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A SYSTEMATIC REVIEW OF MODELING, PLANNING, AND STATISTICAL FRAMEWORKS IN ACCELERATED LIFE TESTING

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Abstract

For highly reliable or long-life systems, conventional testing is often impractical due to the excessive time and cost required. Accelerated Life Testing (ALT) offers a practical and statistically rigorous alternative. It predicts product life and durability under normal-use conditions through data obtained under controlled high-stress conditions. This review systematically examines existing ALT methodologies. It begins with an overview of qualitative and quantitative approaches and key concepts that define the theoretical foundation of ALT assessment. Quantitative ALT is organized into Fully Accelerated Life Testing (FALT) and Partially Accelerated Life Testing (PALT) frameworks. Further, the design principles of ALT and PALT are discussed under constant- and step-stress conditions with emphasis on model formulation, estimation procedures, extrapolation, and model validation. The review also provides a concise discussion of statistical modeling and estimation methodologies. Key elements such as stress-loading, life–stress relationships, and censoring mechanisms are discussed to ensure valid statistical inference and model robustness. The review consolidates theoretical insights and methodological frameworks to advance design optimization and reliability prediction in complex systems. It concludes with perspectives on intelligent, data-driven ALT supported by artificial intelligence and computational analytics.

Keywords: Qualitative Accelerated Life Testing, Quantitative Accelerated Life Testing, Fully & Partially Accelerated Life Testing, Design Framework, Test Plan

I. Introduction

Customer satisfaction depends not only on initial product performance but also on the sustained functionality of the product throughout its intended service life under specified operating conditions. This sustained functionality is quantitatively captured as product reliability, defined as the probability that a system performs its intended function without failure for a given duration under stated conditions [1, 2]. The primary objectives of reliability estimation are to predict product service life, enhance design robustness, and forecast warranty return rates. However, for highly-reliable items or systems, conventional testing is often impractical due to excessive time and cost. ALT addresses this challenge by applying elevated stress levels to expedite failures and facilitate reliability estimation under normal-use conditions within a shorter timeframe. This approach not only enables accurate reliability estimation but also facilitates identification of root

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failure mechanisms, evaluation of alternative components or suppliers, and verification of compliance with reliability specifications.

Key contributions have shaped the development of ALT design and analysis. [2] remains a foundational reference, providing comprehensive coverage of theoretical principles, practical implementation, and real-world case studies. [3] reviews advanced statistical frameworks, including nonparametric, semiparametric, and Bayesian methods, although their industrial adoption is limited due to complexity and lack of accessible tools. [4] provide a comprehensive overview of the ALT methodologies available at the time of their writing. For further detail, [5, 6] discuss reliability fundamentals. [7, 8] present perspectives on survival analysis. [9, 10] cover test planning and stochastic modeling. [11, 12] address advanced life data analysis. [13] provides extensive ALT case studies. [14] offers guidance for software-based implementations. [15] explore probabilistic interpretations of accelerated testing.

The motivation of this review arises from the lack of integration within current ALT literature. This review addresses the fragmented nature of current ALT research by integrating theoretical, methodological, and industrial perspectives. It unifies established and emerging methodologies to provide a coherent framework for theoretical principles, test design, and estimation techniques. The discussion encompasses both qualitative and quantitative approaches, with emphasis on FALTs and PALTs under constant-stress (CS) and step-stress (SS) conditions. The structure of this review is organized as follows. Section 2 introduces fundamental concepts in reliability and ALT. Section 3 classifies major quantitative ALT approaches and methodologies. Section 4 details design frameworks for commonly implemented FALTs, while Section 5 focuses on modeling structures for PALTs. Section 6 concludes with a synthesis of key findings and outlines emerging directions for future ALT research.

II. Key concepts in ALT

ALT is a pivotal methodology within reliability engineering. It is designed to evaluate the failure behavior and durability of components or systems under controlled conditions that simulate long-term usage. By providing insights into product lifespan, failure mechanisms, and reliability, ALT supports improved design, risk mitigation, and operational safety. The following key concepts establish the foundation for understanding and applying ALT in engineering studies:

- Life (Failure) data: In reliability engineering and ALT, life data, or failure data, record the observed times until a product fails or stops functioning.
- Complete (Uncensored) or Censored (Incomplete) data and Censoring: Complete data record the exact failure time of every test unit. Censored data occur when some failure times are unknown. This happens if the test ends before all units fail or if some units are removed during the test and this process is called censoring.
- Failure Time Distribution: Failure time distributions describe the statistical behavior of failure data. Common models include exponential, Weibull, Normal, and gamma distributions, each representing different failure patterns such as random, wear-out, or early-life failures.
- Stress and Stress Loading: In ALT, stress represents external forces or environmental factors applied to test specimens. Stress loading specifies how these factors are applied over time and directly impacts the interpretation of results. Common approaches include constant stress, step stress with discrete increments, and progressive (ramp) stress with continuous increases. These methods apply to single or multiple simultaneous stress variables.
- Life–Stress Relationship: The life-stress relationship defines how the life of a product related to the applied stress. It describes how life characteristics as a function of stress.

Mathematically, life characteristics L is expressed as a function of stress S as $L = f(S)$. Common models include the Arrhenius model for temperature, the inverse power law for voltage or mechanical load, and the Eyring model for multiple stresses.

- Test Planning, Sample Size, Parameter Estimation and Inference: A well-structured test plan is essential for effective life testing. It should define stress levels, number of test units to be assigned to test at each stress level, and test duration. In ALT, statistical estimation provides a foundation for modeling failure behavior and predicting reliability under standard conditions. Core estimation procedures include maximum likelihood estimation (MLE), least squares estimation (LSE), and Bayesian estimation (BE). Supplementary techniques such as weighted least squares estimation (WLSE), percentile estimation (PCE), Cramér–von Mises estimation (CME), Anderson–Darling estimation (ADE), and maximum product of spacing estimation (MPSE) offer flexible alternatives for diverse data scenarios. Graphical and three-point estimation approaches are applied when parameter estimation rely on limited or censored observations. The final choice of estimation framework reflects the nature of the data, censoring mechanism, and computational constraints.
- Extrapolation and Validation: The primary goal of ALT is to extrapolate reliability estimates from high-stress levels to normal operating conditions. This process depends on an appropriate stress–life models and robust statistical estimation. Model adequacy is evaluated through goodness-of-fit tests such as the Kolmogorov–Smirnov test. Final reliability predictions are validated using independent experimental or field data to confirm accuracy and practical relevance.

III. Types of ALTs

The increasing complexity of modern products that integrate multiple materials, subsystems, and technologies introduces diverse failure mechanisms across varied operating environments. These factors create significant challenges in accelerating product life for reliability evaluation. ALT is broadly classified into two categories as follows:

I. Qualitative Accelerated Life Tests

Qualitative ALT is applied during early reliability assessment to identify potential failure mechanisms rather than to measure product lifetime. Test units are exposed to highly elevated stresses such as heat, humidity, vibration, or temperature cycling until failure occurs. This process reveals degradation patterns, structural weaknesses, and dominant failure pathways. The resulting analysis is descriptive and assists in design refinement, corrective measures, and robustness enhancement. Common uses include prototype assessment, material evaluation, and design verification. Techniques include stress screening for early failures, highly ALT, thermal shock testing, environmental stress evaluation, and design margin assessment. Although qualitative ALT does not yield direct life estimates under normal operation, it provides essential guidance for determining stress levels in subsequent quantitative tests.

II. Quantitative Accelerated Life Tests

Quantitative ALT determines product reliability and lifetime characteristics under normal use by analyzing data from controlled accelerated experiments. Test units are subjected to constant or stepwise elevated stresses, and their failure times are recorded. Time-to-failure data are modelled using distributions such as Weibull, exponential, or lognormal, while acceleration models including Arrhenius, Eyring, and inverse power law enable extrapolation to normal conditions.

Quantitative ALT facilitates parameter estimation, model validation, and reliability prediction, providing a foundation for warranty evaluation and design comparison. It encompasses FALT and PALT, both of which integrate statistical modeling with engineering insights into failure mechanisms. A sound ALT plan involves appropriate selection of lifetime distributions and acceleration models to ensure accurate life prediction. Effective design of experiments minimizes cost, optimizes stress levels, and ensures data validity for extrapolation.

IV. Modeling and statistical frameworks for common FALTs

FALT accelerates the occurrence of failures by exposing products to elevated stresses well above operational limits while preserving the same underlying failure mechanisms. Time-to-failure data generated from these tests are modelled through quantitative frameworks such as Arrhenius for temperature acceleration or the Inverse Power Law for mechanical or electrical loading. Extrapolation from high-stress data yields estimates of reliability metrics under normal use conditions. FALT is critical for applications demanding high reliability and reduced development time, such as aerospace, automotive, and medical technologies. It supports design qualification, material evaluation, and lifecycle reliability prediction. However, excessive stress can create non-representative failure mechanisms, thus maintaining the equivalence of failure modes across stress levels is fundamental. When properly designed, FALT provides rapid, cost-effective, and statistically valid insights for product qualification and lifecycle reliability assurance.

I. Design Framework and Test Plan for CSFALT

CSFALT is the most widely used accelerated testing approach. Each unit is subjected to a fixed level of stress that remains constant until failure or termination under a censoring scheme. The failure time distribution is derived from the observed data, with its parameters modeled as functions of the applied stress levels. This classical method reflects actual operating environments for components such as lamps, semiconductors, and microelectronic devices. Its statistical inference has been extensively explored by [16-23].

Before product release, manufacturers must select appropriate testing methods to estimate reliability. A well-designed CSFALT plan provides sufficient data to evaluate performance under operating conditions, shortens test duration, reduces costs, and reveals potential failure modes. The test plan depends on the experimental objective, as emphasized by [24], and must be carefully designed for reliable decision-making. Optimal CSFALT designs have been proposed for multiple lifetime distributions and censoring schemes. [25] developed plans for normal and lognormal distributions. [26] introduced optimal four-level designs under varying censoring times. [27] analyzed the Weibull distribution. [28] designed optimal type-I constant stress tests assuming a generalized logistic distribution. [29-33] discussed optimal designs under periodic inspection and type-I censoring. [34] investigated multiple CSFALT configurations in non-rectangular test regions. [35] studied finite mixture model applications. [36] proposed the geometric process (GP) model for complete and censored exponential data, later extended by [37] to Weibull samples. [38, 39] applied GP to other censored data with different distributions. [40, 41, 42] derived MLEs of Pareto parameters using GP for complete, type-I, and type-II censored data. [43, 44] analyzed Weibull data with GP under type-I, and type-II censoring types. [45] examined Burr Type X distribution using GP. [46] investigated k-level CSFALT under progressive censoring. [47] later analyzed hybrid systems using multiple-level CSFALT with type-II progressive censored masked data.

In general, following steps can be used systematically to outline the complete test procedure and model development under CSFALT:

- Specify Stress Levels, Censoring and Collect Failure Data: Select k constant stress levels ($k \geq 2$), denoted as S_i for $i = 1,2, \ldots, k$. Select a sample of $n = \sum_{i=1}^{k} n_i$ and test n_i, specimens at each level until all units fail or until the test concludes under a censoring plan. Denote the observed life of specimen j ($j = 1,2, \ldots, n$) at stress level i by X_{ij}. Assume lifetimes within each stress level are independent and identically distributed to ensure statistical validity. Record all failure times and censoring times with precision throughout the testing period. Maintain data integrity and completeness to support reliable analysis and inference.
- Select Failure Distribution and Life-Stress Relationship: Identify an appropriate life distribution based on data patterns with underlying probability density, cumulative distribution, and reliability functions as $f(t;\theta)$, $F(t;\theta)$, and $R(t;\theta) = 1 - F(t;\theta)$, where θ denotes the vector of model parameters. Establish an appropriate functional relationship between the life parameter and the applied stress to represent the acceleration mechanism.
- Estimate Parameters: Estimate the parameters θ using an appropriate method among MLE, LSE, WLSE, PCE, CME, ADE, MPSE, BE, graphical, or three-point estimation.
- Extrapolation and Model Validation: Use the fitted model for predictions under normal operating stress and estimate key life characteristics and essential reliability metrics. Assess model adequacy using statistical goodness-of-fit tests, including the Kolmogorov–Smirnov test, and validate predictions with independent experimental or field data to confirm the accuracy, reliability, and predictive strength of the estimated life characteristics.

II. Design Framework and Test Plan for SSFALT

Conventional CSFALTs often require prolonged durations due to wide variability in failure times. This limitation led to the development of SSFALT, which shortens test duration and accelerates failures while maintaining valid stress– life relationships. In SSFALT, each specimen is initially subjected to a defined elevated stress and, after surviving a predetermined interval, progressively exposed to higher stress levels until failure or test termination. This sequential design allows observation of failures across multiple stress levels within a single experiment, enhancing data efficiency and improving the precision of parameter estimation. Recent advancements in SSFALT focus on increased model flexibility, statistical efficiency, and robustness under censoring. Integration of optimal experimental design, Bayesian inference, and computational optimization has further improved methodological performance. These developments reduce experimental duration and cost while maintaining model fidelity and predictive reliability. SSFALT has thus become an essential methodology in modern reliability engineering, particularly when CSFALT is impractical or economically prohibitive.

In the early development of SSFALT, the Cumulative Exposure (CE) model was introduced by [48] and later generalized by [49, 50] to account for the effects of stress variation. Subsequent research extensively investigated optimal designs and inferential procedures across diverse lifetime distributions. Optimal stress-change times under exponential lifetimes were determined by [51] and extended to censored data by [52]. CE models with Weibull lifetimes were examined through inferential procedures [53, 54]. Optimal designs for Khamis–Higgins and lognormal step-stress models were developed by [55, 56]. Exponential lifetimes with log-linear stress–life relationships were studied in [57, 58], while [59] highlighted the reliability of direct exponential inference. Comprehensive SSFALT methods were reviewed by [60], and inference under progressive type-I censoring was addressed by [61]. Design optimization for exponential SSFALT with progressive censoring was performed by [62], and grouped or censored datasets were analyzed by [63]. Models with random stress-change times were investigated by [64]. Exact inference and optimal designs for exponential SSFALT models were derived by [65]. Multi-step SSFALT procedures and analyses of type-II and hybrid censored data were presented in [66–69]. Optimal stress-change times for log-logistic CE models and extensions to the Lomax distribution were reported by [70, 71]. SSFALT plans for Exponentiated Weibull, Power Function, and Rayleigh distributions were developed in [72–74], and comparative analyses of multiple SSFALT models were conducted by [75]. Optimal plans for Mukherjee–Islam and two-parameter Pareto models were formulated in [76, 77], while SSFALT under the Nadarajah–Haghighi distribution, including maximum likelihood estimation of parameters, was studied in [78].

The design of a SSFALT follows a structured and iterative approach to ensure precision, efficiency, and reliability. The complete test procedure and model development can systematically be described in the following stages:

- Specify Stress Levels and Collect Failure Data: Specify k stress levels ($k \geq 2$), denoted as S_i and τ_i for $i = 1,2, \dots, k$. Define the number, magnitude, and order of stress levels according to material limits and desired acceleration factors. Select a sample of n specimens and start testing at S_1 and then change the stress from S_1 to S_{i+1} at time τ_i and test is continued until all item failed or test is terminated. Denote the observed life of specimen j ($j = 1,2, \dots, n$) at stress level i by X_{ij}. Lifetimes at each level are independent and identically distributed.
- Select Failure Distribution and Life-Stress Relationship: Identify an appropriate life distribution based on the physical failure mechanism or empirical data patterns with underlying probability density, cumulative distribution, and reliability functions as $f(t;\theta)$, $F(t;\theta)$, and $R(t;\theta) = 1 - F(t;\theta)$, where θ denotes the vector of model parameters. Establish an appropriate functional relationship between the life parameter and the applied stress to represent the acceleration mechanism.
- Apply CE Model: The CE model assumes that total damage accumulation is a function of the time spent at each applied stress level. When the stress level changes, the model adjusts the equivalent exposure time at the new stress level to account for the damage accumulated under previous stresses. Accordingly, the CDF in SSFALT as per the CE model is given by

$$F(t) = \begin{cases} F_1(t); & 0 \leq t < \tau \\ F_2(t - \tau + \tau'); & \tau \leq t < \infty \end{cases} \tag{1}$$

  Where, the equivalent time τ'at stress S_2 is determined by satisfying the continuity condition of cumulative damage, such that $F_1(\tau) = F_2(\tau')$, where $F_1(t)$and $F_2(t)$ are the CDFs corresponding to stress levels S_1 and S_2, respectively. This ensures that the transition between stress levels maintains consistency in cumulative failure probability, preserving the underlying principles of the CE model framework.

- Estimation and Inference: Employ appropriate method among MLE, LSE, WLSE, etc. to obtain model parameters and reliability measures.
- Optimization of Stress-Change Times: Determine optimal stress change times ($\tau_1, \tau_2, \dots \tau_k$) that minimize estimator variance or maximize test information.
- Extrapolation and Model Validation: Use the fitted model to predict failure distributions under normal operating stress and estimate key life characteristics and reliability metrics. Evaluate model adequacy with goodness-of-fit tests, such as the Kolmogorov–Smirnov test, and validate predictions against independent experimental or field data to ensure accuracy and predictive reliability.

V. Modeling and statistical frameworks for common PALTs

In a FALT, breakdown data obtained under elevated stress conditions are analyzed and extrapolated to normal use levels using an appropriate physical model. However, constructing such a model becomes difficult when the underlying degradation mechanisms are complex or uncertain. In these situations, a partially accelerated life test offers a practical alternative. It combines testing at normal and higher stress levels that accelerate failures without introducing unrealistic failure mechanisms. In this method, the normal stress group operates under baseline use conditions, while the accelerated group functions at higher stress. This setup allows data collection at both normal and accelerated levels in parallel, ensuring a more comprehensive reliability evaluation. The approach balances shorter testing duration with realistic failure behavior, making it suitable for fragile or complex products. It is particularly advantageous when regulatory standards require representative failure mechanisms. Although testing typically lasts longer than FALTs, the resulting life estimates align more closely with field performance. Common implementations include CSPALT and SSPALT, where stress is applied at fixed or incrementally increasing levels to achieve controlled acceleration.

I. Design Framework and Test Plan for CSPALT

In a CSPALT, identical product samples are subjected simultaneously to normal and elevated constant stress levels. Each sample operates exclusively under either the normal-use or accelerated-stress condition. Testing continues until all items fail or test is stopped according to a predefined censoring scheme. Stress acceleration may be induced through temperature, voltage, pressure, vibration, cycling rate, or humidity. CSPALT achieves a balanced trade-off between realism and test duration and has been extensively reported in the literature. [79] analyzed CSPALT data to obtain MLEs of the exponential distribution parameter and the acceleration factor under type-I censoring. [80] extended this work by developing MLEs for the Burr XII distribution under both type-I and type-II censoring. Similarly, [81] applied MLE approach under progressive type-II censoring for the Burr XII distribution, demonstrating the model's flexibility in handling incomplete data. [82] obtained ML estimates for CSPALT under type-II censoring, assuming a Weibull lifetime distribution at design stress. [83] assessed Burr XII parameter estimation in CSPALT through quasi-Newton and EM-based likelihood methods under multiple censoring schemes. [84] derived MLEs for the distribution parameter and acceleration factor assuming a Rayleigh lifetime distribution under type-I censoring. [85] applied type-I censoring to estimate parameters and construct confidence intervals for the Inverted Weibull distribution. [86] extended this work by developing ML estimation and confidence interval procedures for the Power Lindley distribution under multiple-censoring conditions in CSPALT.

The development of a CSPALT model involves a series of systematic steps designed to ensure statistical rigor and practical applicability. The complete testing process and model formulation can be systematically outlined through the following steps:

- Specify Stress Levels and Collect Failure Data: Let k represent the total number of stress levels ($k \geq 2$), denoted as S_i for $i = 0,1,2,\dots,k$, where S_0 represents the normal operating condition. In a simple two-stress CSPALT, n test units are randomly divided into two subgroups of sizes $n(1-r)$ and nr, where r denotes the sample proportion. The first subgroup assigned to normal stress level while the second is subjected to an accelerated stress level to be tested and each unit is monitored until all unit or until the test terminated due to censoring. Let X_{1j} , $j =$

$1,\dots,n(1-r)$ denote the observed lifetimes at normal stress and X_{2j} , $j=1,\dots,nr$ denote the observed lifetimes at accelerated stress.

- Select Failure Distribution and the Time-Scaling Relationship: A suitable lifetime distribution is selected to represent the failure behavior of the product. The observed lifetimes X_{1j} under normal stress are modeled as independent and identically distributed random variables with cumulative distribution function $F_1(X_{1j})$ and probability density function $f_1(X_{1j})$. Lifetimes X_{2j} under accelerated stress retain the same distribution as at normal stress but are modified via the time-scaling transformation given as

$$X_{2j}=\lambda^{-1}X_{1j},\ \lambda>1 \tag{2}$$

where λ denotes the acceleration factor, representing the ratio of characteristic lifetimes between normal and accelerated conditions. This transformation implies that, under accelerated stress, the CDF and its corresponding PDF are given by:

$$F_2(X_{2j})=F_1(\lambda X_{2j}) \tag{3}$$

$$f_2(X_{2j})=\lambda f_1(\lambda X_{2j}) \tag{4}$$

This formulation preserves the stochastic structure of the baseline model while establishing a statistically coherent link between operating and accelerated stress environments. It therefore provides a rigorous analytical foundation for parameter estimation and reliability extrapolation within CSPALT frameworks.

- Parameter estimation: Select and apply an appropriate estimation method, such as MLE, LSE BE etc. to obtain model parameters and reliability measures.
- Extrapolation and Model Validation: Use the fitted model to predictions under normal operating stress and evaluate model adequacy. Validate predictions against independent experimental or field data to ensure accuracy and predictive reliability.

II. Design Framework and Test Plan for SSPALT

In SSPALTs, products are first operated under normal conditions for a set duration. Items that do not fail are subsequently placed under accelerated stress until all items failed or the experiment terminates according to a censoring rule. This sequential stress strategy accelerates failures, preserves realistic failure behavior, and allows efficient estimation of parameters.

For designing SSPALT, [87] introduced the Tampered Random Variable (TRV) and formulated optimal test strategies within a Bayesian framework to estimate the parameters of the exponential distribution under different loss functions. [88] proposed the Tampered Failure Rate (TFR) model, which assumes that stress changes scale the baseline failure rate after the switching time and obtained the MLEs of Weibull parameters. [89] generalized the TFR model to accommodate multiple step-stress levels. Using MLE method, [79] estimated the parameters of the exponential distribution with type-I censored data. [90] extended this approach to estimate Weibull distribution parameters and acceleration factors under both type-I and type-II censoring. [91] conducted a detailed investigation of parameter estimation for the Log-Logistic distribution within the SSPALT framework. [92] performed MLEs of Burr Type XII distribution parameters and corresponding acceleration factors under type-I censoring, highlighting the model's applicability to diverse lifetime distributions. [93] examined SSPALT for Weibull distributions incorporating hybrid-censored data. [94] applied MLE technique to estimate parameters of the generalized inverted Exponential distribution under type-I censoring. [95], along with [96], investigated optimal SSPALT design strategies for censored samples for estimated parameters of the model.

Recently, [97,98] developed inferences for SSPALT under hybrid and adaptive progressive hybrid censoring, assuming lifetimes follow the Nadarajah–Haghighi distribution. [99] extended this framework to estimate parameters and assess reliability of a hybrid system under progressive hybrid censoring with unknown causes. [100] employed the TRV model to analyze multiple censored datasets, assuming lifetimes follow the Nadarajah–Haghighi distribution, and obtained MLEs for model parameters and acceleration factors. [101] proposed a SSPALT framework using adaptive type-II progressive hybrid censoring, assuming lifetimes follow a two-parameter inverted Lomax distribution.

Constructing a SSPALT model requires a series of methodical steps to maintain statistical validity and operational relevance. The entire testing and modeling process can be outlined systematically as follows:

- Stress Level and Censoring Scheme Specification: Specify k stress levels ($k \geq 2$), denoted as S_i and τ_i for $i = 0, 1, 2, \dots, k$, where S_0 represents the normal operating stress. Define the number, magnitude, and order of stress levels according to material limits and desired acceleration factors. The stress changes from S_0 to S_{i+1} at time τ_i. Test a number of specimens at each stress level until all units fail or the test is terminated due to a censoring scheme. Denote the observed life of specimen $j (j = 0, 1, 2, \dots, n)$ at stress level i by X_{ij}. Lifetimes at each level are independent and identically distributed.
- Select Failure Model: Select an appropriate lifetime distribution (e.g., Exponential, Weibull, or Lognormal). The lifetimes of products at each stress level are assumed to be independent and identically distributed according to the selected probability distribution.
- Apply stress change effect Model: In SSPALT, the TRV and TFR models provide complementary frameworks for describing the effect of sequential stress increases on product lifetime. The TRV model, introduced by [87], assumes that a stress change at time τ scales the remaining life of a unit. If T denotes the original lifetime, the remaining life after the stress change is given by

$$T' = \alpha(T - \tau) \tag{5}$$

  In contrast, the TFR model, proposed by [88], assumes that a stress change at time τ scales the failure rate function. If $\lambda(t)$is the failure rate before τ, then the failure rate function after the stress change is given by

$$\lambda'(t) = \alpha\, \lambda(t), \qquad t > \tau \tag{6}$$

  The TRV model directly modifies the lifetime distribution, whereas the TFR model alters the hazard function. Both approaches accelerate failures to enable rapid reliability assessment under normal operating conditions. The TRV model is flexible across multiple stress levels and lifetime distributions. Under specific conditions, TRV and TFR may yield equivalent results. These models provide a rigorous statistical framework for analyzing complex or unknown life–stress relationships.
- Parameter estimation: Select and apply an appropriate estimation technique, such as MLE, LSE, BE etc. to obtain model parameters and reliability measures.
- Optimization of Stress-Change Times: Determine optimal change times $(\tau_0, \tau_1, \tau_2, \dots \tau_k)$ that minimize estimator variance or maximize test information.

- Extrapolation and Model Evaluation and Validation: Use the fitted model to predictions under normal operating stress. Evaluate and validate model adequacy.

VI. Discussion, conclusions, and future directions

This review consolidates conceptual and methodological advancements in ALT, encompassing both fully and partially accelerated designs. Qualitative ALT enables early identification of design weaknesses during product development. Quantitative ALT establishes statistically sound reliability estimates through structured constant- and step-stress experiments. Foundational stress–life models such as Arrhenius, inverse power law, and Eyring remain critical for extrapolating accelerated test data to normal operating conditions. Analytical and estimation techniques including MLE, BE, and GP maintain statistical validity across a wide range of lifetime distributions. Implementation of ALT frameworks is supported by computational platforms such as R, Python, ReliaSoft ALTA, Minitab, and JMP, which promote standardized data analysis, model fitting, and consistency with international reliability standards. These integrated tools ensure reproducibility and accuracy in test planning, parameter estimation, and life prediction within both FALT and PALT environments.

Recent advancements in ALT indicate a transition toward adaptive, hybrid, and AI-integrated modeling frameworks. The incorporation of machine learning and real-time monitoring enhances test efficiency and predictive reliability across complex systems. Modern algorithms are now capable of processing censored, incomplete, or masked datasets while retaining the physical consistency of stress–life relationships. The convergence of traditional statistical models with advanced computational analytics enables more accurate life prediction, optimized test design, and cost-effective reliability evaluation. These developments strengthen the applicability of ALT in industrial environments by providing intelligent, data-driven tools for realistic and efficient reliability assessment.

In conclusion, ALT has matured into a multifaceted, data-driven discipline central to reliability assurance in advanced engineering systems. Its future development will focus on probabilistic modeling, intelligent test design, and digital-twin integration to enable predictive maintenance and design validation. The integration of statistical inference with computational intelligence will shape the next generation of reliability testing frameworks. As engineering systems increase in complexity and demand for high-reliability products grows, ALT will remain essential for ensuring product performance, safety, and operational longevity across diverse industrial sectors.

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