BAYESIAN SINGLE SAMPLING PLANS WITH ZERO-INFLATED BINOMIAL DISTRIBUTION USING COST OPTIMIZATION

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Abstract

The economic design of sampling plans involves determining an optimal plan that minimizes the total cost associated with inspection while ensuring acceptable product quality levels. Bayesian sampling plans are better suited for handling significant between-lot variations in quality, leading to more informed acceptance decisions due to their capability to incorporate prior information. The objective of this article is to design Bayesian single sampling plans considering a Beta prior and a Zero-inflated Binomial (ZIB) distribution, using cost optimization principles by developing an economic model aimed at achieving optimal total costs, considering the Average Total Inspection (ATI). A numerical illustration is provided to demonstrate the selection of Bayesian single sampling plans under the ZIB distribution that minimizes the producer's total cost.

Keywords: Bayesian Single sampling plan, Zero-inflated Binomial distribution, Cost optimization; Average Total Inspection; Average Outgoing Quality

I. Introduction

A sampling plan is a structured statistical approach used to determine whether a batch of products satisfies predefined quality standards without the need for complete inspection. It minimizes the cost and effort of checking every single item by relying on representative samples that reflect the overall quality of the lot. An economic sampling plan is designed to balance the cost of inspection with the potential losses caused by nonconforming products being accepted into the system. It takes into account several factors such as inspection expenses, the cost of nonconformities, production rate, and acceptable risks for both producers and consumers.

A single sampling plan by attributes requires only one sample to be selected and inspected from a given lot of products. By limiting inspection to a single sample, it helps industries to lower manufacturing costs while still maintaining effective quality control. Hsu and Hsu [8] explored optimized single acceptance sampling plans in the context of a two-stage supply chain. Their study focused on achieving economic efficiency by balancing sampling, inspection, and quality failure costs. Similarly, Wimonmas and Pramote [19] used genetic algorithm-based optimization to design double acceptance sampling plans, aiming to reduce sample size while keeping risk at acceptable levels.

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Advances in technology have greatly improved manufacturing processes as well as the quality of finished products. Continuous monitoring during production helps lower the number of nonconforming units, leading to products that are often free from non-conformities. As a result, zero-nonconforming units occur more frequently, making the Zero-inflated Binomial (ZIB) distribution well-suited for modeling such data with excess zeros. This distribution provides a valuable tool for making informed decisions to enhance product quality and minimize nonconforming units.

Hall [7] provided a detailed study of Lambert's Zero-inflated Poisson (ZIP) regression model and later proposed the Zero-inflated Binomial (ZIB) regression model. The ZIB distribution has been widely applied in diverse domains such as medical studies, public health, process monitoring, horticulture, dental research, and industrial manufacturing. Notable contributions using this distribution can be seen in the works of Noorossana et al. [11], Pourhoseingholi et al. [12], Bodromurti et al. [3], Benyue et al. [2], Southwell et al. [16], and Condini et al. [5]. In addition, several researchers, including Noorossana et al. [11], Yawsaeng and Mayureesawan [20], Rakitzis et al. [14], and Alevizakos and Koukouvinos [1] have studied different types of control charts using ZIB distribution.

Conventional sampling plan theory assumes a fixed proportion of nonconforming units in each lot, suggesting that the production process is stable. In practice, however, lots of product produced often display quality variations due to random fluctuations. Such variations are generally classified as within-lot or between-lot variations. When between-lot variation is greater than within-lot variation, the proportion of nonconforming units can fluctuate considerably. Under these circumstances, lot acceptance decisions must account for between-lot variation, making standard sampling approaches less effective. Bayesian sampling plans offer a better alternative by incorporating prior knowledge of process variation into the decision-making framework. The choice of prior distributions for the fraction of nonconforming items in a lot has been investigated by Calvin [4] and Hald [6]. Further studies include Vijayaraghavan et al. [18], who developed Bayesian SSPs using the gamma-Poisson distribution, and Rajagopal et al. [13], who applied the beta-binomial distribution for SSPs by attributes. Additionally, Kaviyarasu and Sivakumar ([9], [10]) proposed Bayesian SSPs under the gamma-ZIP distribution.

When the count data of nonconforming items shows an excess of zeros, the Zero-inflated Binomial (ZIB) distribution offers a more suitable basis for designing sampling plans. The objective of this study is to construct Bayesian Single Sampling Plans (BSSPs) by attributes, applying the ZIB distribution in combination with the Beta distribution as the prior. There is no research has specifically addressed the development of sampling plans using the ZIB distribution. This study contributes to the literature by proposing attribute-based Bayesian single sampling plans (BSSPs) derived from the ZIB distribution under cost optimization. The Bayesian Single Sampling Plan under ZIB distribution is discussed in Section 2. Section 3 presents the formulation of the optimal economic sampling plan. Section 4 provides a sensitivity analysis of the parameters. Section 5 concludes with a summary of the key findings.

II. Bayesian Single sampling plan under ZIB distribution

A single sampling plan (SSP) is a method used to determine the acceptance or rejection of a lot based on the evaluation of one sample. In an attribute-based SSP, three key parameters are specified: N, the total lot size; n, the number of items drawn for inspection; and c, the acceptance number, representing the maximum count of nonconforming units permitted in the sample.

The procedure for applying an attribute-based SSP can be outlined as follows (Stephens [17]):

Step 1: Randomly draw a sample of size n from the lot of size N.

Step 2: Examine each selected item according to the defined quality standards.

Step 3: Record the number of nonconforming units, denoted by x.

Step 4: Accept the lot if $x \leq c$; otherwise, reject it.

Let the random variable X denote the number of nonconforming units in a sample of size n, where $\bar{p}$ represents the average fraction of nonconforming units in a lot of size N. In cases where nonconforming occur mainly as excess zeros, the Zero-inflated Binomial (ZIB) distribution offers a more accurate model. This distribution is especially effective when the number of zeros observed is considerably higher than what would be expected under a standard binomial model. The probability mass function of the ZIB distribution was introduced by Hall [7] and is given as follows.

$$P(X = x|\varphi, n, p) = \begin{cases} \varphi + (1-\varphi)\,(1-p)^n \quad , & \text{when } x = 0 \\ (1-\varphi)\binom{n}{x}\, p^x\,(1-p)^{n-x}, & \text{when } x = 1,2,\ldots,n \end{cases} \tag{1}$$

The key assumption of a stable production process, required for applying the binomial distribution in sampling analysis, does not always hold in real-world scenarios. In such cases, the process average $\bar{p}$ may fluctuate and is better represented as a random variable, denoted by $\bar{P}$. To capture this variation, appropriate probability models must be constructed using historical process data. Since the beta distribution serves as the natural conjugate prior for a Bernoulli population, it is assumed that $\bar{P}$ follows a beta distribution. The probability density function of the beta prior for $\bar{P}$ is expressed as:

$$f_{\bar{P}|\{(s,t)\}}(\bar{p}) = \frac{\Gamma(s+t)}{\Gamma(s)\,\Gamma(t)}(\bar{p})^{s-1}(1-\bar{p})^{t-1}, \qquad 0 \leq \bar{p} \leq 1\,;\, s,t > 0 \tag{2}$$

The probability mass function of the Beta ZIB distribution is derived as follows:

$$P(X = x|\varphi, n, s, t\,) = \begin{cases} \varphi + (1-\varphi)\,\frac{(s+t-1)!(n+t-1)!}{(t-1)!(s+n+t-1)!} \quad , & \text{when } x = 0 \\ (1-\varphi)\binom{n}{x}\,\frac{(s+t-1)!(s+x-1)!(n+t-x-1)!}{(s-1)!(t-1)!(s+n+t-1)!}, & \text{when } x = 1,2,\ldots,n \end{cases} \tag{3}$$

where $t = \frac{s(1-p)}{p}$

The OC function of Beta ZIB SSP is given by,

$$P_a(p) = \sum_{x=0}^{c} P(X = x|\varphi, n, s, t\,) \tag{4}$$

According to Stephens [17], the performance of an SSP can be evaluated using specific metrics. In this context, ATI refers to the Average Total Inspection, while AOQ represents the Average Outgoing Quality.

$$ATI = n + \big(1 - P_a(p)\big)(N-n) \tag{5}$$

$$AOQ = \frac{pP_a(p)(N-n)}{N} \tag{6}$$

Let D_d and D_n be the nonconforming items detected and not detected respectively, (Hsu and Hsu [8]), then

$$D_d = np + \big(1 - P_a(p)\big)(N-n)p \tag{7}$$
$$D_n = pP_a(p)(N-n) \tag{8}$$

III. Designing of optimum economic sampling plan

An economic model is formulated by optimizing the sampling plan to reduce overall quality costs for both producers and consumers, while satisfying their respective quality and risk constraints. The sampling plan is determined using the mathematical model proposed by Hsu and Hsu [8], which is expressed as follows:

$$Minimize \quad TC = C_i \,.ATI + C_f \,.D_d + C_o \,.D_n \tag{9}$$

$$\text{subject to } 1 - P_a(AQL) \leq \alpha \tag{10}$$

$$P_a(LTPD) \leq \beta \tag{11}$$

In this model, C_i, C_f, and C_o correspond to the inspection cost per item, the cost due to internal failure for a nonconforming unit, and the cost of nonconforming units reaching the consumer.

The following input parameters should be considered: N= 1000, φ = 0.01, AQL = 0.01, p = 0.03, α = 0.05, β = 0.10, C_i = 1, C_f = 2 and C_o= 10, the optimum Beta ZIB SSPs for various values of LTPD are obtained using R software and presented in Table 1.

Table 1: *Optimum Beta ZIB SSPs satisfying AQL = 0.01, p = 0.03, α = 0.05, β = 0.10, φ = 0.01, s = 50, C_i = 1, C_f = 2 and C_o= 10*

LTPD	TC	*n*	*c*	AOQ	ATI
0.10	418.43	123	7	0.0253	155.83
0.15	358.59	64	5	0.0277	77.10
0.20	337.13	41	4	0.0285	48.85
0.25	326.38	32	4	0.0290	34.71
0.30	319.79	22	3	0.0292	26.03

I. Illustration

For the set of input parameters N = 1000, φ = 0.01, AQL = 0.01, LTPD = 0.30, p = 0.03, α = 0.05, β = 0.10, C_i = 1, C_f = 2 and C_o=10, the optimal Beta ZIB SSP is n = 22 and c = 3, with a total cost (TC) of 319.79.

IV. Sensitivity analysis of parameters

Table 2 shows that as s increases from 10 to 50, total cost decreases while n and c fluctuate slightly. This means larger s values yield more cost-efficient sampling plans without notably affecting AOQ or ATI, highlighting the benefit of optimizing s.

Table 2: *Optimum Beta ZIB SSPs for AQL=0.01, LTPD = 0.07, p = 0.03, φ = 0.01, α = 0.05, β = 0.10, C_i = 1, C_f = 2 and C_o= 10*

s	TC	*n*	*c*	AOQ	ATI
10	650.98	211	7	0.0161	461.82
20	582.54	210	8	0.0188	371.77
30	558.11	202	8	0.0198	339.62
40	545.22	217	9	0.0203	322.65
50	536.75	214	9	0.0207	311.52

Table 3 shows that as φ increases from 0.01 to 0.09, both sample size and total cost rise, AOQ decreases, and ATI increases. Thus, higher φ improves outgoing quality but at the expense of greater inspection cost.

Table 3: *Optimum Beta ZIB SSPs for AQL = 0.01, LTPD = 0.07, p = 0.03, α = 0.05, β = 0.10, s = 50, C_i = 1, C_f = 2 and C_o = 10*

φ	TC	n	c	AOQ	ATI
0.01	536.75	214	9	0.0207	311.52
0.03	555.02	223	9	0.0199	335.55
0.05	576.99	253	10	0.0191	364.46
0.07	613.28	271	10	0.0176	412.21
0.09	690.10	330	11	0.0146	513.29

Table 4 shows that as p rises from 0.01 to 0.11, total cost and ATI increase, reflecting higher inspection effort and cost. Sample size first grows then drops with a reduced *c*, while AOQ decreases, indicating improved outgoing quality but at greater expense.

Table 4: *Optimum Beta ZIB SSPs for AQL=0.01, LTPD = 0.07, α = 0.05, β = 0.10, φ = 0.01, s = 50, C_i = 1, C_f = 2 and C_o= 10*

p	TC	n	c	AOQ	ATI
0.01	208.04	100	3	0.0088	117.44
0.03	536.75	214	9	0.0207	311.52
0.05	909.67	269	12	0.0159	682.78
0.07	1099.82	79	2	0.0064	908.68
0.09	1169.84	79	2	0.0033	963.71
0.11	1217.89	79	2	0.0019	982.42

Table 5: *Optimum Beta ZIB SSPs for AQL= 0.01, LTPD = 0.07, p = 0.03, α = 0.05, β = 0.10, φ = 0.01, s = 50, C_f = 2 and C_o= 10*

C_i	TC	n	c	AOQ	ATI
0.2	260.00	1000	16	0.0000	1000.00
0.4	349.84	214	9	0.0207	311.52
0.6	412.15	214	9	0.0207	311.52
0.8	474.45	214	9	0.0207	311.52
1.0	536.75	214	9	0.0207	311.52

Table 5 demonstrates the effect of varying C_i on the optimum Beta ZIB SSP. At a very low C_i = 0.2, the plan recommends inspecting all items (n = 1000), achieving zero outgoing nonconforming units but at maximum inspection effort. As C_i increases from 0.4 to 1.0, the plan stabilizes at n = 214 and c = 9, with constant AOQ and ATI, while the total cost rises steadily.

Tables 6 and 7 show the effect of varying C_f with C_i = 1.0 and C_i = 0.2, respectively. In Table 6, changes in C_f only increase the total cost without affecting the sampling plan. In Table 7, when C_i is low (0.2), the internal failure cost (C_f) influences the plan, shifting from full inspection at low C_f to partial sampling at higher C_f.

Table 6: *Optimum Beta ZIB SSPs for AQL=0.01, LTPD = 0.07, p= 0.03, α = 0.05, β = 0.10, φ = 0.01, s = 50, C_i = 1 and C_o= 10*

C_f	TC	n	c	AOQ	ATI
0	518.06	214	9	0.0207	311.52
1	527.41	214	9	0.0207	311.52
2	536.75	214	9	0.0207	311.52
3	546.10	214	9	0.0207	311.52
4	555.44	214	9	0.0207	311.52
5	564.79	214	9	0.0207	311.52

Table 7: *Optimum Beta ZIB SSPs satisfying AQL=0.01, LTPD = 0.07, p= 0.03, α = 0.05, β = 0.10, φ = 0.01, s = 50, C_i = 0.2 and C_o= 10*

C_f	TC	*n*	*c*	AOQ	ATI
0	200.00	1000	16	0.0000	1000.00
2	260.00	1000	16	0.0000	1000.00
4	306.23	214	9	0.0207	311.52
6	324.92	214	9	0.0207	311.52
8	343.61	214	9	0.0207	311.52
10	362.30	214	9	0.0207	311.52

Table 8: *Optimum Beta ZIB SSPs satisfying AQL=0.01, LTPD = 0.07, p= 0.03, α = 0.05, β = 0.10, φ = 0.01, s = 50, C_i = 1 and C_f= 2*

C_o	TC	*n*	*c*	AOQ	ATI
10	536.75	214	9	0.0207	311.52
20	743.30	214	9	0.0207	311.52
30	949.84	214	9	0.0207	311.52
40	1060.00	1000	16	0.0000	1000.00
50	1060.00	1000	16	0.0000	1000.00

Table 8 shows that for C_o values 10–30, plan parameters stay constant while total cost rises. At $C_o \geq 40$, the plan shifts to full inspection, sharply increasing effort to prevent nonconforming units from reaching customers.

V. Conclusion

This paper presents a mathematical model to determine Bayesian Single Sampling Plans (BSSPs) under the Beta ZIB distribution, providing a systematic framework for both producers and consumers. The approach reduces total costs while simultaneously satisfying risk and quality requirements for both producer and consumer. Numerical illustration is presented to demonstrate the selection of Beta ZIB SSPs that minimize the producer's overall cost. The results indicate that the optimum plan is highly dependent on the quality level of the producer's output. Additionally, inspection costs, internal failure costs, and external failure costs are shown to significantly affect the structure of the economic sampling plan. By applying such optimized Bayesian plans, organizations can achieve better cost efficiency, improved allocation of resources, and more reliable quality control. Ultimately, this method strengthens decision-making processes and contributes to higher quality standards and customer satisfaction.

References

[1] Alevizakos, V. and Koukouvinos, C. (2020). Monitoring of zero-inflated binomial processes with a dewma control chart. *Journal of Applied Statistics,* 48 (7): 1319 – 1338.

[2] Benyue, S., Pengpeng, X. and Min, S. (2020). Generalised zero inflated binomial distribution model aimed at air quality data analysis. *Journal of System Simulation,* 32 (11): 2226 – 2234.

[3] Bodromurti, W., Notodiputro, K. A. and Kurnia, A. (2018). Zero inflated binomial model for infant mortality data in Indonesia. *International Journal of Applied Engineering Research*, 13(6): 3139 – 3143.

[4] Calvin T. W. How and When to Perform Bayesian Acceptance Sampling, American Society for Quality Control, Wisconsin, 1990.

[5] Condini, M. V., Oliveira, R. L., Almeida, L. L., Barbosa, E. F., Martins, A. C., Bertoncini, A.

A. and Hostim-Silva, M. (2024). Physicochemical variables influencing the abundance of Atlantic Goliath Grouper Juveniles (Epinephelus Itajara) in a tropical Brazilian mangrove habitat. *Marine Environmental Research*, 197: 1 – 11.

[6] Hald, A. Statistical Theory of Sampling Inspection by Attributes, Academic Press, New York, 1981.

[7] Hall, D. B. (2000). Zero-inflated Poisson and binomial regression with random effects: A case study. *Biometrics,* 56: 1030 – 1039.

[8] Hsu, L.F. and Hsu, J.T. (2012). Economic design of acceptance sampling plans in a two-stage supply chain, *Advances in Decision Sciences*, Vol. 2012, pp. 1-14.

[9] Kaviyarasu, V. and Sivakumar, P. (2019). The implication of production and monitoring techniques in Bayesian single sampling plan using gamma-zero inflated Poisson distribution. *International Journal of Recent Technology and Engineering*. 8(4): 10110 – 10119.

[10] Kaviyarasu, V. and Sivakumar, P. (2021). Optimization of bayesian single sampling plan for the zero inflated poisson distribution involving risk minimization using tangent angle method. *International Journal of Scientific Research in Mathematical and Statistical Sciences*, 8(3): 1 – 11.

[11] Noorossana, R., Fatahi, A. A., Dokouhaki, P. and Babakhani, M. (2011). ZIB–EWMA Control chart for monitoring rare health events. *Journal of Mechanics in Medicine and Biology,* 11 (4): 881 – 895.

[12] Pourhoseingholi, A., Alireza, A. B., Ahmad, R. B., Erfan, G. and Seyed, M. R. (2018). Zero-inflated binomial model for prognosis of the risk factor associated with dmft index in children aged 5-6 years in Tehran. *Iranian Journal of Pediatrics,* 28 (3): 1 – 7.

[13] Rajagopal, K., Loganathan A. and Vijayaraghavan, R., (2009). Selection of bayesian single sampling attributes plans based on polya distribution. *Economic Quality Control,* 24 (2): 179 – 193.

[14] Rakitzis, A. C., Maravelakis, P. E. and Castagliola, P. (2016). CUSUM control charts for the monitoring of zero-inflated binomial processes. *Quality and Reliability Engineering International*, 32: 465 – 483.

[15] Schilling, E. G. and Neubauer, D. V. Acceptance Sampling in Quality Control, CRC Press, Boca Raton, 2009.

[16] Southwell, D. M., Smart, A., Merson, S. D., Selwood, K. E. and Macgregor, N. A. (2024). Using power analysis and spatial prioritization to evaluate the design of a forest bird monitoring programme. *Oryx*, 58(4): 522 – 531.

[17] Stephens, K. S. The Handbook of Applied Acceptance Sampling Plans, Procedures and Principles, ASQ Quality Press, Wisconsin, 2001.

[18] Vijayaraghavan, R., Rajagopal, K. and Loganathan, A. (2008). A procedure for selection of a gamma- poisson single sampling plan by attributes. *Journal of Applied Statistics,* 35 (2): 149 – 160.

[19] Wimonmas, B. and Pramote, C. (2021). Designing of double acceptance sampling plan for zeroinflated and over-dispersed data using multi-objective optimization, *Applied Science and Engineering Progress*, Vol. 14, No. 3, pp. 338-347.

[20] Yawsaeng, B. and Mayureesawan, T. (2012). Control charts for zero-inflated binomial models. *Thailand Statistician*, 10 (1): 107 – 120.