Arulmozhi. V, Narmadha. D, Sivaranjani. R and B. Balachandar
AREA BIASED NEW-TWO PARAMETER SUJATHA DISTRIBUTION

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AREA BIASED NEW-TWO PARAMETER SUJATHA DISTRIBUTION AND ITS APPLICATION TO MEDICAL SCIENCES

Dr. V. Arulmozhi[1], Mrs. D. Narmadha[2], Dr. R. Sivaranjani[3], Mr. B. Balachandar[4]

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1.   Assistant professor of Statistics, Community Medicine, SBMC&Hospital, Chromepet
2.   Tutor Cum Statistician, Community Medicine, SMC& Hospital, Thandalam
3.   Assistant professor of Statistics, Community Medicine, RMC& Hospital, Pennalur
4.   Assistant professor of Mathematics, Humanity and science, MSA.JCE, Chennai
Email: arulmozhiv16@gmail.com, narmadhajana@gmail.com, sivaranjani160692@gmail.com

Abstract

This study proposes an Area Biased new two-parameter distribution based on the Sujatha distribution of the relief times patient data. A proposed distribution's characteristics, such as its entropies, moments, moment generating function, survival function, hazard function, and probability density function, are examined. The behavior of a hazard function and probability density function has been depicted using graphs. The well-known maximum likelihood outcome approach is used to estimate the distribution's parameter. Finally, a comparative analysis is carried out to show the flexibility and superiority of this distribution over other distributions. It demonstrates its importance in reliability engineering by generating a rich class of distributions that can better fit empirical data by capturing a variety of shapes and behaviors. The application of this unique distribution is demonstrated using relief times (In minutes) of twenty analgesic data sets. Furthermore, we compare the performance of our distribution with some other existing distributions.

Keywords: Moments; Moment generating function; Survival function; Hazard function; Entropies; Maximum likelihood Estimation.

I. Introduction

Statistical distributions are essential in a variety of academic disciplines, including economics, engineering, and biomedicine. In statistical data analysis, the exponential and gamma distributions are frequently employed as life-time distributions. One parameter and other statistical characteristics, such the memory-less property and constant hazard function, are features of the exponential distribution. In order to increase the flexibility of the basic models, several variations of these distributions have been suggested in the literature. Al-Aqtash R, et al,.[1] presented the Gumbel-Weibull distribution: Properties and applications. With its statistical characteristics and uses, the Hamza distribution was proposed by Aijaz et al. [3] Recent years have seen a great deal of research on the Lindley distribution, and distributions with one or two parameters have been created for the modelling of complex data. When it comes to bank client wait times, Ghitney et al. [7] looked at the Lindley distribution and found that it performs better than the exponential distribution.

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Additionally, they demonstrate that the mean residual life is decreasing while the hazard rate function of the Lindley distribution is increasing. In recent years, numerous authors have made diverse contributions to the Lindley distribution. Merovci presented and talked about the characteristics of the transmuted Lindley distribution [10]. The Lindley distribution's inverse and other characteristics were proposed by Sharma et al [11]. The Ishita distribution was created by Shanker et al. [12] and has been demonstrated to outperform the Lindley and exponential distributions. Shukla proposed the Pranav distribution and analyzed its different features [13]. The features of transmuted inverse Lindley distributions were proposed by Ahmad et al. [2] This work aims to create a new three-parameter distribution that outperforms current distributions in terms of flexibility and results. The Hamza distribution is a hybrid of the exponential distribution, which has a scale parameter of β, and the gamma distribution, which has a shape parameter of 7 and a combining proportion of $\dfrac{\beta^{c+6}}{\left(6\alpha\beta^5\Gamma c+1+\Gamma c+7\right)}$.

II. Area Biased New Two Parameter Sujatha distribution

The probability density function of New-Two Parameter Sujatha distribution (ABTPSD) is given by

$$f(x;\theta,\alpha) = \frac{\theta^3}{\theta^2 + \alpha\theta + 2}(1 + \alpha x + x^2)e^{-\theta x}; x > 0, \theta > 0, \alpha \geq 0 \tag{1}$$

And the cumulative distribution function of New-Two Parameter Sujatha distribution is given by

$$F(x;\theta,\alpha) = 1 - \left[1 + \frac{\theta x(\theta x + \alpha\theta + 2)}{\theta^2 + \alpha\theta + 2}\right]e^{-\theta x}; x > 0, \theta > 0, \alpha \geq 0 \tag{2}$$

$$f_a(x) = \frac{x^2 f(x)}{E(x)} \tag{3}$$

$$E(x^2) = \int_0^\infty x^2 f(x)dx$$

$$E(x^2) = \int_0^\infty x^2 \frac{\theta^3}{\theta^2 + \alpha\theta + 2}(1 + \alpha x + x^2)e^{-\theta x}dx$$

$$E(x^2) = \frac{\theta^3}{\theta^2 + \alpha\theta + 2}\int_0^\infty x^2(1 + \alpha x + x^2)e^{-\theta x}dx$$

$$= \frac{\theta^3}{\theta^2 + \alpha\theta + 2}\left[\int_0^\infty x^2 e^{-\theta x}dx + \alpha\int_0^\infty x^3 e^{-\theta x}dx + \int_0^\infty x^4 e^{-\theta x}dx\right]$$

$$= \frac{\theta^3}{\theta^2 + \alpha\theta + 2}\left[\int_0^\infty x^{(3)-1} e^{-\theta x}dx + \alpha\int_0^\infty x^{(4)-1} e^{-\theta x}dx + \int_0^\infty x^{(5)-1} e^{-\theta x}dx\right]$$

$$= \frac{\theta^3}{\theta^2 + \alpha\theta + 2}\left[\frac{\Gamma 3}{\theta^3} + \frac{\alpha\Gamma 4}{\theta^4} + \frac{\Gamma 5}{\theta^5}\right]$$

$$= \frac{\theta^3}{\theta^2 + \alpha\theta + 2}\left[\frac{2}{\theta^3} + \frac{6\alpha}{\theta^4} + \frac{24}{\theta^5}\right]$$

$$= \frac{\theta^3}{\theta^2 + \alpha\theta + 2}\left[\frac{2\theta^4 + 6\alpha\theta^3}{\theta^3} + \frac{24}{\theta^5}\right]$$

$$= \frac{\theta^3}{\theta^2 + \alpha\theta + 2}\left[\frac{2\theta^4 + 6\alpha\theta^3 + 24\theta^4}{\theta^{12}}\right]$$

$$= \frac{\theta^3}{\theta^2 + \alpha\theta + 2}\left[\frac{2\theta^2 + 6\alpha\theta + 24}{\theta^{12}}\right]\theta^4$$

$$= \frac{1}{\theta^2 + \alpha\theta + 2}\left[\frac{2\theta^2 + 6\alpha\theta + 24}{\theta^2}\right]$$

$$E(x^2) = \frac{2\theta^2 + 6\alpha\theta + 24}{\theta^2(\theta^2 + \alpha\theta + 2)} \tag{4}$$

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We will now construct the probability density function of the area biased new two parameter Sujatha distribution by substituting equations (1) and (4) in equation (3).

$$f_a(x) = x^2 \frac{\theta^3}{\theta^2 + \alpha\theta + 2}(1 + \alpha x + x^2)e^{-\theta x} \times \frac{\theta^2(\theta^2 + \alpha\theta + 2)}{2\theta^2 + 6\alpha\theta + 24}$$

$$f_a(x) = \frac{x^2\theta^5}{\theta^2 + 6\alpha\theta + 24}(1 + \alpha x + x^2)e^{-\theta x} \tag{5}$$

And due CDF of area biased new two parameter Sujatha distribution can be obtained ass

$$F_a(x) = \int_0^\infty f_a(x)dx$$

$$= \int_0^\infty \frac{x^2\theta^5}{2\theta^2 + 6\alpha\theta + 24}(1 + \alpha x + x^2)e^{-\theta x}dx$$

$$= \frac{1}{2\theta^2 + 6\alpha\theta + 24}\int_0^\infty x^2\theta^5(1 + \alpha x + x^2)e^{-\theta x}dx$$

$$= \frac{1}{2\theta^2 + 6\alpha\theta + 24}\int_0^\infty \left[\theta^5\int_0^\infty x^2\,e^{-\theta x}dx + \alpha\theta^5\int_0^\infty x^3\,e^{-\theta x}dx + \theta^5\int_0^\infty x^4\,e^{-\theta x}dx\right] \tag{6}$$

$$\text{Put } \theta x = t \Rightarrow \theta dx = dt \Rightarrow dx = \frac{dt}{\theta} \text{ also } x = \frac{t}{\theta}$$

$$F_a(x) = \frac{1}{2\theta^2 + 6\alpha\theta + 24}\left[\theta^5\int_0^{\theta x}\left(\frac{t}{\theta}\right)^2 e^{-t}\frac{dt}{\theta} + \alpha\theta^5\int_0^{\theta x}\left(\frac{t}{\theta}\right)^3 e^{-t}\frac{dt}{\theta} + \theta^5\int_0^{\theta x}\left(\frac{t}{\theta}\right)^4 e^{-t}\frac{dt}{\theta}\right]$$

$$= \frac{1}{2\theta^2 + 6\alpha\theta + 24}\left[\theta^2\int_0^{\theta x}t^{3-1}e^{-t}dt + \alpha\theta\int_0^{\theta x}t^{4-1}e^{-t}dt + \int_0^{\theta x}t^{5-1}e^{-t}dt\right]$$

$$F_a(x) = \frac{1}{2\theta^2 + 6\alpha\theta + 24}[\theta^2\gamma(3, \theta x) + \alpha\theta\gamma(4, \theta x) + \gamma(5, \theta x)] \tag{7}$$
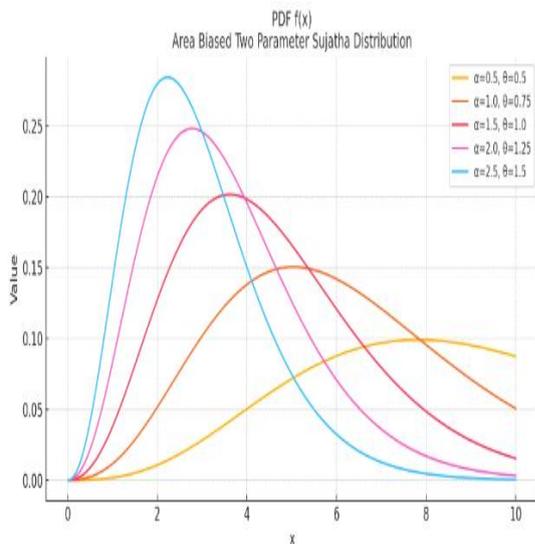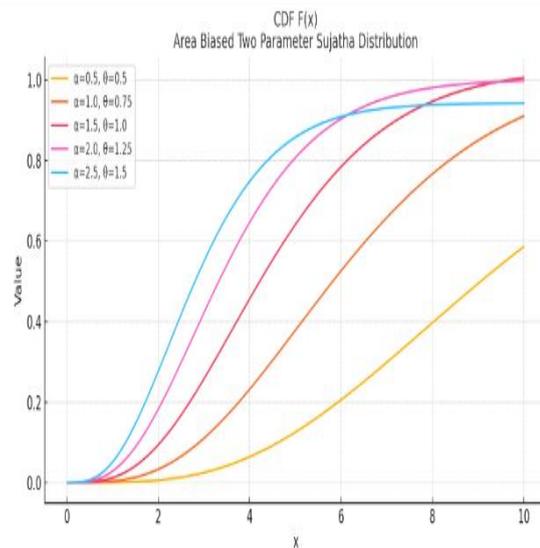

Figure 1: *PDF of the ABTPSD*

Figure 2: *CDF of the ABTPSD*

III. Survival Analysis

The suggested Area biased new two parameter Sujatha distribution's survival function, hazard rate, and reverse hazard rate functions will be covered in this section.

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$$S(x) = 1 - F_a(x)$$

$$S(y) = 1 - \frac{1}{(2\theta^4 + 24\alpha\theta + 120\beta)}[\theta^4\gamma(3,\theta x) + \alpha\theta\gamma(5,\theta x) + \beta\gamma(6,\theta x)]$$

I. The Hazard function

The proposed area-biased new two-parameter Sujatha distribution hazard function is determined as

$$h(x) = \frac{f_a(x)}{s(x)}$$

$$F_a(x) = \frac{x^2\theta^5(1 + \alpha x + x^2)e^{-\theta x}}{(2\theta^2 + 6\alpha\theta + 24) - [\theta^2\gamma(3,\theta x) + \alpha\theta\gamma(4,\theta x) + \gamma(5,\theta x)]}$$

II. Reverse Hazard function

$$h_r(x) = \frac{f_a(x)}{F_a(x)} = \frac{x^2\theta^5(1 + \alpha x + x^2)e^{-\theta x}}{(\theta^2\gamma(3,\theta x) + \alpha\theta\gamma(4,\theta x) + \gamma(5,\theta x))}$$

III. Mills Ratio

$$M.R = \frac{1}{h_r(x)} \frac{(\theta^2\gamma(3,\theta x) + \alpha\theta\gamma(4,\theta x) + \gamma(5,\theta x))}{x^2\theta^5(1 + \alpha x + x^2)e^{-\theta x}}$$
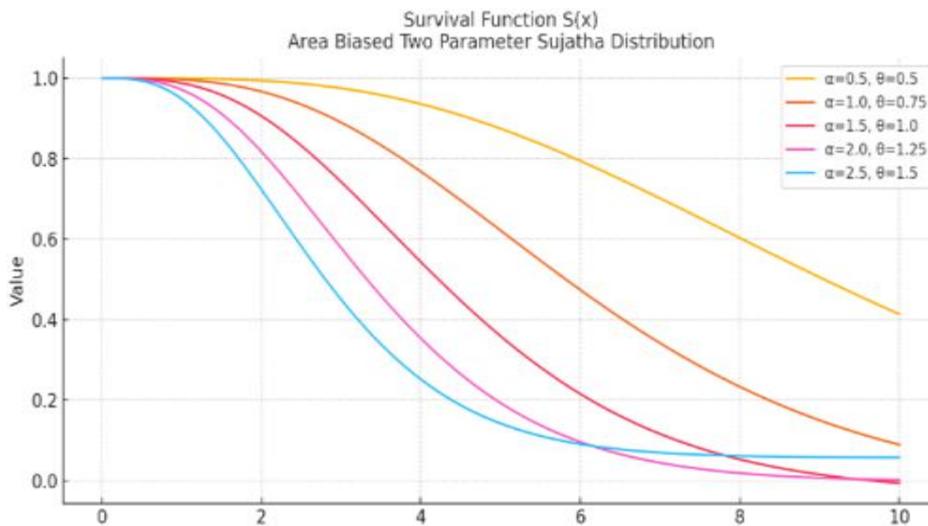

Figure 3: *Survival function ABTPSD*

IV. Statistical Properties

This section discusses the statistical characteristics of the new two-parameter Sujatha distribution that are skewed by Area, including its moments, harmonic mean, MGF, and characteristic function.

I. Order Statistics

Assuming that $X_1, X_2,\ldots, X_n$ are random samples and that $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ are the order statistics of a random sample of size n taken from a continuous population with a probability density function $f_x(x)$ and a cumulative distribution function $F_X(X)$, the probability density function of rth order statistics $X_{(r)}$ can be found using

$$f_{X(r)}(x) = \frac{n!}{(r-1)!\,(n-r)!} f_X(x)[F_X(x)]^{r-1}[1 - F_X(x)]^{n-r}$$

We will now obtain the probability density function of the rth order statistics of the Area Biased Generalized Sauleh distribution by substituting equations (5) and (7) in equation (8).

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$$f_{X(r)}(x) = \frac{n!}{(r-1)!\,(n-r)!} \frac{x^2\theta^6}{(2\theta^4 + 24\alpha\theta + 120\beta)} (\theta + \alpha x^2 + \beta x^3)e^{-\theta x}$$

$$\times \left[\frac{1}{(2\theta^4 + 24\alpha\theta + 120\beta)} (\theta^4\gamma(3,\theta x) + \alpha\theta\gamma(5,\theta x) + \beta\gamma(6,\theta x)) \right]^{r-1}$$

$$\left[\frac{1}{(2\theta^4 + 24\alpha\theta + 120\beta)} (\theta^4\gamma(3,\theta x) + \alpha\theta\gamma(5,\theta x) + \beta\gamma(6,\theta x)) \right]^{n-r}$$

Consequently, the probability density function of the Area Biased Generalized Sauleh distribution higher order statistic $X_{(n)}$ may be found as

$$f_{X(n)}(x) = \frac{nx^2\theta^6}{2\theta^2 + 6\alpha\theta + 2\theta^5} (\theta + \alpha x^2 + \beta x^3)e^{-\theta x}$$

$$\times \left[\frac{1}{(2\theta^4 + 24\alpha\theta + 120\beta)} (\theta^4\gamma(3,\theta x) + \alpha\theta\gamma(5,\theta x) + \beta\gamma(6,\theta x)) \right]^{n-r}$$

Additionally, the probability density function of the Area Biased Generalized Sauleh distribution first order statistic $X_{(1)}$ may be found as

$$f_{X(1)}(x) = \frac{nx^2\theta^6}{(2\theta^4 + 24\alpha\theta + 120\beta)} (\theta + \alpha x^2 + \beta x^3)e^{-\theta x}$$

$$\times \left[1 - \frac{1}{(2\theta^4 + 24\alpha\theta + 120\beta)} (\theta^4\gamma(3,\theta x) + \alpha\theta\gamma(5,\theta x) + \beta\gamma(6,\theta x)) \right]^{n-r}$$

II. Likelihood Ratio Test

Let $X_1, X_{2,\ldots}, Xn$ be a random sample of size n drawn from a new two-parameter Sujatha distribution that is skewed by area. To evaluate the theory

$$H_0: f(x) = f(x;\theta,\alpha) \ against \ H_1: f(x) = f_a(x;\theta,\alpha)$$

To find out if the random sample of size n is from the area-biased new two-parameter Sujatha distribution or the two-parameter Sujatha distribution, the test statistic that follows is utilized.

$$\Delta = \frac{L_1}{L_0} = \prod_{i=1}^{n} \frac{f_a(x;\theta,\alpha)}{f(x;\theta,\alpha)}$$

$$\Delta = \prod_{i=1}^{n} \left[\frac{x_i^2\theta^5}{2\theta^2 + 6\alpha\theta + 24} (1 + \alpha x + x^2)e^{-\theta xi} \times \frac{\theta^2 + \alpha\theta + 2}{\theta^3(1 + \alpha x + x^2)e^{-\theta xi}} \right]$$

$$\Delta = \prod_{i=1}^{n} \left[\frac{\theta^5(\theta^2 + \alpha\theta + 2)}{2\theta^2 + 6\alpha\theta + 24} \right]$$

$$\Delta = \left[\frac{\theta^2(\theta^2 + \alpha\theta + 2)}{2\theta^2 + 6\alpha\theta + 24} \right]^n \prod_{i=1}^{n} x_i^2$$

The null hypothesis should be refused to accept of

$$\Delta = \left[\frac{\theta^2(\theta^2 + \alpha\theta + 2)}{2\theta^2 + 6\alpha\theta + 24} \right]^n \prod_{i=1}^{n} x_i^2 > k \ or \ \Delta^* = \prod_{i=1}^{n} x_i^2 > k \left[\frac{(2\theta^2 + \alpha\theta + 24)}{2\theta^2 + 6\alpha\theta + 2} \right]$$

$$\Delta^* = \prod_{i=1}^{n} x_i^2 > k^* , where \ k^* = k \left[\frac{(2\theta^2 + \alpha\theta + 24)}{2\theta^2 + 6\alpha\theta + 2} \right]^n$$

The chi-square distribution is used to determine the p value for a large sample of size n, where 2log Δ is distributed as a chi-square distribution with one degree of freedom. When the probability value is provided by, we thereby reject the null hypothesis.

$$p(\Delta^* > \beta^*), where \ \beta^* = \prod_{i=1}^{n} x_i^2 \ is \ less \ than \ a \ specified \ level \ of \ significance \ and$$

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$$\prod_{i=1}^{n} x_i^2 \text{ is the observed value of the statistic } \Delta^*$$

II. Moment

Let r^{th} order moment $E(X^r)$ of the Area biased new two parameter Sujatha distribution may be found as follows, where X represents the random variable of the distribution with parameters θ and α.

$$E(X^r) = \mu_r' = \int_0^\infty x^r f_a(x) dx$$

$$= \int_0^\infty x^r \frac{x^2 \theta^5}{2\theta^2 + 6\alpha\theta + 24} (1 + \alpha x + x^2) e^{-\theta x} dx$$

$$= \frac{\theta^5}{2\theta^2 + 6\alpha\theta + 24} \int_0^\infty x^{r+2} (1 + \alpha x + x^2) e^{-\theta x} dx$$

$$= \frac{\theta^5}{2\theta^2 + 6\alpha\theta + 24} \left[\int_0^\infty x^{r+2} e^{-\theta x} dx + \alpha \int_0^\infty x^{r+3} e^{-\theta x} dx + \int_0^\infty x^{r+4} e^{-\theta x} dx \right]$$

$$= \frac{\theta^5}{2\theta^2 + 6\alpha\theta + 24} \left[\int_0^\infty x^{(r+3)-1} e^{-\theta x} dx + \alpha \int_0^\infty x^{(r+4)-1} e^{-\theta x} dx + \int_0^\infty x^{(r+5)-1} e^{-\theta x} dx \right]$$

$$= \frac{\theta^5}{2\theta^2 + 6\alpha\theta + 24} \left[\frac{\theta^{r+4}\theta^{r+5}\Gamma r + 3 + \alpha\theta^{r+3}\theta^{r+5}\Gamma r + 4 + \theta^{r+3}\theta^{r+4}\Gamma r + 5}{\theta^{r+3} \theta^{r+4} \theta^{r+5}} \right]$$

$$\mu_r' = E(X^r) = \frac{\theta^2 \Gamma r + 3 + \alpha\theta\Gamma r + 4 + \Gamma r + 5}{\theta^r (2\theta^2 + 6\alpha\theta + 24)} \tag{9}$$

The mean and the remaining three moments of the area-biased new two-parameter Sujatha distribution will now be obtained by entering r=1, 2, 3, and 4 in equation (9).

$$\mu_1' = E(X) = \frac{6\theta^2 + 24\alpha\theta + 120}{\theta^r (2\theta^2 + 6\alpha\theta + 24)}$$

$$\mu_2' = E(X^2) = \frac{24\theta^2 + 120\alpha\theta + 720}{\theta^2 (2\theta^2 + 6\alpha\theta + 24)}$$

$$\mu_3' = E(X^2) = \frac{120\theta^2 + 720\alpha\theta + 5040}{\theta^3 (2\theta^2 + 6\alpha\theta + 24)}$$

$$\mu_4' = E(X^4) = \frac{720\theta^2 + 5040\alpha\theta + 40320}{\theta^4 (2\theta^2 + 6\alpha\theta + 24)}$$

$$sVariance = \frac{24\theta^2 + 120\alpha\theta + 720}{\theta^2 (2\theta^2 + 6\alpha\theta + 24)} - \left[\frac{6\theta^2 + 24\alpha\theta + 120}{\theta^r (2\theta^2 + 6\alpha\theta + 24)} \right]^2$$

$$S.D(\sigma) = \sqrt{\frac{24\theta^2 + 120\alpha\theta + 720}{\theta^2 (2\theta^2 + 6\alpha\theta + 24)} - \left[\frac{6\theta^2 + 24\alpha\theta + 120}{\theta^r (2\theta^2 + 6\alpha\theta + 24)} \right]^2}$$

IV. Hormonic mean

$$H.M = E\left[\frac{1}{x}\right] = \int_0^\infty \frac{1}{x} f_{a(x)} dx$$

$$= \int_0^\infty \frac{1}{x} \frac{x^2 \theta^5}{\theta^2 + 6\alpha\theta + 24} (1 + \alpha x + x^2) e^{-\theta x} dx$$

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$$= \int_0^\infty \frac{x^2\theta^5}{2\theta^2 + 6\alpha\theta + 24}(1 + \alpha x + x^2)e^{-\theta x}dx$$

$$= \frac{\theta^5}{2\theta^2 + 6\alpha\theta + 24}\int_0^\infty x(1 + \alpha x + x^2)e^{-\theta x}dx$$

$$= \frac{\theta^5}{2\theta^2 + 6\alpha\theta + 24}\left[\int_0^\infty x\,e^{-\theta x}dx + \alpha\int_0^\infty x^2\,e^{-\theta x}dx + \int_0^\infty x^3 e^{-\theta x}dx\right]$$

$$= \frac{\theta^5}{2\theta^2 + 6\alpha\theta + 24}\left[\int_0^\infty x^{(2)-1}\,e^{-\theta x}dx + \alpha\int_0^\infty x^{(3)-1}\,e^{-\theta x}dx + \int_0^\infty x^{(4)-1}e^{-\theta x}dx\right]$$

$$= \frac{\theta^5}{2\theta^2 + 6\alpha\theta + 24}\left[\frac{\Gamma 2}{\theta^2} + \frac{\alpha\Gamma 3}{\theta^3} + \frac{\Gamma 4}{\theta^4}\right]$$

$$= \frac{\theta^5}{2\theta^2 + 6\alpha\theta + 24}\left[\frac{1}{\theta^2} + \frac{2\alpha}{\theta^3} + \frac{6}{\theta^4}\right]$$

$$= \frac{\theta^5}{2\theta^2 + 6\alpha\theta + 24}\left[\frac{\theta^7 + 2\alpha\theta^6 + 6\theta^5}{\theta^9}\right]$$

$$= \frac{\theta^5}{2\theta^2 + 6\alpha\theta + 24}\left[\frac{\theta^7 + 2\alpha\theta + 6}{\theta^9}\right]\theta^5$$

$$H.M = \frac{\theta(\theta^2 + 2\alpha\theta + 6)}{2\theta^2 + 6\alpha\theta + 24}$$

V. Moment generating function & Characteristic function

$$M_X(t) = E[e^{tx}] = \int_0^\infty e^{tx}f_a(x)dx$$

Using Taylor's series, we get

$$M_X(t) = \int_0^\infty\left[1 + tx + \frac{(tx)^2}{2!} + \cdots\right]f_a(x)dx$$

$$= \int_0^\infty\sum_{j=0}^\infty \frac{t^J}{J!}x^J f_a(x)dx$$

$$= \sum_{j=0}^\infty \frac{t^J}{J!}\int_0^\infty x^J f_a(x)dx$$

$$= \sum_{j=0}^\infty \frac{t^J}{J!}\int_0^\infty \mu_j$$

$$= \sum_{j=0}^\infty \frac{t^J}{J!}\left[\frac{\theta^2\Gamma j + 3 + \alpha\theta\Gamma j + 4 + \Gamma j + 5}{\theta^j(2\theta^2 + 6\alpha\theta + 24)}\right]$$

$$M_X(t) = \frac{1}{2\theta^2 + 6\alpha\theta + 24}\sum_{j=0}^\infty \frac{t^J}{J!\,\theta^J}[\theta^2\Gamma j + 3 + \alpha\theta\Gamma j + 4 + \Gamma j + 5]$$

Similarly, the characteristic function of area biased new two parameter Sujatha distribution is given by

$$M_X(t) = \frac{1}{2\theta^2 + 6\alpha\theta + 24}\sum_{j=0}^\infty \frac{it^J}{J!\,\theta^J}[\theta^2\Gamma j + 3 + \alpha\theta\Gamma j + 4 + \Gamma j + 5]$$

VI. Bonferroni and Lorenz curves

$$B(p) = \frac{1}{p\mu_1'}\int_0^q xf_a(x)dx \;\; and \;\; L(p) = PB(p) = \frac{1}{\mu_1'}\int_0^q xf_a(x)dx$$

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$$Here, \ \mu_1' = \frac{6\theta^2 + 24\alpha\theta + 120}{\theta(2\theta^2 + 6\alpha\theta + 24)}$$

$$B(p) = \frac{\theta(2\theta^2 + 6\alpha\theta + 24)}{p(6\theta^2 + 24\alpha\theta + 120)} \int_0^q x \frac{x^2\theta^5}{2\theta^2 + 6\alpha\theta + 24}(1 + \alpha x + x^2)e^{-\theta x} dx$$

$$= \frac{\theta^6}{p(6\theta^2 + 24\alpha\theta + 120)} \int_0^q x^3(1 + \alpha x + x^2)e^{-\theta x}dx$$

$$= \frac{\theta^6}{p(6\theta^2 + 24\alpha\theta + 120)} \left[\int_0^\infty x^3 e^{-\theta x}dx + \alpha \int_0^\infty x^4 e^{-\theta x}dx + \int_0^\infty x^5 e^{-\theta x}dx\right]$$

Put $\theta x = t \Rightarrow \theta dx = dt \Rightarrow dx = \frac{dt}{\theta}$ also $x = \frac{t}{\theta}$

$$B(p) = \frac{\theta^6}{p(6\theta^2 + 24\alpha\theta + 120)} \left[\int_0^{\theta q} \left(\frac{t}{\theta}\right)^3 e^{-t}\frac{dt}{\theta} + \alpha \int_0^{\theta q} \left(\frac{t}{\theta}\right)^4 e^{-t}\frac{dt}{\theta} + \int_0^{\theta q} \left(\frac{t}{\theta}\right)^5 e^{-t}\frac{dt}{\theta}\right]$$

$$= \frac{\theta^6}{p(6\theta^2 + 24\alpha\theta + 120)} \left[\frac{1}{\theta^4}\int_0^{\theta q} t^3 e^{-t}dt + \frac{1}{\theta^5}\int_0^{\theta q} t^4 e^{-t}dt + \frac{1}{\theta^6}\int_0^{\theta q} t^5 e^{-t}dt\right]$$

$$= \frac{\theta^6}{p(6\theta^2 + 24\alpha\theta + 120)} \left[\frac{1}{\theta^4}\int_0^{\theta q} t^{4-1} e^{-t}dt + \frac{1}{\theta^5}\int_0^{\theta q} t^{5-1} e^{-t}dt + \frac{1}{\theta^6}\int_0^{\theta q} t^{6-1} e^{-t}dt\right]$$

$$= \frac{\theta^6}{p(6\theta^2 + 24\alpha\theta + 120)} \left[\frac{1}{\theta^4}\gamma(4,\theta q) + \frac{1}{\theta^5}\gamma(5,\theta q) + \frac{1}{\theta^6}\gamma(6,\theta q)\right]$$

$$= \frac{\theta^6}{p(6\theta^2 + 24\alpha\theta + 120)} \left[\frac{\theta^2\gamma(4,\theta q) + \alpha\theta\gamma(5,\theta q) + \gamma(6,\theta q)}{\theta^9\theta^6}\right]\theta^9$$

$$= \frac{\theta^6}{p(6\theta^2 + 24\alpha\theta + 120)} \left[\frac{\theta^2\gamma(4,\theta q) + \alpha\theta\gamma(5,\theta q) + \gamma(6,\theta q)}{\theta^9\theta^6}\right]$$

$$B(p) = \frac{1}{p(6\theta^2 + 24\alpha\theta + 120)} [\theta^2\gamma(4,\theta q) + \alpha\theta\gamma(5,\theta q) + \gamma(6,\theta q)]$$

$$\therefore L(p) = \frac{1}{(6\theta^2 + 24\alpha\theta + 120)} [\theta^2\gamma(4,\theta q) + \alpha\theta\gamma(5,\theta q) + \gamma(6,\theta q)]$$

V. Maximum likelihood estimation & fisher information matrix

$$L(x) = \prod_{i=1}^n f_a(x)$$

$$= \prod_{i=1}^n \left[\frac{x_i^2 \theta^5}{(2\theta^2 + 6\alpha\theta + 24)}(1 + \alpha x + x^2)e^{-\theta xi}\right]$$

$$= \frac{\theta^5}{(2\theta^2 + 6\alpha\theta + 24)}\prod_{i=1}^n [x_i^2(1 + \alpha x + x^2)e^{-\theta xi}]$$

The log likelihood function is given by

$$\log l = 5n \log \theta - n \log(2\theta^5 + 6\alpha\theta + 24) + 2\sum_{i=1}^n \log x_i + \sum_{i=1}^n \log(1 + \alpha x_i + x_i{}^2) - \theta \sum_{i=1}^n x_i \quad (10)$$

Now differentiating equation (10) write to θ and α.

$$\frac{\partial \log l}{\partial \theta} = \frac{5n}{\theta} - n\left[\frac{4\theta + 6\alpha}{2\theta^2 + 6\alpha\theta + 24}\right] - \sum_{i=1}^n x_i = 0$$

$$\frac{\partial \log l}{\partial \theta} = -n\left[\frac{6\theta}{2\theta^2 + 6\alpha\theta + 24}\right] - \sum_{i=1}^n \left[\frac{x_i}{(1 + \alpha xi + x_i^2)}\right] = 0$$

Fisher information matrix

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$$I(P) = -\frac{1}{n} \begin{bmatrix} E\left[\frac{\partial^2 \log l}{\partial \theta^2}\right] & E\left[\frac{\partial^2 \log l}{\partial \theta^2}\right] \\ E\left[\frac{\partial^2 \log l}{\partial \theta^2}\right] & E\left[\frac{\partial^2 \log l}{\partial \theta^2}\right] \end{bmatrix}$$

Where I(P) in fisher information matrix

$$E\left[\frac{\partial^2 \log l}{\partial \theta^2}\right] = \frac{-5n}{\theta} - n\frac{(2\theta^2 + 6\alpha\theta + 24)(4) - (4\theta + 6\alpha)(4\theta + 6\alpha)}{(2\theta^2 + 6\alpha\theta + 24)^2}$$

$$E\left[\frac{\partial^2 \log l}{\partial \theta^2}\right] = \frac{-5n}{\theta} - n\left[\frac{(6\theta)(6\theta)}{(2\theta^2 + 6\alpha\theta + 24)^2}\right] - \sum_{i=1}^{n}\left[\frac{(x_i)(x_i)}{(1 + \alpha xi + x_i^2)^2}\right]$$

$$E\left[\frac{\partial^2 \log l}{\partial \theta^2}\right] = -n\frac{(2\theta^2 + 6\alpha\theta + 24)(6) - (4\theta + 6\alpha)(6\theta)}{(2\theta^2 + 6\alpha\theta + 24)^2}$$

The highest probability estimate for the parameters θ is obtained from the solution to the nonlinear equation (10). R software was used to calculate the parameter values for the data sets utilized in this investigation.

VI. Results

For the data set under consideration, the widely utilized Bayesian Information Criterion (BIC), Akaike Information Criterion (AIC), and -2lnL were employed as the quality of fit criteria. The most adaptable and ideal distribution for the data set is thought to be the one linked to the lowest AIC, BIC and -2InL. R-Software was used to calculate the MLE and goodness of fit criteria.

I. Dataset:

The data set below shows the relief times (in minutes) of 20 analgesic-using patients who were the subject of a study by Gross and Clarke (1995).

Table 1: *Data regarding Survival times (in months) studied by Susarla and Vanryzins*

1.1	1.4	1.3	1.7	1.9	1.8	1.6	2.2	1.7	2.7
4.1	1.8	1.5	1.2	1.4	3.0	1.7	2.3	1.6	2.0

Table: 2 *Shows parameter estimations, corresponding standard errors, Criterion values and goodness of fit test.*

Distribution	MLE	S.E	- 2logL	AIC	BIC	K-S	P–Value
Area Biased new two parameter Sujatha distribution	$\hat{\theta}$= 0.4013	0.1789	105.54	109.34	112.56	0.0653	0.6798
Two Parameter Sujatha distribution	$\hat{\theta}$= 0.6919	0.2781	110.72	114.72	117.77	0.0848	0.9693
WHD distribution	$\hat{\theta}$ =3.5490	0.9890	128.45	130.23	132.81	0.6092	0.9034
LBWD distribution	$\hat{\theta}$ = 2.3462	0.9821	120.45	122.34	124.23	0.3872	0.8713
Lindley distribution	$\hat{\theta}$= 0.8238	0.1054	112.61	114.61	116.14	0.1326	0.5881
Sujatha distribution	$\hat{\theta}$= 1.1461	0.116	115.54	117.54	119.07	0.164	0.3196

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