RT&A, No 4 (89)
Volume 20, December 2025

VARIABLES CHAIN SAMPLING PLAN BASED ON THE MINIMUM ANGLE METHOD INVOLVING MINIMUM SUM OF RISKS

P. Suthersan[1] and S. Balamurali[2*]

•

[1]Department of Mathematics, Kalasalingam Academy of Research and Education, Krishnankoil – 626126.

[2*] Department of Computer Applications, Kalasalingam Academy of Research and Education, Krishnankoil – 626126.

[1]suthersan8@gmail.com, [2*]sbmurali@rediffmail.com

Abstract

This study presents a designing methodology for a variables based chain sampling plan utilizing the minimum angle criterion. In the context of acceptance sampling, managing both producer's risk and consumer's risk is essential for ensuring consistent product quality, particularly within industrial applications. These risks must be addressed concurrently when making quality related decisions. In this work, both risks are effectively reduced by minimizing the tangent angle formed between two points on the operating characteristic curve: the producer's quality level, associated with the probability of accepting a good lot, and the consumer's quality level, associated with the probability of accepting a bad lot. The variables chain sampling plan provides the underlying framework for this formulation. The proposed design procedures are detailed through step-by-step methodologies to demonstrate the implementation of the approach.

Keywords: **c**onsumer's risk, chain sampling plan, minimum angle method, producer's risk

I. Introduction

In today's highly competitive global marketplace, quality control serves as a critical differentiator among enterprises. Among the various techniques employed to ensure product quality, control charts and acceptance sampling remain fundamental. Acceptance sampling serves as an intermediate strategy between complete inspection of all items and no inspection at all, offering a balance between efficiency and quality assurance. In this approach, a subset of items, referred to as a sample of size n is drawn from a larger batch, or lot, of products. The decision to accept or reject the entire lot is based on the quality assessment of this representative sample. Among various acceptance sampling methodologies, the single-sampling plan is the most commonly adopted in industrial practice, owing to its simplicity, ease of implementation, and suitability for final product inspection. Acceptance sampling plans also play a critical role in quality assurance by simultaneously addressing both producer's risk and consumer's risk. These risks are particularly significant in industrial settings, where maintaining consistent product quality is essential. Therefore, an effective quality assessment strategy must incorporate both risk factors concurrently

DOI: https://doi.org/10.24412/1932-2321-2025-489-1104-1114

during the design and implementation of the sampling plan.

Acceptance sampling can broadly be divided into two categories: attribute sampling plans and variables sampling plans. Extensive discussions on this classification are available in the literature [1, 14, 18]. Attribute sampling evaluates quality characteristics in discrete terms, such as conforming/non-conforming or pass/fail outcomes. In contrast, variables sampling involves the assessment of quality characteristics measured on a continuous scale, such as weight, length, or volume. The main benefit of variables sampling plan (VSP) lies in its ability to achieve the same operating characteristic (OC) curve with a significantly smaller sample size compared to an attribute sampling plan. Consequently, fewer units need to be inspected under a variables sampling framework. Since variables sampling yields quantitative data, it offers more detailed information than the attribute sampling, enabling equivalent quality assurance with a notably reduced sample size. This approach proves especially advantageous in scenarios involving destructive testing, as it minimizes inspection costs while maintaining the desired level of protection. Concerns have been raised about the appropriateness of variables sampling when the goal is to estimate the fraction nonconforming of the incoming lots. Such issues were discussed by Collani [5]. Conversely, later studies have demonstrated that variables sampling remains an optimal approach under various conditions. This was shown by Seidel [19]. The acceptance sampling literature provides extensive documentation and analysis of conventional VSPs [3, 12, 16].

The concept of the chain sampling plan (ChSP) was introduced and designated as the ChSP-1 plan, applicable to both small and large sample sizes. This was first proposed by Dodge [6]. Unlike single sampling plan (SSP), the ChSP-1 utilizes cumulative data from successive samples, thereby addressing limitations such as the potential rejection of an entire lot due to a single nonconforming unit. It also enhances the discriminatory power between acceptable and unacceptable quality levels, particularly improving upon the acceptance criterion where no defective units are permitted. Chain sampling is best suited for scenarios involving continuous production under consistent operating conditions, where product lots are submitted for inspection in the sequence of their manufacture. Such applications are commonly found in the receiving inspection of regularly supplied materials within a manufacturing process. The features and performance attributes of the chain sampling plan have been widely studied, with important insights provided in previous research [8, 21, 22]. Chain sampling is categorized under conditional sampling procedures, alongside methods such as multiple dependent (deferred) sampling plans. This classification has been discussed in the literature [27]. In these approaches, the decision to accept or reject a lot depends not only on the current sample but also on the inspection outcomes of preceding lots. This methodology is particularly suited for Type B scenarios, where inspection is conducted on lots drawn sequentially from a continuous production process assumed to produce uniform quality.

The application of chain sampling to variables inspection has been further developed in subsequent studies [9]. However, their work did not include tabulated values to facilitate practical implementation of the proposed plan. Moreover, their analysis was confined to the case where the standard deviation is known. In response to these limitations, comprehensive tables were developed to support the practical application of chain sampling plans for variables inspection. These tables consider a normally distributed quality characteristic under both known and unknown standard deviation scenarios while aiming to minimize the average sample number (ASN). This development was proposed in the literature [2]. Moreover, the design of existing chain sampling plans and their modified versions is also discussed in the acceptance sampling literature. Examples of such studies include [10, 13, 17, 26].

II. Minimum Angle Method

The minimum angle method is a design approach used in constructing acceptance sampling plans that simultaneously minimizes both the producer's and consumer's risks. This is achieved by

reducing the angle θ formed between two points on the operating characteristic curve: the producer's quality level (PQL) with acceptance probability (1-α) and the consumer's quality level (CQL) with acceptance probability β. Here, α represents the producer's risk and β represents the consumer's risk. Here, $P_a(p_1)$ represents the probability at PQL and $P_a(p_2)$ represents the probability at CQL, as shown in Figure 1.
This method offers two primary advantages:

- It enables the simultaneous minimization of both types of risks.
- The resulting minimized angle approximates the behavior of the ideal OC curve, an ideal that can only be realized through complete inspection, which is often impractical due to time and cost constraints.

Thus, by employing the minimum angle technique, the acceptance decision closely mirrors that of full inspection but with significantly reduced inspection effort and cost.

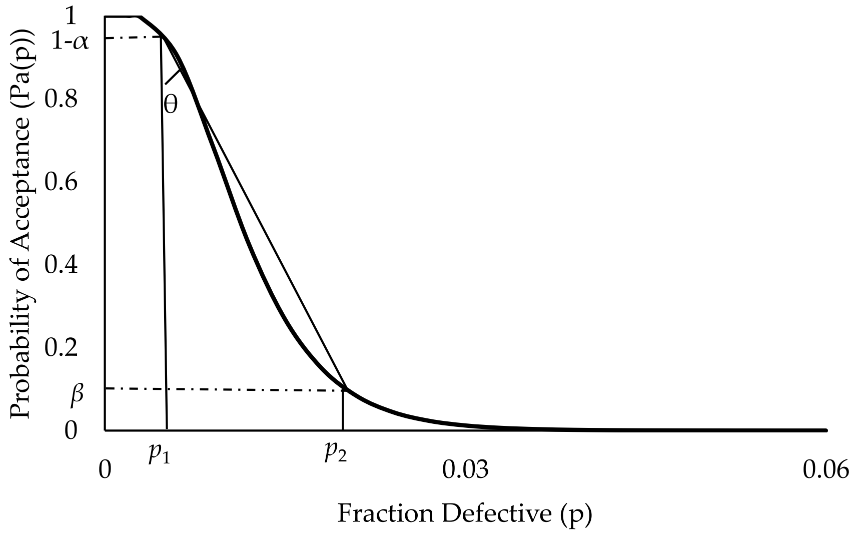

Figure 1: *Tangent angle minimization using p_1 and p_2*

Initially, the development of variables SSP using the minimum angle methodology was proposed in the literature [20]. The orientation of the OC curve has been studied previously, with alternative techniques proposed by Bush [4]. Several studies have extended the use of the minimum angle principle in sampling schemes. Joyce et al. developed mixed sampling schemes based on the tangent angle principle, while Suresh and Vinitha examined generalized two-plan systems [11, 23]. A modified chain sampling plan, formulated using minimum angle optimization, was later proposed by Edna and Joyce [7]. An SkSP-V design using the minimum angle method was later proposed by Nirmala [15]. Further developments include the design of chain sampling plans based on truncated life testing for the log-logistic distribution [25] and a generalized class of group ChSPs [24].

While several sampling plans have been formulated using the minimum angle method, existing research has largely been confined to attribute-based inspection contexts. This highlights a notable gap in the application of the minimum angle approach to variables sampling plans, warranting further investigation. To address this, the present study proposes variables ChSP, designated as VChSP constructed using the minimum angle approach. The structure of the paper is as follows: Section 3 details the operating procedure and key performance measures of the proposed plan. Section 4 describes the development and computational methodology for identifying the optimal design parameters. Finally, Section 5 provides concluding remarks and a summary of the research findings.

III. Variables ChSP

This section outlines the assumptions, operating procedure and two key performance metrics of the proposed VChSP that are particularly pertinent to this study namely, the probability of acceptance and the ASN.

I. Assumptions

- The quality characteristic X is assumed to follow a normal distribution, with the standard deviation being known or unknown.
- The production process under consideration is assumed to generate lots in a sequential manner, each submitted for inspection in the order of production. It is further assumed that the process maintains a constant proportion of nonconforming items across all lots.
- It is presumed that the consumer has confidence in the supplier's quality assurance practices, and there exists no prior reason to suspect that any given lot is of inferior quality compared to its predecessors.

II. Operating Procedure of Known Sigma VChSP

The operating procedure of the proposed known sigma VChSP is described as follows.

Assume that the quality characteristic under consideration follows a normal distribution with an unknown mean, denoted by μ, a known standard deviation, denoted by σ, and is subject to an upper specification limit, denoted by U.

Step 1: Select a random sample of size n_σ from the current lot and calculate the value of $V = \frac{U-\bar{X}}{\sigma}$ where $\bar{X} = \frac{1}{n_\sigma}\sum_{i=1}^{n_\sigma} X_i$.

Step 2: Accept the current lot if $V \geq k_{a\sigma}$ and reject the current lot if $V < k_{r\sigma}$.

Step 3: If $k_{r\sigma} \leq V < k_{a\sigma}$, then accept the current lot provided that preceding i_σ lots have been accepted with the condition that $V \geq k_{a\sigma}$. Otherwise, reject.

If lower specification limit L is specified then, the operating procedure is outlined as follows.

Step 1: Select a random sample of size n_σ from the current lot and calculate the value of $V = \frac{\bar{X}-L}{\sigma}$ where $\bar{X} = \frac{1}{n_\sigma}\sum_{i=1}^{n_\sigma} X_i$.

Step 2: Accept the current lot if $V \geq k_{a\sigma}$ and reject the current lot if $V < k_{r\sigma}$.

Step 3: If $k_{r\sigma} \leq V < k_{a\sigma}$, then accept the current lot provided that preceding i_σ lots have been accepted with the condition that $V \geq k_{a\sigma}$. Otherwise, reject.

III. Operating Procedure of Unknown Sigma VChSP

Assume that the quality characteristic under consideration follows a normal distribution with an unknown mean, denoted by μ, a unknown standard deviation, denoted by S, and is subject to an upper specification limit, denoted by U

Step 1: Select a random sample of size n_s from the current lot and calculate the value of $V = \frac{U-\bar{X}}{S}$ where $\bar{X} = \frac{1}{n_S}\sum_{i=1}^{n_S} X_i$ and $= \sqrt{\sum(X_i - \bar{X})^2/(n_S - 1)}$.

Step 2: Accept the current lot if $V \geq k_{aS}$ and reject the current lot if $V < k_{rS}$.

Step 3: If $k_{rS} \le V < k_{aS}$, then accept the current lot provided that preceding i_S lots have been accepted with the condition that $V \ge k_{aS}$. Otherwise, reject.

If lower specification limit L is specified then, the operating procedure is outlined as follows.

Step 1: Select a random sample of size n_S from the current lot and calculate the value of $V = \frac{\bar{X}-L}{S}$ where $\bar{X} = \frac{1}{n_S}\sum_{i=1}^{n_S} X_i$ and $S = \sqrt{\sum(X_i-\bar{X})^2/(n_S-1)}$.

Step 2: Accept the current lot if $V \ge k_{aS}$ and reject the current lot if $V < k_{rS}$.

Step 3: If $k_{rS} \le V < k_{aS}$, then accept the current lot provided that preceding i_S lots have been accepted with the condition that $V \ge k_{aS}$. Otherwise, reject.

The VChSP is fully characterized by four key parameters: n_σ (or n_S) denoting the sample size, i_σ (or i_S) denoting the number of preceding lots considered, $k_{a\sigma}$ (in the case of known sigma) or k_{aS} (unknown sigma) as the acceptance criterion, and $k_{r\sigma}$ (known sigma) or k_{rS} (unknown sigma) as the rejection criterion. The plan reduces to the conventional SSP when the acceptance and rejection thresholds are equal.

IV. Performance Measures

The effectiveness of any sampling plan can be evaluated through its performance metrics. Among these, the OC function and the ASN function are widely regarded as the most critical indicators of performance. The OC and ASN functions for the proposed VChSP are formulated as outlined below:

The probability of acceptance for the VChSP for known sigma and unknown sigma respectively are defined by

$$P_a(p) = P(V \ge k_{a\sigma}) + [P(V \ge k_{r\sigma}) - P(V \ge k_{a\sigma})][P(V \ge k_{a\sigma})]^{i_\sigma} \quad (1)$$

$$P_a(p) = P(V \ge k_{aS}) + [P(V \ge k_{rS}) - P(V \ge k_{aS})][P(V \ge k_{aS})]^{i_S} \quad (2)$$

The ASN function for the VChSP is defined as

$$ASN(p) = n \quad (3)$$

IV. Development of a VChSP Based on Tangent Angle Method Using PQL and CQL

This section presents a systematic procedure for constructing a VChSP indexed through the PQL and CQL, utilizing the tangent angle minimization method. The core objective is to identify the sampling plan that minimizes both α and β risks concurrently by geometrically optimizing the angle formed on the OC curve.

I. Design Methodology

The procedure for designing the sampling plan is outlined below:

Step 1: Specification of OC Curve Coordinates:

Identify two points on the OC curve corresponding to desired quality levels:

p_1: fraction defective at PQL, with an acceptance probability of $1-\alpha$

p_2: fraction defective at CQL, with an acceptance probability of β

Step 2: Determine sample size n:

Fix the sample size n based on operational or economic constraints.

Step 3: Compute acceptance probabilities:

For the selected values p_1 and p_2, calculate the probability of acceptance: $P_a(p_1)$ and $P_a(p_2)$

Step 4: Evaluate the tangent angle θ:

Determine the tangent angle between the two points on the OC curve using the formula:

$$\tan\theta = \frac{p_2 - p_1}{P_a(p_1) - P_a(p_2)} \quad (4)$$

Step 5: Risk assessment:

Based on the chosen coordinates, calculate the associated risks α and β, and determine the cumulative risk $\alpha+\beta$.

Thus the optimal parameters and risk minimum of VChSP are determined by solving the following optimization problem:

$$\min_{\{n,k_a,k_r,i|p_1,p_2,\alpha,\beta\}} \tan\theta \tag{5}$$

$$\text{Subject to}$$
$$P_a(p_1) \geq 1-\alpha$$
$$P_a(p_2) \leq \beta$$
$$n > 1, k_a > k_r > 0, i \geq 1$$

This geometric formulation aims to derive a sampling plan where the OC curve closely approximates the ideal scenario (i.e., a step function). The tangent angle minimization ensures that both types of errors (producer's and consumer's) are kept as low as possible, resulting in a plan that is both statistically efficient and practically implementable.

Tables are generated through this methodology enable users such as quality engineers and decision-makers in manufacturing or service sectors to choose appropriate plans tailored to their operational quality requirements.

II. Computational Methodology

We solve the nonlinear optimization problem presented in equation (5) to identify the optimal parameter values for the VChSP. In the case of known sigma, these parameters are the acceptance criterion, rejection criterion, and the number of preceding lots denoted by $k_{a\sigma}$, $k_{r\sigma}$ and i_{σ}, respectively. For unknown, the corresponding parameters are k_{aS}, k_{rS} and i_S. In addition, the minimum angle ($(\tan\theta)_{min}$) and minimum sum of risks ($(\alpha+\beta)_{min}$) are obtained for different combinations of p_1, p_2 and n. The minimum tangent and sum of risks can be obtained by fixing the sample size, along with p_1 and p_2, whereas the other optimal parameters (k_a, k_r and i) are determined through a search procedure, where the parameter ranges are specified as follows: k_a ranging from 0.1 to 3.5, k_r ranging from k_a+0.01 to 3.5, and i ranging from 1 to 10.

Tables 1 and 2 show the optimal values of the parameters along with $(\tan\theta)_{min}$ and $(\alpha+\beta)_{min}$ for the different combinations of p_1, p_2, $n_\sigma(n_S)$ for the fixed α = 5% and β = 10% for the known sigma and unknown sigma respectively. When we are analyzing the optimal values provided in Table 1 (that is for known sigma case), we observe that for the given p_1 = 0.01, p_2 = 0.05 and n_σ = 13, the optimal parameters are ($k_{a\sigma}$, $k_{r\sigma}$, i_σ) = (1.656, 2.102, 1) along with $(\tan\theta)_{min}$ = 0.045517 and $(\alpha+\beta)_{min}$ = 0.12120 which is less than 0.15 ($\alpha+\beta$). Similarly for the given p_1 = 0.03, p_2 = 0.08 and n = 62, Table 2 yields, the optimal parameters are (k_{aS}, k_{rS}, i_S) = (1.448, 1.718, 1) along with $(\tan\theta)_{min}$ = 0.05741 and $(\alpha+\beta)_{min}$ = 0.12904 which is less than 0.15 ($\alpha+\beta$).

III. Comparative Study

In this section, we compare the minimum sum of risks and sample sizes for the proposed VChSP to the SSP based on the minimum angle method.

The comparative results in Table 3 clearly demonstrate the superiority of the proposed VChSP over the conventional SSP when evaluated under the minimum angle method. Firstly, it is observed that for every pair of quality levels (p_1, p_2), the proposed VChSP consistently yields smaller values of the minimum sum of risks $(\alpha+\beta)_{min}$ compared to the SSP. For instance, at p_1 = 0.001 and p_2 = 0.01, the total risk decreases from 0.1310 under SSP to 0.09498 under VChSP, reflecting a substantial improvement in discriminatory power between acceptable and unacceptable lots. A similar pattern is maintained across all other pair of quality levels, where reductions in total risk range from approximately 12% to 30%. (see Figure 2)

Secondly, in addition to lowering the overall risks, the VChSP achieves this with smaller required sample sizes in all the cases. For example, at (p_1, p_2) = (0.0025, 0.025), the SSP requires a sample size of 12, whereas the VChSP attains lower risk with only sample size of 10. Likewise, for (p_1, p_2) = (0.005, 0.035), the SSP again requires 15 sample size, while the VChSP uses 12 sample size with markedly reduced risk. This indicates that the VChSP provides both statistical and operational efficiency, enabling risk control with fewer observations. (see Figure 3)

Thirdly, the reduction in sample size without compromising the risk structure reflects the practical advantage of the chain procedure. By incorporating information from preceding lots, the VChSP leverages conditional sampling to achieve a more powerful decision rule than the SSP, which treats each lot independently. This structural improvement explains the VChSP is more effective under the minimum angle criterion, as it sharpens the separation between the producer's quality level and the consumer's quality level. Therefore, the results substantiate that the VChSP not only adheres to the theoretical foundations of the minimum angle principle but also offers significant practical advantages. This establishes the proposed plan as a more reliable and efficient alternative to the conventional SSP for real-world acceptance sampling applications.

It is important to note that the present comparison is carried out for the case of known sigma, however, a similar trend is observed in the case of unknown sigma as well.

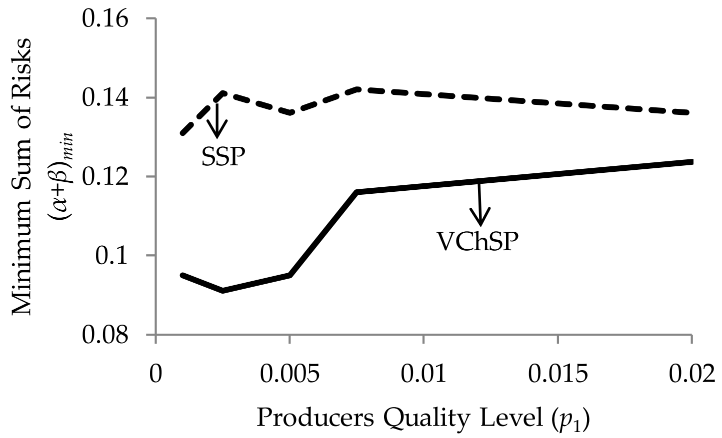

Figure 2: *Minimum sum of risks for SSP and VChSP*

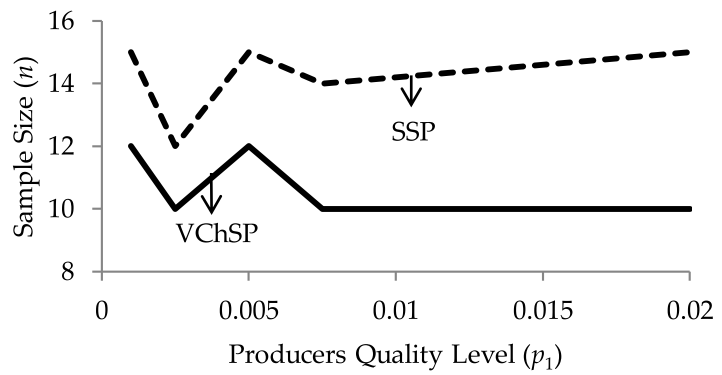

Figure 3: *Sample size for SSP and VChSP*

Table 1: *The optimal plan parameters of the VChSP based on minimum angle method for the given values of p_1, p_2 and n_σ when sigma is known*

p_1	p_2	n_σ	$k_{r\sigma}$	$k_{a\sigma}$	i_σ	$(tan\ \theta_\sigma)_{min}$	α_{min}	β_{min}	$(\alpha+\beta)_{min}$
0.001	0.002	119	2.809	3.000	2	0.001172	0.04927	0.09768	0.14694
	0.0025	68	2.715	2.971	2	0.001750	0.04936	0.09352	0.14288
	0.003	46	2.630	2.946	2	0.002339	0.05000	0.09500	0.14499
	0.004	31	2.655	2.945	1	0.003425	0.04992	0.07419	0.12411
	0.01	12	2.370	2.845	1	0.009945	0.04418	0.05081	0.09498
0.0025	0.004	230	2.602	2.742	2	0.001745	0.04889	0.09144	0.14034
	0.006	64	2.504	2.706	1	0.004063	0.04994	0.08862	0.13856
	0.01	26	2.328	2.649	1	0.008544	0.04993	0.07228	0.12221
	0.02	12	2.093	2.567	1	0.019435	0.04648	0.05310	0.09958
	0.025	10	2.012	2.534	1	0.024754	0.04243	0.04864	0.09106
0.005	0.0075	259	2.384	2.515	2	0.002933	0.04977	0.09763	0.14761
	0.01	86	2.245	2.470	2	0.005860	0.04967	0.09711	0.14678
	0.02	24	2.077	2.411	1	0.016785	0.04971	0.05665	0.10636
	0.03	14	1.916	2.355	1	0.027800	0.04714	0.05357	0.10071
	0.035	12	1.856	2.331	1	0.033148	0.04435	0.05062	0.09497
0.0075	0.01	505	2.327	2.396	1	0.002888	0.04986	0.08435	0.13421
	0.02	43	2.064	2.309	1	0.014159	0.04999	0.06715	0.11714
	0.03	19	1.876	2.247	1	0.025868	0.04995	0.08024	0.13019
	0.04	13	1.755	2.209	1	0.036977	0.04999	0.07110	0.12109
	0.05	10	1.665	2.177	1	0.048076	0.04998	0.06600	0.11598
0.01	0.015	248	2.172	2.275	1	0.005686	0.04979	0.07087	0.12066
	0.02	74	1.922	2.213	2	0.011643	0.05000	0.09108	0.14108
	0.03	21	1.731	2.138	2	0.023430	0.05000	0.09638	0.14638
	0.05	13	1.656	2.102	1	0.045517	0.04999	0.07121	0.12120
	0.07	9	1.511	2.058	1	0.067202	0.04997	0.05720	0.10717
0.02	0.03	187	1.874	1.995	1	0.011593	0.04998	0.08745	0.13743
	0.04	59	1.742	1.948	1	0.023884	0.04998	0.09474	0.14472
	0.06	22	1.540	1.881	1	0.046632	0.04995	0.09226	0.14221
	0.08	14	1.404	1.838	1	0.068666	0.04995	0.07626	0.12621
	0.1	10	1.282	1.799	1	0.091293	0.04999	0.07371	0.12370
	0.15	6	1.056	1.725	1	0.147018	0.04998	0.06578	0.11575
0.03	0.04	341	1.749	1.837	1	0.011502	0.04973	0.08083	0.13056
	0.06	52	1.448	1.745	2	0.034882	0.04986	0.09010	0.13996
	0.08	26	1.407	1.722	1	0.057238	0.04991	0.07654	0.12646
	0.1	17	1.291	1.685	1	0.079458	0.04993	0.06910	0.11904
	0.15	10	1.088	1.609	1	0.132171	0.04294	0.04915	0.09208
	0.2	6	0.884	1.552	1	0.190587	0.05000	0.05802	0.10802
0.04	0.06	145	1.553	1.683	1	0.023218	0.04992	0.08870	0.13862
	0.08	50	1.410	1.636	1	0.045631	0.04988	0.07353	0.12341
	0.1	27	1.285	1.595	1	0.068526	0.04993	0.07449	0.12442
	0.12	18	1.181	1.560	1	0.091284	0.04997	0.07364	0.12361
	0.15	11	1.020	1.507	1	0.127415	0.04996	0.08672	0.13668
	0.2	8	0.884	1.463	1	0.17868	0.04886	0.05569	0.10455
0.05	0.07	188	1.421	1.573	2	0.023422	0.04914	0.09698	0.14611
	0.09	60	1.242	1.518	2	0.046472	0.04938	0.08988	0.13926
	0.12	26	1.172	1.486	1	0.080617	0.04996	0.08174	0.13170
	0.15	17	1.056	1.449	1	0.112773	0.04997	0.06329	0.11326

Table 2: *The optimal plan parameters of the VChSP based on minimum angle method for the given values of p_1, p_2 and n_S when sigma is unknown*

p_1	p_2	n_S	k_{rS}	k_{aS}	i_S	$(tan\ \theta_S)_{min}$	α_{min}	β_{min}	$(\alpha+\beta)_{min}$
0.001	0.002	650	2.823	3.000	2	0.00117	0.04978	0.09819	0.14797
	0.004	147	2.549	2.906	2	0.00350	0.04999	0.09756	0.14755
	0.006	84	2.418	2.850	2	0.00580	0.04996	0.08851	0.13846
	0.008	62	2.475	2.858	1	0.00803	0.04998	0.07825	0.12823
	0.01	45	2.395	2.819	1	0.01050	0.04999	0.09258	0.14258
0.0025	0.004	1131	2.656	2.754	1	0.00173	0.04983	0.08313	0.13296
	0.006	282	2.443	2.682	2	0.00410	0.04998	0.09697	0.14695
	0.01	106	2.355	2.640	1	0.00870	0.04998	0.08758	0.13756
	0.03	29	2.011	2.479	1	0.03100	0.04189	0.07149	0.11338
	0.04	21	1.906	2.427	1	0.04250	0.04183	0.07574	0.11757
0.005	0.0075	1113	2.391	2.516	2	0.00291	0.04999	0.09078	0.14077
	0.02	75	1.967	2.356	2	0.01751	0.04988	0.09356	0.14344
	0.03	43	1.956	2.336	1	0.02877	0.04977	0.08093	0.13089
	0.04	31	1.856	2.288	1	0.03983	0.04595	0.07542	0.12137
	0.05	25	1.780	2.244	1	0.05065	0.04082	0.07065	0.11146
0.0075	0.01	1817	2.296	2.387	2	0.00294	0.04908	0.09956	0.14865
	0.02	133	1.960	2.272	2	0.01466	0.04999	0.09717	0.14716
	0.04	41	1.822	2.198	1	0.03738	0.04998	0.08054	0.13052
	0.05	28	1.566	2.096	2	0.04959	0.04770	0.09519	0.14290
	0.06	26	1.679	2.121	1	0.05909	0.04121	0.07028	0.11149
0.01	0.015	786	2.129	2.260	2	0.00585	0.04895	0.09591	0.14485
	0.03	93	1.909	2.172	2	0.01753	0.05000	0.09437	0.14437
	0.05	39	1.722	2.094	1	0.04582	0.04999	0.07695	0.12694
	0.07	26	1.596	2.025	1	0.06753	0.04138	0.07008	0.11146
	0.08	21	1.536	1.998	1	0.07932	0.04271	0.07477	0.11748
0.02	0.03	528	1.836	1.980	2	0.01173	0.04922	0.09810	0.14732
	0.05	88	1.568	1.880	2	0.03519	0.04994	0.09748	0.14742
	0.07	44	1.523	1.852	1	0.05798	0.04997	0.08772	0.13770
	0.09	27	1.262	1.750	2	0.08170	0.04819	0.09503	0.14322
	0.1	22	1.202	1.723	2	0.09394	0.04934	0.09910	0.14844
0.03	0.04	1027	1.759	1.839	1	0.01120	0.04863	0.05877	0.10740
	0.06	126	1.487	1.742	2	0.03519	0.04965	0.09786	0.14752
	0.08	62	1.448	1.718	1	0.05741	0.04995	0.07909	0.12904
	0.1	36	1.333	1.671	1	0.08122	0.04993	0.08824	0.13817
	0.2	12	1.005	1.501	1	0.19171	0.03913	0.07409	0.11322
0.04	0.06	339	1.506	1.668	2	0.02334	0.04920	0.09386	0.14306
	0.08	102	1.335	1.604	2	0.04690	0.04995	0.09709	0.14704
	0.1	53	1.201	1.551	2	0.07044	0.04999	0.09733	0.14732
	0.15	23	1.117	1.503	1	0.12680	0.04996	0.08252	0.13248
	0.2	15	0.977	1.424	1	0.18014	0.04028	0.07150	0.11178
0.05	0.07	427	1.431	1.574	2	0.02332	0.04999	0.09251	0.14250
	0.09	127	1.276	1.517	2	0.04662	0.04948	0.09258	0.14205
	0.12	49	1.086	1.445	2	0.08218	0.05000	0.09825	0.14824
	0.16	25	0.912	1.370	2	0.12819	0.04885	0.09303	0.14188
	0.2	17	0.945	1.363	1	0.17167	0.04711	0.07912	0.12623

Table 3: *Sample size and minimum sum of risks of the SSP and VChSP for the specified values of p_1 and p_2*

p_1	p_2	SSP		VChSP	
		n_σ	$(\alpha+\beta)_{min}$	n_σ	$(\alpha+\beta)_{min}$
0.001	0.01	15	0.1310	12	0.09498
0.0025	0.025	12	0.1411	10	0.09106
0.005	0.035	15	0.1361	12	0.09497
0.0075	0.05	14	0.1420	10	0.11598
0.02	0.10	15	0.1361	10	0.12370

V. Conclusions

The proposed VChSP designed using the minimum angle method offers a robust and statistically efficient framework for acceptance sampling. By minimizing the angle between the OC curve coordinates corresponding to the PQL and CQL, this method ensures a balanced trade-off between α and β risks. The minimum angle approach not only enhances the discriminative power of the sampling plan but also provides improved lot sentencing decisions when compared to conventional techniques. Moreover, the proposed VChSP is particularly advantageous in continuous production scenarios where minimizing inspection cost while maintaining product quality is critical. The design tables developed in this study further enhance the practical utility of the plan by offering ready-to-use solutions for industrial applications. Thus, the integration of the minimum angle method into the VChSP framework significantly improves both theoretical performance and real-world applicability, making it a compelling alternative to classical sampling strategies.

Acknowledgement

This research was supported by Department of Science and Technology – Science and Engineering Research Board (DST – SERB) of India through the project (CRG/2020/005078).

References

[1] Allen, T. T. Introduction to engineering statistics and six sigma: Statistical quality control and design of experiments and systems, Springer Science & Business Media, 2006.

[2] Balamurali, S. and Usha, M. (2013). Optimal designing of variables chain sampling plan by minimizing the average sample number. *International Journal of Manufacturing Engineering*, 2013(1):751–807.

[3] Bender Jr, A. (1975). Sampling by variables to control the fraction defective: Part II. *Journal of Quality Technology*, 7(sup1):71–75.

[4] Bush, N. A. Method of discrimination for single and double sampling OC curves: Utilizing the tangent at the point of inflection, Chemical Corps Engineering Agency, 1953.

[5] Collani, V. E. (1991). A note on acceptance sampling for variables. *Metrika*, 38(1):19–36.

[6] Dodge, H. F. (1977). Chain sampling inspection plan. *Journal of Quality Technology*, 9(3):139–142.

[7] Edna, K. R. J. and Joyce, V. J. (2018). Selection of modified chain sampling plan through minimum angle criteria. *International Journal of Mechanical Engineering and Technology*, 9(1):1102–1105.

[8] Frischman, F. (1960). An extended chain sampling plan. *Industrial Quality Control*, 17(1):10–12.

[9] Govindaraju, K. and Balamurali, S. (1998). Chain sampling plan for variables inspection. *Journal of Applied Statistics*, 25(1):103–109.

[10] Govindaraju, K. and Lai, C. D. (1998). A modified ChSP-1 chain sampling plan, MChSP-1, with very small sample sizes. *American Journal of Mathematical and Management Sciences*, 18(3-4):343–358.

[11] Joyce, V. J. Devaarul, S. and Edna, K. R. J. (2013). Designing and selection of mixed sampling plans based on Tangent angle. *International Journal of Mathematics and Computations*, 3(1):217–222.

[12] Lieberman, G. J. and Resnikoff, G. J. (1955). Sampling plans for inspection by variables. *Journal of the American Statistical Association*, 50(270):457–516.

[13] Luca, S. (2018). Modified chain sampling plans for lot inspection by variables and attributes. *Journal of Applied Statistics*, 45(8):1447–1464.

[14] Montgomery, D. C. Statistical Quality Control, 6th Edition, New York: Wiley, 2009.

[15] Nirmala, V. (2019). Designing of skip-lot sampling plan - V with minimum angle criteria. *Journal of Statistics and Mathematic Engineering*, 5(1):27–33.

[16] Owen, D. B. (1967). Variables sampling plans based on the normal distribution. *Technometrics*, 9(3):417–423.

[17] Raju, C. (1991). Three-stage chain sampling plans. *Communications in Statistics-Theory and Methods*, 20(5-6):1777–1801.

[18] Schilling, E. G. and Neubauer, D. V. Acceptance sampling in quality control, 2nd Edition, Chapman and Hall/CRC, 2009.

[19] Seidel, W. (1997). Is sampling by variables worse than sampling by attributes? A decision theoretic analysis and a new mixed strategy for inspecting individual lots. *Sankhyā: The Indian Journal of Statistics, Series B*, 96–107.

[20] Soundararajan, V. (1978). Procedures and tables for construction and selection of chain sampling plans (ChSP-1)-Part I. *Journal of Quality Technology*, 10(2):56–60.

[21] Soundararajan, V. (1978). Procedures and tables for construction and selection of chain sampling plans (ChSP-1) Part 2: Tables for selection of chain sampling plans. *Journal of Quality Technology*, 10(3):99–103.

[22] Soundararajan, V. and Christina, A. L. (1997). Selection of single sampling variables plans based on the minimum angle. *Journal of Applied Statistics*, 24(2):207–218.

[23] Suresh, K. K. and Vinitha, K. X. (2014). Selection of generalized two plan system using minimum angle method. *Mathematical Journal of Interdisciplinary Sciences*, 3(1):65–72.

[24] Teh, M. A. P. Aziz, N. and Zain, Z. (2022). Generalized family of group chain sampling plans using minimum angle method (MAM). *Mathematics and Statistics*, 10(2):314–319.

[25] Tharani, K. and Ramaswamy, A. S. (2019). Designing chain sampling plan based on truncated life test for Log-Logistic distribution using minimum angle method. *International Journal of Mathematics Trends and Technology*, 65(7):368–374.

[26] Tripathi, H. Al-Omari, A. I. Saha, M. and Alanzi, A. R. (2021). Improved attribute chain sampling plan for darna distribution. *Computer Systems Science and Engineering*, 38(3):381–392.

[27] Wortham, A. W. and Baker, R. C. (1976). Multiple deferred state sampling inspection. *The International Journal of Production Research*, 14(6):719–731.