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A REVIEW OF COMPETING AND SEMI-COMPETING RISKS METHODS IN SURVIVAL ANALYSIS

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Abstract

Recent advancements in competing and semi-competing risk models have significantly contributed to fields like biomedical, reliability studies (on items related to machines), epidemiological studies (treatment outcomes, disease progression, cause-specific deaths), and so on, with new emerging models. The existing frameworks are also being extended to enhance the theoretical robustness. This article provides a comprehensive review from the very first model to the latest developments, aiming to consolidate the diverse methodologies and offering insights into the most suitable approaches for various practical scenarios. The methodologies adopted in different contexts and the steps that form the base for the study are explained for a better understanding. The methodologies are adopted on the account to the nature of the problem which we are dealing with and in this article the summary of the review is incorporated so that the practitioners can decide which methodology to adopt along with the common pitfalls for a specified case. Domain-specific applications are also incorporated, which helps the practitioners to understand the real-life application of these concepts as well as it paves the way to make further contributions to society in the related fields. By examining both foundational and contemporary contributions, this review seeks to guide researchers and practitioners in selecting effective strategies for handling competing and semi-competing risks in survival analysis.

Keywords: statistical models, survival analysis, competing methods, semi- competing risks

1. Introduction

Survival analysis is a collection of statistical procedures for analyzing data where the outcome variable is the time until an event occurs. The time can be years, months, weeks, days or even the age of an individual and the event can mean death, recovery, or any experience or interest of an individual. One of the important concepts in the survival analysis is of censoring. Censoring occurs when we only have some information about the survival time and not the exact time. There are different types of censoring: Right censoring, left censoring, Interval censoring and many more progressive ones. The reasons for censoring include: withdrawal of the person from the study, the person lost to follow-up, or the person does not experience the event before the study ends. When the person is withdrawn from the study or lost to follow-up it results in right censoring. If we know that the individual experienced the event between the time 0 and t but we do not know the exact time it is left censoring. If we know that the event occurred in between a certain time interval it is interval censoring. While performing a survival analysis, the objective is to find the survival function and the hazard function. Survival function gives the probability that a person survives longer than some specified time t whereas hazard function gives the instantaneous potential per unit time for the event to occur given that the individual has survived up to the time t. When a subject is under study for cancer, and gets the event of death, then the person might not be dead because of cancer alone. There can be multiple covariates that

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would have increased the chance of the event occurring, or the event might have occurred solely because of some other reason apart from cancer, like a stroke or an accident. Such a case can be considered as a competing risk where at least two covariates can lead to the occurrence of the event in the same subject. There are multiple factors affecting the variable under consideration. This is where the concept of competing risk comes into the picture. There are different methods through which we can proceed with the analysis, but the results thus obtained might not be reliable, as we deal with multiple factors under consideration simultaneously. The survival curves plotted in this scenario will be difficult to interpret. Even though there were models built to deal with the censoring data and multiple covariates, Dr. Robert J Gray and Jason P Fine [1] were the first to acknowledge competing risk by introducing a proportional hazards model for the subdistribution of a competing risk, known as the Fine-Gray model. This model estimated and made inferences on a finite- dimensional regression parameter, considering multiple censoring scenarios. The partial likelihood principle and weighting techniques were used to derive the model. But in 2020 [2] revisited the Fine-Gray model and worked on the drawbacks that the model holds. The comparison of the Fine-Gray model and the cause-specific approaches is made to eventually develop a reduction factor that can be used while estimating cumulative incidence functions.

Besides proportional hazards, additive hazard models are used in cases with competing risks. An additive hazard regression model was developed by [3] based on the modified Weibull distribution. Discussing the cumulative incidence function and cause-specific hazard opens a new perspective. Once the framework is decided, we go for testing, where we can either test the effect by ignoring the competing risks or the test can be conducted after incorporating the risks. The most widely used tests for the same are the log-rank and the Gray's test. The variability in the inferences made using these two different tests in clinical cancer research was explained by [4]. Here, the cause-specific hazards are analyzed using the Cox proportional hazards model, assuming that the competing risks are independent, censoring is non-informative, the ratio of hazard ratio remains constant over time and the treatment effects are homogeneous for different groups. The estimation procedure includes both the maximum likelihood and Bayesian estimation techniques in the case of non-informative prior. The idea of a non-informative prior will add challenges as the Bayesian inference is very sensitive to the choice of the prior distributions. The appropriate sample size is calculated before the testing, and this is done based on the work in [5]. The inability to decide on what inference to choose from multiple tests adds complexity as different tests give different results, and it is up to the researchers and statisticians to decide on where to lean on. [6] studied data on ICU patients, where the difference in the exposure time and the censoring concepts are not considered effectively; for example, the discharged patients are considered to be censored. [7] have applied these techniques to real-time data on diabetes patients, where the competing risks considered are end-stage renal disease (ESRD) and death without the disease. [8] addressed the common competing risk of recurrence and death but considering two aspects: expected death and excess death where the excess death can be associated with the disease directly or indirectly. [9] took the data on a clinical trial for the cancer patients. [10] came up with a flexible model to obtain the baseline hazard function which uses the basic concepts and incorporates methodologies that are data and case-specific. [11] inferred the mortality risks among under-five Bangladeshi children using the Bangladesh Demographic and Health Survey 2011 data. [12] found the important risk factors for the patients' cardiovascular and cerebrovascular death while suffering from Kidney cancer. [13] used the Fine-Gray model to find the predictors which are significant to the risk associated with the microinvasive cutaneous squamous cell carcinoma (misSCC).

On the other hand, semi-competing risk model analysis has drawn a lot of attention, particularly in reliability and medical research. In many real-world situations, a non-terminal event like the progression or recurrence of a disease may occur before a terminal event like death. However, after the terminal event, it is no longer possible to observe the non-terminal event, which means that another event will censor it. More complex statistical procedures are required since traditional survival analysis methods frequently assume independent censoring, which may not apply in such situations [14]. Researchers have created a number of modeling techniques in recent decades to account for the connection between events, such as joint modeling frameworks, copula-based techniques, and frailty models. Significant progress has been made, although handling informative filtering and precisely calculating parameters still present difficulties, making this an active area of research.

There are two kinds of events in a semi-competing risks problem: terminal events (death) and non- terminal events. Although the non-terminal event is typically the focus of the study, the terminal event's occurrence may prevent the non-terminal event from occurring [15]. In order to tackle this, we provide and illustrate the semi-competing risks paradigm, which is applicable to situations in which a non-terminal event (like readmission) is the main emphasis, but its occurrence might be impacted by a terminal event (like death). Over time, patients progress through several health states according to the semi-competing hazards paradigm. Patients begin in the dismissal condition in the Medicare data (Figure 1A). After that, they can either die or be readmitted to the hospital. In the event of a readmission, they still run the risk of eventually changing into the deceased state. Nevertheless, no more transitions are feasible after a patient enters the deceased condition. Since readmission can happen before death but not the other way around, this structure effectively depicts the relationship between readmission and death [16].

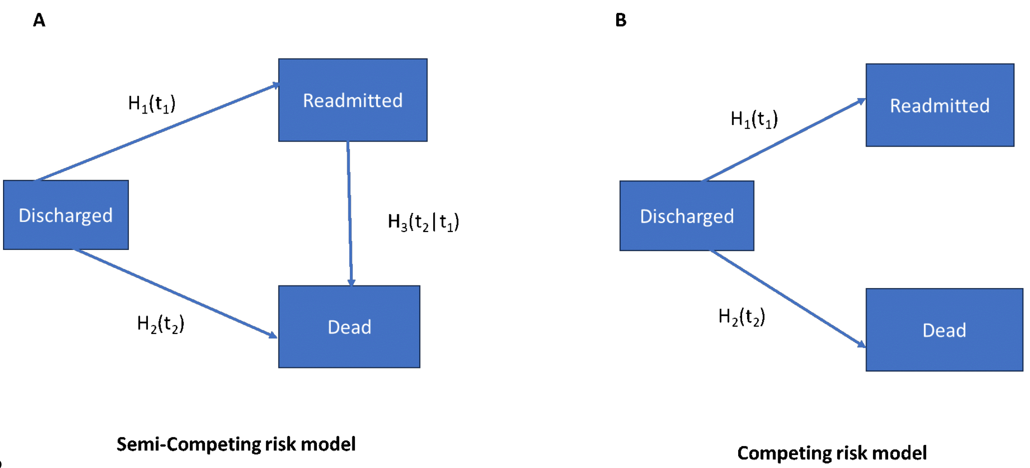

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Figure 1: *Graphical representation of the states that patients can transition between under the semi-competing risk (A) Competing risk (B)*

The semi-competing risks analysis framework properly considers the terminal event as a competing risk while incorporating the dependence between non-terminal and terminal events within the model specification [17, 18]. To formally explain the structure of semi-competing risks data, let T_1 and T_2 represent the times to the non-terminal and terminal events, respectively. From a modeling perspective, the primary goal in the semi-competing risks setting is to characterize the distribution of T_1 and its potential relationship with T_2, i.e., the joint distribution of (T_1 and T_2). A subject may transition from an initial state (e.g., transplantation) to either the non-terminal or terminal state over time. If a transition occurs to the non-terminal state, the subject may later move to the terminal state, but not vice versa. A key limitation of the competing risks framework in studying the non-terminal event is that it disregards information about the occurrence and timing of the terminal event following the non- terminal event. This information, however, could provide valuable insights into the dependence between the two events. Numerous studies have explored causal interpretation in survival data with competing risks [19, 20, 21]. However, the use of semi-competing risks in causal inference remains relatively unexplored. The paper is organized as follows: the primitive competing risk models developed are discussed in the second section. How the censoring affects these primitive models is explained in Section 3. Section 4 deals with the semi-competing risk models. The next section comprises the domain-specific summary table, the measures to be adopted, and the pitfalls awaiting while analysing. Section 6 gives some real-life applications and future directions for the same. Conclusion is given in Section 7.

2. Competing Risk Models

2.1. Cause-specific Hazard Models

These models can be used to analyze time-to-event data in the presence of competing risks. The cause-specific hazard function gives the instantaneous rate of failure from a particular cause within a small interval of time provided that the subject has not achieved the event yet.

$$h_t = \frac{P\big(t \leq T_c < t + \Delta t \mid T_c \geq t\big)}{\Delta t} \tag{1}$$

where h_t is the cause-specific hazard function, the time interval is given between t and $t + \Delta t$, and T_c = time-to-failure from event c.

2.2. Proportional Hazards Model

Fine and Gray [1] developed a subdistribution hazard model with proportional hazards assumption in 1999. The random variables considered here are the failure times, T and the censoring times C. The main assumption of this model is that the ratio of the hazard rates for two subjects remains constant over time. The cumulative incidence function is directly estimated using the concept of partial likelihood for a single failure type in the presence of competing risks. The inclusion of the idea of partial likelihood helps in estimating the regression coefficients by incorporating different censoring techniques. The cumulative incidence function is given by [1],

$$F(t; Z) = 1 - \exp\left[-\int_0^t \lambda_{10}(s)\ \exp\big(Z(s)^\top \beta_0\big)\, ds\right] \tag{2}$$

where $F(t;Z)$ is the cumulative incidence function, λ_{10} is a nonnegative function in t. It gives the probability of failure due to the failure type i by time t with the covariates Z with the small increment fo time ds.

2.3. Cox PH model

If we are concerned only about a single failure type, considering the other failure types as censored and proceed with the evaluation of the hazard rate function regarding the single failure type. The hazard function is given as,

$$h(t, X) = h_0(t) \exp\left(\sum_{i=1}^{p} \beta_i X_i\right) \tag{3}$$

$h_o(t)$ is the baseline hazard function, β_i is the regression coefficient for the ith predictor, and X=X_1, X_2, ..., X_n are the n explanatory variables which are independent of time.

3. Competing Risk Model with Censored Data

3.1. Competing risk model with dependent left-truncation

A cause-specific hazard function is developed where the variables considered are the time to event, T, and the truncation time, L. While conducting a study the subject i is observed only if the subject experiences the event after entering the study whereas the subjects who achieve the event before entering the study are not observed. In essence, the subjects with $L < T$ are included in the study, whereas the subjects with $T \leq L$ are not included in the sample. It is assumed that L and T are independent. The dependence of L and T can, in principle, be examined by considering L as a covariate in the regression model of the hazard function of T. However, in practice, it is difficult to identify the reasons leading to L, and this may lead to biased results as the general methodologies assume independence between L and T. To overcome this drawback, we proceed with a new approach named Inverse Probability of Left-truncation Weight (IPLW).

In the Inverse Probability Weighting (IPW) approach, selection bias can be adjusted by weighting the subjects in the study by the inverse of their probability of being selected. In contrast, in IPLW, the probability of entering the study at a particular time is considered. Thus, the model developed is given as [22],

$$\alpha_{oj}\big(t \mid L = l\big)\, dt := P\big(T \in [t, t + dt],\ X_T = j \mid T \geq t,\ L = l\big) \tag{4}$$

where $\alpha_{oj}\big(t \mid L = l\big)$ is the instantaneous transition rate at time t conditioned to the latent variable, which can be found by the Cox proportional hazards model where each transition type X_t is represented by j.

The cumulative incidence function conditional on L=l is given by,

$$P\big(T \leq t,\ X_T = j \mid L = l\big) := \int_0^t P\big(T > u- \mid L = l\big)\, \alpha_{oj}\big(u \mid L = l\big)\, du, \qquad j = 1, 2, 3 \tag{5}$$

Here, u is the lower time limit and L is considered as a latent variable.
After careful simplifications the predicted cumulative cause-specific hazards for individual i with entry time l_i is [22],

$$\hat{A_{0j}}(t \mid L = l_i) = \hat{A_{0j;0}}(t) \exp\left(\hat{\beta}_{0j} l_i\right), \quad j = 1, 2, 3 \tag{6}$$

where $\hat{A_{0j;0}}(t)$ gives the estimate of the baseline cumulated hazard, $\exp\left(\hat{\beta}_{0j} l_i\right)$ is the scaling factor on the baseline hazard function for each transition j. The baseline hazard function proposed here can be estimated using the Breslow estimator [22],

$$\sum_{s \leq t} \frac{\Delta N_{0j}(s)}{\sum_{k=1}^{n} Y_k(s) \exp \hat{\beta}_{0j} l_i} \tag{7}$$

Here, $\Delta N_{0j}(s)$ gives the number of events occurred that are of type j at time s, $Y_k(s)$ is a risk factor indicator for the subject k that takes binary inputs like 1, if the subject is at risk at time s and 0 at no risk.

The conditional survival probability and the conditional cumulative incidence functions can be derived from the above equations. As, we are experiencing dependent left-truncation, all the events cannot be observed. It is possible that only n events are observed when actually m events have occurred. Now, the value m is unknown and we need to find the estimate and thus the concept of IPW is used to proceed with the further derivation. [22] came up with the equation given below as a support to these concerns.

$$Q = P(T > L) \approx \frac{n}{m} \tag{8}$$

The estimator of m is given as [22], which gives the probability of the subject being included in the study:

$$\hat{m} = \sum_{i=1}^{n} \frac{1}{\hat{P}(T > l_i | L = l_i)} \tag{9}$$

The distribution of the left-truncation can thus be given as [22],

$$\hat{F}_L(l) = \frac{\hat{Q}}{n} \sum_{s \leq t} \frac{1(l_i \leq l)}{\hat{P}(T_i > l_i \mid L_i = l_i)} \tag{10}$$

The estimator of the marginal cumulative incidence function derived after considering all the above equations is [22],

$$\hat{P}(T \leq t,\, X_T = j) = \frac{\hat{Q}}{n} \sum_{l_k} \sum_{i=1}^{n} \hat{P}(T \leq t,\, X_T = j \mid L = l_i) \frac{\mathbf{1}(l_k \leq l_i)}{\hat{P}(T > l_i \mid L_i = l_i)} \tag{11}$$

3.2. Competing risk model with Type-I progressive censoring

In Adaptive Type-I progressive censoring, the number of subjects to be censored is pre-determined initially, but the time at which censoring occurs is not planned prior. The decision on the same is taken throughout the study after observing the failures. [23] developed a competing risk model to be applied in such a scenario. The lifetimes in the model are assumed to follow Xgamma distribution, which leads to a better understanding of the time-to-event data. Thus, the pdf for the subject k is given by [23],

$$f_k(y;\beta_k) = \frac{\beta_k^2 e^{-\beta_k y}}{1+\beta_k} \left(1 + 0.5\beta_k y^2\right), \quad y > 0,\ \beta_k > 0 \tag{12}$$

where y is the random variable corresponding to the lifetime, k, k=1, 2 indicates two subpopulations whereas β_k is a scale parameter for k.

The hazard rate function in the given scenario is [23],

$$h(y;\boldsymbol{\beta}) = \sum_{k=1}^{2} \frac{\beta_k^2 \left(1 + 0.5\beta_k y^2\right)}{1 + \beta_k(1+y) + 0.5(\beta_k y)^2} \tag{13}$$

3.3. Additive Generalized Linear-Exponential competing risk model

The AGLE model is a more robust framework that can be applied while handling cancer data where the treatment response and the disease progression differ from person to person. It stands out because of its flexibility. Multiple distributions can be accommodated into the model by adjusting the parameters involved. Let $X_1, X_2, \ldots, X_n$ be the lifetimes of the subjects under study, which follow the Generalized Linear Exponential (GLE) distribution with rate parameters $\theta > 0$ and $\lambda > 0$, and shape parameter $k_j > 0$. Then, the survival function is given by [24],

$$Pr(X_{ij} > x) = S_j(x;\theta,\lambda,k_j) = \exp\left\{-\left[\frac{\theta x^2}{2} + \lambda x\right]^{k_j}\right\}, \quad x > 0,\ j = 1,2 \tag{14}$$

If X_i, the minimum of X_{i1} and X_{i2}, is the lifetime of the object i under study, then the survival function of $X_1, X_2, \ldots, X_n$ is given as follows [24]:

$$Pr(X_i > x) = S(x;\theta) = \exp\left\{-\left[\frac{\theta x^2}{2} + \lambda x\right]^{k_1} + \left[\frac{\theta x^2}{2} + \lambda x\right]^{k_2}\right\}, \quad x > 0 \tag{15}$$

Thus, the model developed is the AGLE distribution. Here, the model considers only two independent latent causes. This method can be used widely in medical data while [3] can be made use for engineering problems.

4. Semi-Competing Risk Models

Let T_1 represent the potential happening time of a nonterminal event, and T_2 denotes the potential time of death. The censoring time is given by C. The observed event times are defined as follows [15]:

$$Y_1 = \min(C, T_1, T_2), \quad Y_2 = \min(C, T_2) \tag{16}$$

The indicators for event occurrences are [15]:

$$\delta_1 = \mathbf{1}\{T_1 = Y_1\}, \quad \delta_2 = \mathbf{1}\{T_2 = Y_2\} \tag{17}$$

Additionally, X represents a vector of fixed covariates, while $V(t)$ denotes a marker process, where $0 \leq t \leq Y_2$. One of the primary objectives in semi-competing risks analysis is to infer the distribution of the failure time for the non-terminal event T_1. Ideally, observing T_{1i} for each individual $i = 1, \ldots, n$ would facilitate direct inference on T_1. However, in practical scenarios, only n independent and identically distributed copies of $\{Y_{1i}, Y_{2i}, \delta_{1i}, \delta_{2i}, X_i, V_i(t)\}$ are available for observation.

4.1. Semi-Competing Risks Through Illness-Death Models

The three-state illness-death model is widely used as a standard framework for executing regression analysis in semi-competing risk data. To set up the structure of semi-competing risk data, let T_1 represent the time to the non-terminal event and T_2 the time to the terminal event. A key obstacle in analyzing such data is constructing an interpretable model for T_1 while accounting for the possible dependence between T_1‹ and T_2. Recently, various statistical methods have been proposed in the literature for the analysis of semi-competing risk data. To officially explain the illness“death process, let us revisit Figure 1A. By considering T_1 and T_2 as time-to-event variables, the rates at which patients move between the three states can be specified using three distinct transition-specific hazard functions [14, 16, 18, 25, 26].

$$H_1(t_1) = \lim_{\Delta \to 0} \left[\frac{P(t_1 \leq T_1 < t_1 + \Delta \mid T_1 \geq t_1, T_2 \geq t_1)}{\Delta} \right], \quad t_1 > 0$$

$$H_2(t_2) = \lim_{\Delta \to 0} \left[\frac{P(t_2 \leq T_2 < t_2 + \Delta \mid T_1 \geq t_2, T_2 \geq t_2)}{\Delta} \right], \quad t_2 > 0$$

$$H_3(t_2 \mid t_1) = \lim_{\Delta \to 0} \left[\frac{P(t_2 \leq T_2 < t_2 + \Delta \mid T_1 = t_1, T_2 \geq t_2)}{\Delta} \right], \quad 0 < t_1 < t_2$$

The first hazard rate represents the likelihood of readmission after discharge at a specific time t_1, assuming that neither readmission nor death has occurred before t_1. The second hazard rate describes the probability of death following discharge at time t_2, given that neither readmission nor death occurred before t_2. Lastly, $h_3(t_2 \mid t_1)$ denotes the hazard rate for death after readmission at time t_2, conditional on readmission having occurred at $T_1 = t_1$ and death not having occurred before t_2. The Cox model is the most widely used framework. In the context of semi-competing risks, the standard Cox model can be adapted and extended as follows:

$$H_1(t_1 \mid \gamma_i, X_i) = \gamma_i H_{01}(t_1) \exp\left(X_i^\top \beta_1\right), \quad t_1 > 0$$

$$H_2(t_2 \mid \gamma_i, X_i) = \gamma_i H_{02}(t_2) \exp\left(X_i^\top \beta_2\right), \quad t_2 > 0$$

$$H_3(t_2 \mid t_1, \gamma_i, X_i) = \gamma_i H_{03}(t_2 \mid t_1) \exp\left(X_i^\top \beta_3\right), \quad 0 < t_1 < t_2$$

Here, γ_i represents a frailty term that is specific to each patient, and X_i denotes a vector of covariates specific to each patient. $H_{01}(t_1)$ represents the baseline hazard function (i.e., the hazard for a population of patients when $X = 0$) for readmission after discharge, where β_1 represents the log hazard rate of the covariate vector. Likewise, $H_{02}(t_2)$ represents the baseline hazard function for death following discharge. The components of $\exp(\beta_2)$ can be interpreted as hazard ratios *(HRs)* for death associated with the covariates in X, given that a readmission event has not taken place. Lastly, $H_{03}(t_2 \mid t_1)$ corresponds to the conditional baseline hazard function for death, provided that a readmission event occurred at time t_1, while $\exp(\beta_3)$ represents the corresponding HR parameters.

A semi-competing risks model involves three stochastic processes, as presented in Figure 1: the time until the occurrence of a nonterminal event, the time until a terminal event without undergoing the nonterminal event, and the time until a terminal event after experiencing the nonterminal event. Unlike traditional competing risk models, semi-competing risk data provide at least some insight into the joint distribution of nonterminal and terminal events.

For instance, in an illness–death model, the age at disease diagnosis serves as the nonterminal event, while the age at death represents the terminal event. In this context, two key challenges arise:

(a) The age at death following disease diagnosis is left-truncated by the age at diagnosis.

(b) In most real-world applications, assuming conditional independence between age at diagnosis and age at death after diagnosis, given the observed covariates, is often unrealistic.

4.2. Cox Model

Two frailty-based illness-death models are presented below [18, 27]: one follows the conditional approach, where inference is made by conditioning on the unobserved frailty variate, and the other adopts the marginalized approach. In the conditional approach, the regression coefficient of the covariate reflects a within-cluster effect, whereas in the marginalized approach, it represents a population-averaged effect. It is well established that both approaches yield the exact estimate in a linear model. However, in a nonlinear model, they differ, making the distinction between them practically significant. The choice between these methods depends on the objectives of the analysis.

5. Summary

Table 1: *Summary Table*

Domain	Actual estimand	Suggested model	Common pitfalls
Transplant (graft failure) where death acts as a competing event)	Cumulative Incidence Function of graft failure taking death under consideration.	Cumulative incidence function incorporating the covariate effects on the Fine-Gray subdistribution hazards model	Ignoring the effect of covariate and misinterpreting the model framed.
Readmission (non-terminal) and death (terminal)	Hazard function of readmission before attaining death.	Illness-death multi-state semi-competing risk model	Death shouldn't be considered as an ordinary competing risk like readmission.
Aging studies like dementia and Death	Cumulative incidence function and the transition probabilities can be found in the Illness-death models.	Semi-competing risk or Illness-death models can be used. Cause-specific and joint frailty models to model the dependence between terminal and non-terminal deaths.	The data should be of ample size so that the complexity of the model can be reduced.
Left-truncation of a dependent entry	Marginal cumulative incidence function	Inverse Probability of Left-Truncation Weighting (IPLW)	Incorect weight assigning and considering dependent entries as independent.
Progressive Type-I adaptive censoring in engineering and radiological	Cause-specific cumulative incidence functio with censoring schemes can be used.	Maximum likelihood estimation and bayesian estimation with progressive censoring scheme is suggested.	Complex likelihood derivation and requirement of simulation for assurance.
Clinical trials	Reasonable inference based on the treatment effects.	Cumulative incidence function can be found as per the Fine-Gray model and the instantaneous hazard can be found through the cause-specific Cox method.	assuming false information and using different tests leading to misinterpretations.

6. Applications and Future Directions

6.1. Medical Field

The competing and semi-competing risk models can be made useful in many medical fields. Competing risk analysis plays a major role in the fields like Oncology, Organ transplant, Cardiology, Infectious diseases and so on. In all these cases the death can be achieved not just because of the disease, there can be multiple side effects or infections that can be a part of it which can lead to attainment of the event. Studies can be conducted to understand more about the cause-specific deaths to the deaths due other reasons. We can analyze graft failures versus death, failure of antibiotic treatments, evaluate risks associated with the post-transplant. Similarly, semi-competing risk models can be used to investigate disease recurrence and what effects it has on the survival. The progression of a chronic disease can be considered while the terminal event considered is death. The estimated thus made are much more reliable compared to the primitive ones. This information can lead to a better development in the treatment strategies and risk management.

6.2. Epidemiological Studies

The mortality data collected often serves as a key to understand many important facts regarding the human life and their surroundings. Through the analysis of this data, using the models discussed we will be able to find the root cause for the deaths which dominates over the others. This will help us in developing policies relating to the cause. There are times when longitudinal studies are performed. Over the analysis, we might encounter the diseases and various risks associated with a disease or complications pre or post recovery. It also helps us validate the health programmes and facilities offered in the area under study and take necessary actions if something is not working effectively. In essence, the models bring out a better study design, better understanding considering multiple causal pathways, provide better estimates, and in turn provide better planning in the health sector and clinical decision-making.

6.3. Social Science and Economics

These models can be used not only in medical fields we could use them to study the timing of significant life events like marriage, childbirth etc. considering other life events like death, retirement etc. Studies can be made on the migration patterns, where there exists competition on which country to go to. Similarly, who are the countries with whom we might be competing when others have to make a choice about migrating to India? They can also be used to study the job market, education sector, family dynamics and so on. The study can be done on the duration of a person being employed while having risks of unemployment, changes in the job, retirement etc. whereas the education sector has risks like detention, suspension, dropping out etc. In family, there can be death of a spouse, divorce or any other unfortunate incidents. Analysis on loan defaults and bankruptcy risk is another financial aspect that we might overlook using competing and semi-competing risk models. As we have seen competing and semi-competing risk models can be made use in various fields. Similar developments can be made in the existing models as per the requirement and make more reliable and meaningful models applicable for the problems still prevailing in the research world. To begin with, there are various censoring patterns which are less explored and such schemes can be modelled and applied in appropriate situations. This will be a breakthrough to tackle various problems in the areas of survival analysis, reliability, econometrics, finance and so on.

7. Conclusions

Competing and semi-competing risks are one of the most important issues to be addressed in the domain of survival analysis. There can be multiple risks associated with a variable under study and it is important to acknowledge the presence of the same. The models thus developed give a robust and reliable estimate, thus improving the overall perception on the study. In this paper, from the primitive models to the latest ones that can be used in various scenarios are given, along with their corresponding estimate calculation equations of the hazard function, survival function, and cumulant incidence function, which are the sole concepts to be addressed while performing a survival analysis. Censoring is another concept that goes hand in hand with survival analysis and accommodating the same increases the proximity of performing a study in real life. Incorporating it into the study rather than ignoring it reduces the amount of bias and thus provides a better picture of the results. It also increases the statistical power of the analysis as the sample size increases when we incorporate the censored data along with the completely observed ones. The pre-existing models are thus modified based on different censoring patterns and the way in which the models change is also explained in detail. By, making use of these models, survival analysis will result in a more detailed, robust, and informative. They can be made useful in various fields other than medical as we have discussed which makes the model versatile.

References

[1] Fine, J. P. and Gray, R. J. (1999). A Proportional Hazards Model for the Subdistribution of a Competing Risk. *Journal of the American Statistical Association*, 94(446):496-509.

[2] Putter, H., Schumacher, M., and van Houwelingen, H. C. (2020). On the relation between the cause-specific hazard and the subdistribution rate for competing risks data: The Fine-Gray model revisited. *Biometrical Journal*, 62(3):790-807.

[3] Rehman, H., Chandra, N., and Abuzaid, A. H. (2023). Analysis and modelling of competing risks survival data using modified Weibull additive hazards regression approach. *Hacettepe Journal of Mathematics and Statistics*, 52(5):1263-1281.

[4] Dignam, J. J. and Kocherginsky, M. N. (2008). Choice and interpretation of statistical tests used when competing risks are present. *Journal of Clinical Oncology*, 26(24):4027-4034.

[5] Latouche, A., Porcher, R., and Chevret, S. (2004). Sample size formula for proportional hazards modelling of competing risks. *Statistics in Medicine*, 23:3263-3274. doi:10.1002/sim.1915.

[6] Resche-Rigon, M., Azoulay, E., and Chevret, S. (2006). Evaluating mortality in intensive care units: contribution of competing risks analyses. *Critical Care*, 10(1):R5. doi:10.1186/cc3921.

[7] Lim, H. J., Zhang, X., Dyck, R., and Osgood, N. (2010). Methods of competing risks analysis of end-stage renal disease and mortality among people with diabetes. *BMC Medical Research Methodology*, 10:97. doi:10.1186/1471-2288-10-97.

[8] Belot, A., Abrahamowicz, M., Remontet, L., and Giorgi, R. (2010). Flexible modeling of competing risks in survival analysis. *Statistics in Medicine*, 29(23):2453-2468. doi:10.1002/sim.4005.

[9] Dutz, A., and Löck, S. (2019). Competing risks in survival data analysis. *Radiotherapy and Oncology*, 130:185–189. doi:10.1016/j.radonc.2018.09.007.

[10] Belot, A., Remontet, L., Launoy, G., Jooste, V., and Giorgi, R. (2011). Competing risk models to estimate the excess mortality and the first recurrent-event hazards. *BMC Medical Research Methodology*, 11:78. doi:10.1186/1471-2288-11-78.

[11] Mohammad, K. A., Fatima-Tuz-Zahura, M., and Bari, W. (2017). Fine and Gray competing risk regression model to study the cause-specific under-five child mortality in Bangladesh. *BMC International Health and Human Rights*, 17(1):3. doi:10.1186/s12914-017-0112-8.

[12] Mo, X., Zhou, M., Yan, H., Chen, X., and Wang, Y. (2021). Competing risk analysis of cardiovascular/cerebrovascular death in T1/2 kidney cancer: a SEER database analysis. *BMC Cancer*, 21(1):13. doi:10.1186/s12885-020-07718-z.

[13] Zheng, S., Xie, S., Yu, H., Duan, X., He, Y., Ho, C., Wan, Y., Hang, T., Chen, W., Lyu, J., and Deng, L. (2023). Competing-risks analysis for evaluating the prognosis of patients with microinvasive cutaneous squamous cell carcinoma based on the SEER database. *BMC Medical Research Methodology*, 23(1):286. doi:10.1186/s12874-023-02109-x.

[14] Andersen, P. K., Borgan, ., Gill, R. D., and Keiding, N. (2012). Statistical models based on counting processes. *Springer Science & Business Media*.

[15] Varadhan, R., Xue, Q. L., and Bandeen-Roche, K. (2014). Semicompeting risks in aging research: Methods, issues and needs. *Lifetime Data Analysis*, 20(4):538-562.

[16] Haneuse, S. and Lee, K. H. (2016). Semi-competing risks data analysis. *Circulation: Cardiovascular Quality and Outcomes*, 9(3):322-331.

[17] Fine, J. P., Jiang, H., and Chappell, R. (2001). On semi-competing risks data. *Biometrika*, 88(4):907–919. doi:10.1093/biomet/88.4.907.

[18] Xu, J., Kalbfleisch, J. D., and Tai, B. (2010). Statistical analysis of illness–death processes and semicompeting risks data. *Biometrics*, 66(3):716–725. doi:10.1111/j.1541-0420.2009.01340.x.

[19] Young, J. G., Stensrud, M. J., Tchetgen Tchetgen, E. J., and Hernn, M. A. (2020). A causal framework for classical statistical estimands in failure-time settings with competing events. *Statistics in Medicine*, 39:1199-1236.

[20] Stensrud, M. J., Hernn, M. A., Tchetgen Tchetgen, E. J., Robins, J. M., Didelez, V., and Young, J. G. (2021). A generalized theory of separable effects in competing event settings. *Lifetime Data Analysis*, 27(4):588-631.

[21] Stensrud, M. J., Young, J. G., Didelez, V., Robins, J. M., and Hernn, M. A. (2022). Separable effects for causal inference in the presence of competing events. *Journal of the American Statistical Association*, 117(537):175-183.

[22] Stegherr, R., Allignol, A., Meister, R., Schaefer, C., and Beyersmann, J. (2020). Estimating cumulative incidence functions in competing risks data with dependent left-truncation. *Statistics in Medicine*, 39(4):481-493.

[23] Alotaibi, R., Nassar, M., and Elshahhat, A. (2024). Analysis of Xgamma distribution using adaptive Type-I progressively censored competing risks data with applications. *Journal of Radiation Research and Applied Sciences*, 17(3):101051.

[24] Alhidairah, S., Alam, F. M. A., and Nassar, M. (2024). A new competing risks model with applications to blood cancer data. *Journal of Radiation Research and Applied Sciences*, 17(3):101001.

[25] Putter, H., Fiocco, M., and Geskus, R. B. (2007). Tutorial in biostatistics: competing risks and multi-state models. *Statistics in Medicine*, 26(11):2389–2430. doi:10.1002/sim.2712.

[26] Lee, C., Gilsanz, P., and Haneuse, S. (2021). Fitting a shared frailty illness-death model to left-truncated semi-competing risks data to examine the impact of education level on incident dementia. *BMC Medical Research Methodology*, 21(1):18. doi:10.1186/s12874-020-01203-8.

[27] Gorfine, M., and Zucker, D. M. (2023). Shared frailty methods for complex survival data: a review of recent advances. *Annual Review of Statistics and Its Application*, 10(1):51-73.