TWO COMPONENT MIXTURE OF LEFT TRUNCATED THREE PARAMETER LOGISTIC TYPE DISTRIBUTION WITH APPLICATIONS TO MANPOWER MODELING

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Abstract

The logistic distribution serves as an alternative to normal distribution due to its is similarity in shape with heavier tails. Several extensions and generalizations of the logistic distribution have been proposed and explored in research papers published from 2020 onwards, particularly for modeling data with varying characteristics like skewness and kurtosis, or for specific applications in areas like finance and survival analysis. Some important distributions include the skew logistic distribution, the extended log-logistic distribution, two parameter logistic type distribution, three parameter logistic type distribution and the beta log-logistic distribution. These extensions offer increased flexibility in fitting data compared to the standard logistic distribution. Recently Usha et al (2025) have studied truncated three parameter Logistic distribution. Based on this distribution, we propose a new model called the two component mixture of left truncated tree parameter logistics type distribution for describing manpower data. It involves various specialized models, such as mixture of truncated logistic and a mixture of two-parameter logistic type distributions, among several others, so it's possibly better model for analyzing skewed data. The new density function can be expressed as a weighted combination of truncated tree parameter logistic-type densities. Various mathematical properties of the new distribution including explicit expressions for the moments are derived. We discuss the maximum likelihood method to estimate the model parameters for heterogeneous data. The superiority of the proposed manpower model is illustrated.

Keywords: Three parameter logistic type distribution, Mixture of distributions, Distributional properties, Manpower modeling, Recruitment policies.

I. Introduction

Probability distributions play a vital role in analyzing the random phenomenon through data sets. One such probability distribution is Logistic distribution which was used by several authors to analyse various data sets in different areas of applications. Extensions of logistic distribution are studied by many researcher to develop various models that suits the data sets. Dubey [1] has derived logistic distribution as a special case of compound generalized extreme value distribution by compounding generalized extreme value distribution with gamma distribution. Olapade [2]

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discussed characterizations of the half logistic distribution. He proved theorems that characterize the half logistic distribution. Nassar et al. [3] studied the beta generalized logistic distribution. Gupta et al. [4] discussed the properties of the two generalizations of the logistic distributions, which can be used to model the data exhibiting unimodal density with some skewness. Alzaatreh et al. [5] developed a method for generating families of continuous distributions. Adeyinka et al. [6] studied the four parameters generalized log-logistic distribution using quadratic rank transmutation. Aljarrah et al. [7] considered a generalized asymmetric logistic distribution. Satheesh Kumar et al. [8] studied a range of generalized logistic distribution (GGLD) with asymmetric datasets. They discussed about some of the distribution characteristics. Alizadeh [9] developed a new goodness of fit test for the logistic distribution based on a new estimate of kullback-Leibler information. The properties of the test statistic are presented.

Usha H et al. [10] developed and analyzed a left truncated three parameter logistic type distribution. They have studied the properties of singly and doubly truncated three-parameter logistic-type distribution. These distributions are useful only when the population under consideration is homogenous. In many practical situations, the population under consideration is heterogeneous, arising from various sources of generation. For example, manpower modeling the complete length of service of employee in an organization should be considered as a mixture of two types of employee namely: committed and non-committed, each having different distribution. To analyze such situations, one needs to consider the random variable under study as a two-component mixture of distributions. The complete length of employee in an organization has finite values. In this chapter, we develop the two-component mixture of left-truncated three-parameter logistic type distribution.

The properties of the proposed distribution, such as mean, distribution function, moments, skewness, kurtosis, survival function, and hazard function, are derived. The inferential aspects of the parameters are examined through the likelihood function, and the distribution utility in analyzing the manpower situation is also evaluated. Two promotion policies, namely promotion by seniority and promotion by random, are also studied.

II. Two Component Mixture of Left Truncated Three Parameter Logistic Type Distribution

A continuous random variable X is said to have a two component mixture of left truncated three parameter logistic type distribution if its p.d.f is expressed in the following form:

$$f(x) = p\, f_1(x) + (1-p) f_2(x) \qquad 0 < p < 1$$

Hence the probability density function of two component mixture of left truncated three parameter logistic type distribution

$$\begin{aligned} f(x;\mu_1,\mu_2,\sigma_1,\sigma_2,s_1,s_2,p) &= p\left[C_1\left[s_1+\left(\frac{x-\mu_1}{\sigma_1}\right)^2\right]\frac{e^{-\left(\frac{x-\mu_1}{\sigma_1}\right)}}{\sigma_1\left[1+e^{-\left(\frac{x-\mu_1}{\sigma_1}\right)}\right]^2}\right] \\ &+ (1-p)\left[C_2\left[s_2+\left(\frac{x-\mu_2}{\sigma_2}\right)^2\right]\frac{e^{-\left(\frac{x-\mu_2}{\sigma_2}\right)}}{\sigma_2\left[1+e^{-\left(\frac{x-\mu_2}{\sigma_2}\right)}\right]^2}\right] \end{aligned} \tag{1}$$

$$0 < x < \infty;\ \ \mu_1,\mu_2 > 0;\ \ \sigma_1,\sigma_2 > 0;\ \ s_1,s_2 > 0;\ 0 < p < 1.$$

This implies,

$$C_1 = \left[\left(s_1 + \frac{\mu_1^2}{\sigma_1^2}\right)\frac{e^{\mu_1/\sigma_1}}{[1+e^{\mu_1/\sigma_1}]} - \frac{2\mu_1}{\sigma_1}\log\left(1+e^{\mu_1/\sigma_1}\right) - 2Li_2\left(-e^{\mu_1/\sigma_1}\right)\right]^{-1}$$

$$C_2 = \left[\left(s_2 + \frac{\mu_2^2}{\sigma_2^2}\right)\frac{e^{\mu_2/\sigma_2}}{[1+e^{\mu_2/\sigma_2}]} - \frac{2\mu_2}{\sigma_2}\log\left(1+e^{\mu_2/\sigma_2}\right) - 2Li_2\left(-e^{\mu_2/\sigma_2}\right)\right]^{-1}$$

where, $Li_2(\cdot)$ is Dilogarithm Function

Here, p is the mixing parameter, μ_1, μ_2 are the location parameters, σ_1, σ_2 are the scale parameters and s_1, s_2 are the drift parameters.

Making transformation $y_1 = \frac{x-\mu_1}{\sigma_1}$ *and* $y_2 = \frac{x-\mu_2}{\sigma_2}$ in the Eq. (1), we get

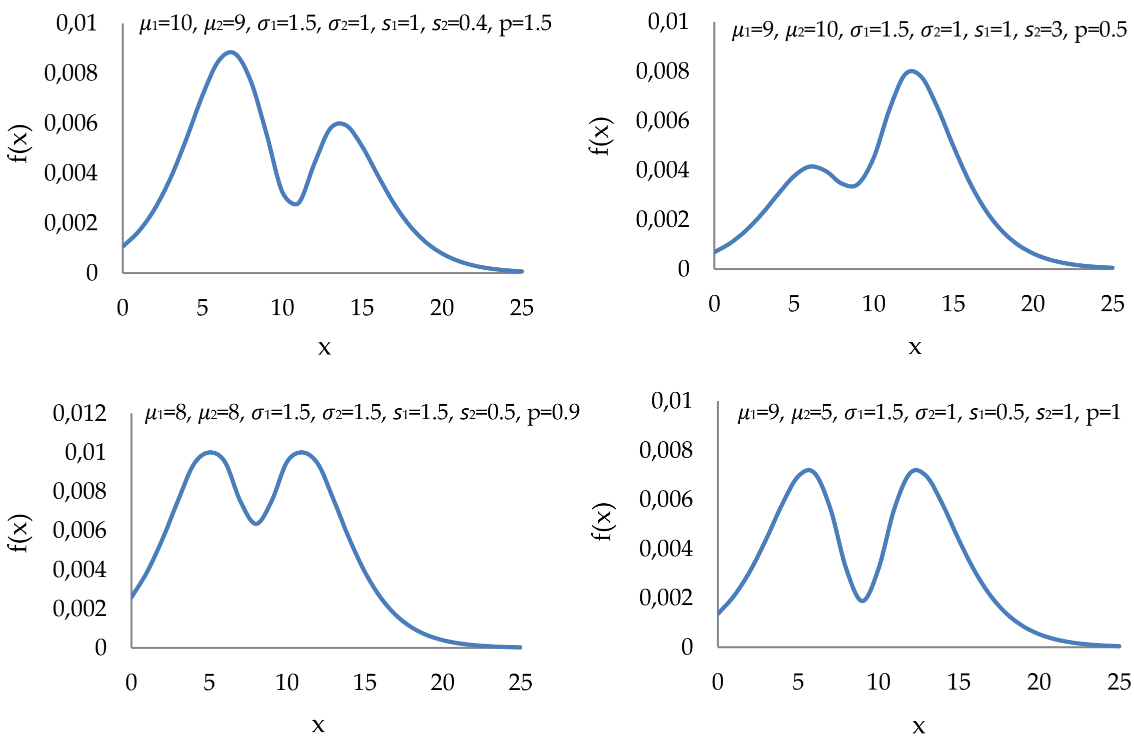
$$f(x;s) = p\left[C_1[s_1 + y_1^2]\frac{e^{-y_1}}{[1+e^{-y_1}]^2}\right] + (1-p)\left[C_2[s_2 + y_2^2]\frac{e^{-y_2}}{[1+e^{-y_2}]^2}\right] \quad (2)$$

Which may be called the standard two-component mixture of left truncated three parameter logistic type distribution.

III. Distributional Properties

This section presents the different distributional properties of the two component mixture of left truncated three parameter logistic type distribution.

Different shapes of the frequency curves for given values of the parameters are shown in Figure 1



Figures 1 *Two component mixture of left truncated three parameter logistic type distribution frequency curves for different values of parameters* μ_1, μ_2, σ_1, σ_2, s_1, s_2 *and p.*

The distribution function of the random variable X is

$$F_X(x) = \int_0^x f(t)dt$$

After simplifying, we get

$$F(x) = pC_1\left[\left(s_1+\frac{\mu_1^2}{\sigma_1^2}\right)\frac{e^{\mu_1/\sigma_1}}{[1+e^{\mu_1/\sigma_1}]}-\frac{2\mu_1}{\sigma_1}\log\left(1+e^{\mu_1/\sigma_1}\right)-2Li_2\left(-e^{\mu_1/\sigma_1}\right) - \frac{\left[s_1+\left(\frac{x-\mu_1}{\sigma_1}\right)^2\right]e^{-\left(\frac{x-\mu_1}{\sigma_1}\right)}}{\left[1+e^{-\left(\frac{x-\mu_1}{\sigma_1}\right)}\right]} - 2\left(\frac{x-\mu_1}{\sigma_1}\right)\log\left[1+e^{-\left(\frac{x-\mu_1}{\sigma_1}\right)}\right] + 2Li_2\left(-e^{-\left(\frac{x-\mu_1}{\sigma_1}\right)}\right)\right] + (1-p)C_2\left[\left(s_2+\frac{\mu_2^2}{\sigma_2^2}\right)\frac{e^{\mu_2/\sigma_2}}{[1+e^{\mu_2/\sigma_2}]}-\frac{2\mu_2}{\sigma_2}\log\left(1+e^{\mu_2/\sigma_2}\right)-2Li_2\left(-e^{\mu_2/\sigma_2}\right) - \frac{\left[s_2+\left(\frac{x-\mu_2}{\sigma_2}\right)^2\right]e^{-\left(\frac{x-\mu_2}{\sigma_2}\right)}}{\left[1+e^{-\left(\frac{x-\mu_2}{\sigma_2}\right)}\right]} - 2\left(\frac{x-\mu_2}{\sigma_2}\right)\log\left[1+e^{-\left(\frac{x-\mu_2}{\sigma_2}\right)}\right] + 2Li_2\left(-e^{-\left(\frac{x-\mu_2}{\sigma_2}\right)}\right)\right] \tag{3}$$

where, C_1 and C_2 are as specified in Eq. (1) and $Li_2(\cdot)$ is Dilogarithm function

Mean of the distribution is

$$E(X) = \int_0^\infty x\, f(x)dx$$

After simplifying, we get

$$E(X) = pC_1\left[\left(\frac{\mu_1^2+s_1\sigma_1^2}{\sigma_1}\right)\log\left(1+e^{\mu_1/\sigma_1}\right)+4\mu_1 Li_2\left(-e^{\mu_1/\sigma_1}\right)-6\sigma_1 Li_3\left(-e^{\mu_1/\sigma_1}\right)\right] + (1-p)C_2\left[\left(\frac{\mu_2^2+s_2\sigma_2^2}{\sigma_2}\right)\log\left(1+e^{\mu_2/\sigma_2}\right)+4\mu_2 Li_2\left(-e^{\mu_2/\sigma_2}\right) - 6\sigma_2 Li_3\left(-e^{\mu_2/\sigma_2}\right)\right] \tag{4}$$

where, C_1 and C_2 are as specified in Eq. (1) and $Li_n(\cdot)$ is Polylogarithm function of order n (n ≥ 2)

Second raw moment of the distribution is

$$\mu_2' = \int_0^\infty x^2\, f(x)\, dx$$

After simplifying, we get

$$\mu_2' = p\left[2C_1\left[-(\mu_1^2+s_1\sigma_1^2)Li_2\left(-e^{\mu_1/\sigma_1}\right)+6\mu_1\sigma_1 Li_3\left(-e^{\mu_1/\sigma_1}\right)-12\sigma_1^2 Li_4\left(-e^{\mu_1/\sigma_1}\right)\right]\right] + (1-p)\left[2C_2\left[-(\mu_2^2+s_2\sigma_2^2)Li_2\left(-e^{\mu_2/\sigma_2}\right)+6\mu_2\sigma_2 Li_3\left(-e^{\mu_2/\sigma_2}\right) - 12\sigma_2^2 Li_4\left(-e^{\mu_2/\sigma_2}\right)\right]\right] \tag{5}$$

where, C_1 and C_2 are as specified in Eq. (1) and $Li_n(\cdot)$ is Polylogarithm function of order n (n ≥ 2)
Third raw moment of the distribution is

$$\mu_3' = \int_0^\infty x^3\, f(x)\, dx$$

After simplifying, we get

$$\mu_3' = p\left[6C_1\sigma_1\left[-(\mu_1^2+s_1\sigma_1^2)Li_3\left(-e^{\mu_1/\sigma_1}\right)+8\mu_1\sigma_1 Li_4\left(-e^{\mu_1/\sigma_1}\right)-20\sigma_1^2 Li_5\left(-e^{\mu_1/\sigma_1}\right)\right]\right] + (1-p)\left[6C_2\sigma_2\left[-(\mu_2^2+s_2\sigma_2^2)Li_3\left(-e^{\mu_2/\sigma_2}\right)+8\mu_2\sigma_2 Li_4\left(-e^{\mu_2/\sigma_2}\right) - 20\sigma_2^2 Li_5\left(-e^{\mu_2/\sigma_2}\right)\right]\right] \tag{6}$$

where, C_1 and C_2 are as specified in Eq. (1) and $\mathrm{Li}_n(\cdot)$ is Polylogarithm function of order n (n ≥ 2)

Fourth raw moment of the distribution is

$$\mu_4' = \int_0^\infty x^4 f(x)\,dx$$

After simplifying, we get

$$\mu_4' = p\left[24C_1\sigma_1^2\left[-(\mu_1^2+s_1\sigma_1^2)Li_4\left(-e^{\mu_1/\sigma_1}\right)+5\sigma_1(6-4\mu_1)Li_5\left(-e^{\mu_1/\sigma_1}\right) - 30\sigma_1^2 Li_6\left(-e^{\mu_1/\sigma_1}\right)\right]\right] + (1-p)\left[24C_2\sigma_2^2\left[-(\mu_2^2+s_2\sigma_2^2)Li_4\left(-e^{\mu_2/\sigma_2}\right)+5\sigma_2(6-4\mu_2)Li_5\left(-e^{\mu_2/\sigma_2}\right) - 30\sigma_2^2 Li_6\left(-e^{\mu_2/\sigma_2}\right)\right]\right] \tag{7}$$

where, C_1 and C_2 are as specified in Eq. (1) and $\mathrm{Li}_n(\cdot)$ is Polylogarithm function of order n (n ≥ 2)

The Second central moment of the distribution is

$$\mu_2 = p\left[2C_1\left[-(\mu_1^2+s_1\sigma_1^2)Li_2\left(-e^{\mu_1/\sigma_1}\right)+6\mu_1\sigma_1 Li_3\left(-e^{\mu_1/\sigma_1}\right)-12\sigma_1^2 Li_4\left(-e^{\mu_1/\sigma_1}\right)\right]-C_1^2K_1^2\right] + (1-p)\left[2C_2\left[-(\mu_2^2+s_2\sigma_2^2)Li_2\left(-e^{\mu_2/\sigma_2}\right)+6\mu_2\sigma_2 Li_3\left(-e^{\mu_2/\sigma_2}\right) - 12\sigma_2^2 Li_4\left(-e^{\mu_2/\sigma_2}\right)\right]-C_2^2K_2^2\right] \tag{8}$$

where, C_1 and C_2 are as specified in Eq. (1)

$$K_1 = p\left[\left(\frac{\mu_1^2+s_1\sigma_1^2}{\sigma_1}\right)\log\left(1+e^{\mu_1/\sigma_1}\right)+4\mu_1 Li_2\left(-e^{\mu_1/\sigma_1}\right)-6\sigma_1 Li_3\left(-e^{\mu_1/\sigma_1}\right)\right]$$

$$K_2 = (1-p)\left[\left(\frac{\mu_2^2+s_2\sigma_2^2}{\sigma_2}\right)\log\left(1+e^{\mu_2/\sigma_2}\right)+4\mu_2 Li_2\left(-e^{\mu_2/\sigma_2}\right)-6\sigma_2 Li_3\left(-e^{\mu_2/\sigma_2}\right)\right]$$

and $\mathrm{Li}_n(\cdot)$ is Polylogarithm function of order n (n ≥ 2)
Third central moment of the distribution is

$$\begin{aligned}\mu_3 = p\big[&6C_1\sigma_1\big[-(\mu_1^2+s_1\sigma_1^2)Li_3\big(-e^{\mu_1/\sigma_1}\big)+8\mu_1\sigma_1 Li_4\big(-e^{\mu_1/\sigma_1}\big)-20\sigma_1^2 Li_5\big(-e^{\mu_1/\sigma_1}\big)\big]\\ &-6C_1^2K_1\big[-(\mu_1^2+s_1\sigma_1^2)Li_2\big(-e^{\mu_1/\sigma_1}\big)+6\mu_1\sigma_1 Li_3\big(-e^{\mu_1/\sigma_1}\big)\\ &-12\sigma_1^2 Li_4\big(-e^{\mu_1/\sigma_1}\big)\big]+2C_1^3K_1^3\big]\\ &+(1\\ &-p)\big[6C_2\sigma_2\big[-(\mu_2^2+s_2\sigma_2^2)Li_3\big(-e^{\mu_2/\sigma_2}\big)+8\mu_2\sigma_2 Li_4\big(-e^{\mu_2/\sigma_2}\big)\\ &-20\sigma_2^2 Li_5\big(-e^{\mu_2/\sigma_2}\big)\big]\\ &-6C_2^2K_2\big[-(\mu_2^2+s_2\sigma_2^2)Li_2\big(-e^{\mu_2/\sigma_2}\big)+6\mu_2\sigma_2 Li_3\big(-e^{\mu_2/\sigma_2}\big)\\ &-12\sigma_2^2 Li_4\big(-e^{\mu_2/\sigma_2}\big)\big]+2C_2^3K_2^3\big]\end{aligned} \tag{9}$$

where, C_1 and C_2 are as specified in Eq. (1)

$$K_1 = p\left[\left(\frac{\mu_1^2+s_1\sigma_1^2}{\sigma_1}\right)\log\big(1+e^{\mu_1/\sigma_1}\big)+4\mu_1 Li_2\big(-e^{\mu_1/\sigma_1}\big)-6\sigma_1 Li_3\big(-e^{\mu_1/\sigma_1}\big)\right]$$

$$K_2 = (1-p)\left[\left(\frac{\mu_2^2+s_2\sigma_2^2}{\sigma_2}\right)\log\big(1+e^{\mu_2/\sigma_2}\big)+4\mu_2 Li_2\big(-e^{\mu_2/\sigma_2}\big)-6\sigma_2 Li_3\big(-e^{\mu_2/\sigma_2}\big)\right]$$

and $Li_n(\cdot)$ is Polylogarithm function of order n ($n \geq 2$)

Fourth central moment of the distribution is

$$\begin{aligned}\mu_4 = p\big[&24C_1\sigma_1^2\big[-(\mu_1^2+s_1\sigma_1^2)Li_4\big(-e^{\mu_1/\sigma_1}\big)+5\sigma_1(6-4\mu_1)Li_5\big(-e^{\mu_1/\sigma_1}\big)\\ &-30\sigma_1^2 Li_6\big(-e^{\mu_1/\sigma_1}\big)\big]\\ &-24C_1^2K_1\sigma_1\big[-(\mu_1^2+s_1\sigma_1^2)Li_3\big(-e^{\mu_1/\sigma_1}\big)+8\mu_1\sigma_1 Li_4\big(-e^{\mu_1/\sigma_1}\big)\\ &-20\sigma_1^2 Li_5\big(-e^{\mu_1/\sigma_1}\big)\big]\\ &-12C_1^3K_1^2\big[-(\mu_1^2+s_1\sigma_1^2)Li_2\big(-e^{\mu_1/\sigma_1}\big)+6\mu_1\sigma_1 Li_3\big(-e^{\mu_1/\sigma_1}\big)\\ &-12\sigma_1^2 Li_4\big(-e^{\mu_1/\sigma_1}\big)\big]-3C_1^4K_1^4\big]\\ &+(1\\ &-p)\big[24C_2\sigma_2^2\big[-(\mu_2^2+s_2\sigma_2^2)Li_4\big(-e^{\mu_2/\sigma_2}\big)+5\sigma_2(6-4\mu_2)Li_5\big(-e^{\mu_2/\sigma_2}\big)\\ &-30\sigma_2^2 Li_6\big(-e^{\mu_2/\sigma_2}\big)\big]\\ &-24C_2^2K_2\sigma_2\big[-(\mu_2^2+s_2\sigma_2^2)Li_3\big(-e^{\mu_2/\sigma_2}\big)+8\mu_2\sigma_2 Li_4\big(-e^{\mu_2/\sigma_2}\big)\\ &-20\sigma_2^2 Li_5\big(-e^{\mu_2/\sigma_2}\big)\big]\\ &-12C_2^3K_2^2\big[-(\mu_2^2+s_2\sigma_2^2)Li_2\big(-e^{\mu_2/\sigma_2}\big)+6\mu_2\sigma_2 Li_3\big(-e^{\mu_2/\sigma_2}\big)\\ &-12\sigma_2^2 Li_4\big(-e^{\mu_2/\sigma_2}\big)\big]-3C_2^4K_2^4\big]\end{aligned} \tag{10}$$

where, C_1 and C_2 are as specified in Eq. (1)

$$K_1 = p\left[\left(\frac{\mu_1^2+s_1\sigma_1^2}{\sigma_1}\right)\log\big(1+e^{\mu_1/\sigma_1}\big)+4\mu_1 Li_2\big(-e^{\mu_1/\sigma_1}\big)-6\sigma_1 Li_3\big(-e^{\mu_1/\sigma_1}\big)\right]$$

$$K_2 = (1-p)\left[\left(\frac{\mu_2^2+s_2\sigma_2^2}{\sigma_2}\right)\log\big(1+e^{\mu_2/\sigma_2}\big)+4\mu_2 Li_2\big(-e^{\mu_2/\sigma_2}\big)-6\sigma_2 Li_3\big(-e^{\mu_2/\sigma_2}\big)\right]$$

and $Li_n(\cdot)$ is Polylogarithm function of order n ($n \geq 2$)

The skewness of the distribution is

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$\beta_1 = \frac{\begin{array}{c}[p[6C_1\sigma_1[-(\mu_1^2+s_1\sigma_1^2)Li_3(-e^{\mu_1/\sigma_1})+8\mu_1\sigma_1 Li_4(-e^{\mu_1/\sigma_1})-20\sigma_1^2 Li_5(-e^{\mu_1/\sigma_1})] \\ -6C_1^2K_1[-(\mu_1^2+s_1\sigma_1^2)Li_2(-e^{\mu_1/\sigma_1})+6\mu_1\sigma_1 Li_3(-e^{\mu_1/\sigma_1})-12\sigma_1^2 Li_4(-e^{\mu_1/\sigma_1})] \\ +2C_1^3K_1^3]+(1-p)[6C_2\sigma_2[-(\mu_2^2+s_2\sigma_2^2)Li_3(-e^{\mu_2/\sigma_2})+8\mu_2\sigma_2 Li_4(-e^{\mu_2/\sigma_2})-20 \\ \sigma_2^2 Li_5(-e^{\mu_2/\sigma_2})]-6C_2^2K_2[-(\mu_2^2+s_2\sigma_2^2)Li_2(-e^{\mu_2/\sigma_2})+6\mu_2\sigma_2 Li_3(-e^{\mu_2/\sigma_2})- \\ 12\sigma_2^2 Li_4(-e^{\mu_2/\sigma_2})]+2C_2^3K_2^3]^2\end{array}}{\begin{array}{c}[p[2C_1[-(\mu_1^2+s_1\sigma_1^2)Li_2(-e^{\mu_1/\sigma_1})+6\mu_1\sigma_1 Li_3(-e^{\mu_1/\sigma_1})-12\sigma_1^2 Li_4(-e^{\mu_1/\sigma_1})] \\ -C_1^2K_1^2]+(1-p)[2C_2[-(\mu_2^2+s_2\sigma_2^2)Li_2(-e^{\mu_2/\sigma_2})+6\mu_2\sigma_2 Li_3(-e^{\mu_2/\sigma_2}) \\ -12\sigma_2^2 Li_4(-e^{\mu_2/\sigma_2})]-C_2^2K_2^2]]^3\end{array}} \tag{11}$$

where, C_1 and C_2 are as specified in Eq. (1)

$$K_1 = p\left[\left(\frac{\mu_1^2+s_1\sigma_1^2}{\sigma_1}\right)\log(1+e^{\mu_1/\sigma_1})+4\mu_1 Li_2(-e^{\mu_1/\sigma_1})-6\sigma_1 Li_3(-e^{\mu_1/\sigma_1})\right]$$

$$K_2 = (1-p)\left[\left(\frac{\mu_2^2+s_2\sigma_2^2}{\sigma_2}\right)\log(1+e^{\mu_2/\sigma_2})+4\mu_2 Li_2(-e^{\mu_2/\sigma_2})-6\sigma_2 Li_3(-e^{\mu_2/\sigma_2})\right]$$

and $Li_n(\cdot)$ is Polylogarithm function of order n ($n \geq 2$)

The kurtosis of the distribution is

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\beta_2 = \frac{\begin{array}{c}p[24C_1\sigma_1^2[-(\mu_1^2+s_1\sigma_1^2)Li_4(-e^{\mu_1/\sigma_1})+5\sigma_1(6-4\mu_1)Li_5(-e^{\mu_1/\sigma_1})-30\sigma_1^2 Li_6 \\ (-e^{\mu_1/\sigma_1})]-24C_1^2K_1\sigma_1[-(\mu_1^2+s_1\sigma_1^2)Li_3(-e^{\mu_1/\sigma_1})+8\mu_1\sigma_1 Li_4(-e^{\mu_1/\sigma_1})-20 \\ \sigma_1^2 Li_5(-e^{\mu_1/\sigma_1})]-12C_1^3K_1^2[-(\mu_1^2+s_1\sigma_1^2)Li_2(-e^{\mu_1/\sigma_1})+6\mu_1\sigma_1 Li_3(-e^{\mu_1/\sigma_1}) \\ -12\sigma_1^2 Li_4(-e^{\mu_1/\sigma_1})]]-3C_1^4K_1^4]+(1-p)[24C_2\sigma_2^2[-(\mu_2^2+s_2\sigma_2^2)Li_4(-e^{\mu_2/\sigma_2}) \\ +5\sigma_2(6-4\mu_2)Li_5(-e^{\mu_2/\sigma_2})-30\sigma_2^2 Li_6(-e^{\mu_2/\sigma_2})]-24C_2^2K_2\sigma_2[-(\mu_2^2+s_2\sigma_2^2) \\ Li_3(-e^{\mu_2/\sigma_2})+8\mu_2\sigma_2 Li_4(-e^{\mu_2/\sigma_2})-20\sigma_2^2 Li_5(-e^{\mu_2/\sigma_2})]-12C_2^3K_2^2[-(\mu_2^2+s_2\sigma_2^2) \\ Li_2(-e^{\mu_2/\sigma_2})+6\mu_2\sigma_2 Li_3(-e^{\mu_2/\sigma_2})-12\sigma_2^2 Li_4(-e^{\mu_2/\sigma_2})]-3C_2^4K_2^4]\end{array}}{\begin{array}{c}[p[2C_1[-(\mu_1^2+s_1\sigma_1^2)Li_2(-e^{\mu_1/\sigma_1})+6\mu_1\sigma_1 Li_3(-e^{\mu_1/\sigma_1})-12\sigma_1^2 Li_4(-e^{\mu_1/\sigma_1})] \\ -C_1^2K_1^2]+(1-p)[2C_2[-(\mu_2^2+s_2\sigma_2^2)Li_2(-e^{\mu_2/\sigma_2})+6\mu_2\sigma_2 Li_3(-e^{\mu_2/\sigma_2}) \\ -12\sigma_2^2 Li_4(-e^{\mu_2/\sigma_2})]-C_2^2K_2^2]]^2\end{array}} \tag{12}$$

where, C_1 and C_2 are as specified in Eq. (1)

$$K_1 = p\left[\left(\frac{\mu_1^2+s_1\sigma_1^2}{\sigma_1}\right)\log(1+e^{\mu_1/\sigma_1})+4\mu_1 Li_2(-e^{\mu_1/\sigma_1})-6\sigma_1 Li_3(-e^{\mu_1/\sigma_1})\right]$$

$$K_2 = (1-p)\left[\left(\frac{\mu_2^2+s_2\sigma_2^2}{\sigma_2}\right)\log(1+e^{\mu_2/\sigma_2})+4\mu_2 Li_2(-e^{\mu_2/\sigma_2})-6\sigma_2 Li_3(-e^{\mu_2/\sigma_2})\right]$$

and $Li_n(\cdot)$ is Polylogarithm Function of order n ($n \geq 2$)

IV. Estimation of Parameters

The estimation of the parameters for the two component mixture of left truncated three parameter logistic type distribution is considered in this section.

Method of Moments

This approach determines the estimators of the parameters by equating the population and sample moments accordingly. Suppose that a sample of size "n" is selected from a population whose probability density function has the following form:

$$f(x;\mu,\sigma,s) = p\left[C_1\left[s_1+\left(\frac{x-\mu_1}{\sigma_1}\right)^2\right]\frac{e^{-\left(\frac{x-\mu_1}{\sigma_1}\right)}}{\sigma_1\left[1+e^{-\left(\frac{x-\mu_1}{\sigma_1}\right)}\right]^2}\right] + (1-p)\left[C_2\left[s_2+\left(\frac{x-\mu_2}{\sigma_2}\right)^2\right]\frac{e^{-\left(\frac{x-\mu_2}{\sigma_2}\right)}}{\sigma_2\left[1+e^{-\left(\frac{x-\mu_2}{\sigma_2}\right)}\right]^2}\right]$$

$$0<x<\infty;\ \ \mu_1,\mu_2>0;\ \ \sigma_1,\sigma_2>0;\ \ s_1,s_2>0.$$

Where, C_1 and C_2 are as specified in Eq. (1) and $Li_2(\cdot)$ is Dilogarithm function

The distribution parameters $\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,s_1$ and s_2. Therefore, we consider the first two raw moments of the both the sample and the population, which results in the following equation.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\bar{x} = pC_1\left[\left(\frac{\mu_1^2+s_1\sigma_1^2}{\sigma_1}\right)\log\left(1+e^{\mu_1/\sigma_1}\right)+4\mu_1 Li_2\left(-e^{\mu_1/\sigma_1}\right)-6\sigma_1 Li_3\left(-e^{\mu_1/\sigma_1}\right)\right] + (1-p)C_2\left[\left(\frac{\mu_2^2+s_2\sigma_2^2}{\sigma_2}\right)\log\left(1+e^{\mu_2/\sigma_2}\right)+4\mu_2 Li_2\left(-e^{\mu_2/\sigma_2}\right) - 6\sigma_2 Li_3\left(-e^{\mu_2/\sigma_2}\right)\right] \tag{13}$$

$$\frac{1}{n}\sum_{i=1}^{n} x_i^2 = p\left[2C_1\left[-(\mu_1^2+s_1\sigma_1^2)Li_2\left(-e^{\mu_1/\sigma_1}\right)+6\mu_1\sigma_1 Li_3\left(-e^{\mu_1/\sigma_1}\right)-12\sigma_1^2 Li_4\left(-e^{\mu_1/\sigma_1}\right)\right]\right] + (1-p)\left[2C_2\left[-(\mu_2^2+s_2\sigma_2^2)Li_2\left(-e^{\mu_2/\sigma_2}\right)+6\mu_2\sigma_2 Li_3\left(-e^{\mu_2/\sigma_2}\right) - 12\sigma_2^2 Li_4\left(-e^{\mu_2/\sigma_2}\right)\right]\right] \tag{14}$$

where, C_1 and C_2 are as specified in Eq. (1) and $Li_n(\cdot)$ is Polylogarithm function of order n (n ≥ 2)

When we simultaneously solve Eqs. (13) and (14), we obtain the moment estimates of $\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,s_1$ and s_2.

Maximum Likelihood Method of Estimation

Let $x_1, x_2, x_3, \ldots, x_n$ be a sample of size n selected from a population. Whose p.d.f of this population is shown in Eq. (1). Accordingly the likelihood function for this sample is expressed as follows

$$L = p^n C_1^n \sigma_1^{-n}\prod_{i=1}^{n}\left\{\left[s_1+\left(\frac{x_i-\mu_1}{\sigma_1}\right)^2\right]\frac{e^{-\left(\frac{x_i-\mu_1}{\sigma_1}\right)}}{\left[1+e^{-\left(\frac{x_i-\mu_1}{\sigma_1}\right)}\right]^2}\right\} + (1-p)^n C_2^n \sigma_2^{-n}\prod_{i=1}^{n}\left\{\left[s_2+\left(\frac{x_i-\mu_2}{\sigma_2}\right)^2\right]\frac{e^{-\left(\frac{x_i-\mu_2}{\sigma_2}\right)}}{\left[1+e^{-\left(\frac{x_i-\mu_2}{\sigma_2}\right)}\right]^2}\right\} \tag{15}$$

where, C_1 and C_2 are as specified in Eq. (1) and $Li_2(\cdot)$ is Dilogarithm function
The log likelihood function of the sample is

$$\begin{aligned}\log L = n\log p - n\log\sigma_1 \\ - n\log\left[\left(s_1+\frac{\mu_1^2}{\sigma_1^2}\right)\frac{e^{\mu_1/\sigma_1}}{[1+e^{\mu_1/\sigma_1}]}-\frac{2\mu_1}{\sigma_1}\log\left(1+e^{\mu_1/\sigma_1}\right)-2Li_2\left(-e^{\mu_1/\sigma_1}\right)\right] \\ +\sum_{i=1}^{n}\log\left[s_1+\left(\frac{x_i-\mu_1}{\sigma_1}\right)^2\right]-\sum_{i=1}^{n}\left(\frac{x_i-\mu_1}{\sigma_1}\right)-2\sum_{i=1}^{n}\log\left[1+e^{-\left(\frac{x_i-\mu_1}{\sigma_1}\right)}\right] \\ + n\log(1-p) - n\log\sigma_2 \\ - n\log\left[\left(s_2+\frac{\mu_2^2}{\sigma_2^2}\right)\frac{e^{\mu_2/\sigma_2}}{[1+e^{\mu_2/\sigma_2}]}-\frac{2\mu_2}{\sigma_2}\log\left(1+e^{\mu_2/\sigma_2}\right)-2Li_2\left(-e^{\mu_2/\sigma_2}\right)\right] \\ +\sum_{i=1}^{n}\log\left[s_2+\left(\frac{x_i-\mu_2}{\sigma_2}\right)^2\right]-\sum_{i=1}^{n}\left(\frac{x_i-\mu_2}{\sigma_2}\right)-2\sum_{i=1}^{n}\log\left[1+e^{-\left(\frac{x_i-\mu_2}{\sigma_2}\right)}\right]\end{aligned} \tag{16}$$

For determine the M.L.E. of the parameters, it is necessary to maximum L or Log L w.r.t these concerning the parameters $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, s_1$ and s_2.

$$\begin{aligned} nC_1\left\{\left(s_1+\frac{\mu_1^2}{\sigma_1^2}\right)\frac{e^{\mu_1/\sigma_1}}{[1+e^{\mu_1/\sigma_1}]^2}-2\left(1+\frac{\sigma_1}{e^{\mu_1/\sigma_1}}\right)\log\left(1+e^{\mu_1/\sigma_1}\right)\right\} \\ +2\sum_{i=1}^{n}\frac{\left(\frac{x_i-\mu_1}{\sigma_1}\right)}{\left[s_1+\left(\frac{x_i-\mu_1}{\sigma_1}\right)^2\right]}+2\sum_{i=1}^{n}\frac{1}{\left[1+e^{\left(\frac{x_i-\mu_1}{\sigma_1}\right)}\right]}=n\end{aligned} \tag{17}$$

$$\begin{aligned} nC_2\left\{\left(s_2+\frac{\mu_2^2}{\sigma_2^2}\right)\frac{e^{\mu_2/\sigma_2}}{[1+e^{\mu_2/\sigma_2}]^2}-2\left(1+\frac{\sigma_2}{e^{\mu_2/\sigma_2}}\right)\log\left(1+e^{\mu_2/\sigma_2}\right)\right\} \\ +2\sum_{i=1}^{n}\frac{\left(\frac{x_i-\mu_2}{\sigma_2}\right)}{\left[s_2+\left(\frac{x_i-\mu_2}{\sigma_2}\right)^2\right]}+2\sum_{i=1}^{n}\frac{1}{\left[1+e^{\left(\frac{x_i-\mu_2}{\sigma_2}\right)}\right]}=n\end{aligned} \tag{18}$$

$$\begin{aligned} -nC_1\left\{-\frac{\mu_1}{\sigma_1}\left(s_1+\frac{\mu_1^2}{\sigma_1^2}\right)\frac{e^{\mu_1/\sigma_1}}{[1+e^{\mu_1/\sigma_1}]^2}-\frac{2\mu_1}{\sigma_1^2}\left(\frac{1}{\sigma_1^2}-\mu_1\right)\frac{e^{\mu_1/\sigma_1}}{[1+e^{\mu_1/\sigma_1}]} \right. \\ \left. +2\left(\frac{\mu_1}{\sigma_1}-\frac{\sigma_1}{e^{\mu_1/\sigma_1}}\right)\log\left(1+e^{\mu_1/\sigma_1}\right)\right\} \\ +\sum_{i=1}^{n}\left(\frac{x_i-\mu_1}{\sigma_1}\right)-2\sum_{i=1}^{n}\frac{\left(\frac{x_i-\mu_1}{\sigma_1}\right)^2}{\left[s_1+\left(\frac{x_i-\mu_1}{\sigma_1}\right)^2\right]}-2\sum_{i=1}^{n}\frac{\left(\frac{x_i-\mu_1}{\sigma_1}\right)}{\left[1+e^{\left(\frac{x_i-\mu_1}{\sigma_1}\right)}\right]}=n\end{aligned} \tag{19}$$

$$\begin{aligned} -nC_2\left\{-\frac{\mu_2}{\sigma_2}\left(s_2+\frac{\mu_2^2}{\sigma_2^2}\right)\frac{e^{\mu_2/\sigma_2}}{[1+e^{\mu_2/\sigma_2}]^2}-\frac{2\mu_2}{\sigma_2^2}\left(\frac{1}{\sigma_2^2}-\mu_2\right)\frac{e^{\mu_2/\sigma_2}}{[1+e^{\mu_2/\sigma_2}]} \right. \\ \left. +2\left(\frac{\mu_2}{\sigma_2}-\frac{\sigma_2}{e^{\mu_2/\sigma_2}}\right)\log\left(1+e^{\mu_2/\sigma_2}\right)\right\} \\ +\sum_{i=1}^{n}\left(\frac{x_i-\mu_2}{\sigma_2}\right)-2\sum_{i=1}^{n}\frac{\left(\frac{x_i-\mu_2}{\sigma_2}\right)^2}{\left[s_2+\left(\frac{x_i-\mu_2}{\sigma_2}\right)^2\right]}-2\sum_{i=1}^{n}\frac{\left(\frac{x_i-\mu_2}{\sigma_2}\right)}{\left[1+e^{\left(\frac{x_i-\mu_2}{\sigma_2}\right)}\right]}=n\end{aligned} \tag{20}$$

$$-nC_1\left(\frac{e^{\mu_1/\sigma_1}}{[1+e^{\mu_1/\sigma_1}]}\right)+\sum_{i=1}^{n}\frac{1}{\left[s_1+\left(\frac{x_i-\mu_1}{\sigma_1}\right)^2\right]}=0 \tag{21}$$

$$-nC_2\left(\frac{e^{\mu_2/\sigma_2}}{[1+e^{\mu_2/\sigma_2}]}\right)+\sum_{i=1}^{n}\frac{1}{\left[s_2+\left(\frac{x_i-\mu_2}{\sigma_2}\right)^2\right]}=0 \tag{22}$$

where, C_1 and C_2 are as specified in Eq. (1) and $Li_2(\cdot)$ is Dilogarithm function

It is possible to obtain the parameter estimates of $\hat{\mu}_1, \hat{\mu}_2, \hat{\sigma}_1^2, \hat{\sigma}_2^2, \hat{s}_1$ and $\hat{s}_2$ by simultaneously solving Eqs. (17), (18), (19), (20), (21) and (22) using numerical methods like Newton Rapson method.

V Manpower Planning Model with Two Component Mixture of Left Truncated Three Parameter Logistic Type Distribution as CLS

In this section examines the application of the developed distribution to the manpower planning models. Assume that CLS of an employee in an organization follows a two component mixture of left truncated three parameter logistic type distribution with parameters $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, s_1$ and s_2. Its probability density function, distribution function and expected length of service are given in the Eqs. (1), (3) and (4) respectively.

The probability that an individual survives in the organization for the length of time 't' is G(t), which is the survival function of the employee in the organization. It can be defined as the complement of the distribution function F(t). Then

$$G(t)=1-F(t)$$

$$G(t)=pC_1\int_t^{\infty}\left[s_1+\left(\frac{t-\mu_1}{\sigma_1}\right)^2\right]\frac{e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}}{\sigma_1\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right]^2}\,dt+(1-p)C_2\int_t^{\infty}\left[s_2+\left(\frac{t-\mu_2}{\sigma_2}\right)^2\right]\frac{e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}}{\sigma_2\left[1+e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right]^2}\,dt$$

Making the necessary transformations and integration, we obtain

$$G(t)=pC_1\left\{\frac{\left[s_1+\left(\frac{t-\mu_1}{\sigma_1}\right)^2\right]e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}}{\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right]}+2\left(\frac{t-\mu_1}{\sigma_1}\right)\log\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right]-2Li_2\left(-e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right)\right\}+(1-p)C_2\left\{\frac{\left[s_2+\left(\frac{t-\mu_2}{\sigma_2}\right)^2\right]e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}}{\left[1+e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right]}+2\left(\frac{t-\mu_2}{\sigma_2}\right)\log\left[1+e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right]-2Li_2\left(-e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right)\right\} \tag{23}$$

where, C_1 and C_2 are as specified in Eq. (1) and $Li_2(\cdot)$ is Dilogarithm function
Different shapes of survival function G(t) for specified values of the parameters are shown in Figure 2, 3 and 4.

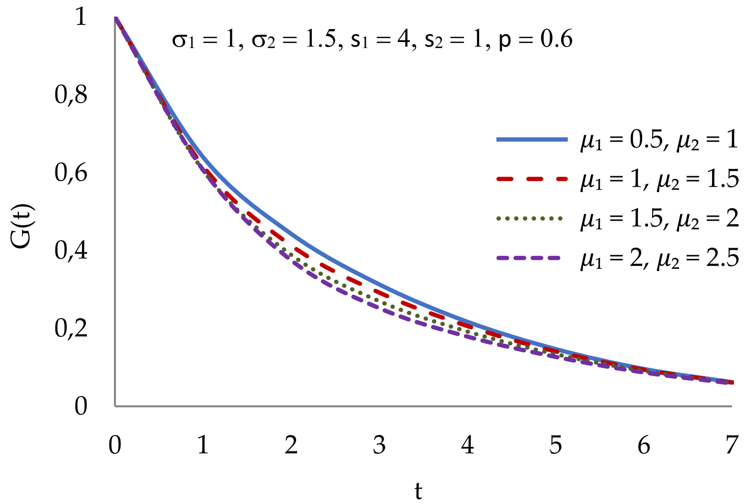


Figure 2: *Shapes of the Survival Function for different values of parameters μ_1 and μ_2*

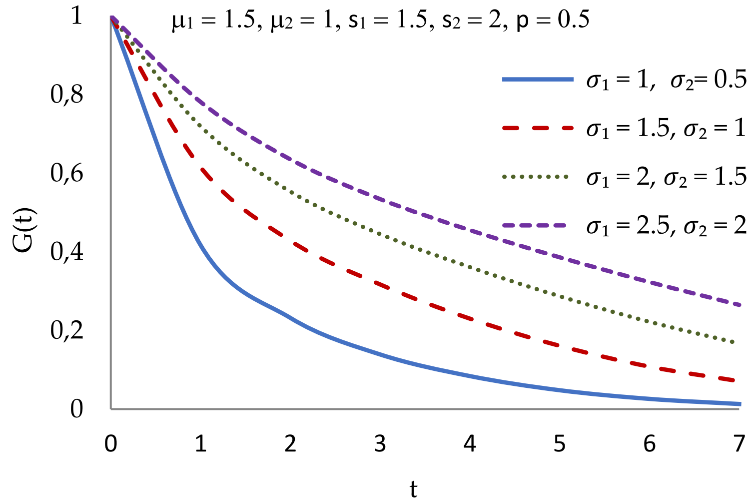


Figure 3: *Shapes of the Survival Function for different values of parameters σ_1 and σ_2*

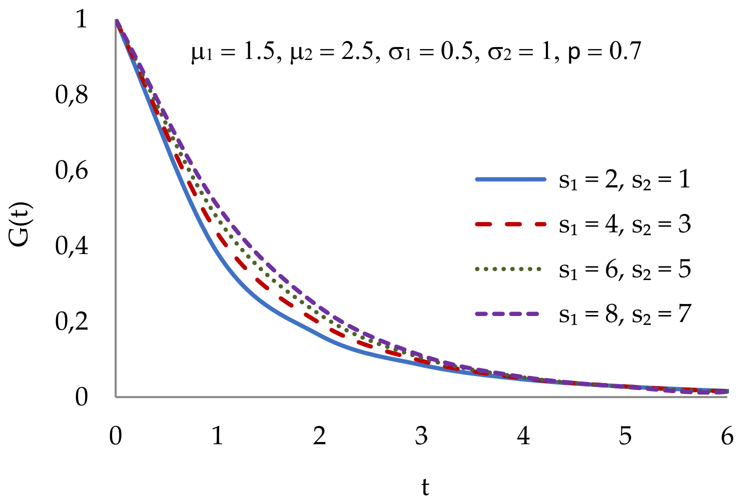


Figure 4: *Shapes of the Survival Function for different values of parameters s_1 and s_2*

The force of separation, also known as loss of intensity or rate of labour wastage, can be obtained as

$$L(t) = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{G(t)}$$

When the functional form of f(t) and G(t) from the Eqs. (1) and (23), we obtain

$$L(t)=\frac{p\frac{\left[s_1+\left(\frac{t-\mu_1}{\sigma_1}\right)^2\right]e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}}{\sigma_1\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right]^2}+(1-p)\frac{\left[s_2+\left(\frac{t-\mu_2}{\sigma_2}\right)^2\right]e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}}{\sigma_2\left[1+e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right]^2}}{p\left\{\frac{\left[s_1+\left(\frac{t-\mu_1}{\sigma_1}\right)^2\right]e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}}{\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right]}+2\left(\frac{t-\mu_1}{\sigma_1}\right)\log\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right]-2Li_2\left(-e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right)\right\}+(1-p)\left\{\frac{\left[s_2+\left(\frac{t-\mu_2}{\sigma_2}\right)^2\right]e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}}{\left[1+e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right]}+2\left(\frac{t-\mu_2}{\sigma_2}\right)\log\left[1+e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right]-2Li_2\left(-e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right)\right\}} \tag{24}$$

where, $Li_2(\cdot)$ is Dilogarithm function

Different values of parameters the shapes of labour wastage L(t) is shown in Figures 5, 6 and 7

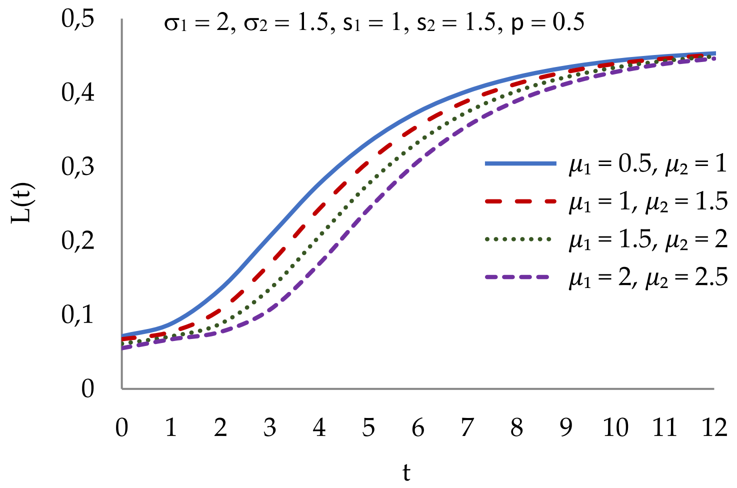


Figure 5: *Loss of Intensity curve for different values of parameters μ_1 and μ_2*

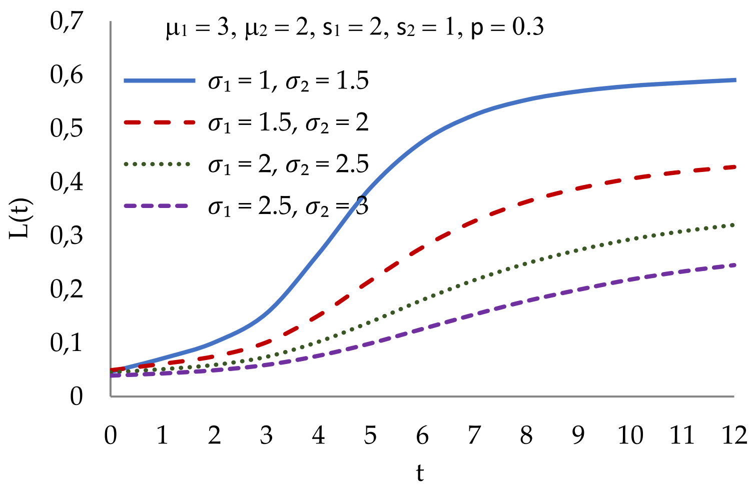


Figure 6: *Loss of Intensity curve for different values of parameters σ_1 and σ_2*

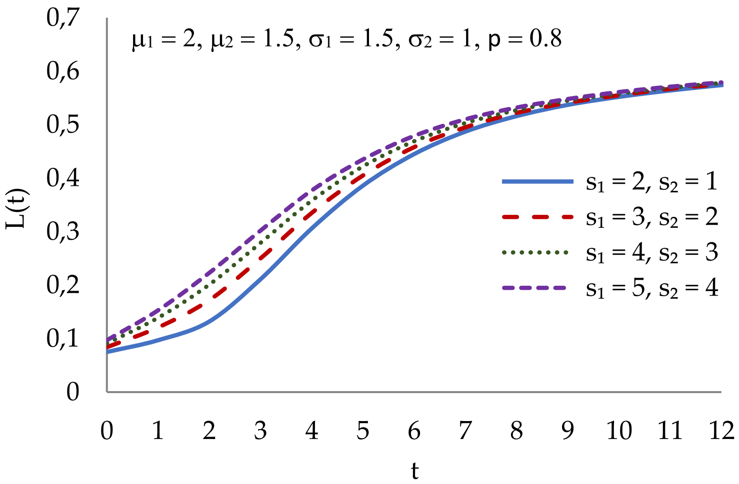


Figure 7: *Loss of Intensity curve for different values of parameters s_1 and s_2*

The renewal density of this model is

$$h(t) = f(t) + \int_0^t G(t-x)\, f(x)dx$$

By substituting the expressions for f(t) and G(t) from the Eqs. (1) and (23) and simplifying, we obtain

$$\begin{aligned} h(t) = {} & pC_1\left[s_1 + \left(\frac{t-\mu_1}{\sigma_1}\right)^2\right]\frac{e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}}{\sigma_1\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right]^2} + (1 \\ & - p)C_2\left[s_2 + \left(\frac{t-\mu_2}{\sigma_2}\right)^2\right]\frac{e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}}{\sigma_2\left[1+e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right]^2} \\ & + \int_0^t \left[pC_1\left[s_1 + \left(\frac{x-\mu_1}{\sigma_1}\right)^2\right]\frac{e^{-\left(\frac{x-\mu_1}{\sigma_1}\right)}}{\sigma_1\left[1+e^{-\left(\frac{x-\mu_1}{\sigma_1}\right)}\right]^2} + (1 \right. \\ & - p)C_2\left[s_2 \right. \\ & \left.\left. + \left(\frac{x-\mu_2}{\sigma_2}\right)^2\right]\frac{e^{-\left(\frac{x-\mu_2}{\sigma_2}\right)}}{\sigma_2\left[1+e^{-\left(\frac{x-\mu_2}{\sigma_2}\right)}\right]^2}\right]\left\{pC_1\left[\frac{\left[s_1 + \left(\frac{t-x-\mu_1}{\sigma_1}\right)^2\right]e^{-\left(\frac{t-x-\mu_1}{\sigma_1}\right)}}{\left[1+e^{-\left(\frac{t-x-\mu_1}{\sigma_1}\right)}\right]} \right.\right. \\ & \left. + 2\left(\frac{t-x-\mu_1}{\sigma_1}\right)\log\left[1+e^{-\left(\frac{t-x-\mu_1}{\sigma_1}\right)}\right] - 2Li_2\left(-e^{-\left(\frac{t-x-\mu_1}{\sigma_1}\right)}\right)\right] + (1 \\ & - p)C_2\left[\frac{\left[s_2 + \left(\frac{t-x-\mu_2}{\sigma_2}\right)^2\right]e^{-\left(\frac{t-x-\mu_2}{\sigma_2}\right)}}{\left[1+e^{-\left(\frac{t-x-\mu_2}{\sigma_2}\right)}\right]} \right. \\ & \left.\left. + 2\left(\frac{t-x-\mu_2}{\sigma_2}\right)\log\left[1+e^{-\left(\frac{t-x-\mu_2}{\sigma_2}\right)}\right] - 2Li_2\left(-e^{-\left(\frac{t-x-\mu_2}{\sigma_2}\right)}\right)\right]\right\}dx \end{aligned} \tag{25}$$

where, C_1 and C_2 are as specified in Eq. (1) and $Li_2(\cdot)$ is Dilogarithm function

Different values of the parameters the shape of renewal density function h(t) is shown in Figures 8, 9 and 10

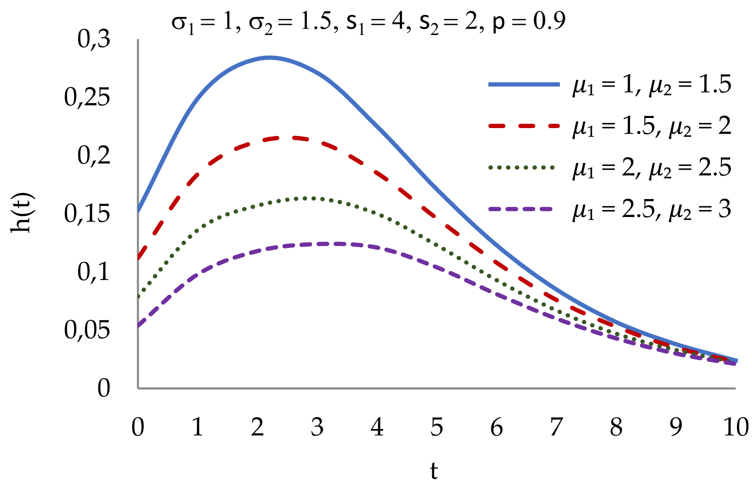


Figure 8: *Shapes of Renewal density function h(t) for different values of parameters μ_1 and μ_2*

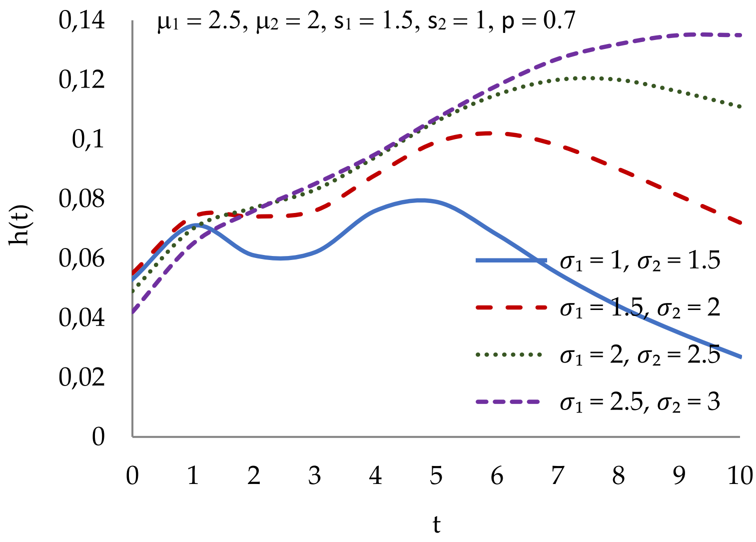


Figure 9: *Shapes of Renewal density function h(t) for different values of parameters σ_1 and σ_2*

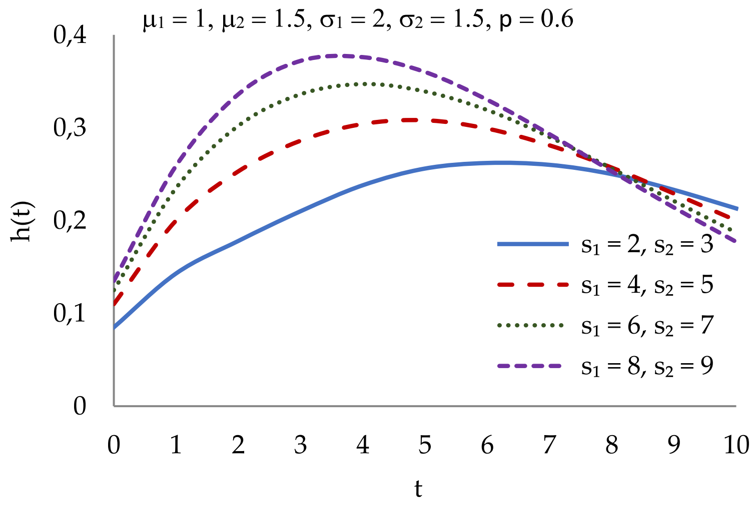


Figure 10: *Shapes of Renewal density function h(t) for different values of parameters s_1 and s_2*

VI Recruitment Policies for Manpower Models with Two Component Mixture of Left Truncated Three Parameter Logistic Type Distribution

When a CLS follows the two component mixture of left truncated three parameter logistic type distribution, the survival rate of labour waste and renewal rates are examined in the previous section. This section discusses an organization various recruitment procedures, which are crucial to operating system plans. An organization can be thought of as having two grades: training grade (Grade I) and the second grade is organization grade (Grade II). The training grade was implemented to provide experienced men to fill vacancies that occur within an origination grade, Naturally, the organization will generally have a hierarchical structure. The movement between grades within an organization was taken into account, nevertheless. The organization as a whole is viewed as a single entity.

Let N_1 and N_2 be the sizes of the training grade and the organization grades respectively, Further $N = N_1+N_2$. Where, N_2 is assumed to be known and constant. The issue at hand determining the appropriate duration of the training grade such that, on average, individuals spend a specified period 't' in it prior to advancement to the organization grade.

Two rules apply in this study i) Promotion by seniority i.e. where by the training grade most senior (and thus most experienced) member gets promoted and ii) Promotion by random based on duration of service. In other words, promotion is granted based on other criteria, such as competence, personal attributes, qualifications, specialized training, and so forth.

The individual loss rate in training grade (Grade I) is represented by W_1, the individual loss rate in organization grade (Grade II) is represented by W_2, and P is the promotion rate.

Given that the input and output of each grade must equilibrate, for grade II, it can be expressed as

$$N_1 P = N_2 W_2 \tag{26}$$

Hence, in equilibrium the system expected input per unit time is N/m, where m is the mean length of completed service. As a result, for Grade I,

$$\frac{N}{m} = N_1(P + W_1)$$
$$= N_1 W_1 + N_2 W_2 \tag{27}$$

In general, W_1, W_2 and P are functions of time. Bartholomew [11] that P, W_1, W_2 tend to equilibrium values which are independent of the age of the system.

Let m_1 be the average amount of time spent in Grade I. When the system is in equilibrium, the expected number of vacancies in this grade per unit time is

$$\frac{N_1}{m_1} = N_1(P + W_1)$$

This implies

$$\frac{1}{m_1} = P + W_1 \tag{28}$$

From Eqs. (27) and (28)

$$\frac{N}{m} = \frac{N_1}{m_1} \tag{29}$$

This equation states that the through-put for the whole system is equal to the number of vacancies in Grade I

VII Promotion by Seniority when the CLS Follows Two Component Mixture of Left Truncated Three Parameter Logistic Type Distribution

Let $a(t/T)$ represent the age distribution of the system at time T, assuming the system was initiated at T = 0. Hence, $a(t/T)\delta t$ denotes the probability that a randomly selected individual chosen at time 'T' has a length of service within the interval $(t, t+\delta t)$.

Thus

$$a\left(\frac{t}{T}\right)\delta t = \text{Prob }\{\text{Individual joined in}(T-t, T-t+\delta t) \text{ and remained for time } t\}$$

$$= h(T-t)\,\delta t\, G(t)$$

where, h(t) is the renewal density of the whole system

Hence

$$a\left(\frac{t}{T}\right)\delta t = h(T-t)\delta t\, G(t)$$

This holds for t<T we have,

$$\lim_{T\to\infty} h(T) = \frac{1}{m}$$

Now

$$\lim_{T\to\infty} G(T) = 0$$

$$\lim_{T\to\infty} a\left(\frac{t}{T}\right) = \frac{G(T)}{m}$$

$$= a(t)$$

This is the equilibrium age distribution.

A threshold value exists at t_1 such that all individuals with a length of service less than that t_1 are in Grade I because a loss from Grade II is replaced by the most senior member of Grade I. That means that at any given time, every individual in Grade II has a length of service at least as long as any individual in Grade I. Although "t_1" is random variable, the approximation calculation for its expected value is

$$\int_{t_1}^{\infty} a(t)\, dt = \frac{N_2}{N_1 + N_2} \tag{30}$$

As a result, in equilibrium, the expected number of promotions that should occur per unit of time will be the number of new recruits who have served to the t_1 threshold length of service.

i.e., t_1

$$N_1 P = G(t_1)\frac{N}{m} \tag{31}$$

Let m_L be the average length of time spent in Grade I by those who leave while still in Grade I, and let m_P be the average length of time spent in Grade I by those who are finally promoted to Grade II. After that, consider about the challenge of selecting N_1 in order to give   a fixed value. Eqs. (30) and (31) are valid in this instance since m_P is equal to the average value that was previously introduced.

Thus one can have

$$\int_{m_P}^{\infty} a(t)\, dt = \frac{N_2}{N}$$

i.e.,

$$\frac{1}{m}\int_{m_P}^{\infty} G(t)\,dt = \frac{N_2}{N} \tag{32}$$

And

$$N_1 P = G(m_P)\frac{N}{m}$$

Eq. (32) presents $R = \frac{N_1}{N_2}$ as a function of m_P; hence, one may calculate N_1 for a given value of m_P by knowing the size N of the organization.

This implies

$$\frac{1}{m}\int_{m_P}^{\infty} G(t)\,dt = \frac{1}{1+R} \tag{33}$$

It is assumed that the CLS of an employee in the organization follow a two component mixture of left truncated logistic type distribution. Eq. (23) can be substituted in (33), we obtain

$$\frac{pC_1}{m}\int_{m_P}^{\infty}\left\{\frac{\left[s_1+\left(\frac{t-\mu_1}{\sigma_1}\right)^2\right]e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}}{\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right]}+2\left(\frac{t-\mu_1}{\sigma_1}\right)\log\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right]-2Li_2\left(-e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right)\right\}dt$$
$$+\frac{(1-p)C_2}{m}\int_{m_P}^{\infty}\left\{\frac{\left[s_2+\left(\frac{t-\mu_2}{\sigma_2}\right)^2\right]e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}}{\left[1+e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right]}+2\left(\frac{t-\mu_2}{\sigma_2}\right)\log\left[1+e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right]\right.$$
$$\left.-2Li_2\left(-e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right)\right\}dt=\frac{1}{1+R}$$

where, C_1 and C_2 are as specified in Eq. (1) and $\mathrm{Li}_2(\cdot)$ is Dilogarithm function

where, $\mathrm{Li}_n(\cdot)$ is Polylogarithm function of order n (n ≥ 2)

Making necessary transformations and by integration, we get

$$\frac{pC_1\sigma_1}{m}\left\{\left[s_1+\left(\frac{m_P-\mu_1}{\sigma_1}\right)^2\right]\log\left[1+e^{-\left(\frac{m_P-\mu_1}{\sigma_1}\right)}\right]-4\left(\frac{m_P-\mu_1}{\sigma_1}\right)Li_2\left(-e^{-\left(\frac{m_P-\mu_1}{\sigma_1}\right)}\right)-6Li_3\left(-e^{-\left(\frac{m_P-\mu_1}{\sigma_1}\right)}\right)\right\}$$
$$+\frac{(1-p)C_2\sigma_2}{m}\left\{\left[s_2+\left(\frac{m_P-\mu_2}{\sigma_2}\right)^2\right]\log\left[1+e^{-\left(\frac{m_P-\mu_2}{\sigma_2}\right)}\right]\right.$$
$$\left.-4\left(\frac{m_P-\mu_2}{\sigma_2}\right)Li_2\left(-e^{-\left(\frac{m_P-\mu_2}{\sigma_2}\right)}\right)-6Li_3\left(-e^{-\left(\frac{m_P-\mu_2}{\sigma_2}\right)}\right)\right\}=\frac{1}{1+R}$$

m_P is the average length of time spent in Grade I by those who are eventually promoted to Grade II.
On Simplification,

$$R = \frac{m\left\{\left(s_1 + \frac{\mu_1^2}{\sigma_1^2}\right)\frac{e^{\mu_1/\sigma_1}}{[1+e^{\mu_1/\sigma_1}]} - \frac{2\mu_1}{\sigma_1}\log\left(1+e^{\mu_1/\sigma_1}\right) - 2Li_2\left(-e^{\mu_1/\sigma_1}\right)\right\}}{p\sigma_1\left\{\left[s_1 + \left(\frac{m_P-\mu_1}{\sigma_1}\right)^2\right]\log\left[1+e^{-\left(\frac{m_P-\mu_1}{\sigma_1}\right)}\right] - 4\left(\frac{m_P-\mu_1}{\sigma_1}\right)Li_2\left(-e^{-\left(\frac{m_P-\mu_1}{\sigma_1}\right)}\right) - 6Li_3\left(-e^{-\left(\frac{m_P-\mu_1}{\sigma_1}\right)}\right)\right\}} + \frac{m\left\{\left(s_2 + \frac{\mu_2^2}{\sigma_2^2}\right)\frac{e^{\mu_2/\sigma_2}}{[1+e^{\mu_2/\sigma_2}]} - \frac{2\mu_2}{\sigma_2}\log\left(1+e^{\mu_2/\sigma_2}\right) - 2Li_2\left(-e^{\mu_2/\sigma_2}\right)\right\}}{(1-p)\sigma_2\left\{\left[s_2 + \left(\frac{m_P-\mu_2}{\sigma_2}\right)^2\right]\log\left[1+e^{-\left(\frac{m_P-\mu_2}{\sigma_2}\right)}\right] - 4\left(\frac{m_P-\mu_2}{\sigma_2}\right)Li_2\left(-e^{-\left(\frac{m_P-\mu_2}{\sigma_2}\right)}\right) - 6Li_3\left(-e^{-\left(\frac{m_P-\mu_2}{\sigma_2}\right)}\right)\right\}} - 1 \quad (34)$$

where, $Li_n(\cdot)$ is Polylogarithm function of order n ($n \geq 2$)

For various values of m, m_P, μ_1, μ_2, σ_1, σ_2, s_1 and s_2 corresponding values of R are computed using Eq. (34) and presented in Table 1.

Table 1: *Values of R for different input parameters under Promotion by Seniority*

m	m_P	p	μ_1	μ_2	σ_1	σ_2	s_1	s_2	R
8	4	0.5	2	2.5	**1**	**1.5**	2	3	83.319
8	4	0.5	2	2.5	**1.5**	**2**	2	3	28.846
8	4	0.5	2	2.5	**2**	**2.5**	2	3	15.663
8	4	0.5	2	2.5	**2.5**	**3**	2	3	10.244
8	4	0.5	**1**	**1.5**	1.5	2	2	3	28.326
8	4	0.5	**1.5**	**2**	1.5	2	2	3	28.383
8	4	0.5	**2**	**2.5**	1.5	2	2	3	28.846
8	4	0.5	**2.5**	**3**	1.5	2	2	3	29.62
8	4	0.5	2	2.5	1.5	2	**4**	**3**	32.633
8	4	0.5	2	2.5	1.5	2	**6**	**5**	37.787
8	4	0.5	2	2.5	1.5	2	**8**	**7**	42.356
8	4	0.5	2	2.5	1.5	2	**10**	**11**	47.65
8	4	**0.1**	2	2.5	1.5	2	2	3	99.668
8	4	**0.2**	2	2.5	1.5	2	2	3	53.128
8	4	**0.3**	2	2.5	1.5	2	2	3	38.337
8	4	**0.4**	2	2.5	1.5	2	2	3	31.754
8	**5**	0.5	2	2.5	1.5	2	2	3	41.38
8	**6**	0.5	2	2.5	1.5	2	2	3	61.636
8	**7**	0.5	2	2.5	1.5	2	2	3	94.563
8	**8**	0.5	2	2.5	1.5	2	2	3	148.563
9	4	0.5	2	2.5	1.5	2	2	3	32.576
10	4	0.5	2	2.5	1.5	2	2	3	36.307
11	4	0.5	2	2.5	1.5	2	2	3	40.038
12	4	0.5	2	2.5	1.5	2	2	3	43.768

From Table 1, it is observed that as σ_1 and σ_2 increases, the value of R is decreasing. μ_1 and μ_2 increases the value of R is increasing and s_1 and s_2 increases the value of R is increasing and for p increases the R value is decreasing. m_P increases the R value is increasing. An increase in m, results increase in R.

VIII Promotion by Random when the CLS Follows Two Component Mixture of Left Truncated Three Parameter Logistic Type Distribution

Let $F_1(t)$ be the probability that an individual stays in the system for a period of time 't' without being promoted and let n_t be the number promotions in $(0, t)$. Then

$$F_1(t) = \text{Prob \{Individual not promoted in } (0,t)/\text{does not leave in } (0,t)\}$$
$$* \text{Prob\{does not leave in } (0,t)\}$$

$$F_1(t) = \left(1 - \frac{1}{N_1}\right)^{n_t} G(T)$$

The excepted value of is N_1P_t

Thus as a first approximation one can take

$$F_1(t) = G(t)\left[1 - \frac{1}{N_1}\right]^{N_1P_t}$$

$N_1 \to \infty$ gives

$$F_1(t) = G(t)\, e^{-P_t}$$

Then the average time spent in Grade I is

$$m_1 = \int_0^\infty F_1(t)\, dt = \int_0^\infty G(t)\, e^{-P_t}\, dt$$

But from Eq. (29)

$$\frac{m_1}{m} = \frac{N_1}{N}$$

Thus

$$\frac{1}{m}\int_0^\infty G(t)e^{-P_t}\, dt = \frac{N_1}{N} \tag{35}$$

Assuming that an individual is promoted before leaving, let $q(t)$ be the probability density function of the time interval between his entry into Grade I and his promotion to Grade II in equilibrium.

$$\text{Let } Q(t) = \int_t^\infty q(x)\, dx$$

Since $Q(t) = \text{prob\{Not promoted in a period of length t / doesnot leave in that period\}}$

$$= \left(1 - \frac{1}{N_1}\right)^{n_t}$$

Where, n_t is the number of promotions in a interval of length t.

Replacing n_t can be replaced by its expected value, N_1P_t. We get the

$$Q(t) = \left(1 - \frac{1}{N_1}\right)^{N_1P_t}$$

Therefore

$$m_p = \int_0^\infty Q(t)\, dt = \int_0^\infty e^{-P_t}\, dt = \frac{1}{P} \tag{36}$$

Using this in conjunction with Eq. (35) gives N_1 in terms of m_P, and hence

$$\frac{N_1}{N} = \frac{1}{m}\int_0^\infty G(t)\, e^{-t/m_P}\, dt \tag{37}$$

The expression for the value "R" is computed from the following equation.

$$\frac{1}{m}\int_0^\infty G(t)\, e^{-t/m_P}\, dt = \frac{R}{R+1} \tag{38}$$

Substituting G(t) as given in the Eq. (23), we get

$$\frac{pC_1}{m}\int_0^\infty e^{-t/m_P}\left\{\frac{\left[s_1+\left(\frac{t-\mu_1}{\sigma_1}\right)^2\right]e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}}{\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right]}+2\left(\frac{t-\mu_1}{\sigma_1}\right)\log\left[1+e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right]-2Li_2\left(-e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)}\right)\right\}dt$$
$$+\frac{(1-p)C_2}{m}\int_0^\infty e^{-t/m_P}\left\{\frac{\left[s_2+\left(\frac{t-\mu_2}{\sigma_2}\right)^2\right]e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}}{\left[1+e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right]}+2\left(\frac{t-\mu_2}{\sigma_2}\right)\log\left[1+e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right]-2Li_2\left(-e^{-\left(\frac{t-\mu_2}{\sigma_2}\right)}\right)\right\}dt=\frac{R}{1+R} \quad (39)$$

where, C_1 and C_2 are as specified in Eq. (1) and $Li_2(\cdot)$ is Dilogarithm function

By solving the (39), for various values of m, m_P, μ_1, μ_2, σ_1, σ_2, s_1 and s_2 the R values are computed and presented in Table 2.

Table 2: *Values of R for different input parameters under Promotion by Random*

m	m_P	p	μ_1	μ_2	σ_1	σ_2	s_1	s_2	R
6	2	0.5	1	1.5	**1**	**1.5**	2	1	0.2159
6	2	0.5	1	1.5	**1.5**	**2**	2	1	0.2656
6	2	0.5	1	1.5	**2**	**2.5**	2	1	0.3006
6	2	0.5	1	1.5	**2.5**	**3**	2	1	0.3265
2	3	0.4	**0.5**	**1**	2	1	4	2	2.3791
2	3	0.4	**1**	**1.5**	2	1	4	2	2.1788
2	3	0.4	**1.5**	**2**	2	1	4	2	2.0822
2	3	0.4	**2**	**2.5**	2	1	4	2	2.0699
5	1	0.6	2	2.5	1.5	2	**2**	**1**	0.1711
5	1	0.6	2	2.5	1.5	2	**4**	**3**	0.1782
5	1	0.6	2	2.5	1.5	2	**6**	**5**	0.1827
5	1	0.6	2	2.5	1.5	2	**8**	**7**	0.1858
2	4	**0.1**	2	1	1	1.5	3	2	5.8249
2	4	**0.2**	2	1	1	1.5	3	2	4.8560
2	4	**0.3**	2	1	1	1.5	3	2	4.1280
2	4	**0.4**	2	1	1	1.5	3	2	3.5610
3	**2**	0.3	1.5	2	1.5	2.5	4	3	0.3860
3	**3**	0.3	1.5	2	1.5	2.5	4	3	0.8149
3	**4**	0.3	1.5	2	1.5	2.5	4	3	1.4038
3	**5**	0.3	1.5	2	1.5	2.5	4	3	2.2025
7	6	0.2	2.5	3	2	2.5	5	2	0.2926
8	6	0.2	2.5	3	2	2.5	5	2	0.2470
9	6	0.2	2.5	3	2	2.5	5	2	0.2137
10	6	0.2	2.5	3	2	2.5	5	2	0.1624

From Table 2, it is observed that the recruit policy is significantly increased by CLS distribution parameters. As the shape parameter σ_1 and σ_2 increases the value of R is increasing contrary to the policy of promotion by seniority when other parameters remaining fixed. It is also observed that as μ_1 and μ_2 increases the value of R is decreasing and also observed that as s_1 and s_2 increases the value of R is increasing, this increases very small. p increases the value of R is decreasing. When m_P is increases the value of R increasing. It is also observed that m increases the value of R is decreasing.

VIII Conclusion

In this paper, we introduced a two component mixture of left truncated three parameter logistic type distribution. The various distributional properties, such as distribution function, mean, four moments, hazard function, and survival function of the distribution, are derived. It is observed that the hazard function is increasing or decreasing depending on the truncation parameters. The three parameter logistic type distribution is highly applicable in analyzing various datasets as an alternative to the gaussian distribution were the variable under study is leptokurtic. The utility of this distribution in manpower modeling, in particular deriving the recruitment/promotion policies of the organization, are also discussed. It is observed that promotion by random is more effective than promotion by seniority. It is also possible to obtain other inferential aspects, such as testing of hypothesis which requires further study. This distribution is useful for analyzing several datasets in management sciences, finance quality control and agricultural experiments etc.

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Declarations

The authors state that there are no conflicts of interest to declare.

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