RISK MODELING WITH THE NEW LOGARITHMIC TANGENT WEIBULL DISTRIBUTION: APPLICATIONS IN INSURANCE AND NET OFFICIAL DEVELOPMENT ASSISTANCE

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Abstract

The Weibull distribution is widely used for modeling lifetime and reliability data but often struggles with complex hazard structures and heavy-tailed behaviors. To address these limitations, this study proposes the New Logarithmic Tangent Weibull (NLT-Wei) distribution, which integrates logarithmic and tangent transformations into the Weibull model, introducing an additional shape parameter to enhance flexibility. Key functions, including the PDF, CDF, survival, and hazard functions, are derived, along with quantiles and reliability measures. Parameters are estimated using maximum likelihood estimation (MLE). The model is applied to two real datasets: insurance survival data and Net Official Development Assistance (Net ODA) data. Comparative analyses with Weibull, Exponentiated Weibull, Flexible Weibull, NGLog-Weibull, and Kumaraswamy Weibull distributions are conducted using information criteria (AIC, BIC, CAIC, HQAIC) and goodness-of-fit tests (KS, Cramér–von Mises, Anderson–Darling). Results show that the NLT-Wei consistently achieves lower information criteria, smaller test statistics, and higher p-values, confirming superior fit. Its flexibility allows effective modeling of monotone, bathtub, and inverted-bathtub hazard rates, as well as heavy-tailed behaviors. The study demonstrates that the NLT-Wei is not only robust for reliability and survival analysis but also highly suitable for actuarial and financial applications, offering a practical tool for risk assessment, insurance modeling, and lifetime data analysis across diverse domains.

Keywords: Weibull distribution, New Logarithmic Tangent Weibull, financial modelling and reliability analysis.

I. Introduction

The Weibull distribution is one of the most widely used lifetime models in survival analysis, reliability engineering and risk assessment due to its ability to accommodate constant, increasing and decreasing hazard rates. This adaptability has led to its effective application in a variety of

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sectors, including actuarial research, medical sciences, engineering dependability, and climatology. However, complicated hazard structures and heavy-tailed behaviors seen in practice are frequently missed by the traditional Weibull distribution. A number of generalizations have been created to get around these restrictions, including the transmuted Weibull, Marshall–Olkin Weibull, and exponentiated Weibull families, which add extra parameters to enhance hazard rate behavior and tail flexibility. Actuarial risks are the financial hazards associated with insurance and pension plans. One important technique that actuaries employ to understand and quantify these risks is statistical analysis. Actuaries utilize probability distributions to simulate the probability of a range of occurrences, such as policy cancellations, deaths, and claims. The Poisson, Weibull, and exponential distributions are often used in actuarial science. Stochastic modelling is used to represent random processes, such as insurance cancellations and claims. This approach is used to estimate the expected value of future claims and determine how variable these estimates are. Loss distributions describe the distribution of losses caused by events such as insurance cancellations and claims. This technique is used to calculate the expected value of future losses as well as the risk associated with them. Actuaries use statistical techniques, such portfolio optimization and hedging methods, to assess and manage financial risks. In conclusion, statistical analysis is an essential technique used by actuaries to understand and manage financial risks in insurance and pension systems. Among the most important fields in which we statisticians are interested in measuring risk are the insurance and reinsurance sectors, as well as the actuarial sciences in general. In addition to insurance, Net Official Development Assistance (Net ODA), which involves uncertain resource distribution, project payments, and financial inflows, also requires similar probabilistic modeling. For policymakers and development organizations to make well-informed choices, reduce risk, and maximize resource usage, accurate modeling of these uncertainties is crucial.

Recent research has also emphasized the use of logarithmic and trigonometric transformations to construct new distribution families with enhanced adaptability. For example, Chakraborty et al. [1, 22] developed a new skew logistic distribution, Jayakumar and Babu [2] proposed the Marshall–Olkin extended Weibull and its double/asymmetric variations, applying them to stock exchange data. Ahmad et al. [3] further refined Weibull extensions through algebraic generalizations. Trigonometric-based approaches have also emerged: Chesneau et al. [4] applied tangent-based modifications, Silveira et al. [5] introduced the Normal-Tangent-G family, and Alomair et al. [6, 18] developed the Type-I Cosine Exponentiated-X family, including the cosine-exponentiated Weibull. Similarly, Kamal et al. [7, 19, 20] proposed the Sine-G family, while Benchiha et al. [8] introduced sine-based generalizations that improved elasticity in modeling lifetime data.

The Logarithmic Weibull approach to applications in engineering was presented by Zhao et al. [9] on the logarithmic side and The NGLog-Weibull distribution, which belongs to the New Generalized Logarithmic-X family, was proposed by Shah et al. [10] for biological data. By demonstrating its usefulness in risk modeling, Abubakar and Sabri [11, 21] further emphasized the significance of these models in actuarial science and finance. The New Logarithmic Tangent-U (NLT-U) family of distributions, which combines logarithmic and tangent transformations, was recently presented by Alsolmi [12], building on these advancements. The New Logarithmic Tangent Weibull (NLT-Wei) distribution is a prominent particular instance. It retains the flexibility of the Weibull baseline but adds an additional parameter that permits a wide range of hazard shapes, such as monotone expanding, decreasing, bathtub, and inverted bathtub patterns.

This study proposes and investigates the NLT-Wei distribution as a powerful extension of the Weibull model. The main contributions of this paper are as follows: This study presents the NLT-Wei distribution, a versatile expansion of the Weibull family, to address these types of problems. We define the NLT-Wei distribution and present its fundamental functions, including the cumulative distribution function (CDF), probability density function (PDF), survival function (SF), and hazard function (HF). We derive key statistical properties such as quantiles and reliability measures to characterize the model. Parameter estimation is addressed using MLE and the practical utility of the NLT-Wei is demonstrated using real lifetime datasets, with comparisons to competing models based on goodness-of-fit statistics and graphical diagnostics especially in finance and insurance.

The remainder of this paper is organized as follows: Section 2 introduces the NLT-Wei distribution and its basic properties. Section 3 discusses parameter estimation methods. Section 4 presents real data applications. Section 5 concludes the study with final remarks.

II. NLT-Weibull Distribution

In this section, we use the derived method and suggest a new trigonometric and logarithmic version of the classic Weibull model. Let's suppose a RV (random variable)W ($\in R^{+}$) follows the CDF $U(W;\mu) = 1 - e^{-\sigma w^{\alpha}}$ and PDF $u(w;\mu) = \sigma\alpha w^{\alpha-1}e^{-\sigma w^{\alpha}}$ of the Weibull model with parameters $\sigma > 0$, and $\alpha > 0$, then the CDF $G(w;\theta,\mu)$ of the NLT-Wei distribution is expressed by

$$G(w;\theta,\mu) = 1 - \frac{log\left(e^{\theta}+\left(1-e^{\theta\, tan\left(\frac{\pi}{4}\left(1-e^{-\sigma w^{\alpha}}\right)\right)}\right)\right)}{\theta}, w \geq 0 \quad (1)$$

Link to Eq. (1), the $S(w;\theta,\mu)$SF of the NLT-Wei distribution is provided by

$$S(w;\theta,\mu) = \frac{log\left(e^{\theta}+\left(1-e^{\theta\, tan\left(\frac{\pi}{4}\left(1-e^{-\sigma w^{\alpha}}\right)\right)}\right)\right)}{\theta}, w \geq 0 \quad (2)$$

Corresponding to Eq. (1) and Eq. (2), the PDF $g(w;\theta,\mu)$, and $h(w;\theta,\mu)$ HF of the NLT-Wei istribution are given, respectively, by

$$g(w;\theta,\mu) = \frac{\pi\sigma\alpha w^{\alpha-1}e^{-\sigma w^{\alpha}}\, sec^{2}\left(\frac{\pi}{4}\left(1-e^{-\sigma w^{\alpha}}\right)\right)e^{\theta\, tan\left(\frac{\pi}{4}\left(1-e^{-\sigma w^{\alpha}}\right)\right)}}{4\left(e^{\theta}+\left(1-e^{\theta\, tan\left(\frac{\pi}{4}\left(1-e^{-\sigma w^{\alpha}}\right)\right)}\right)\right)}, w > 0, \quad (3)$$

$$h(w;\theta,\mu) = \frac{\pi\theta\sigma\alpha w^{\alpha-1}e^{-\sigma w^{\alpha}}\, sec^{2}\left(\frac{\pi}{4}\left(1-e^{-\sigma w^{\alpha}}\right)\right)e^{\theta\, tan\left(\frac{\pi}{4}\left(1-e^{-\sigma w^{\alpha}}\right)\right)}}{4\left\{log\left(e^{\theta}+\left(1-e^{\theta\, tan\left(\frac{\pi}{4}\left(1-e^{-\sigma w^{\alpha}}\right)\right)}\right)\right)\right\}\left(e^{\theta}+\left(1-e^{\theta\, tan\left(\frac{\pi}{4}\left(1-e^{-\sigma w^{\alpha}}\right)\right)}\right)\right)}, w > 0, \quad (4)$$

Similarly, corresponding to Eq. (1), Eq. (2), and Eq. (3), the RHF τ (w; θ, μ), and CHF H (w; θ, μ) of the NLT-Wei distribution is presented by

$$\tau(w;\theta,\mu) = \frac{\pi\theta\sigma\alpha w^{\alpha-1}e^{-\sigma w^{\alpha}}\, \sec^{2}\left(\frac{\pi}{4}\left(1-e^{-\sigma w^{\alpha}}\right)\right)e^{\theta\tan\left(\frac{\pi}{4}\left(1-e^{-\sigma w^{\alpha}}\right)\right)}}{4\left(e^{\theta}+\left(1-e^{\theta\tan\left(\frac{\pi}{4}\left(1-e^{-\theta w^{\alpha}}\right)\right)}\right)\right)\left\{\theta-\log\left(e^{\theta}+\left(1-e^{\theta\tan\left(\frac{\pi}{4}\left(1-e^{-\sigma w^{\alpha}}\right)\right)}\right)\right)\right\}}, w > 0,$$

$$H(w;\theta,\mu) = -\log\left(\frac{\log\left(e^{\theta}+\left(1-e^{\theta\tan\left(\frac{(\pi)}{2}\left(1-e^{-\sigma w^{\alpha}}\right)\right)}\right)\right)}{\theta}\right), w > 0.$$

Furthermore, let W be a random variable that follows the NLT-Wei distribution with $q \in (0,1)$, ther IF, say $Q_w(q)$ can be expressed in the following form

$$Q_w(q) = \left[-\frac{1}{\sigma}\log\left(1 - \frac{4}{\pi}\tan^{-1}\left(\frac{\log\left(e^{\theta} - e^{(1-q)\theta} + 1\right)}{\theta}\right)\right)\right]^{\frac{1}{\sigma}}$$

I. Statistical Properties

Here, we derive a few basic properties of the NLT-U family, such as the rth moments, skewness, kurtosis, quantile function, quartiles, and moment generating functions. These qualities' primary goal is to investigate the characteristics and makeup of the suggested probability distributions. For example, skewness is used to verify if the distribution is symmetrical or asymmetrical. The mesokurtic, platykurtic, and leptokurtic distributions are examples of peaked distributions that are typically measured using kurtosis.

II. Quantile Function

W follows the NLT-U family of distributions, then, the quartile function (QF), denoted as $Q(q)$. given by

$$Q(q) = U^{-1}\left(\frac{4}{\pi}\tan^{-1}\left(\frac{\log\left(1 - e^{\theta(1-q)} + e^{\theta}\right)}{\theta}\right)\right), \tag{5}$$

III. Moments

Let W follow a random variable of the NLT-U family of distribution linked to the PDFg$(w;\delta,\Omega)$, then the rth moment, say μ', is calculated by

$$\mu' = E(w^r) = \int_{-\infty}^{\infty} w^r g(w;\theta,\mu)\partial w \tag{6}$$

$$\mu' = \frac{\pi}{4}\sum_{a=0}^{\infty}\sum_{b=0}^{c}\sum_{k=1}^{\infty}\frac{e^{(1-a)\theta}(-1)^{a+b}(1+d)^k\theta^k}{k!}\binom{a}{b}\varphi_{r,a,b,k}(w;\mu)$$

Additionally, the NLT-U family of distributions' moment generating function (MGF), denoted as $M_w(t)$, is calculated as

$$M_w(t) = \int_{-\infty}^{\infty} e^{wt} g(w;\theta,\mu)\partial w$$

By using exponential series,

$$M_w(t) = \frac{\pi}{4}\sum_{a=0}^{\infty}\sum_{b=0}^{c}\sum_{k=1}^{\infty}\sum_{r=0}^{\infty}\frac{e^{(1-a)\theta}(-1)^{a+b}(1+d)^k\theta^k w^r}{k!\,r!}\binom{a}{b}\varphi_{r,a,b,k}(w;\mu)$$

III. MLE Estimation

In the present section, the model parameters of the NLT-U method are estimated by applying the maximum likelihood estimation (MLE) approach i.e. MLEs $\left(\hat{\theta}_{\text{MLE}}, \hat{\mu}_{\text{MLE}}\right)$ of the parameters (θ, μ). Let's suppose that a set of RS (random-samples) of size n, say $W_1, W_2, W_3, \dots . W_n$ are taken from the proposed family with a PDF $g(w, \theta, \mu)$. The then the likelihood-function (LF) associated with Eq 1, is given by

$$T\left(\frac{\Theta}{w_1, w_2, \dots w_n}\right) = \prod_{i=1}^{n} \frac{\pi u(w;\mu) \sec^2\left(\frac{\pi}{4} U(w;\mu)\right) e^{\theta \tan\left(\frac{\pi}{4} U(w;\mu)\right)}}{4\left(e^{\theta} + \left(1 - e^{\theta \tan\left(\frac{\pi}{4} U(w-\mu)\right)}\right)\right)}. \tag{7}$$

Where $\Theta = (\theta, \mu)^T$. The log LF (LLF), say $\ell(\Theta)$ and taking derivative W.R.T θ and μ, are respectively given by

$$\frac{\partial}{\partial\theta}\ell(\Theta) = \sum_{i=1}^{n} \tan\left(\frac{\pi}{4} U(w_i;\mu) - \sum_{i=1}^{n} \frac{\frac{\partial\left(e^{\theta} + \left(1 - e^{\theta \tan\left(\frac{\pi}{4} U(w_i-\mu)\right)}\right)\right)}{\partial\theta}}{\left(e^{\theta} + \left(1 - e^{\theta \tan\left(\frac{\pi}{4} U(w_i;\mu)\right)}\right)\right)}\right.,$$

and

$$\frac{\partial}{\partial\mu}\ell(\Theta) = \sum_{i=1}^{n} \frac{\frac{\partial u(w_i;\mu)}{\partial\mu}}{u(w_i\mu)} - \sum_{i=1}^{n} \frac{\pi}{4} \frac{\tan\left(\frac{\frac{\pi}{4} U(w_i;\mu)\partial U(w_i\mu)}{\partial\mu}\right.}{\sec^2\left(\frac{\pi}{4} U(w_i;\mu)\right)} + \theta \sum_{i=1}^{n} \frac{\pi}{4} \frac{\sec^2\left(\frac{\pi}{4} U(w_i;\mu)\, \partial u(w_i;\mu)/\partial\mu\right)}{\tan\left(\frac{\pi}{4} U(w_i;\mu)\right)}$$

$$-\sum_{i=1}^{n} \frac{\frac{\partial\left(e^{\theta} + \left(1 - e^{\theta \tan\left(\frac{\pi}{4} U(w_i-\mu)\right)}\right)\right)}{\partial\theta}}{\left(e^{\theta} + \left(1 - e^{\theta \tan\left(\frac{\pi}{4} U(w_i;\mu)\right)}\right)\right)}$$

Setting $\frac{\partial}{\partial\delta}\ell(\Theta) = 0$, and $\frac{\partial}{\partial\mu}\ell(\Theta) = 0$, and solving simultaneously, we get MLEs $\left(\hat{\theta}, \hat{\mu}\right)$ of (θ, μ).

IV. Results and Discussion

I. Applications using Real data

After selecting existing Weibull-type distributions as competing models, the next step involves evaluating model performance using analytical measures and p-values to identify the most suitable distribution among the NLT-Wei and other candidates. The selected comparative tools include: (I) Akaike Information Criterion (AIC), (II) Bayesian Information Criterion (BIC), (III) Consistent Akaike Information Criterion (CAIC), (IV) Hannan-Quinn Information Criterion (HQIC), (V) Cramér–von Mises (CVM) statistic, (VI) Anderson–Darling (AD) statistic, and (VII) Kolmogorov–Smirnov (KS) statistic.

The mathematical expressions of these criteria and goodness-of-fit statistics are provided below, facilitating a clear comparison of model adequacy and selection of the most optimal distribution for the datasets.

$$AIC = 2q - 2\ell(\varphi)$$

$$BIC = \mathrm{q}\log(n) - 2\ell(\varphi)$$

$$CAIC = \frac{2qn}{q-n-1} - 2\ell(\varphi)$$

$$HQIC = 2q\log[\log(n)] - 2\ell(\varphi)$$

$$CRMI = \sum_{p=1}^{n}\left(G(w_p) - \frac{2p-1}{2n}\right)^2 + \frac{1}{12n},$$

$$ANDA = -n - \frac{1}{n}\sum_{p=1}^{n}(2p-1)\times\left[\log\left(1-G(w_{(p-n+1)})\right) + \log\left(G(w_{(p)})\right)\right)$$

$$\mathrm{KOSM} = \mathrm{Max}_{p=1,23\ldots n}\left(\left(\frac{p}{n} - G(w_{(p)})\right), \left(G(w_{(p)}) - \frac{p-1}{n}\right)\right),$$

II. Dataset I: Insurance Data

The first dataset consists of 48 observations of insurance coverage, collected from the World Development Indicators (WDI). Table 1 displays summary statistics for the Insurance Data. A slightly right-skewed distribution is indicated by the data's mean of 3.232, variance of 2.218, mild positive skewness (0.554), and kurtosis of 2.294. Figure 1 provides a visual inspection utilizing boxplots, violin plots, and TTT-plots, demonstrating modest tail behavior and variability.

Table 1: *The summary statistics for the insurance data*

Parameter	N	Min	Q1	Q3	Mean	Max	Skewness	Kurtosis	Variance
Dataset1	48	1.287	2.04	4.13	3.232	6.531	0.5535	2.2936	2.2936

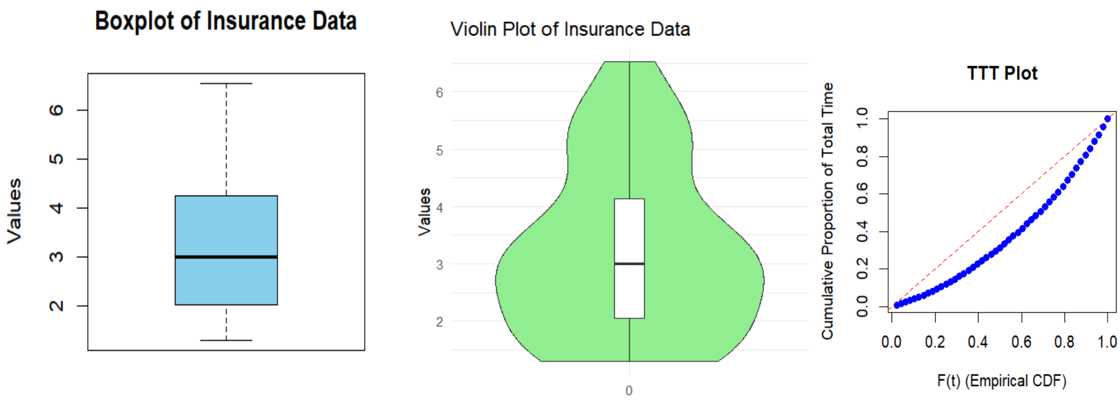


Figure 1: *Box Plot, Violin Plot, and Total Time on Test (TTT) Plot for insurance data*

Maximum likelihood estimation (MLE) was used to fit six Weibull-type distributions: Weibull, NLT-Wei, E-Wei, F-Wei, NGL-Wei, and K-Wei. Table 2 provides a summary of the estimated parameters (α, β, γ, a, b).

Table 2: *The MLEs values of the α, β, γ, a, b*

Distribution	α	β	γ	a	b
Weibull	2.3686078	3.6627564			
NLT-Wei	11.249502	2.247537	3.360695		
E-Wei	5.0589052	1.111906	0.6455213		
F-Wei	-	0.3020747	4.0368493		
NGL-Wei	0.1343426	5.5132409	1.7675898		
K-Wei		2.063135	1.195248	2.4013	0.1131656

The AIC, BIC, CAIC, HQAIC, Kolmogorov-Smirnov (KS), Cramer-von Mises (C), and Anderson-Darling (AD) tests were used to assess the model's adequacy. Table 3 shows the findings.

Table 3: *The numerical values of the model adequacy measures of the fitted distributions for Insurance Data.*

Distribution	AIC	BIC	CAIC	HQAIC	KS	C	AD
Weibull	172.478	176.2208	178.2208	173.892	0.094	0.0746	0.5633
					(0.746)	(0.725)	(0.725)
NLT-Wei	168.415	172.158	174.158	169.829	0.0905	0.0559	0.4235
					(0.793)	(0.841)	(0.824)
E-Wei	172.755	178.3687	181.3687	174.876	0.0773	0.0467	0.448
					(0.915)	(0.898)	(0.798)
F-Wei	175.304	180.9176	183.9176	177.425	0.1110	0.0979	0.660
					(0.716)	(0.785)	(0.788)
NGL-Wei	170.564	170.5643	179.1779	172.685	0.079	0.0469	0.4506
					(0.896)	(0.896)	(0.796)
K-Wei	173.654	181.1396	174.5851	176.483	0.078	0.0545	0.4503
					(0.908)	0.1131	(0.797)

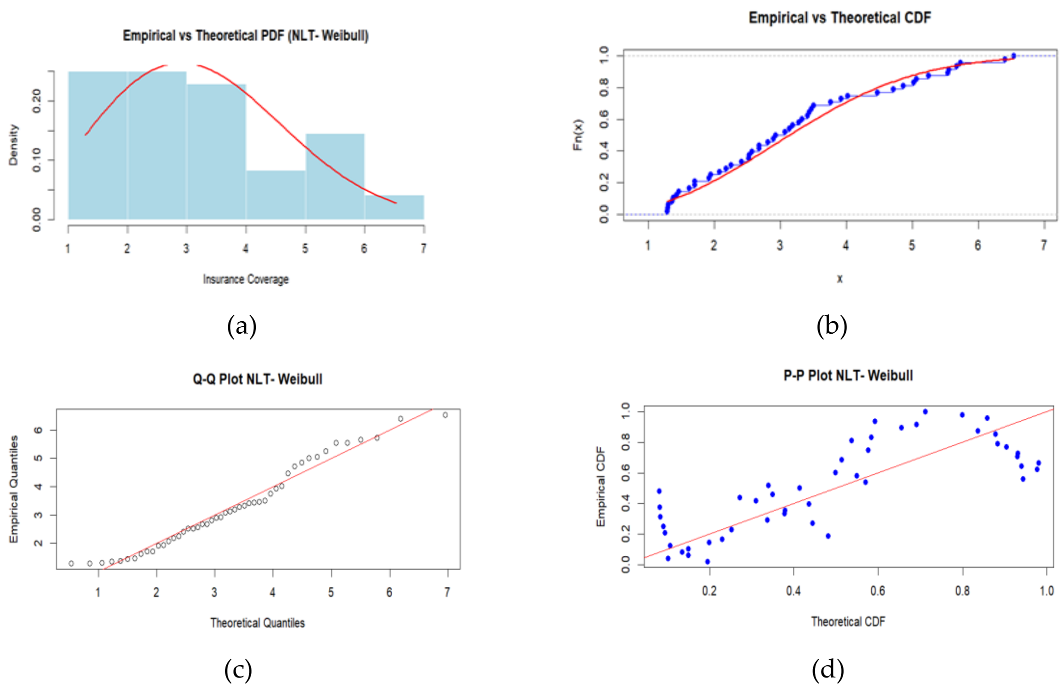


Figure 2: *The empirical and theoretical PDFs, empirical and theoretical CDFs, Q-Q plots and p-p plot for new tangent Logarithmic Weibull distribution for the insurance data.*

The comparative analysis of Dataset 1, which is summarized in Table 3, demonstrates the significant advantage of the proposed NLT-Wei distribution over the classical and extended Weibull distributions. Figure 2 displays the fitted PDFs, CDFs, Q–Q plot, and P–P plot for the insurance data. The NLT-Wei model's precision and effectiveness in fitting the data are demonstrated by the lowest information criteria values it obtained: AIC = 168.4156, BIC = 172.1580, CAIC = 174.1580, and HQAIC = 169.8298. The goodness-of-fit statistics, which demonstrate that the NLT-Wei generated KS = 0.0905 (p = 0.7932), C = 0.0559 (p = 0.8419), and AD = 0.4235 (p = 0.8244), all of which were lower than those of the other models, provide additional evidence for this result. When compared to other competing models, such as Weibull, E-Wei, F-Wei, NGL-Wei, and K-Wei, which yield greater information criteria and comparatively weaker test results, the corresponding high p-values further show that the null hypothesis of statistical suitability cannot be rejected for the NLT-Wei. When compared to other well-known Weibull-type distributions, the NLT-Wei distribution offers the best fit to the dataset, proving its resilience, adaptability, and appropriateness for modeling lifetime data. This is supported by both statistical evidence and numerical measurements.

III. Dataset II: Net ODA Data.

The second dataset consists of 50 observations of net official development assistance (ODA) from the WDI. The Net ODA Data's summary statistics are shown in Table 4. A strongly right-skewed distribution is indicated by the dataset's mean of 0.482, variance of 0.163, moderate positive skewness (0.793), and kurtosis of 3.978. Using box plots, violin plots, and TTT-plots, Figure 3 presents a visual summary of the observations' variability and tail behavior.

Table 4: *The summary statistics for the Net ODA data*

Parameter	N	Min	Q1	Q3	Mean	Max	Skewness	Kurtosis	Variance
Dataset2	50	0.068	0.14	0.793	0.481	1.703	1.1308	3.978	0.1628

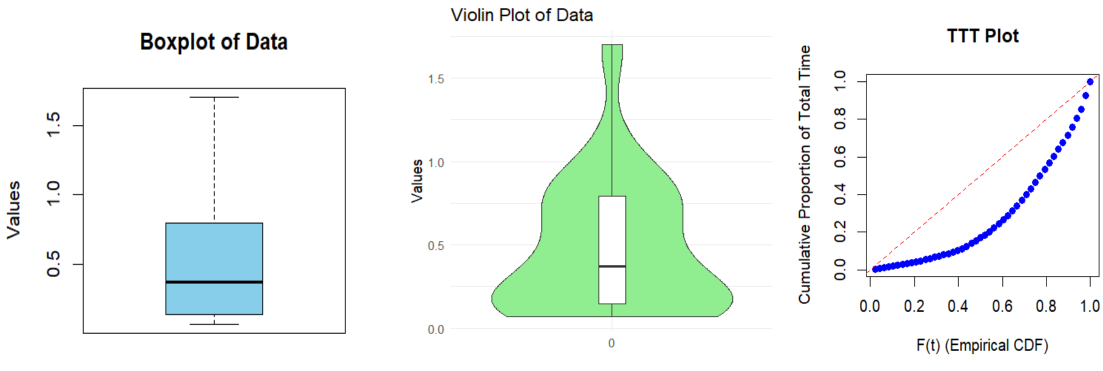


Figure 3: *Box Plot, Violin Plot, and Total Time on Test (TTT) Plot for Net ODA data*

Table 5: *The MLEs values of the α, β, γ, a, b*

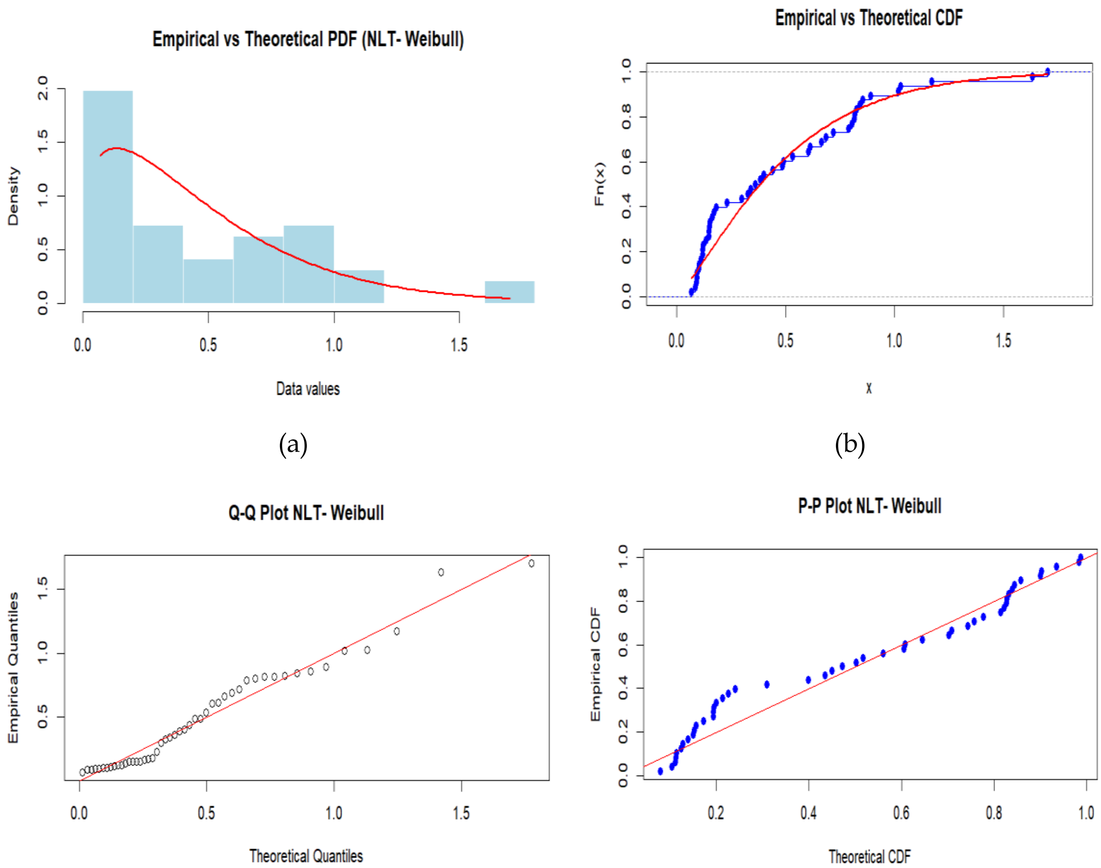
Distribution	α	β	γ	a	b
Weibull	1.2310	0.5169			
NLT-Wei	14.866	1.1805	0.4392		
E-Wei	8.1145	0.4819	20.1005		
F-Wei	-	0.9817	0.2262		
NGL-Wei	60.837	7.227	0.259		
K-Wei		0.2238	0.0363	14.130	13.0155

Maximum likelihood estimation (MLE) was used to fit six Weibull-type distributions: Weibull, NLT-Wei, E-Wei, F-Wei, NGL-Wei, and K-Wei. Table 5 provides a summary of the estimated parameters (α, β, γ, a, b).

The AIC, BIC, CAIC, HQAIC, Kolmogorov-Smirnov (KS), Cramer-von Mises (C), and Anderson-Darling (AD) tests were used to assess the model's adequacy. Table 6 shows the findings.

Table 6: *The numerical values of the model adequacy measures of the fitted distributions for Net ODA Data.*

Distribution	AIC	BIC	CAIC	HQAIC	KS	C	AD
Weibull	26.749	30.491	32.492	28.164	0.155	0.171	1.1159
					(0.175)	(0.330)	(0.301)
NLT-Wei	15.678	19.420	21.420	17.092	0.106	0.118	0.715
					(0.614)	(0.503)	(0.545)
E-Wei	27.319	32.933	35.933	29.441	0.1412	0.221	1.321
					(0.267)	(0.229)	(0.225)
F-Wei	28.957	34.570	37.570	31.078	0.1708	1.1196	0.159
					(0.152)	(0.048)	(0.875)
NGL-Wei	27.982	33.595	36.595	30.103	0.155	0.186	1.176
					(0.175)	(0.295)	(0.276)
K-Wei	29.603	37.088	30.533	32.432	0.1526	0.2015	1.2408
					(0.192)	(0.265)	(0.252)



(a) (b)

(c) (d)

Figure 4: *The empirical and theoretical PDFs, empirical and theoretical CDFs, Q-Q plots and p-p plot for new tangent Logarithmic Weibull distribution for the Net ODA data.*

As demonstrated by the model comparison for Dataset 1 in Table 6 and the fitted PDFs, CDFs, Q–Q plot, and P–P plot for the Net ODA data in Figure 4, the proposed NLT-Wei distribution outperforms the other competing models. The NLT-Wei information criteria values (AIC = 15.6783, BIC = 19.4207, CAIC = 21.4207, and HQAIC = 17.0925) are consistently lower than those of the Weibull, E-Wei, F-Wei, NGL-Wei, and K-Wei distributions, indicating better accuracy and overall fit. The goodness-of-fit results, which demonstrate that the NLT-Wei produced the smallest KS (0.1061, p = 0.6144), C (0.1184, p = 0.5036), and AD (0.7151, p = 0.5456) values, further support the above conclusion. For the NLT-Wei model, the comparatively higher p-values verify that the null hypothesis of adequacy cannot be disproved. Conversely, the competing models show lower p-values, greater test statistics, and bigger information criteria, indicating relatively poorer performance.

V. Conclusion

In this study, we proposed and analyzed the New Logarithmic Tangent Weibull (NLT-Wei) distribution, a novel extension of the classical Weibull model that integrates logarithmic and tangent transformations. By introducing an additional shape parameter, the NLT-Wei achieves greater flexibility in capturing diverse hazard rate patterns, including monotone, bathtub, and inverted-bathtub forms. We derived its key statistical properties, developed estimation procedures via maximum likelihood, and applied the model to real datasets. The empirical applications to insurance survival data and Net Official Development Assistance (Net ODA) data confirmed the superior performance of the NLT-Wei compared to competing models such as Weibull, Exponentiated Weibull, Flexible Weibull, NGLog-Weibull, and Kumaraswamy Weibull, as it consistently produced lower information criteria values, smaller goodness-of-fit statistics, and higher p-values verifying adequacy. Unlike many extensions restricted mainly to engineering reliability, the NLT-Wei demonstrated effectiveness in actuarial science, finance, and international development modeling. Its ability to handle heavy tails and complex hazard structures makes it especially suitable for applications in finance, including risk forecasting, insurance claim analysis, and financial time series modeling. Overall, the findings establish the NLT-Wei as a robust, adaptable, and practical model for lifetime data analysis across engineering, medical, financial, and economic domains.

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