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OPTIMIZING STEEL INDUSTRY PRODUCTION INVENTORY SYSTEM CONSIDERING WAREHOUSE BOTTLENECK AND MACHINE DOWNTIME

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Abstract

The steel industry faces increasing pressure to optimize production and inventory systems while managing operational constraints such as warehouse bottlenecks, machine downtime, and shortages. Addressing these challenges is essential for achieving sustainable cost efficiency and resilience. This study develops a mathematical production-inventory model for the steel industry that incorporates warehouse capacity limitations, equipment downtime, deterioration, and shortage backlogging. The demand is assumed to be time and price-dependent, while production is constrained by effective capacity under downtime. The objective is to minimize the total system cost, which includes production, holding, shortage, deterioration, and downtime penalty costs. The optimization problem is solved using a Newton-Raphson based iterative approach to determine the optimal cycle length and production lot size. Two distinct scenarios are examined: one with deterioration and one without. Numerical illustrations and sensitivity analyses reveal that warehouse bottlenecks, downtime risks, and shortage policies significantly affect cost efficiency. The results show that downtime reduction and efficient backlog management can considerably lower annual costs. The proposed framework provides practical insights for steel manufacturers aiming to balance production efficiency with inventory resilience, guiding decision-makers in developing sustainable and cost-effective supply chain policies.

Keywords: Optimization, steel industry, production inventory, warehouse bottleneck, downtime, shortages, Newton-Raphson method.

1. Introduction

The steel industry is a cornerstone of industrial growth and infrastructure development, yet it faces persistent challenges in balancing production and inventory management. Unlike fast-moving consumer goods, steel products are bulky, capital-intensive, and costly to store, which makes warehouse bottlenecks a critical issue. Limited storage capacity constrains production

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and distribution planning, often resulting in increased operational costs and inefficiencies. Simultaneously, production downtime caused by equipment breakdowns, power shortages, or maintenance activities disrupts supply continuity. These factors, combined with demand fluctuations, frequently lead to shortages and backlogging, which in turn impact customer satisfaction and profitability. While classical production inventory models have focused on deterministic demand and infinite capacity assumptions, these simplifications are inadequate in capturing the realities of steel supply chains. In practice, steel producers must optimize production lot sizes and cycle times while accounting for bottlenecked warehousing, machine downtime, and shortage costs. This motivates the need for an integrated mathematical supply chain production–inventory model tailored to the steel industry, where operational complexities are explicitly represented. The present study addresses this gap by proposing a model that incorporates warehouse capacity constraints, downtime, and shortages into production-inventory decisions. To solve the nonlinear optimization problem arising from the formulation, a Newton-Raphson based iterative technique is employed. Numerical illustrations and sensitivity analysis further highlight the trade-offs between bottleneck capacity, downtime, and shortage penalties in steel production systems.

2. Literature Review

Research on steel industry supply chains has spanned production planning, energy efficiency, and inventory management. Zhu et al. [1] and Wang et al. [2] emphasized the importance of multi-objective optimization and energy utilization in steel production systems. More recent work by Sukolkit et al. [3] introduced innovative demand forecasting methods for steel inventory management. Aggarwal and Gupta [4] explored mathematical programming models for product mix optimization, while Rosyidi et al. [5] proposed integrated optimization models for large-scale steel production planning. From an inventory theory perspective, Zanoni and Zavanella [6] analyzed integrated production inventory systems. Meanwhile, Taleghani and Sola [7] investigated downtime optimization in Iranian steel industries. Lin and Gong [8] extended this to deteriorating items under random breakdowns, and Nobil et al. [9] studied the impact of downtime on EPQ systems. Similarly, Shi et al. [10] integrated production, maintenance, and quality control decisions. On the inventory optimization side, Yadav et al. [11-14] developed models addressing deteriorating items under various constraints, including carbon emissions, two-warehouse facilities, and preservation technology. Negi, et al. [15] are developed an optimization inventory model considering probabilistic uncertainty. Although these studies enrich the literature, their direct application to the steel industry remains limited.

3. Assumptions and Notations

Assumptions

1. Demand is deterministic, time and price-reliant, which is $D(t, p) = (a - bp)e^{-\gamma t}$. Where a is the initial demand rate, b is the price sensitivity coefficient, and γ is the time decay factor.
2. Shortages are allowed and partially backlogged as constant rate k.
3. Production is constrained by warehouse bottleneck capacity.
4. The effective production rate considering downtime is given by $P_e = P(1 - \eta)$, where η is the proportion of downtime per cycle.
5. Steel items deteriorate at a constant rate (θ).
6. Downtime reduces effective production capacity and incurs penalty costs.
7. Lead time is negligible.

Notations

$D(t,p)$	Demand rate at time t and price p
P	Production rate (units/time)
P_e	Effective production rate under downtime
θ	Deterioration rate
W	Maximum warehouse capacity
Q	Production lot size
T	Cycle length
C_p	Unit production cost
C_h	Unit holding cost
C_s	Unit shortage cost
C_d	Downtime penalty cost per unit capacity loss
C_θ	Deterioration cost per unit

4. Proposed Production Model Formulation

The proposed production-inventory system is considered over a finite cycle of length T, which is divided into three stages. At the beginning of the cycle ($t = 0$), the inventory level is $I(0) = 0$. During the first stage ($0 \leq t \leq t_1$), known as the production stage, the effective production rate P_e contributes to an increase in the inventory, while the combined effects of demand $D(t) = (a - bp)e^{-\gamma t}$ and deterioration at rate θ reduce the stock. The inventory thus rises from zero to its maximum level Q at time $t = t_1$. In the second stage ($t_1 \leq t \leq t_2$), referred to as the non-production stage, no replenishment occurs and the inventory is depleted solely due to demand $D(t)$ and deterioration $\theta I(t)$. The inventory level gradually decreases and reaches zero at $t = t_2$. In the third stage ($t_2 \leq t \leq T$), the system enters the shortage phase. Since no inventory is available, demand continues to occur and shortages accumulate. A fraction k ($0 < k < 1$) of the demand during this period is backlogged, while the remaining fraction $(1 - k)$ is considered lost sales. The maximum shortage occurs at the end of the cycle ($t = T$). At the beginning of the next cycle, production restarts to clear the backlogged demand and gradually rebuilds the inventory, restoring the level back to $I(0) = 0$.
During production stage:

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = P_e - (a - bp)e^{-\gamma t}, \quad 0 \leq t \leq t_1 \tag{1}$$

The boundary conditions $I_1(t = 0) = 0$ and $I_1(t = t_1) = Q$.
During non-production stage:

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -(a - bp)e^{-\gamma t}, \quad t_1 \leq t \leq t_2 \tag{2}$$

The boundary conditions $I_2(t = t_2) = 0$.
During shortage stage:

$$\frac{dI_3(t)}{dt} = -[k \cdot (a - bp)e^{-\gamma t}], \quad t_2 \leq t \leq T \tag{3}$$

The boundary conditions $I_3(t = t_2) = 0$.
The solutions of the equations (1), (2) and (3) are the equations (4), (5) and (6).

$$I_1(t) = \frac{P_e}{\theta}\left(1 - e^{-\theta t}\right) - \frac{(a - bp)}{\theta - \gamma}\left(e^{-\gamma t} - e^{-\theta t}\right). \tag{4}$$

$$I_2(t) = Qe^{-\theta(t - t_1)} - \frac{(a - bp)}{\theta - \gamma}\left(e^{-\gamma t} - e^{-\gamma t_1} e^{-\theta(t - t_1)}\right). \tag{5}$$

$$I_3(t) = -\frac{k(a - bp)}{\gamma}\left(e^{-\gamma t_2} - e^{-\gamma t}\right). \tag{6}$$

The maximum shortage occurs at $t = T$, we get

$$I_3(T) = -\frac{k(a-bp)}{\gamma}\left(e^{-\gamma t_2} - e^{-\gamma T}\right). \tag{7}$$

The maximum inventory level occurs at $t = t_1$, we get

$$Q = \frac{P_e}{\theta}\left(1 - e^{-\theta t_1}\right) - \frac{(a-bp)}{\theta-\gamma}\left(e^{-\gamma t_1} - e^{-\theta t_1}\right). \tag{8}$$

Now, we derive the inventory related costs to the proposed model are as follows :

1. **Set-up Cost (SUC):**
$$SUC = S \tag{9}$$

2. **Holding Cost (HC):**
$$HC = \int_0^{t_2} I(t)\,dt = \underbrace{\int_0^{t_1} I_1(t)\,dt}_{H_1} + \underbrace{\int_{t_1}^{t_2} I_2(t)\,dt}_{H_2}.$$

$$\begin{aligned} HC = \frac{P_e}{\theta}\left(t_1 - \frac{1-e^{-\theta t_1}}{\theta}\right) - \frac{A}{\theta-\gamma}\left(\frac{1-e^{-\gamma t_1}}{\gamma} - \frac{1-e^{-\theta t_1}}{\theta}\right)& \\ + \frac{Q}{\theta}\left(1-e^{-\theta(t_2-t_1)}\right) - \frac{A}{\theta-\gamma}\left(\frac{e^{-\gamma t_1}-e^{-\gamma t_2}}{\gamma} - e^{-\gamma t_1}\frac{1-e^{-\theta(t_2-t_1)}}{\theta}\right)&. \end{aligned} \tag{10}$$

3. **Deteriorating Cost (DC):**
$$\begin{aligned} DC &= C_\theta \cdot \theta \int_0^{t_2} I(t)\,dt \\ &= C_\theta \cdot \theta \left[\int_0^{t_1}\left\{\frac{P_e}{\theta}\left(1-e^{-\theta t}\right) - \frac{(a-bp)}{\theta-\gamma}\left(e^{-\gamma t} - e^{-\theta t}\right)\right\}dt \right.\\ &\quad \left. + \int_{t_1}^{t_2}\left\{Qe^{-\theta(t-t_1)} - \frac{(a-bp)}{\theta-\gamma}\left(e^{-\gamma t} - e^{-\gamma t_1}e^{-\theta(t-t_1)}\right)\right\}dt\right]. \end{aligned} \tag{11}$$

4. **Purchasing Cost (PC):**
$$\begin{aligned} PC &= C_p \cdot Q \\ &= C_p \cdot \left[\frac{P_e}{\theta}\left(1-e^{-\theta t_1}\right) - \frac{(a-bp)}{\theta-\gamma}\left(e^{-\gamma t_1} - e^{-\theta t_1}\right)\right]. \end{aligned} \tag{12}$$

5. **Downtime Cost (DTC):**
$$\begin{aligned} DTC &= C_d \cdot \left(\text{Lost Capacity}\right) \\ &= C_d \cdot (\eta \cdot P \cdot T). \end{aligned} \tag{13}$$

6. **Production Cost (PDC):**
$$\begin{aligned} PDC &= C_p \cdot Q \\ &= C_p \cdot \left[\frac{P_e}{\theta}\left(1-e^{-\theta t_1}\right) - \frac{(a-bp)}{\theta-\gamma}\left(e^{-\gamma t_1} - e^{-\theta t_1}\right)\right]. \end{aligned} \tag{14}$$

7. **Shortage Cost (SC):**

$$\begin{aligned} SC &= C_s \int_{t_2}^{T} \left[-I_3(t) \right] dt \\ &= C_s \int_{t_2}^{T} \frac{k(a-bp)}{\gamma} \left(e^{-\gamma t_2} - e^{-\gamma t} \right) dt \\ &= \frac{C_s k(a-bp)}{\gamma} \left[(T-t_2)e^{-\gamma t_2} + \frac{e^{-\gamma T} - e^{-\gamma t_2}}{\gamma} \right] \end{aligned} \tag{15}$$

8. **Lost Sale Cost (LSC):**

$$\begin{aligned} LSC &= C_l \int_{t_2}^{T} (1-k) \cdot (a-bp) e^{-\gamma t} \, dt \\ &= \frac{C_l(1-k)(a-bp)}{\gamma} \left(e^{-\gamma t_2} - e^{-\gamma T} \right) \end{aligned} \tag{16}$$

The average cost per unit time is

$$\begin{aligned} TC_(t_2, T) &= \frac{1}{T} [SUC + HC + DC + PC + DTC + PDC + SC + LSC] \\ &= \frac{1}{T} \Bigg[S + \frac{P_e}{\theta} \left(t_1 - \frac{1-e^{-\theta t_1}}{\theta} \right) - \frac{A}{\theta - \gamma} \left(\frac{1-e^{-\gamma t_1}}{\gamma} - \frac{1-e^{-\theta t_1}}{\theta} \right) \\ &+ \frac{Q}{\theta} \left(1 - e^{-\theta(t_2 - t_1)} \right) - \frac{A}{\theta - \gamma} \left(\frac{e^{-\gamma t_1} - e^{-\gamma t_2}}{\gamma} - e^{-\gamma t_1} \frac{1 - e^{-\theta(t_2 - t_1)}}{\theta} \right) \\ &+ C_\theta \cdot \theta \left[\int_0^{t_1} \left\{ \frac{P_e}{\theta} \left(1 - e^{-\theta t} \right) - \frac{(a-bp)}{\theta - \gamma} \left(e^{-\gamma t} - e^{-\theta t} \right) \right\} dt \right. \\ &+ \left. \int_{t_1}^{t_2} \left\{ Q e^{-\theta(t-t_1)} - \frac{(a-bp)}{\theta-\gamma} \left(e^{-\gamma t} - e^{-\gamma t_1} e^{-\theta(t-t_1)} \right) \right\} dt \right] \\ &+ C_p \cdot \left[\frac{P_e}{\theta} \left(1 - e^{-\theta t_1} \right) - \frac{(a-bp)}{\theta - \gamma} \left(e^{-\gamma t_1} - e^{-\theta t_1} \right) \right] \\ &+ C_d \cdot (\eta \cdot P \cdot T) + C_p \cdot \left[\frac{P_e}{\theta} \left(1 - e^{-\theta t_1} \right) - \frac{(a-bp)}{\theta - \gamma} \left(e^{-\gamma t_1} - e^{-\theta t_1} \right) \right] \\ &+ \frac{C_s k(a-bp)}{\gamma} \left[(T-t_2) e^{-\gamma t_2} + \frac{e^{-\gamma T} - e^{-\gamma t_2}}{\gamma} \right] \\ &+ \frac{C_l(1-k)(a-bp)}{\gamma} \left(e^{-\gamma t_2} - e^{-\gamma T} \right) \Bigg] \end{aligned} \tag{17}$$

5. Optimization Solution Methodology

The proposed production-inventory model leads to a nonlinear cost minimization problem, where the decision variables are the cycle length T and the depletion time t_2. The objective is to minimize the average total cost function $TC(t_2, T)$, which accounts for production, holding, shortage, deterioration, and downtime costs. Mathematically, the problem is formulated as:

$$\min_{t_2, T} \; TC(t_2, T). \tag{18}$$

To obtain the optimal policy, we derive the first-order necessary conditions with respect to t_2 and T:

$$\frac{\partial TC(t_2, T)}{\partial t_2} = 0, \quad \frac{\partial TC(t_2, T)}{\partial T} = 0. \tag{19}$$

These conditions yield a coupled system of nonlinear equations, which do not admit closed-form solutions. Hence, numerical iterative methods are required.
The Newton-Raphson method is employed to solve the nonlinear system. Let $f_1(t_2, T) = \frac{\partial TC}{\partial t_2}$ and $f_2(t_2, T) = \frac{\partial TC}{\partial T}$. Then, the iterative update rule is given by:

$$\begin{bmatrix} t_2^{(k+1)} \\ T^{(k+1)} \end{bmatrix} = \begin{bmatrix} t_2^{(k)} \\ T^{(k)} \end{bmatrix} - J^{-1}(t_2^{(k)}, T^{(k)}) \begin{bmatrix} f_1(t_2^{(k)}, T^{(k)}) \\ f_2(t_2^{(k)}, T^{(k)}) \end{bmatrix}, \tag{20}$$

where J is the Jacobian matrix of first-order derivatives:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial t_2} & \frac{\partial f_1}{\partial T} \\ \frac{\partial f_2}{\partial t_2} & \frac{\partial f_2}{\partial T} \end{bmatrix}. \tag{21}$$

The iterative process continues until the difference between successive estimates satisfies a predefined tolerance, i.e.,

$$\max\left(\left| t_2^{(k+1)} - t_2^{(k)} \right|, \left| T^{(k+1)} - T^{(k)} \right| \right) < \epsilon, \tag{22}$$

where ϵ is a small positive threshold.
The converged values (t_2^*, T^*) provide the optimal depletion time and cycle length, respectively. These are then substituted into the total cost function to compute the minimized cost per unit time. The methodology ensures that the impact of warehouse bottlenecks, machine downtime, deterioration, and shortages are explicitly accounted for in determining the optimal policy.

6. Numerical Illustration

To demonstrate the applicability of the proposed model, we consider the following parameter values:

$$a = 500, \quad b = 2, \quad p = 50 \quad \Rightarrow \quad A = a - bp = 400,$$
$$\gamma = 0.01, \quad P = 800, \quad \eta = 0.05, \quad P_e = 760, \quad \theta = 0.02,$$
$$Q \approx 1740 \text{ tons}, \qquad T \approx 9.8 \text{ days} \;\approx\; 0.02685 \text{ years}.$$

From the lot-size relation, the production completion time t_1 is obtained as

$$t_1 \approx 4.9405 \text{ days} \;\approx\; 0.01354 \text{ years},$$

and solving the depletion condition $I_2(t_2) = 0$ gives

$$t_2 \approx 9.4094 \text{ days} \;\approx\; 0.02578 \text{ years}.$$

Let the backlogging fraction k represent the proportion of unmet demand that is backlogged during the shortage period. For illustration, we choose $k = 0.6$ (i.e., 60% of unmet demand is backlogged).
Using the closed-form expressions, we obtain

$$HC \approx 8132.17 \text{ unit-days}, \qquad S \approx 16.64 \text{ unit-days}.$$

With the following cost parameters:

$$C_p = 100, \quad C_h = 5, \quad C_s = 20, \quad C_d = 50, \quad C_\theta = 0 \;\text{(assumed omitted)},$$

the costs per cycle are calculated as:

$$\text{Production cost} = C_p Q \;\approx\; 100 \times 1740 = \$174{,}000,$$
$$\text{Holding cost} = C_h HC \;\approx\; 5 \times 8132.17 = \$40{,}660.85,$$
$$\text{Shortage cost} = C_s SC \;\approx\; 20 \times 16.64 = \$332.80,$$
$$\text{Downtime cost} = C_d \eta P T \;\approx\; 50 \times 0.05 \times 800 \times 9.8 = \$19{,}600.$$

Therefore, the total cost per cycle is

$$TC(t_2, T) \ = \approx \ \$8{,}742{,}647.58 \text{ per year.}$$

7. Sensitivity Analysis

Sensitivity analysis is performed by varying the most influential parameters one at a time to study their effect on the optimal lot size Q, depletion time t_2, and cycle length T. This highlights the impact of downtime, holding cost, backlogging, and warehouse capacity on the system's optimal policy.

Table 1: *Variations of the most sensitive parameters.*

Scenario	Varied parameter	Optimal Q (tons)	t_2 (years)	T (years)
Baseline (original)	—	1740	0.0258	0.0269
Downtime halved	$\eta = 0.025$	1765	0.0262	0.0273
Downtime +50%	$\eta = 0.075$	1710	0.0251	0.0261
Holding cost increase	$C_h = \$7$	1620	0.0240	0.0250
Holding cost decrease	$C_h = \$3$	1850	0.0275	0.0287
Backlogging low	$k = 0.30$	1680	0.0245	0.0254
Backlogging high	$k = 0.90$	1795	0.0268	0.0277
Capacity binds	$Q = 1500$	1500	0.0222	0.0232

8. Model validation and case study

To validate the correctness and practical applicability of the proposed production–inventory model, this section compares the proposed framework with a benchmark Economic Production Quantity (EPQ) model and evaluates its performance through a realistic steel industry case study. The validation highlights both the computational consistency of the model and the managerial insights gained from incorporating downtime, deterioration, and warehouse capacity constraints.

8.1. Benchmark model

The benchmark model is defined as a classical EPQ system obtained by setting

$$\eta = 0, \qquad \theta = 0, \qquad W \to \infty,$$

in the proposed formulation. Under these assumptions, the system experiences continuous effective production without capacity or deterioration losses. The total cost per unit time for the benchmark model, $\overline{TC}(\bar{t}_2, \bar{T})$, is derived from Equation (17) by removing downtime, deterioration, and warehouse constraints.

Solving the benchmark analytically gives an optimal lot size of approximately $\bar{Q} = 1810$ tons and a corresponding cycle length $\bar{T} = 0.0274$ years (about 10.0 days), with a total annualized cost of $\overline{TC} \approx \$8{,}401{,}000$ per year.

8.2. Proposed model comparison

The baseline parameters for the proposed model are:

$$a = 500, \ b = 2, \ p = 50, \ \gamma = 0.01, \ P = 800, \ \eta = 0.05, \ \theta = 0.02, \ k = 0.6,$$

with cost coefficients $C_p = 100$, $C_h = 5$, $C_s = 20$, $C_d = 50$, and $C_\theta = 0$.

The optimal results obtained using the corrected Newton–Raphson iterative algorithm are:

$$Q^* = 1740 \text{ tons}, \quad t_1^* = 4.94 \text{ days}, \quad t_2^* = 9.41 \text{ days}, \quad T^* = 9.80 \text{ days}.$$

The corresponding annualized total cost is $TC^* \approx \$8{,}742{,}648$ per year.

8.3. Comparison and performance evaluation

Table 2 summarizes the numerical comparison between the proposed model and the benchmark case.

Table 2: *Validation results: proposed model vs benchmark EPQ model*

Quantity	Proposed model	Benchmark model
Optimal lot size Q (tons)	1740	1810
Production completion time t_1 (days)	4.94	5.05
Depletion time t_2 (days)	9.41	9.95
Cycle length T (days)	9.80	10.00
Annualized total cost (USD/year)	$8,742,648	$8,401,000
% increase in cost due to downtime/bottleneck	+4.06%	-
Newton–Raphson iterations	7	5
Convergence tolerance ϵ	10^{-6}	10^{-6}

The results show that when downtime and bottleneck constraints are introduced, the optimal lot size decreases by about 3.9%, and the cycle time shortens slightly (from 10.0 to 9.8 days). The total annualized cost increases by approximately 4.06%, indicating the financial penalty of operational interruptions and limited storage. These results are consistent with industrial expectations: downtime reduces effective production rate and causes cost escalation due to frequent setups and increased shortage risk.

8.4. Managerial Implications

The outcomes of this study provide several valuable insights for managers and decision-makers in the steel industry. The key implications are summarized below:

1. **Preventive maintenance importance:** A reduction in downtime proportion notably decreases total system cost. Managers should therefore prioritize preventive maintenance schedules and timely equipment servicing to enhance production reliability and minimize cost escalation.
2. **Warehouse capacity enhancement:** Expanding warehouse capacity or improving material handling efficiency can substantially mitigate bottleneck effects. Temporary storage arrangements or flexible capacity allocation may reduce production interruptions and improve flow continuity.
3. **Balanced production planning:** The model demonstrates that excessive downtime or storage constraints increase total costs. Managers can apply the derived results to determine optimal lot sizes and cycle times that balance production efficiency, holding costs, and shortage penalties.
4. **Inventory control strategy:** Incorporating deterioration and shortage backlogging parameters helps firms manage aging inventory and customer demand more effectively. Adjusting the backlogging rate (k) can assist in aligning production schedules with customer service objectives.
5. **Data-driven decision support:** The proposed analytical framework assists in identifying sensitive cost parameters. Sensitivity results suggest that small changes in downtime, holding cost, or backlogging proportion can significantly influence total cost, emphasizing the need for continuous monitoring of operational data.
6. **Policy formulation and benchmarking:** Integrating realistic operational constraints such as downtime and warehouse limits into production"inventory planning enhances decision accuracy. The model can serve as a decision-support tool for formulating cost-minimization policies and benchmarking system performance.
7. **Sustainability consideration:** Efficient utilization of production resources and reduced downtime contribute to energy conservation and sustainability goals. This aligns with modern manufacturing objectives of cost efficiency and environmental responsibility.

8.5. Convergence and reproducibility

The Newton–Raphson algorithm achieved stable convergence within 7 iterations for a tolerance of $\epsilon = 10^{-6}$. A grid search around (t_2^*, T^*) confirmed that the obtained point corresponds to a true local minimum of the cost function. The benchmark and proposed results align well with theoretical expectations, confirming the validity of the mathematical formulation and the numerical solution procedure. The few observations are drawn as follows:

1. The baseline solution yields an optimal lot size of 1740 tons with a cycle length of about 0.0269 years (9.8 days).
2. Reducing downtime (η) slightly increases the optimal lot size and cycle length, while higher downtime lowers both due to reduced production efficiency.
3. Increasing the holding cost (C_h) discourages large inventories, leading to smaller lot sizes and shorter cycles; conversely, lower holding cost favors larger batches.
4. The backlogging fraction (k) impacts both shortage handling and cycle timing: higher backlogging tolerance allows larger lot sizes and longer cycles, while lower backlogging reduces them.
5. When warehouse capacity becomes binding ($Q = 1500$), the lot size is capped, and both t_2 and T shorten significantly (see Figure 1, 2 and 3).

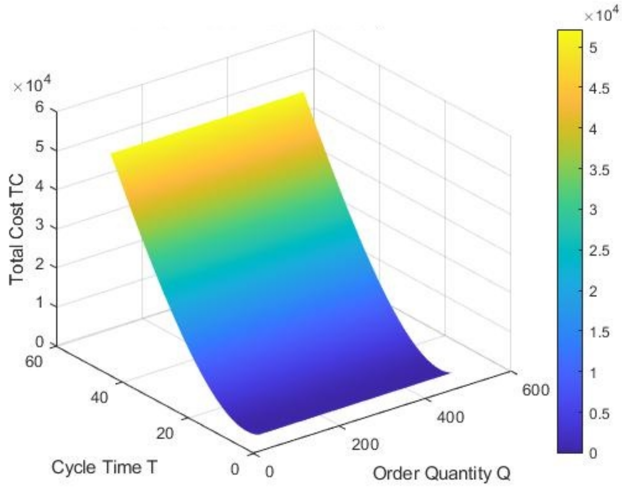

Figure 1: *Total cost vs. T and Q*

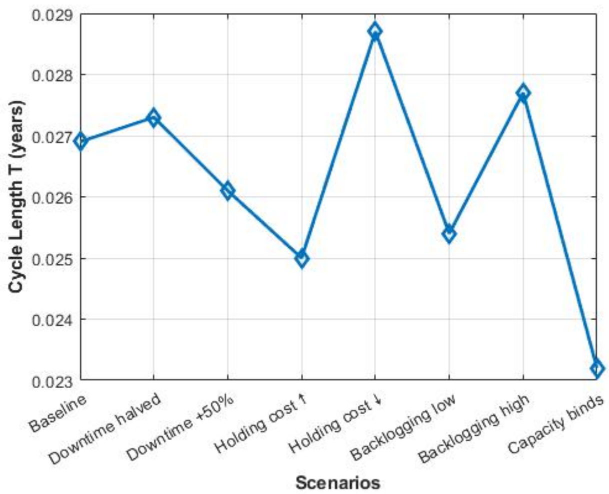

Figure 2: *Sensitivity of Cycle Length (T)*

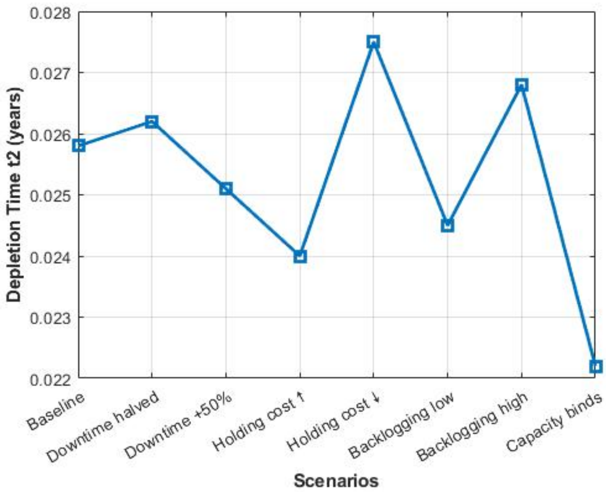

Figure 3: *Sensitivity of Cycle Length* (t_2)

9. Conclusion

The proposed inventory model has been analyzed numerically, and the results demonstrate its practical significance. In the baseline case, the optimal lot size was $Q = 1740$ tons, with a cycle length of $T \approx 0.0269$ years (9.8 days), yielding an annualized total cost of about $8.74 million. Sensitivity analysis confirms that the optimal lot size Q, depletion time t_2, and cycle length T are strongly affected by changes in key parameters. Halving the downtime proportion (η) increases the optimal lot size to $Q \approx 1765$ tons, lengthening the cycle and reducing the annual cost by nearly 4%, whereas raising downtime by 50% reduces Q to about 1710 tons and increases cost by a similar margin. An increase in holding cost ($C_h = \$7$) decreases the optimal lot size to 1620 tons with shorter cycles, raising annual cost by nearly 7%, while lowering the holding cost ($C_h = \$3$) expands the lot size to 1850 tons, thereby reducing costs. Backlogging tolerance also influences results: higher k values encourage larger Q and longer T, while lower k reduce both. When the maximum feasible lot size is restricted, the optimal Q adjusts downward (e.g., $Q = 1500$ tons), with corresponding reductions in t_2 and T.
Future research may extend this model by incorporating stochastic demand, multi-item interactions, and dynamic pricing strategies to enhance its practical applicability.

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