel. SOCAR Proceedings Special Issue 1 (2022) 001-005 http://proceedings.socar.az., 2022, 5 p., DOI: 10.5510/OGP2022SI100690.

[10] A.S. İmanov, I.A. Khalilov, A.G. Aliyev New approach to calculation of transition curves on curved roads. Proceedings of the International Conference on Problems of Logistics, Management and Operation in the East-West Transport Corridor (PLMO), Baku, Azerbaijan. Oktobmer 27-29, 2021, 6 p.

[11] A.S. Imanov, P.S. Abdullayev. Profiling of flat aviation blades based on the differential equation of curvature. *Vestnik of Engine Engineering*, Issue No. 2. Zaporizhzhia, JSC. «Мотор Сич», 2015, 6 ст, ISSN 1727-0219.

[12] A.S. İmanov, I.A. Khalilov Kinematic and Dynamic Calculation of profiled Cam Mechanism Based on New Equation. International Symposium on Unmanned Systems: AI, Design, and Efficiency was held in Baku, Azerbaijan on 22nd of May and 24th of May. ISBN : 978-9952-582-04-8 DOI : 10.30546/2224.978-9952-582-04-8.Springer.