RT&A, No 4 (89)
Volume 20, December 2025

OPTIMIZATION OF A FUZZY SUPPLY CHAIN INVENTORY SYSTEM FOR DETERIORATING ITEMS UNDER ENVIRONMENTAL SUSTAINABILITY

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Abstract

The growing emphasis on environmental sustainability and operational reliability has encouraged researchers to extend classical inventory models using fuzzy logic to better handle uncertainty. This study proposes a sustainable inventory framework for deteriorating items, where demand is influenced by both selling price and greening initiatives. The model incorporates investments in preservation technology and green practices under trapezoidal fuzzy environments. Shortages are permitted, and the problem is solved through signed distance defuzzification method, supported by MATLAB for computation and analysis. To demonstrate the applicability, numerical examples are presented, followed by a comparative analysis between crisp and fuzzy settings. The results show that the fuzzy-based model consistently reduces overall costs, with an average decrease of 4.72%, while also lowering cycle time and reorder-to-shortage intervals by 4.04% and 5.52%, respectively. Sensitivity analysis further reveals that total cost is most sensitive to fluctuations in green technology and procurement expenses, whereas preservation and shortage costs have comparatively smaller effects. The findings highlight that adopting fuzzy-based decision approaches not only enhances financial performance but also supports ecological objectives, offering a more reliable pathway for sustainable supply chain management.

Keywords: Optimization, Sustainable Inventory, Deteriorating items, Price and Greening dependent demand, Preservation, and Green technology (PT & GT), Carbon emissions.

1. Introduction and literature review

Global habitability has declined markedly in recent decades, driven by escalating air pollution from industrial activities, transportation, and inadequate waste management. Rising carbon emissions and global warming have further intensified this deterioration, highlighting the urgent need to foster greater environmental awareness among consumers. In response, governments and non-governmental organizations are promoting sustainable growth strategies, such as imposing higher taxes on polluting industries and enforcing stricter regulations on transportation. To address these challenges, organizations worldwide must integrate sustainability into their inventory and supply chain practices to curb carbon emissions and reduce ecological damage. Such initiatives are vital not only for preserving the environment for future generations but also for ensuring long-term sustainable development.

DOI: https://doi.org/10.24412/1932-2321-2025-489-905-920

1.1. Sustainable Supply Chain Inventory

In recent years, sustainable supply chain inventory modeling has become a vital research area due to rising environmental concerns and the increasing demand for green products. Ahmed et al. [1] introduced a two-warehouse inventory framework by combining selling price decisions with investment in green technology, while also considering carbon emissions for deteriorating items. Singh et al. [2] investigated pharmaceutical inventory systems under time-dependent demand for products with fixed lifetimes, incorporating sustainability and deterioration, thereby highlighting challenges in balancing healthcare efficiency with ecological responsibility. Kumar et al. [3] proposed a sustainable preservation model where temperature-controlled storage was used to reduce spoilage, emphasizing how preservation technology can simultaneously lower costs and improve product quality. Padiyar et al. [4] expanded this work by integrating reverse logistics with preservation investments in a fuzzy environment, demonstrating the importance of addressing uncertainty in sustainable inventory systems. Chaudhary et al. [5] presented an integrated production"inventory model that considered imperfect production processes, green initiatives, and carbon emissions, showing the influence of environmental consciousness on production strategies. Singh and Mishra [6] advanced this direction by analyzing substitutable deteriorating items under both industrial and transportation-related emissions, stressing the necessity of managing multiple emission sources. Verma and Mishra [7] emphasized sustainability in remanufacturing processes under emission constraints, while Kaushik [8] developed a linear inventory model linking green investment with regulatory carbon policies. Similarly, Suvetha et al. [9] analyzed production"inventory systems with power-pattern demand and finite shelf life, incorporating carbon emissions into decision-making. Hakim et al. [10] explored pricing decisions for deteriorating items under green-level dependent demand, providing evidence that environmentally sensitive pricing directly shapes consumer preferences. Collectively, these studies illustrate a gradual progression from basic deterioration modeling to comprehensive approaches that integrate preservation, carbon policies, reverse logistics, and green initiatives into inventory frameworks.

1.2. Fuzzy Environment in Inventory Models

The presence of uncertainty in real-world decision-making has led to the widespread use of fuzzy set theory in supply chain and inventory research. Zadeh [29] laid the foundation for fuzzy set theory, introducing membership functions to capture imprecision in data, while Zimmermann [31] expanded its application to optimization and decision problems. Kaufmann and Gupta [34] provided systematic methods for fuzzy arithmetic, enabling researchers to build more practical fuzzy optimization models. In inventory management, Yao and Lee [30] first applied trapezoidal fuzzy numbers to order quantity models with and without backordering, highlighting the adaptability of fuzzy logic. El-Wakeel and Al-Yazidi [35] extended this by formulating probabilistic inventory systems constrained by trapezoidal fuzzy parameters, demonstrating how uncertainty can be managed for more reliable decisions. Dutta and Kumar [38] further studied deteriorating items with shortages under complete backlogging in a fuzzy framework, reinforcing the role of fuzzy systems in perishable product scenarios. Recent developments have explored more complex fuzzy structures. Sindhuja et al. [32] applied neutrosophic fuzzy trapezoidal numbers to energy-optimized inventory management, introducing multi-valued logic for sustainability analysis. Kumar and Paikray [33] examined deteriorating items under trapezoidal fuzzy demand with full backlogging, comparing crisp and fuzzy outcomes to show how fuzziness changes cost structures. Dhandapani et al. [36] employed trapezoidal and pentagonal fuzzy numbers for manufacturing inventory, highlighting cost optimization in uncertain environments. Negi and Singh [37] proposed a probabilistic model for deteriorating products under pentagonal fuzzy conditions, defuzzified using signed distance method. These works collectively demonstrate the evolution of fuzzy set applications in inventory systems from simple trapezoidal membership functions to advanced neutrosophic and pentagonal fuzzy numbers, reflecting the growing demand for robust modeling under uncertainty.

1.3. Combined Perspective: Sustainability, Carbon, and Fuzziness

Bringing together sustainability and fuzzy modeling, several researchers have attempted to construct inventory frameworks that integrate both environmental considerations and uncertainty. Shah et al. [11] examined deteriorating inventory models with greening efforts under price-sensitive and stock-dependent demand, illustrating how consumer behavior interacts with environmental responsibility. Ali et al. [12] combined preservation investment with demand sensitivity and solved the resulting framework using a marine predator™s algorithm, showing the potential of metaheuristic methods for sustainable optimization. Rani et al. [13] proposed a fuzzy green supply chain inventory model under variable demand and inflation, linking macroeconomic uncertainty with sustainability objectives. Mahato et al. [14] studied deteriorating products with fixed lifetimes and partial backordering, embedding carbon emission policies into decision-making. Sen et al. [15] developed a consignment-based inventory model with price- and green-sensitive demand, emphasizing responsiveness to market and environmental pressures. Banerjee and Agrawal [16] incorporated freshness- and price-dependent demand into sustainable systems, providing practical guidelines for discounting and ordering. Ahmad et al. [17] introduced preservation technology within green inventory models under emission constraints, while Yadav et al. [18] optimized deteriorating inventory with selling price and carbon emission considerations, reflecting the importance of joint economic environmental strategies. Das et al. [19] applied teaching-learning-based optimization to inventory problems involving green-level dependent demand and payment strategies, illustrating the role of intelligent algorithms. Kundu et al. [20] solved deteriorating green inventory models with hybrid algorithms, highlighting computational advances in sustainability research. Malik et al. [21] integrated learning effects, inflation, trade credit, and preservation into a green supply chain model for decaying items, while Shah et al. [22] considered quadratic price-dependent demand with preservation technology and trade credit. Rani et al. [23] presented a fuzzy inventory model under carbon-sensitive demand, reinforcing the link between environmental and uncertainty factors. Katariya and Shukla [24] designed pricing and inventory strategies under greening and discount policies for non-instantaneous deteriorating products, later extending this to markdown strategies in an EPQ setting [25]. These studies collectively highlight how sustainability, carbon emission regulations, and fuzzy uncertainty are increasingly interlinked in modern supply chain research, shaping the foundation for advanced models like the one proposed in this study.

2. Research Gap and Contributions

Most existing studies on sustainable supply chain inventory models have primarily examined deterioration, pricing, or carbon emissions in isolation. However, these models often neglect the combined effects of selling price and greening level dependent demand, preservation technology, and carbon emissions under uncertain environments. Moreover, while metaheuristic approaches are widely applied, limited research has explored analytical optimization methods to obtain more reliable optimal solutions in fuzzy sustainable supply chain contexts. To address these gaps, the present study makes the following contributions:

1. A sustainable supply chain inventory model is developed for deteriorating items with demand influenced by both selling price and greening level, integrating carbon emission considerations and partial shortages.
2. The model incorporates preservation technology as a strategic investment to mitigate deterioration and extend product shelf life, reducing both economic loss and environmental impact.
3. A trapezoidal fuzzy environment is employed to capture uncertainty in demand, deterioration, and cost parameters, thereby enhancing the model's realism.
4. The signed distance method is used for effective defuzzification, ensuring precise transformation of fuzzy parameters into crisp equivalents.

5. An analytical optimization method is applied for cost minimization, yielding exact and more reliable optimal solutions compared to heuristic-based approximate approaches.

3. Definitions and Preliminaries

1. **Fuzzy set:** A fuzzy set allows for partial membership in which an element can belong to a set to a certain degree or degree of membership, which is represented by a value between 0 and 1. Mathematically, a fuzzy set defined on a universe of discourse $X = \{x_1, x_2, x_3, \ldots, x_n\}$ is given by $A = \{(x, \mu_A(x)) : x \in X\}$, where $\mu_A(x)$ is the membership function of A given by $\mu_A(x) : X \to [0, 1]$.

2. **Fuzzy Number:** A fuzzy number is a fuzzy set on the real line $\mathbb{R}$, if its membership function $\mu_A(x)$ has the following properties:

   (a) $\mu_A(x)$ is upper semi-continuous.

   (b) There exist real numbers a_1, a_2, a_3, a_4 such that $a_1 \le a_2 \le a_3 \le a_4$, $\mu_A(x)$ is increasing on $[a_1, a_2]$, equal to 1 on $[a_2, a_3]$, and decreasing on $[a_3, a_4]$.

   (c) $\mu_A(x) = 0$ outside the interval $[a_1, a_4]$.

3. **Trapezoidal Fuzzy Number:** A trapezoidal fuzzy number is specified by the quadruple (a_1, a_2, a_3, a_4) and defined by its membership function $\mu_A(x) : X \to [0, 1]$ as follows:

$$\mu_A(x) = \begin{cases} 0, & x \le a_1, \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2, \\ 1, & a_2 \le x \le a_3, \\ \dfrac{a_4 - x}{a_4 - a_3}, & a_3 \le x \le a_4, \\ 0, & x \ge a_4. \end{cases}$$

4. **α-cut of a Trapezoidal Fuzzy Number:**

   Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number. The α-cut of $\tilde{A}$, denoted by $\tilde{A}_\alpha$, is defined as the crisp interval:

$$\tilde{A}_\alpha = [\, A^L_\alpha,\, A^R_\alpha \,], \quad \alpha \in [0, 1],$$

   where

$$A^L_\alpha = a_1 + \alpha(a_2 - a_1), \qquad A^R_\alpha = a_4 - \alpha(a_4 - a_3).$$

   Thus,

$$\tilde{A}_\alpha = \left[\, a_1 + \alpha(a_2 - a_1),\; a_4 - \alpha(a_4 - a_3) \,\right].$$

5. **L–R Representation of a Trapezoidal Fuzzy Number:**

   A trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ can also be expressed in terms of its L–R representation as

$$\tilde{A} = (m, \alpha, \beta)_{LR},$$

   where

$$m = \frac{a_2 + a_3}{2}, \quad \alpha = m - a_1, \quad \beta = a_4 - m.$$

Here m is the center of the fuzzy number, α is the left spread, and β is the right spread. The membership function in L–R form is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq a_1, \\ L\left(\dfrac{m-x}{\alpha}\right), & a_1 \leq x \leq m, \\ R\left(\dfrac{x-m}{\beta}\right), & m \leq x \leq a_4, \\ 0, & x \geq a_4, \end{cases}$$

where L and R are non-increasing functions with $L(0) = R(0) = 1$.

4. Notations and assumptions

The development of the mathematical model is based on the following assumptions and notations, which are introduced for clarity and consistency.

4.1. Notations

Table 2 presents the notations used in the developed model.

Table 1: *Notations, their descriptions, and units used in the model*

Notation	Description	Unit
p	Selling price per unit	\$/unit
g	Greening level (investment in green technology)	\$
θ	Deterioration rate	proportion per unit time
ξ	Preservation technology investment	\$
Q	Order quantity per cycle	units
T	Cycle length	Years
B	Maximum backorder level	units
OC	Ordering cost per cycle	\$/cycle
DC	Deterioration cost	\$/cycle
HC	Holding cost	\$/unit/time
PC	Procurement cost	\$/unit
$PTIC$	Preservation technology investment cost	\$
$GTIC$	Green technology investment cost	\$
SC	Shortage cost	\$/cycle
LSC	Lost sales cost	\$/cycle
CEC	Carbon emission cost	\$/cycle
TC	Total cost per cycle	\$/cycle
C_1	Procurement cost per unit	\$/unit
C_2	Holding cost per unit per time	\$/unit/time
C_3	Shortage cost per unit short	\$/unit
C_4	Lost sales cost per unit	\$/unit
C_5	Carbon emission cost per unit	\$/unit
C_6	Preservation technology cost per unit	\$/unit
C_7	Green technology cost per unit	\$/unit
C_8	Carbon emission cost per unit	\$/unit

4.2. Assumptions

The proposed inventory model is developed under the following assumptions:

1. The demand rate is a function of selling price and greening level, given by $D(p, g) = (\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g)$, where $\beta_1, \beta_2, \gamma_1, \gamma_2 > 0$.
2. The deterioration rate is dependent on preservation technology investment, given by $\theta(\xi) = \alpha(1 - \eta\xi)$, where $0 \leq \xi \leq 1$.
3. Shortages are permissible, as partially backlogged, and that is considered as $B(t) = \frac{\rho}{1+\delta(T-t)}$, where $0 \leq \rho \leq 1$.
4. The holding cost is considered constant throughout the planning horizon.
5. Lead time is assumed to be negligible or zero.
6. The single warehouse is employed.

5. Crisp Mathematical Model Formulation and Solution

At the beginning of the cycle, the retailer places an order of Q units of the deteriorating item. Thus, the inventory level at $t = 0$ is equal to Q. Over time, the stock in the retailer™s warehouse decreases due to customer demand as well as product deterioration. However, the impact of deterioration is partially mitigated through investment in preservation technology, which helps extend the shelf life of items. The inventory level continues to decline and eventually becomes zero at $t = t_1$. Beyond this point, from $t = T$, shortages occur, and partial backlogging is allowed. The overall dynamics of the system are depicted in Figure 1.

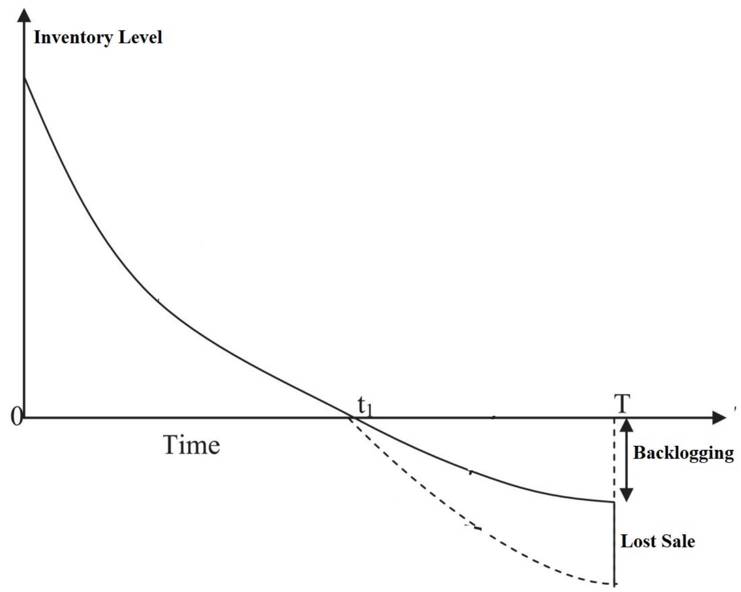


Figure 1: *Graphical illustration of the deteriorating inventory system with shortages.*

The inventory level $I(t)$ at any time t in the interval $[0, T]$ is represented by the following differential equations:

$$\frac{dI(t)}{dt} + \alpha(1 - \eta\xi)I(t) = -[(\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g)] \quad \text{for } \ 0 \leq t \leq t_1 \tag{1}$$

With the boundary conditions $I(t = 0) = Q$ and $I(t = t_1) = 0$.

$$\frac{dI_s(t)}{dt} = -[(\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g)] \cdot \left[\frac{\rho}{1 + \delta(T - t)}\right] \quad \text{for} \;\; t_1 \le t \le T \tag{2}$$

With the boundary condition $I_s(t = t_1) = 0$.
The solutions of differential equations 1 and 2 with boundary conditions are as follows:

$$I(t) = \frac{[(\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g)]}{\alpha(1 - \eta\xi)} \left(e^{\alpha(1-\eta\xi)(t_1 - t)} - 1\right) \tag{3}$$

$$I_s(t) = \frac{(\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g)\rho}{\delta} \left[\delta(t_1 - t) - \frac{\delta^2}{2}\left((T - t)^2 - (T - t_1)^2\right)\right] \tag{4}$$

Using the boundary condition $I(t = 0) = Q$ in equation (3), we get (5).

$$Q = \frac{[(\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g)]}{\alpha(1 - \eta\xi)} \left(e^{\alpha(1-\eta\xi)t_1} - 1\right) \tag{5}$$

The following are the cost elements incorporated in the sustainable supply chain inventory model:

1. **Ordering cost (OC):**

$$OC = A \tag{6}$$

2. **Deterioration cost (DC):**

$$DC = C_1 \cdot \theta(\xi) \int_0^{t_1} I(t)\, dt$$

$$DC = C_1 \cdot (\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g) \left[\frac{\left(e^{\alpha(1-\eta\xi)t_1} - 1\right)}{\alpha(1 - \eta\xi)} - t_1\right] \tag{7}$$

3. **Holding cost (HC):**

$$HC = C_2 \int_0^{t_1} I(t)\, dt$$

$$HC = C_2 \cdot \frac{[(\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g)]}{\alpha(1 - \eta\xi)} \left[\left(e^{\alpha(1-\eta\xi)t_1} - 1\right) - \alpha(1 - \eta\xi)t_1\right] \tag{8}$$

4. **Procurement cost (PC):**

$$PC = C_3 \cdot Q$$

$$PC = C_3 \cdot \frac{[(\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g)]}{\alpha(1 - \eta\xi)} \left(e^{\alpha(1-\eta\xi)t_1} - 1\right) \tag{9}$$

5. **Preservation technology investment cost (PTIC):**

$$PTIC = C_4 \cdot \xi^2 \tag{10}$$

6. **Green technology investment cost (GTIC):**

$$GTIC = C_5 \cdot g^2 \tag{11}$$

7. **Shortage cost (SC):**

$$SC = C_6 \int_{t_1}^{T} B(t) \cdot D(p, g)\, dt$$

$$SC = C_6(\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g)\rho \left[t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2} + \frac{\delta}{3}(T - t_1)^3\right] \tag{12}$$

8. **Lost sales cost (LSC):**

$$LSC = C_7 \int_{t_1}^{T} \left(1 - B(t)\right) \cdot D(p, g)\, dt$$

$$LSC = C_7(\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g) \left[(T - t_1) - \rho \left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2} + \frac{\delta}{3}(T - t_1)^3 \right) \right] \tag{13}$$

9. **Carbon emission cost (CEC):**

$$CEC = C_8 \left\{ \xi_p Q + \xi_1 \int_0^{t_1} I(t)\, dt + \xi_3 \int_{t_1}^{T} B(t) D(p, g)\, dt \right\}$$

$$\begin{aligned} CEC = C_8 \Bigg\{ & \frac{(\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g)}{\alpha(1-\eta)\xi} \left[\xi_p \left(e^{\alpha(1-\eta)t_1} - 1 \right) + \xi_1 \left(\frac{e^{\alpha(1-\eta)t_1} - 1}{\alpha(1-\eta)\xi} - 1 \right) \right] - t_1 \\ & + \xi_3 (\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g) \rho \left[t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2} + \frac{\delta}{3}(T - t_1)^3 \right] \Bigg\} \end{aligned} \tag{14}$$

The total cost per cycle (TC), along with the decision variables t_1 and T, is expressed as follows:

$$TC(t_1, T) = \frac{1}{T} \Big[OC + DC + HC + PC + PTIC + GTIC + SC + LSC + CEC \Big]$$

$$\begin{aligned} TC(t_1, T) = \frac{1}{T} \Bigg\{ & A + C_1(\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g) \left[\frac{(e^{\alpha(1-\eta)t_1} - 1)}{\alpha(1-\eta)\xi} \right] - t_1 + \frac{C_2(\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g)}{\alpha(1-\eta)\xi} \\ & + \frac{(e^{\alpha(1-\eta)t_1} - 1)}{\alpha(1-\eta)\xi} - t_1 + C_3 \left[\frac{(\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g)}{\alpha(1-\eta)\xi} \right] (e^{\alpha(1-\eta)t_1} - 1) \\ & + C_4 \cdot \xi_2^2 + C_5 \cdot g^2 \\ & + C_6 \rho (\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g) \left[t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2} + \frac{\delta}{3}(T - t_1)^3 \right] \\ & + C_7(\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g) \left[(T - t_1) - \rho \left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2} + \frac{\delta}{3}(T - t_1)^3 \right) \right] \\ & + C_8 \frac{(\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g)}{\alpha(1-\eta)\xi} \left[\xi_p (e^{\alpha(1-\eta)t_1} - 1) + \xi_1 \left(\frac{e^{\alpha(1-\eta)t_1} - 1}{\alpha(1-\eta)\xi} - 1 \right) \right] - t_1 \\ & + \xi_3 (\beta_1 - \beta_2 p)(\gamma_1 + \gamma_2 g) \rho \left[t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2} + \frac{\delta}{3}(T - t_1)^3 \right] \Bigg\} \end{aligned} \tag{15}$$

6. Fuzzy Mathematical Model Formulation and Solution

We consider the model in fuzzy environment. Due to fuzziness, it is not easy to define all the parameters precisely. Let $\widetilde{C}_1 = (C_{11}, C_{12}, C_{13}, C_{14})$, $\widetilde{C}_2 = (C_{21}, C_{22}, C_{23}, C_{24})$, $\widetilde{C}_3 = (C_{31}, C_{32}, C_{33}, C_{34})$, $\widetilde{C}_6 = (C_{61}, C_{62}, C_{63}, C_{64})$, $\widetilde{C}_7 = (C_{71}, C_{72}, C_{73}, C_{74})$ be trapezoidal fuzzy numbers in L-R form.

Then, the total cost of the system per unit time in fuzzy sense is given by

$$\begin{aligned}\widetilde{TC}_{ds}(t_1,T) = \frac{1}{T}\Bigg\{ & A + \widetilde{C}_1 \frac{(\beta_1-\beta_2 p)(\gamma_1+\gamma_2 g)}{\alpha(1-\eta\xi)}\left(\frac{e^{\alpha(1-\eta\xi)t_1}-1}{\alpha(1-\eta\xi)}\right) - t_1 \\ & + \widetilde{C}_2 \frac{(\beta_1-\beta_2 p)(\gamma_1+\gamma_2 g)}{\alpha(1-\eta\xi)}\left(\frac{e^{\alpha(1-\eta\xi)t_1}-1}{\alpha(1-\eta\xi)}\right) - t_1 \\ & + \widetilde{C}_3 \frac{(\beta_1-\beta_2 p)(\gamma_1+\gamma_2 g)}{\alpha(1-\eta\xi)}\left(\frac{e^{\alpha(1-\eta\xi)t_1}-1}{\alpha(1-\eta\xi)}\right) + C_4\,\xi^2 + C_5\,\delta^2 \\ & + \widetilde{C}_6(\beta_1-\beta_2 p)(\gamma_1+\gamma_2 g)\rho\left(\frac{t_1 T}{2}-\frac{T^2}{2}+\frac{t_1^2}{2}+\frac{\delta}{3}(T-t_1)^3\right) \\ & + \widetilde{C}_7(\beta_1-\beta_2 p)(\gamma_1+\gamma_2 g)(T-t_1)\rho\left(\frac{t_1 T}{2}-\frac{T^2}{2}+\frac{t_1^2}{2}+\frac{\delta}{3}(T-t_1)^3\right) \\ & + \widetilde{C}_8 \frac{(\beta_1-\beta_2 p)(\gamma_1+\gamma_2 g)}{\alpha(1-\eta\xi)}\left(\xi_p \frac{e^{\alpha(1-\eta\xi)t_1}-1}{\alpha(1-\eta\xi)} + \xi_h \frac{e^{\alpha(1-\eta\xi)t_1}-1}{\alpha(1-\eta\xi)}\right) - t_1 \\ & + \widetilde{C}_9\,\xi_s(\beta_1-\beta_2 p)(\gamma_1+\gamma_2 g)\rho\left(\frac{t_1 T}{2}-\frac{T^2}{2}+\frac{t_1^2}{2}+\frac{\delta}{3}(T-t_1)^3\right)\Bigg\} \end{aligned} \tag{16}$$

$$\begin{aligned}\widetilde{TC}_{ds}(t_1,T) = \frac{1}{T}\Bigg[& A + (C_{11},C_{12},C_{13},C_{14})\cdot \frac{(\beta_1-\beta_2 p)(\gamma_1+\gamma_2 g)}{\alpha(1-\eta\xi)}\left(\frac{e^{\alpha(1-\eta\xi)t_1}-1}{\alpha(1-\eta\xi)}\right) - t_1 \\ & + (C_{21},C_{22},C_{23},C_{24})\cdot \frac{(\beta_1-\beta_2 p)(\gamma_1+\gamma_2 g)}{\alpha(1-\eta\xi)}\left(\frac{e^{\alpha(1-\eta\xi)t_1}-1}{\alpha(1-\eta\xi)}\right) - t_1 \\ & + (C_{31},C_{32},C_{33},C_{34})\cdot \frac{(\beta_1-\beta_2 p)(\gamma_1+\gamma_2 g)}{\alpha(1-\eta\xi)}\left(\frac{e^{\alpha(1-\eta\xi)t_1}-1}{\alpha(1-\eta\xi)}\right) + C_4\xi^2 + C_5\delta^2 \\ & + (C_{61},C_{62},C_{63},C_{64})(\beta_1-\beta_2 p)(\gamma_1+\gamma_2 g)\rho\left(\tfrac{t_1 T}{2}-\tfrac{T^2}{2}+\tfrac{t_1^2}{2}+\tfrac{\delta}{3}(T-t_1)^3\right) \\ & + (C_{71},C_{72},C_{73},C_{74})(\beta_1-\beta_2 p)(\gamma_1+\gamma_2 g)(T-t_1)\rho\left(\tfrac{t_1 T}{2}-\tfrac{T^2}{2}+\tfrac{t_1^2}{2}+\tfrac{\delta}{3}(T-t_1)^3\right) \\ & + C_8 \frac{(\beta_1-\beta_2 p)(\gamma_1+\gamma_2 g)}{\alpha(1-\eta\xi)}\left[\xi_p\left(\frac{e^{\alpha(1-\eta\xi)t_1}-1}{\alpha(1-\eta\xi)}\right) + \xi_h\left(\frac{e^{\alpha(1-\eta\xi)t_1}-1}{\alpha(1-\eta\xi)}\right) - t_1\right] \\ & + \xi_s(\beta_1-\beta_2 p)(\gamma_1+\gamma_2 g)\rho\left(\tfrac{t_1 T}{2}-\tfrac{T^2}{2}+\tfrac{t_1^2}{2}+\tfrac{\delta}{3}(T-t_1)^3\right)\Bigg]. \end{aligned} \tag{17}$$

Let $\widetilde{TC}_{ds}(t_1,T) = (W,X,Y,Z)$. The α–cut of trapezoidal fuzzy number $\widetilde{TC}_{ds}(t_1,T)$ are $C_L(\psi)$ and $C_R(\psi)$ and are given by $C_L(\psi) = W + (X-W)\psi, \quad C_R(\psi) = Z - (Z-Y)\psi$.

The defuzzified value of the fuzzy total cost $\widetilde{TC}_{ds}(t_1,T)$, derived through the signed distance method, is expressed as:

$$\widetilde{TC}_{ds}(t_1,T) = \frac{1}{2}\int_0^1 \left[C_L(\psi)+C_R(\psi)\right]d\psi \tag{18}$$

7. Optimal Solution Procedure

The nonlinear nature of the objective function requires minimizing the total cost functions per unit time, $TC(t_1, T)$ and $\widetilde{TC}_{ds}(t_1, T)$, to identify the optimal decision variables t_1 and T. These values are obtained by solving the below equations (19), and (20). Substituting these values into equation (15), and (18) yields the minimum total cost per unit time for the system. Given the complexity of the model, an algorithmic procedure has been designed and implemented in MATLAB. The following sections report numerical examples and sensitivity analysis, along with graphical demonstrations.

$$\frac{\partial TC(t_1, T)}{\partial t_1} = 0, \qquad \frac{\partial TC(t_1, T)}{\partial T} = 0 \tag{19}$$

$$\frac{\partial \widetilde{TC}_{ds}(t_1, T)}{\partial t_1} = 0, \qquad \frac{\partial \widetilde{TC}_{ds}(t_1, T)}{\partial T} = 0 \tag{20}$$

Provided that the above total cost function in equations (15) and (18) for crisp and fuzzy model respectively, satisfies the following necessary and sufficient conditions:

$$\left(\frac{\partial^2 TC(t_1, T)}{\partial t_1^2}\right)\left(\frac{\partial^2 TC(t_1, T)}{\partial T^2}\right) - \left(\frac{\partial^2 TC(t_1, T)}{\partial t_1 \partial T}\right)^2 > 0, \quad \text{and} \quad \left(\frac{\partial^2 TC(t_1, T)}{\partial t_1^2}\right) > 0 \tag{21}$$

$$\left(\frac{\partial^2 \widetilde{TC}_{ds}(t_1, T)}{\partial t_1^2}\right)\left(\frac{\partial^2 \widetilde{TC}_{ds}(t_1, T)}{\partial T^2}\right) - \left(\frac{\partial^2 \widetilde{TC}_{ds}(t_1, T)}{\partial t_1 \partial T}\right)^2 > 0, \quad \text{and} \quad \left(\frac{\partial^2 \widetilde{TC}_{ds}(t_1, T)}{\partial t_1^2}\right) > 0 \tag{22}$$

Algorithm:

Step 1 : *Start*

Step 2 : *Input the values.*

Step 3 : *Find* $\frac{\partial TC(t_1,T)}{\partial t_1}$ and $\frac{\partial TC(t_1,T)}{\partial T}$.

Step 4 : *Solve the simultaneous equations* $\frac{\partial TC(t_1,T)}{\partial t_1}$ and $\frac{\partial TC(t_1,T)}{\partial T}$.

Step 5 : *Evaluate the value of* t_1 *and* T.

Step 6 : *Check the optimality conditions as in equation (21).*

Step 7 : *Find* $TC(t_1, T)$.

Step 8 : *Stop.*

Note that, similarly an algorithm is employed for fuzzy cost function.

8. Numerical Illustrations

To illustrate and validate the proposed model, a hypothetical system is considered by assigning specific values to different parameters. The corresponding examples are presented as follows.

Example 1 (Crisp Model)
To illustrate the proposed method, let us consider the following input data:
Let us suppose, $A = 120$ \$ per unit, $C_1 = 12$ \$ per unit, $\alpha = 0.06$, $\eta = 0.25$, $\xi = 0.32$, $\beta_1 = 110$, $\beta_2 = 10$, $\gamma_1 = 1.25$, $\gamma_2 = 0.5$, $g = 0.62$, $p = 0.08$, $C_2 = 6$ \$ per unit, $C_3 = 8.00$ \$ per unit, $C_4 = 7.15$ \$ per unit, $C_5 = 5.39$ \$ per unit, $C_6 = 9$ \$ per unit, $C_7 = 8$ \$ per unit, $C_8 = 5.82$ \$ per unit,

$\rho = 0.021$, $\delta = 0.03$.
Therefore, we obtain following optimal solutions:
$t_1 = 0.7245$ year, $T = 0.9642$ year and $TC(t_1, T) = 527.38$ \$ per cycle.

Example 2 (Fuzzy Model)
To illustrate the proposed method, let us consider the following input data:
Let us suppose, $\tilde{C}_1 = (9, 11, 13, 15)$, $\tilde{C}_2 = (3.5, 5.5, 7.5, 9.5)$, $\tilde{C}_3 = (5, 7, 9, 11)$, $\tilde{C}_6 = (6, 8, 10, 12)$ and $\tilde{C}_7 = (5.5, 7.5, 9.5, 11.5)$.
Therefore, we obtain following optimal solutions:
$t_1 = 0.6845$ year, $T = 0.9252$ year and $\widetilde{TC}_{ds}(t_1, T) = 502.48$ \$ per cycle.

9. Sensitivity Analysis

To examine the impact of variations in the system parameters, a sensitivity analysis has been conducted. The corresponding results are presented in the following tables.

Table 2: *Comparison of Optimal Results*

Model	t_1 (Year)	T (Year)	$TC(t_1, T)$ (\$)
Crisp Model	0.7245	0.9642	527.38
Fuzzy Model	0.6845	0.9252	502.48

Table 3: *Sensitivity Analysis on parameter C_1*

Defuzzify value of C_1	Fuzzify value of C_1	t_1 (Year)	T (Year)	$TC(t_1, T)$ (\$)	$\widetilde{TC}_{ds}(t_1, T)$ (\$)
10	(10.8, 13.2, 15.6, 18)	0.7349	1.1102	478.0	465.976
11	(9.9, 12.1, 14.3, 16.5)	0.7023	0.9508	497.5	482.728
12	(9, 11, 13, 15)	0.6845	0.9252	520.0	502.480
13	(8.1, 9.9, 11.7, 13.5)	0.6648	0.8827	542.5	526.232
14	(7.2, 8.8, 10.4, 12)	0.6472	0.7302	570.0	549.984

Table 4: *Sensitivity Analysis on parameter C_2*

Defuzzify value of C_2	Fuzzify value of C_2	t_1 (Year)	T (Year)	$TC(t_1, T)$ (\$)	$\widetilde{TC}_{ds}(t_1, T)$ (\$)
4	(4.2, 6.6, 9, 11.4)	0.6914	0.9502	486.0	471.976
5	(3.85, 6.05, 8.25, 10.45)	0.6897	0.9377	502.0	486.728
6	(3.5, 5.5, 7.5, 9.5)	0.6845	0.9252	520.0	502.480
7	(3.15, 4.95, 6.75, 8.55)	0.6761	0.9127	537.0	519.232
8	(2.8, 4.4, 6, 7.6)	0.6676	0.9002	556.0	537.984

Table 5: *Sensitivity Analysis on parameter C_3*

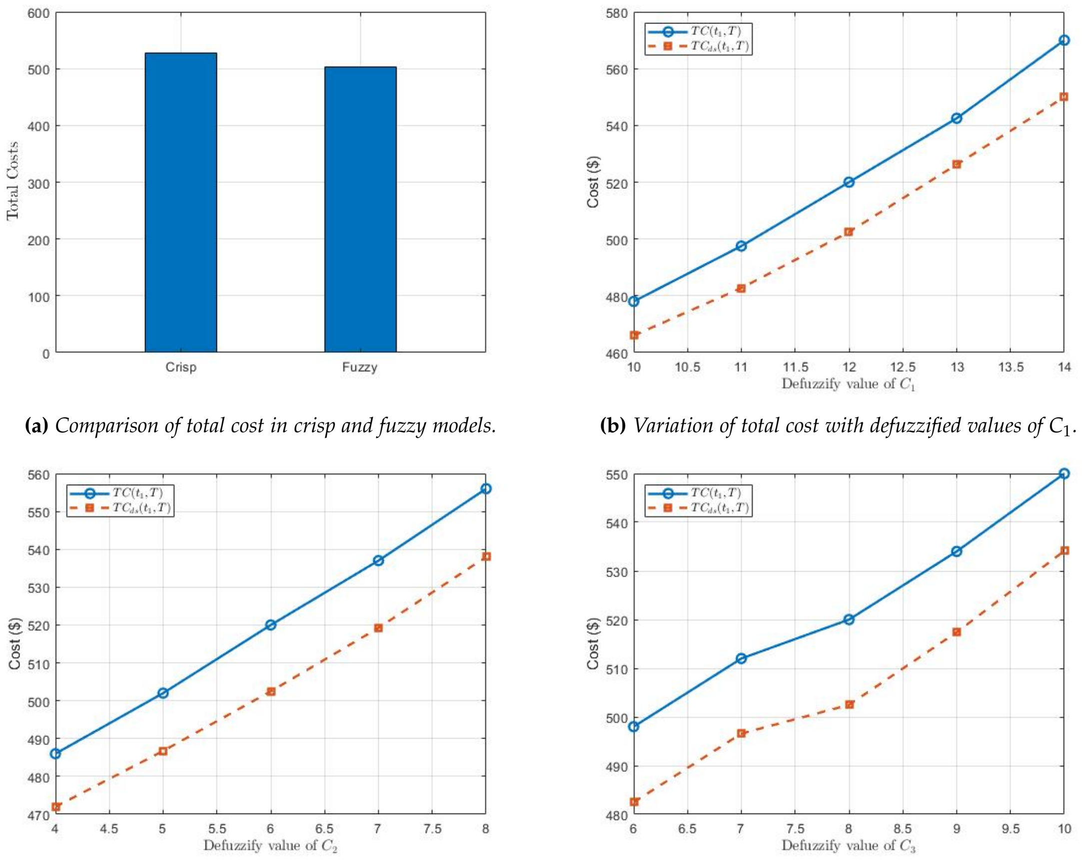
Defuzzify value of C_3	Fuzzify value of C_3	t_1 (Year)	T (Year)	$TC(t_1, T)$ (\$)	$\widetilde{TC}_{ds}(t_1, T)$ (\$)
6	(1, 3, 5, 7)	0.7128	0.9607	498.0	482.472
7	(3, 5, 7, 9)	0.6947	0.9467	512.0	496.579
8	(5, 7, 9, 11)	0.6845	0.9252	520.0	502.480
9	(7, 9, 11, 13)	0.6748	0.9129	534.0	517.405
10	(9, 11, 13, 15)	0.6676	0.8945	550.0	534.107

Table 6: *Sensitivity Analysis on parameter C_6*

Defuzzify value of C_6	Fuzzify value of C_6	t_1 (Year)	T (Year)	$TC(t_1, T)$ (\$)	$\widetilde{TC}_{ds}(t_1, T)$ (\$)
7	(2, 4, 6, 8)	0.6967	0.9402	498.0	482.472
8	(4, 6, 8, 10)	0.6804	0.9297	512.0	496.579
9	(6, 8, 10, 12)	0.6845	0.9252	520.0	502.480
10	(8, 10, 12, 14)	0.6657	0.9146	534.0	517.405
11	(10, 12, 14, 16)	0.6479	0.9047	550.0	534.107

Table 7: *Sensitivity Analysis on parameter C_7*

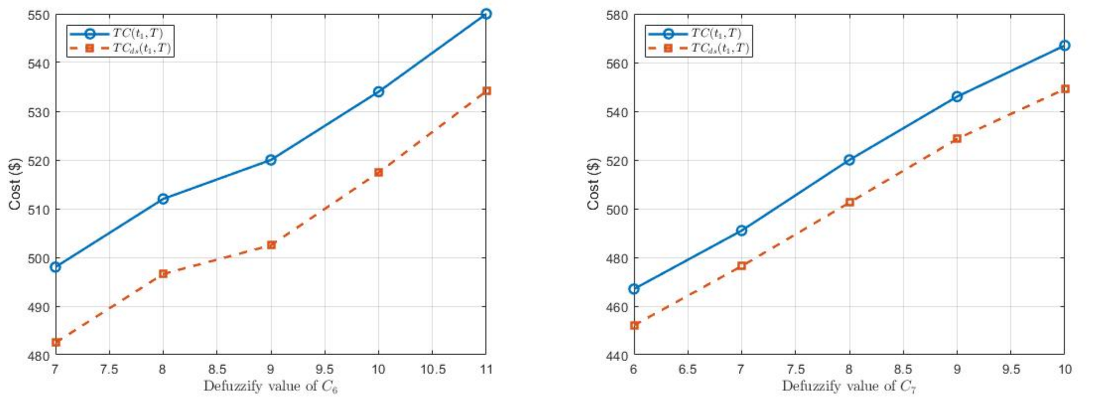
Defuzzify value of C_7	Fuzzify value of C_7	t_1 (Year)	T (Year)	$TC(t_1, T)$ (\$)	$\widetilde{TC}_{ds}(t_1, T)$ (\$)
6	(2, 4, 6, 8)	0.8469	1.4389	467.0	452.106
7	(4, 6, 8, 10)	0.7435	0.9603	491.0	476.413
8	(5.5, 7.5, 9.5, 11.5)	0.6845	0.9252	520.0	502.480
9	(8, 10, 12, 14)	0.6136	0.8531	546.0	528.649
10	(10, 12, 14, 16)	0.5619	0.7946	567.0	549.121



(a) *Comparison of total cost in crisp and fuzzy models.* **(b)** *Variation of total cost with defuzzified values of C_1.*

(c) *Variation of total cost with defuzzified values of C_2.* **(d)** *Variation of total cost with defuzzified values of C_3.*

Figure 2: *Total costs variation under crisp and fuzzy models on most sensitive parameters.*



(a) *Variation of total cost with defuzzified values of* C_6.

(b) *Variation of total cost with defuzzified values of* C_7.

Figure 3: *Total costs variation under crisp and fuzzy models on most sensitive parameters.*

The following key observations are derived from the sensitivity analysis:

1. Optimal decision times shrink under the fuzzy solution: the reorder-to-shortage instant t_1 decreases from 0.7245 to 0.6845 and the cycle length T falls from 0.9642 to 0.9252. These changes indicate that uncertainty-aware (defuzzified) solutions favour slightly shorter cycles.

2. For every parameter tested (procurement cost C_1, holding cost C_2, shortage cost C_3, preservation cost C_6, green-tech cost C_7) the total cost increases monotonically with the defuzzified parameter value, while the optimal t_1 and T decrease. This pattern is consistent across Tables 4-8.

3. The green-technology cost C_7 is the most influential. Shortage cost C_3 and preservation cost C_6 produce similar, moderate effects. These percentages are computed from the minimum-to-maximum defuzzified cost values in Tables 3-7.

4. The total cost values $TC(t_1, T)$ reported alongside the defuzzified fuzzy costs $\tilde{T}C_{ds}(t_1, T)$ are consistently higher on average TC is about 3.22% larger than the corresponding $\tilde{T}C_{ds}$ across the sensitivity experiments. This small but systematic gap highlights that defuzzified fuzzy estimates are slightly optimistic relative to the corresponding totals in the tested scenarios.

5. Graphical results show near-linear upward trends of total cost with parameter increases (especially for C_1 and C_7); any local nonlinearity is mild and does not change the monotonic character. This supports simple ranked prioritization for managerial action.

10. Conclusion and Future Research Directions

This study developed a sustainable fuzzy inventory model for deteriorating items, incorporating price and greening sensitive demand, as well as costs associated with preservation and green technology investment. The comparative assessment between crisp and fuzzy scenarios demonstrates that the fuzzy-based approach not only offers greater robustness in uncertain environments but also achieves superior economic outcomes. The fuzzy framework reduces the overall cost by 4.72% relative to the crisp model, while also shortening the cycle time and shortage interval by 4.04% and 5.52%, respectively. Sensitivity analysis indicates that green technology cost and procurement cost exert the strongest influence on the total cost, leading to variations of 21.46% and 18.03%, whereas preservation, holding, and shortage costs play comparatively moderate roles. From a managerial standpoint, the results suggest that strategic investment in advanced preservation techniques and green technologies can provide both financial savings and ecological advantages. Moreover, fuzzy-based decision frameworks enable firms to address uncertainties more effectively, improve operational reliability, and achieve sustainable supply chain outcomes. The applicability of the proposed model extends to industries where deterioration and eco-sensitivity are critical, such as food processing, pharmaceuticals, agro-products, chemicals, and healthcare supplies.

Future research could extend this work in several directions. One promising avenue is to consider probabilistic deterioration rates or product quality defects to reflect more realistic market conditions. Trade credit policies, carbon taxation, time-dependent demand, and multi-warehouse structures may also be incorporated to broaden practical relevance.

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