RT&A, No 4 (89)
Volume 20, December 2025

OPTIMIZATION OF INDUSTRIAL SYSTEM MAINTENANCE USING THE MIXED WEIBULL DISTRIBUTION

R. Zellagui[1], L. Khammar[2]
•
[1]University of Constantine 1, Algeria
[2]National Polytechnic School of Constantine, Algeria
[1]zellaguiredouane25@gmail.com, [2]khammar_leila@yahoo.fr

Abstract

This work is part of an approach to optimizing industrial maintenance in a modern and automated production context. It focuses on the in-depth study of the maintenance system implemented in the company IRIS TYRES, specialized in the manufacturing of tires in Algeria, more precisely in the production unit located in Sétif. The main objective of this work is to evaluate the performance of the maintenance applied in the company, based on the fundamental concepts of Reliability, Maintainability and Availability (RAM), as well as modern analytical tools such as the ARROW NOVI CMMS software, the Pareto method. In this article, the Weibull mixture model is proposed to model the failure times of an electrical system in operation with different failure modes and analyze reliability. The estimation of the parameters of the reliability law is done by the maximum likelihood method. Based on IRIS TYRES' system lifetime data, we analyzed and compared the performance of our model with the classic two-parameter Weibull distribution. Strictly speaking, it is necessary to verify which of the two models correctly fits the data. The results of this evaluation show that the machine with a very high failure rate was the curing press, its subsystems (3-bar sensors, valves), and led to the development of action plans to improve equipment reliability, reduce unplanned downtime, and strengthen the efficiency of the maintenance department. The results show that the Weibull mixture distribution tends to overestimate the reliability of the sensors and therefore to overestimate the frequency of preventive maintenance which reduces maintenance costs and the probability of failure.

Keywords: Reliability, weibull mixed model, steelelastic machine, iris tyres, likelihood method, arrow novi software

I. Introduction

Modern maintenance is no longer limited to simple equipment repair. Today, it constitutes a strategic pillar of industrial management, aiming to anticipate failures, improve the lifespan of facilities, and ensure the safety of people and property, and support production under optimal conditions. The emergence of new digital tools, particularly Computerized Maintenance Management Systems (CMMS), has profoundly transformed traditional practices, introducing more rigorous, analytical, and proactive management. Since Pearson's first attempt at analyzing a mixture model [1], the study of mixture distributions has become a separate field of modern statistics. Numerous reference works exist on the subject, the most recent being McLachlan and Peel [2], which provides a "state of the art" review of the various approaches developed to date. For a detailed overview of non-Bayesian analysis

DOI: https://doi.org/10.24412/1932-2321-2025-489-1150-1158

techniques, see Titterington et al. [3], as well as McLachlan and Basford [4]. Mixtures of distributions have provided a set of mathematical approaches for the statistical modeling of a wide variety of random phenomena.

Given their usefulness as flexible modeling methods, mixture models have continued to enjoy increasing popularity and interest from both a practical and theoretical perspective.

Fields in which mixture models have been successfully explained include astronomy, biology, genetics, medicine, psychiatry, economics, engineering, and marketing. Mixtures of distributions are a powerful (a wide variety of distributions can be modeled in this form) and parsimonious (the distribution thus modeled is described by a reduced number of parameters) approach [5,6,7].

Moreover, the composition of the distribution as a mixture can often be exploited to identify a hidden structure in the dataset, i.e. to determine interesting subpopulations from the point of view of the interpretation of the dataset. Heterogeneity in a distributed operating context is manifested by a mixture of failure probabilities $f_j(t)$ resulting from the behavior of the equipment in operation on each environment j. When the same equipment is operated on a number m of distinct environments (organization, climatic, production conditions) where the wear process is likely to be modified by the operating variables, then the lifetime data are a mixture. The resulting model of the mixture is a weighted mixture of failure probabilities [8]. Therefore, the model is a sum of failure laws $f_j(t)$ weighted by a weight p_j representing the percentage of lifetime data from each environment j.

Therefore, the model is a sum of failure laws $f_j(t)$ weighted by a weight pj representing the percentage of lifetime data from each environment j [9,10]. Lesobre R and Beovard K, presented an approach to optimize the maintenance strategy and evaluate the system design based on the concept of MFP (maintenance free period) [11]. Arfa M and Mohammad A studied the mixed Weibull distribution, involving two shape parameters, two scale parameters and one proportionality parameter, is simulated using the statistical software MINITAB [12]. Zhiman HE et al gave a new mixed Weibull probability distribution model for the reliability evaluation of paper-oil insulation. The breakdown voltage, furfural and six other characteristic parameters were selected to reflect the reliability of paper-oil insulation. This new mixed Weibull probability distribution model was established to evaluate the reliability of paper-oil insulation..[13]. *Benaicha. H and Chaker. A*. proposed a maximum likelihood algorithm for a mixture of two Weibull distributions. The mixed distribution and the traditional Weibull distribution are applied to the historical failure time data of power transformers of the National Electricity and Gas Company (SONELGAZ) in western Algeria. A comparison of the reliability analysis through their respective probability distributions is presented in his study. Finally, the Akaike Information Criterion (AIC) is used to select the optimal distribution for modeling these data.[14]

II. Mixed Weibull model

I. Analytic model

The mixed Weibull analytical model is a method of combining two or more Weibull distributions with different parameters (eg: machine composed of two or more subsystems). In addition to the simple weibull distribution, which is characterized by the shape (β), scale (η) and position (γ) parameters, a mixture or proportion (Wi) parameter is added to represent the importance of each subsystem on the overall behavior of the system. The mixed weibull distribution is expressed by the probability density as follows:

$$f(t) = \sum_{j=1}^{m} w_j f_j(t) \tag{1}$$

With : $f_j(t)$ represents the probability density of failure given by:

$$f_j(t) = \frac{\beta_j}{\eta_j} * \left(\left(\frac{t-\gamma_j}{\eta_j}\right)^{\beta_j - 1}\right) * e^{-\left(\left(\frac{t-\gamma_j}{\eta_j}\right)^{\beta_j}\right)} \tag{2}$$

With: β: Shape parameter, η: Scale parameter, γ : position parameter
Similarly, the function of the mixing failure rate can be given by:

$$h(t) = \sum_{j=1}^{n} w_j(t) h_j(t) \tag{3}$$

Where the failure rate

$$h_j(t) = \frac{\beta_j}{\eta_j}\left(\frac{t-\gamma_j}{\eta_j}\right)^{\beta_j - 1} \tag{4}$$

$$w_j(t) = \frac{w_j . R_j(t)}{\sum_{j=1}^{n} w_j . R_j(t)} \tag{5}$$

And the reliability function is:

$$R(t) = e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta-1}} \tag{6}$$

For a mixture of two distributions we will have:

$$f(t) = w f_1(t) + (1-w) f_2(t) \tag{7}$$

$$h(t) = w h_1(t) + (1-w) h_2(t) \tag{8}$$

By replacement we will have:

$$h(t) = \frac{w . R_1(t)}{w . R_1(t) + (1-w) R_2(t)} * h_1(t) + \frac{(1-w) . R_2(t)}{w . R_1(t) + (1-w) R_2(t)} * h_2(t) \tag{9}$$

II. Estimation of the partition coefficient w

The maximum likelihood method gives an efficient estimator of the mixing parameter w, the likelihood formula of a series of failure times given by the following formula:

$$L = \prod_{i=1}^{n} f(t_i) = f_1(t) . f_2(t) \dots f_n(t) \tag{10}$$

For a problem of two functions, the partial directivity of the function (log L) becomes:

$$\frac{dLL}{dw} = \sum_{i=1}^{n} \frac{f_1(t)-f_2(t)}{wf_1(t)+(1-w)f_2(t)} \tag{11}$$

III. Case study

I. Pareto method

The Iris Tyers factory has implemented a structured maintenance system with the main aim of guaranteeing the optimum reliability and availability of its production equipment; To achieve this goal, it relies on a Computerized Maintenance Management System (CMMS) software, specifically ARROW NOVI, which allows for the total centralization of all maintenance interventions, thus ensuring precise tracking of each corrective or preventive action carried out on the machines, including the date, the nature of the work performed, the spare parts used, the identification of the technician responsible, and the intervention time. This facilitates disciplinary planning of preventive maintenance operations based on calendars or usage thresholds, optimizes the management of intervention requests issued by employees, identify areas for improvement, and ultimately contribute to smoother communication between the various stakeholders involved in the maintenance process within the factory. Based on the ARROW NOVI software reports, the Pareto method was applied to the factory's machines to identify the most critical machines.

Table 1: *Classification of machines according to the total breakdowns time.*

Machines	Breakdowns time	Accrual	% accrual
Curing press	885,01	885,01	38,70%
Steelastic	245,12	1130,13	49,42%
Zf machine	129,14	1259,27	55,07%
Tpcs	124,78	1384,05	60,53%
Tbm	106,72	1490,77	65,19%
Cimcorp b	95,82	1586,59	69,38%
Quintoplex	77,06	1663,65	72,65%
Bead apex	73,3	1736,95	75,65%
Prodicon & antitack	57,36	1794,31	78,47%
Colorservice	57,09	1851,4	80,96%
Bartell	51,46	1902,86	83,22%
Maxi slitter	48,09	1950,95	85,32%
Innerliner	47,22	1998,17	87,38%
Mixer master batch	45	2043,17	89,35%
Cimcorp a	42,83	2086	91,22%
Mini slitter	42,64	2128,64	93,09%
Ilmberger	37,32	2165,96	94,72%
Comerio	36,56	2202,52	96,32%
Laboratory	26,09	2228,61	97,46%
Nuovaciba	24,4	2253,01	98,35%
Rolling machine	14,88	2267,89	99,18%
Utility	8,07	2275,96	99,53%
Machine co2	5,54	2281,5	99,77%
Cimcorp c	2,53	2284,03	99,87%
Mixer final batch	2,51	2286,54	100%
Storage swisslog	0	2286,54	100%
	2286,54	2286,54	100%

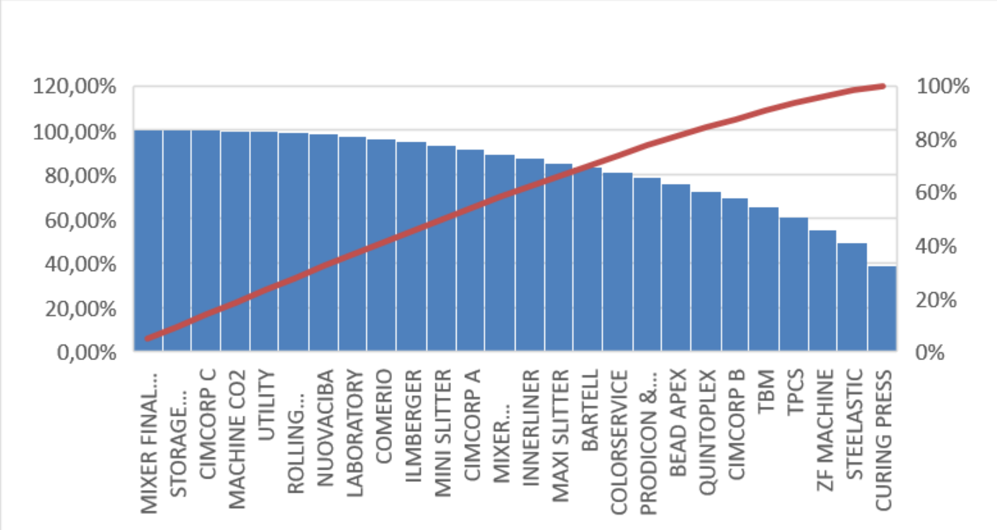

Figure 1: *PARETO diagram*

The mean time of breakdowns is the rate between breakdown time and breakdown number, the calculation of the breakdowns time during the year 2024 gave the following table, the classification of machines in relation to production shows that the most penalizing are: curing press, steel elastic.

II. Mixture Model

Mixture distributions have provided a set of mathematical approaches for the statistical modeling of a wide variety of random phenomena.

Given their usefulness as a flexible modeling method, mixture models have continued to enjoy increasing popularity and interest from both practical and theoretical perspectives. The Weibull mixture is the most widely used model in practice due to its high flexibility. It is recommended for modeling the lifetimes of units with more than one cause of failure. The mixture of two Weibull distributions allows the behavior of a system to be modeled by assigning each subsystem a mixture proportion, denoted Wj.

In part of work, we used a failure history to estimate the mixture parameter for two subsystems: the Herbert Curing Press, based on the Maximum Likelihood estimation method. For our work, we divided the machine into two main subsystems.

In Figure 2, we show the mixing curve of two Weibull distributions which fits the data much better. Indeed, it is able to take into account mixing, competition as well as the change in failure mode; depending on the operating age of the equipment unlike the Weibull model. We then speak of the heterogeneity of the lifetime data.

The distribution of points linearly reflects the performance of the subgroup. The shape parameters β, scale η, and position γ (for the complete system, subsystem 1, and subsystem 2) are obtained by linear regression.

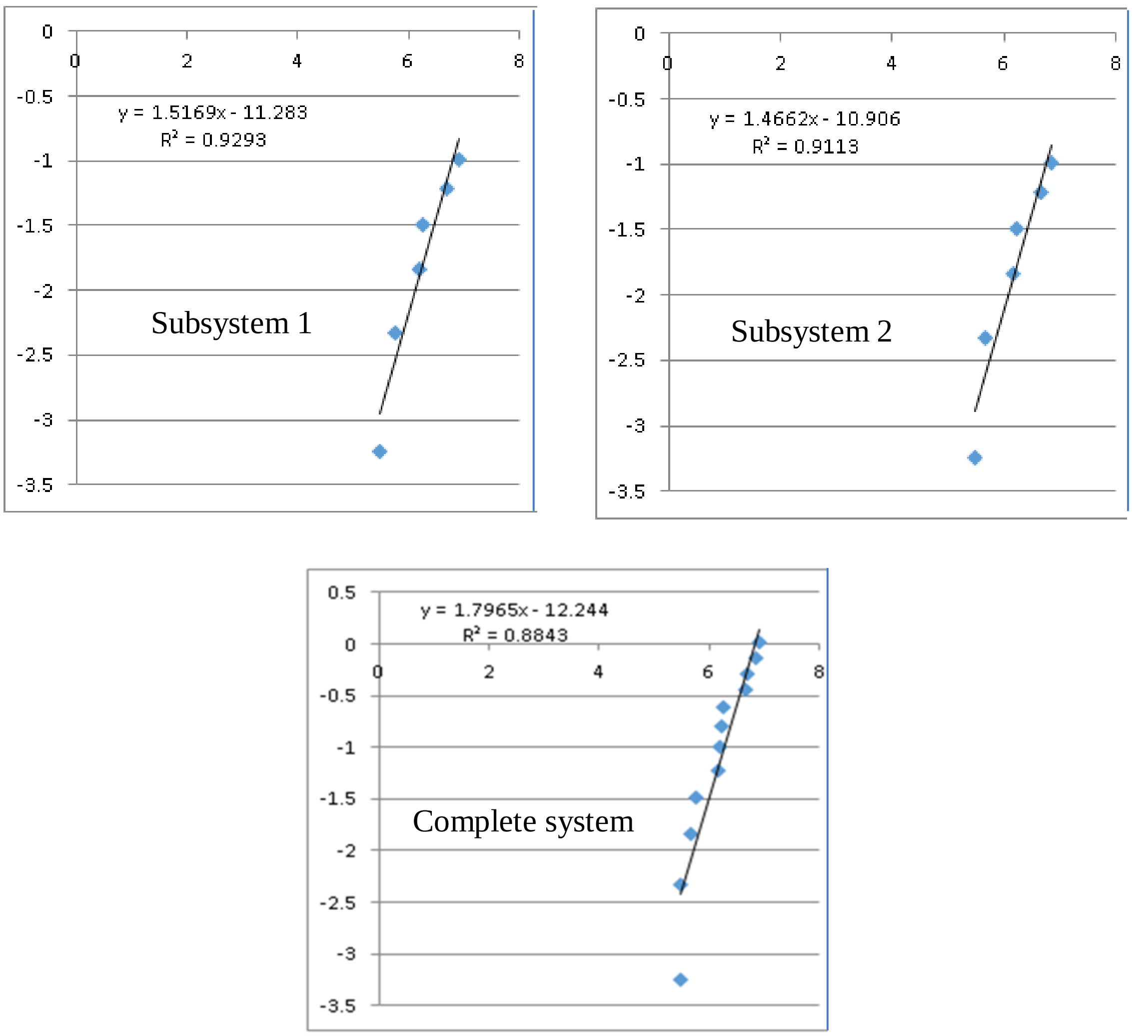

Figure 2: *Weibull distribution.*

Table 2: *The reliability function of the complete system without mixing ($R_w(t)$) and mixed system ($R_m(t)$)*

Nombre de la défaillance	TBFi(h)	$R_w(t)$ (without mixture)	$R_m(t)$ (mixed model)
1	237.75	0.914969	0.000403
2	243.05	0.911241	0.931889
3	289.1	0.880812	0.91252
4	319,12	0.859374	0.899248
5	472,33	0.73612	0.82587
6	493,23	0.718003	0.815266
7	507,1	0.705836	0.808147
8	526,61	0.689233	0.798432
9	782,3	0.468215	0.665006
10	792,8	0.460082	0.659816
11	950,85	0.340817	0.579376
12	983,92	0.31838	0.563062

To highlight the impact of each subsystem on the system's behavior, we used the likelihood method, explained previously, which allows us to estimate the value of the partition coefficient w, and we found its value w = 0.6. Therefore, w1 = 60% and w2 = (1 - w1) = 40%.

IV. Results and Discussion

The following graphs present the different shapes of the failure rates, failure probability densities, and reliability functions.

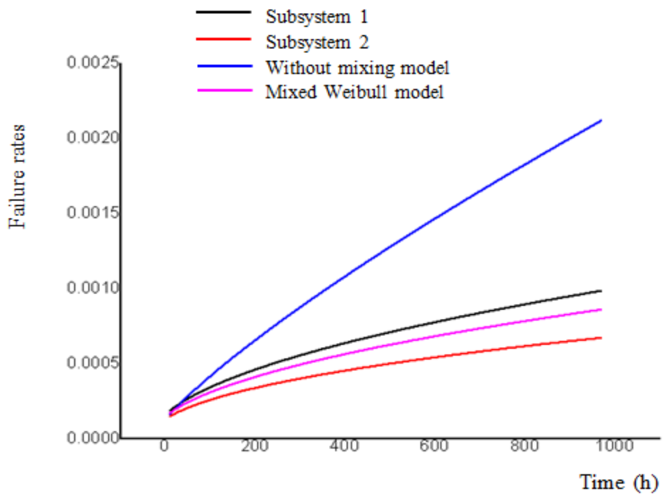

Figure 3: *Failure rates.*

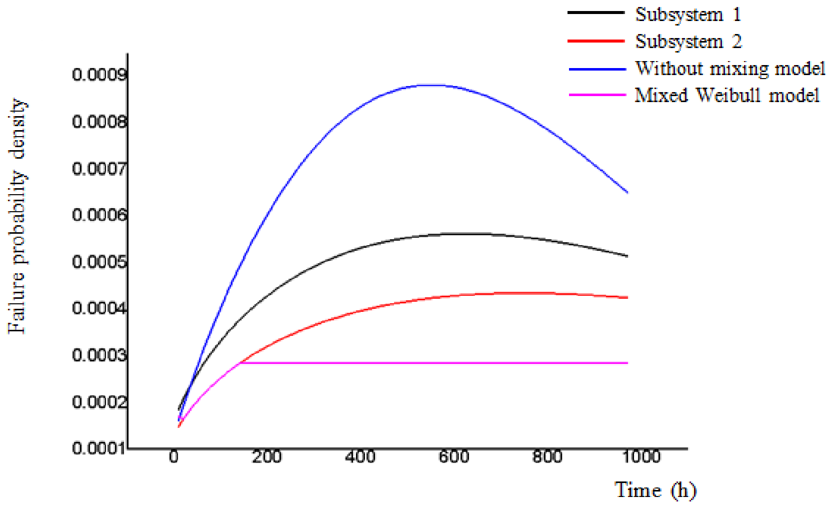

Figure 4: *Failure probability density*

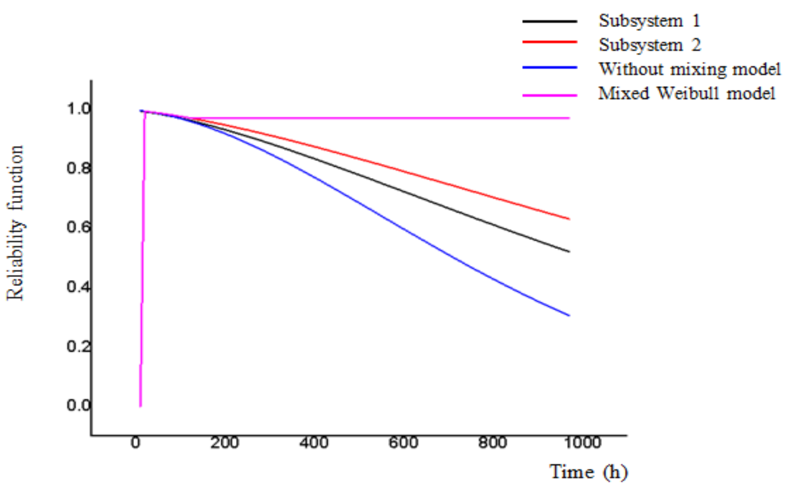

Figure 5: *Reliability function.*

From the graphs above, we can see that the two subsystems follow the same aging (fatigue) regime because β>1.

The partition coefficient w_1=60% indicates that the first subsystem is predominant. Therefore, the failure rate must follow the behavior of this predominant subsystem. The modeling of reliability parameters using the general Weibull model (without mixing) shows a very large deviation from the mixed Weibull model. With the usual modeling of the general Weibull distribution, we also note, for all reliability parameters, an overestimation of the failure rate and probability density values.
The mixed Weibull model allows for estimating system reliability by better approximating reality than the conventional Weibull model. Based on Table 2, the reliability function of power transformers can then be written as follows:

$$R(t) = 0.6e^{-\left(\frac{4}{1327.7}\right)^{11.5169}} + 0.4e^{-\left(\frac{4}{1699.81}\right)^{1.4662}} \tag{12}$$

The reliability formula R(t) can be used to determine the optimal preventive maintenance frequency for power transformers.
Table 3 shows the different preventive maintenance frequencies based on the reliability values calculated by the two respective laws.

Table 3: *Preventive maintenance periods based on Reliability*

Reliability	Preventive maintenance frequency	
	Weibull law	Weibull mixing law
90%	250h	320h
80%	400h	540h
70%	520h	740h

The results of the previous table show that the Weibull mixture distribution tends to overestimate the reliability of the sensors and therefore to overestimate the frequency of preventive maintenance which reduces maintenance costs and the probability of failure.

V. Conclusion

In this study we have discussed the characteristics of the mixed Weibull model using a real history of IRIS TYRES Company. This notion's mixing allows to express the importance of each subsystem by a partition coefficient W by giving a predominance of subsystem 1 (sensors) with a value of 0.60. This mixing parameter, along with the other Weibull parameters, used to obtain the behavior of the system in the form of different plots of failure rate and failure probability density. The results found by this mixture modeling can guide maintenance managers to integrate into their maintenance policy a particular intension to subsystem 1 (sensors) in order to guarantee the availability of the system.
Through this study, we advise IRIS TRYS maintenance managers to consider the following points:

- The components responsible for the high system failure rate.
- The implementation of a preventive maintenance period to avoid any risk of downtime.

References

[1] McLachlan G. and Peel, D. (2000). Finite mixture models. Wiley-Interscience, New York.

[2] Titterington D. M. Smith, A. F. M. Makov, U. E. (1985). Statistical analysis of finite mixture distributions.

[3] McLachlan G. J. and Basford, K. E. (1988). Mixture models, volume 84 of Statistics: Textbooks and Monographs. Marcel Dekker Inc

[4] Murthy D. Xie, M. Jiang, R .(2004). Weibull Models. John Wiley and Sons, Hoboken, New Jersey.

[5] Chapouille P. (1999). Fiabilité et maintenabilité. Les techniques de l'ingénieur : traité de l'entreprise industrielle. Edition Eyrolles.

[6] Pauline B. Optimisation des plans d'essais accélérés Application à la tenue en fatigue de pièces métalliques de liaison au sol. Doctoral thesis, University of Nantes Angers.

[7] Bruneau, P. et al (2010). Parsimonious reduction of gaussian mixture models with a variational-bayes approach. Pattern Recognition, 43(3):850–858.

[8] Bruneau P. et al, (2010). Aggregation of probabilistic pca mixtures with a variational-bayes technique over parameters. In 20th International Conference on Pattern Recognition, 702–705.

[9] Nielsen F et al. (2009). Gaussian mixture models via entropic quantization. In: European Signal Processing Conference, 2012–2016.

[10] Lawless F. (2002). Statistical Models and Methods for Lifetime Data. second ed, Wiley.

[11] Lesobre R. Beovard, K. (2014). Dynamic maintenance policy for a multi-component system integrating the monotoring information. 19th risk management and operational safety congress, Dijon 21-23.

[12] Arfa, M. Mohammad, A. (2008). A comparative study to estimate the parameters of mixed-Weibull distribution. Pak.j.stat.oper.res, 4(1): 1-8.

[13] Zhiman, HE. Youyuan, W. Jian, L. Senlian, G. Stanislaw, G. (2013). New Mixed Weibull Probability Distribution Model for Reliability Evaluation of Paper-oil Insulation. PRZEGLĄD ELEKTROTECHNICZNY, 89:202-206.

[14] Benaicha, H. Chaker, A. (2014). Weibull Mixture Model for Reliability Analysis. International Review of Electrical Engineering, 9(5):986-990.