RT&A, No 4 (89)
Volume 20, December 2025

PERFORMANCE AND COST ANALYSIS OF A GI/M(a,b)/1 QUEUE WITH COMPULSORY AND EXTENDED REPAIRS DURING MULTIPLE WORKING VACATIONS

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Abstract

The paper analyses a single-server queuing model with general distribution arrival and batch service under multiple working vacations, incorporating both compulsory and extended repairs. Customers receive service in groups following a general bulk service approach, instead of being served one at a time. Each service batch requires a minimum of 'a' and maximum of 'b' customers. The study considers two types of repairs—compulsory and extended—and derives the steady-state equations, performance measures, and expected cost of the model. Finally, the impact of various parameters is illustrated in numerical analysis section and graphical representation

Keywords: General Distribution Arrival, Batch Service, Multiple Working Vacations, Compulsory Repair, Extended Repair, Performance Measures.

I. Introduction

The study of queuing systems with working vacations, batch service, and server repairs has attracted significant attention in recent years due to its wide range of real-world applications in manufacturing, telecommunications, healthcare systems, transportation, computer networks, cloud computing, banking services, call centers, and Maintenance & Repair Operations. Researchers have extensively examined the effect of bulk service schemes and repair mechanisms in various queuing environments with multiple working vacations.

Queueing Processes with Bulk Service is discussed in [16]. The concept of working vacations in queuing systems was introduced by [20]. Extensions of this idea were further explored by [21] who considered M/M/1 queue model with a single working vacation and analyzed system performance under varying arrival and service conditions. Non- Markovian Queuing System Mx/G/1 with Server Breakdown and Repair Times was analyzed in [11]. Cost Analysis of MAP/G(a,b)/1/N Queue with Multiple Vacations and Closedown Times, Quality Technology and Quantitative Management was examined by [19].

Queue with Breakdowns and interrupted Repairs was carried by [22]. Analysis of batch arrival bulk service queue with multiple vacation closedown essential and optional repair was examined

DOI: https://doi.org/10.24412/1932-2321-2025-489-881-894

by [3]. M/G(a,b)/1 Queueing System with Breakdown and Repair, Stand-By Server, Multiple Vacation and Control Policy on Request for Re-Service was studied in [4]. M[X]/G (a, b)/1 queue with breakdown and delay time to two phase repairs under multiple vacation was analyzed in [6]. Bulk service queue with sever breakdown and repairs was analyzed by [9].

A study on M/G/1 feedback retrial, queue with subject to server breakdown and repair under multiple working vacation policy by [17]. Batch Arrival Poisson Queue with Breakdown and Repairs was developed by [8]. A queueing model with server breakdowns, repairs, vacations, and backup server includes the work of [10] who examined systems with server breakdowns and repairs. Performance Analysis of Retrial Queueing Model with Working Vacation, Interruption, Waiting Server, Breakdown and Repair was analyzed by [12]. Analytic and Computational Analysis of GI/Ma,b/c Queueing System was examined in [15].

Steady-state distributions and performance metrics such as mean queue length, server availability and system cost in Cost optimization of a heterogeneous server queuing system with breakdown was evaluated by [1]. Phase type Queuing model of server vacation, Repair and Degrading service with Breakdown, starting failure and breakdown was studied in [5]. Strategic Analysis of Retrial Ques with setup Times, Breakdown and Repairs was developed by [18]. Performance of a queuing system with heterogeneous arrivals and various types of breakdowns under multiple working vacations was studied by [13]. Performance of Queuing Model with the Busy Period Breakdown was analyzed by [14]. Cost Analysis of a Finite capacity Queue with Server Failures, Balking and Threshold -Driven Recovery Policy was evaluated in [2]. Batch arrival bulk service priority queue with multiple vacation, close down, setup, differentiate breakdown, and repair was analyzed in [7].

Based on previous research this paper analyzes a general distribution arrival batch service queuing model incorporating compulsory and extended repairs with working vacations and server breakdown.

II. Model Description

In the proposed model, the server processes customers in batches according to the General Bulk Service Rule (GBSR), originally introduced by Neuts (1967). Under this rule, the server initiates service only when there is at least ‘a’ customer in the system, and it can serve up to a maximum of ‘b’ customers in a single service cycle. Upon completing a service, if the number of waiting customers is less than or equal to ‘b’, the server serves all of them as one batch. If the number exceeds ‘b’, only the first ‘b’ customers are selected for the next service batch. The service time for a batch of size k where a ≤ k ≤ b is assumed to be independent and identically distributed. When the system has fewer than ‘a’ customer, the server enters a working vacation mode. During this period, if the number of customers reaches or exceeds ‘a’, the server resumes service at a reduced rate, denoted by μ_v which follows an exponential distribution. If the number of customers is still below ‘a’, the server initiates another working vacation, forming a multiple working vacation (MWV) structure. Customer arrivals follow a general inter arrival (GI) distribution with arrival rate λ. The service time in the regular busy period is exponentially distributed with rate μ_r , while vacation durations service rate is exponentially distributed with rate η. Server breakdowns may occur in both operational modes, during working vacations, breakdowns follow a Poisson process with rate β_v. By considering let the breakdowns during regular busy periods occur follows at the rate of β_r . Once a breakdown occurs, the server undergoes one of two types of repair. A compulsory repair is performed immediately after the breakdown during regular busy period with a parameter ψ, regardless of system state. An extended repair is initiated during working vacation periods at parameter φ under specific conditions, such as after a failure during service continuation. Both types of repair are assumed to follow an exponential distribution with mean for compulsory and extended repairs. The system is analyzed using a Discrete-Time Markov Chain (DTMC) framework observed at pre-arrival epochs. Under these assumptions, key performance metrics such as steady-state equations, performance measures, and expected cost of the model. are derived and evaluated.

III Discrete –Time Markov Chain Pre-Arrival Queue Length

Let t_n= 1, 2, 3, 4… represent the nth customer's arrival time. If the system is initially empty, let t_0= 0. The Laplace Stieltjes Transform of G(t) is represented as G*(θ), where θ ≥ 0. The inter arrival times { t_n, n = 1,2,3, 4….} are assumed to be independent and identically distributed with a general distribution function G(t) having a mean of 1/λ We examine a GI/M (a, b)/1 queue with multiple working vacations (MWV) and two types of server breakdowns. Let $N_q(t)$ be the number of customers in the queue at time t. Let Jn(t) = 0, 1 and 2 where server is in idle vacation, working vacation and regular busy period respectively. In this model the process {(Nq (t_n - 0), Jn); n ≥ 1} is an embedded Markov chain with state space Ꞇ={(n≥0); J=1,2} U {0 ≤n ≤ a-1; J=0}.

At time t the system reaches steady state queue size probabilities given by

$$P_n^r(t)=\lim_{k\to\infty} Pr\{\text{Nq}\ (t_k-0)\ =\text{n},\ J_n\ =1\},\ \text{n}\geq 0$$

$$Q_n^v(t)=\lim_{k\to\infty} Pr\{\text{Nq}\ (t_k-0)\ =\text{n},\ J_n\ =2\},\ \text{n}\geq 0$$

$$R_n^i(t)=\lim_{k\to\infty} Pr\{\text{Nq}\ (t_k-0)\ =\text{n},\ J_n\ =0\},\ 0\leq \text{n}\leq \text{a-1}$$

Assuming the existence of steady-state probabilities P_n^r, Q_n^v *and* R_n^i which represents the probabilities that the server is in a regular busy period, working vacation period and idle respectively. Then

$$P_n^r=\lim_{t\to\infty} P_n^r(t);\ Q_n^v=\lim_{t\to\infty} Q_n^v(t);\ R_n^i=\lim_{t\to\infty} R_n^i(t)$$

During working vacation and regular busy period there will be n number of customers in the queue and the system has k (a ≤ k ≤ b) customers in service, whereas in idle period the number of customers in the queue and in the system are same.

We define b_k^r as the probability that k batches will be completed during an inter-arrival time at regular service rate μ_r as follows.

$$b_k^r\ =\int_0^\infty e^{-\mu_r t}\ \frac{(\mu_r t)^k}{k!}\ d\ G(t)\qquad\qquad k\geq 0$$

The probability that a working vacation time is longer than the interarrival time and k batches are served at a rate of μ_v during an inter-arrival time is defined as c_k^v where

$$c_k^v=\int_0^\infty e^{-\eta t}\ \ \frac{(e^{-\mu_v t})(\mu_v\ t)^k}{k!}\ dG(t)\qquad k\geq 0$$

Let d_k^{ia} represents the probability that the server comes back from vacation during the inter-arrival period and completes k services during that period at a rate of η. Then

$$d_k^{ia}=\int_0^\infty \sum_{i=0}^k \left\{\int_0^t \eta e^{-\eta x}\ \frac{(\mu_v\ x)^t}{i!}\ e^{-\mu_v\ x}\ \frac{(\mu_r\ (t-x))^{k-i}}{(k-i)!}\ e^{-\mu_r(t-x)}\ dx\right\} dG(t)\quad k\geq 0$$

Taking e_k^v as the probability that a breakdown occurs at the rate of β_v while the server is in a working vacation period. Thereafter

$$e_k^v = \int_0^\infty e^{-\beta_v t} \frac{(\beta_v t)^k}{k!} dG(t), \qquad k \geq 0$$

Assuming f_k^r represents the probability that the server experiences a breakdown at the rate of β_r during a regular busy period. Then

$$f_k^r = \int_0^\infty e^{-\beta_r t} \frac{(\beta_r t)^k}{k!} dG(t), \qquad k \geq 0$$

Based on the previously defined probabilities steady state equations are formulated and the steady state solution at the pre-arrival time is computed by applying the displacement between the Markov chain states.

IV. Steady State Equations

I. During Regular Busy Period

$$P_n^r = \sum_{k=0}^\infty P_{kb+n-1}^r (b_k^r + f_k^r + \psi) + \sum_{k=0}^\infty Q_{kb+n-1}^v (d_k^{ia}) \quad n \geq 1 \tag{1}$$

$$P_0^r = \sum_{k=1}^\infty \sum_{j=a-1}^{b-1} P_{(k-1)b+j}^r (b_k^r + f_k^r + \psi) + \sum_{k=0}^\infty \sum_{j=a-1}^{b-1} Q_{(k-1)b+j}^v (d_k^{ia}) + R_{a-1}^i d_0^{ia} \tag{2}$$

II. During Working Vacation Period

$$Q_n^v = \sum_{k=0}^\infty Q_{kb+n-1}^v (c_k^v + e_k^v + \varphi) \qquad n \geq 1 \tag{3}$$

$$Q_0^v = \sum_{k=1}^\infty \sum_{j=a-1}^{b-1} Q_{(k-1)b+j}^v (c_k^v + e_k^v + \varphi) + R_{a-1}^i c_0^v \tag{4}$$

III. During Idle Period

$$R_n^i = R_{n-1}^i + \sum_{k=0}^\infty Q_{kb+n-1}^v (1 - \sum_{i=0}^k (c_i^v + d_i^{ia} + e_i^v + \varphi)) + \sum_{k=0}^\infty P_{kb+n-1}^r (1 - \sum_{i=0}^k (b_i^r + f_i^r + \psi)) \qquad 1 \leq n \leq a-1 \tag{5}$$

$$R_0^i = \sum_{k=1}^\infty \sum_{j=a-1}^{b-1} Q_{(k-1)b+j}^v (1 - \sum_{i=0}^k (c_i^v + d_i^{ia} + e_i^v + \varphi + \sum_{k=1}^\infty \sum_{j=a-1}^{b-1} P_{(k-1)b+j}^r (1 - \sum_{i=0}^k (b_i^r + f_i^r + \psi))) + R_{a-1}^i (1 - (d_0^{ia} + c_0^v)) \tag{6}$$

Steady state solution is obtained with the help of the steady state equation.

V. Steady State Solution

Steady state solution is obtained by using forward shifting operator E on P_n^r *and* Q_n^v.

$$E(P_n^r) = P_{n+1}^r \quad \text{and} \quad E(Q_n^v) = Q_{n+1}^v$$

Hence the Equation (1) gives the non- homogeneous equation:

$$(E - \sum_{k=0}^{\infty} E^{kb}(b_k^r + f_k^r + \psi)) \ P_n^r = \sum_{k=0}^{\infty} Q_{kb+n}^v \ (d_k^{ia}) \qquad n \geq 0 \qquad (7)$$

Where the Equation (3) gives the homogeneous equation:

$$(E - \sum_{k=0}^{\infty} E^{kb} \ (c_k^v + e_k^v + \varphi)) \ Q_n^v = 0 \qquad , \ n \geq 0 \qquad (8)$$

Now to obtain P_n^r, Q_n^v *and* R_n^i, we consider $B^r(z^b)$, $C^v(z^b)$, $E^{Bv}(z^b)$ *and* $F^{Br}(z^b)$ to express the Probability Generating Function of b_k^r , c_k^v , e_k^v *and* f_k^r as follows:

$$B^r(z^b) = \sum_{k=0}^{\infty} b_k^r \, z^{kb} = G^*(\mu_r(1 - z^b)) \qquad (9)$$

$$C^v(z^b) = \sum_{k=0}^{\infty} c_k^v \, z^{kb} = G^*\big(\eta + \mu_{\ v}(1 - z^b)\big) \qquad (10)$$

$$E^{Bv}(z^b) = \sum_{k=0}^{\infty} e_k^v \, z^{kb} = G^*(\eta + \beta_v(1 - z^b)) \qquad (11)$$

$$F^{Br}(z^b) = \sum_{k=0}^{\infty} f_k^r \, z^{kb} = G^*(\beta_r(1 - z^b)) \qquad (12)$$

If $\rho = (\frac{\lambda}{b\mu}) < 1$, then the equation z= B (zb) has a unique root r_R inside (0,1). This result follows from the corresponding result of GI/M/1 model of Gross and Harris (1985), B^r (r_Rb) = r_R with 0< r_R< 1.
If η > 0 then the equation $G^*\big(\eta + \mu_v(1 - z^b)\big)$ has a unique root r_V in the interval (0,1), $(i.e)$
C^v $({r_V}^b) = r_V$ with 0< r_v <1
If $0 < \beta_v < 1$ then the equation $E(z^b) =$ z has unique root $r_{\beta V}$ inside (0,1) then we have
E^{Bv} $\big(r_{\beta v}$ b$\big) = \beta_v$ with 0< β_v <1.
If $0 < \beta_r < 1$ then the equation F$(z^b) =$ z has unique root $r_{\beta R}$ inside (0,1) then we have
F^{Br} $\big(r_{\beta R}$ b$\big) = \beta_r$ with 0< β_r <1.

Hence the homogeneous difference equation (7) has a solution

$$Q_n^v = {r_v}^n \, Q_0^v \qquad n \geq 0 \qquad (13)$$

And the non-homogeneous difference equation (8) has a solution as

$$P_n^r = (L \, {r_R}^n + M \, {r_V}^n) \, Q_0^v \qquad n \geq 0 \qquad (14)$$

where $M = \frac{D({r_V}^b)}{r_V - B({r_V}^b) - E({r_V}^b)}$ and $D{r_V}^b = \sum_{k=0}^{\infty} {r_V}^{kb} \ (d_k^{ia}) = M(r_V - B \ ({r_V}^b) - E({r_V}^b))$

(15)

Thus we get $P_n^r = \left[L{r_R}^n + \left(\sum_{k=0}^{\infty} {r_V}^{kb} \ (d_k^{ia}) \frac{{r_V}^n}{r_V - B({r_V}^b) - E({r_V}^b)}\right] Q_0^v \qquad where \ r_V \neq r_R$ (16)

The expression R_n^i is obtained by substituting Q_n^v and P_n^r from equations (13) & (14)

$$R_n^i = \left[L \left(\frac{1 - r_R - \beta_r + \psi}{1 - r_R}\right)\left(\frac{{r_R}^{a-1} - {r_R}^n}{1 - {r_R}^b}\right) + (M+1)\left(\left(\frac{1 - r_V - \beta_v + \varphi}{1 - r_V}\right)\left(\frac{{r_V}^{a-1} - {r_V}^n}{1 - {r_V}^b}\right)\right) + \frac{{r_V}^{a-1} - {r_V}^n}{{r_V}^b \, (1 - r_V)} + \frac{{r_V}^b - {r_V}^a}{c_0^v \ {r_V}^b \, (1 - r_V)} \right] Q_0^v \qquad 0 \leq n \leq a-1 \qquad (17)$$

Thus, the steady state queue size probabilities are expressed in terms of Q_0^v and L.

Now L can be determined by substituting Q_n^v and P_n^r in Equation (2):
Thus, we have

$$L \left(\frac{{r_R}^a - {r_R}^b + [(\beta_r + \psi) + (b_0^r - (\beta_r + \psi))]({r_R}^b - {r_R}^{a-1})}{{r_R}^b(1 - r_R)}\right) = \frac{d_0^{ia}}{c_0^v} \left(\frac{{r_V}^a - {r_V}^b}{{r_V}^b(1 - r_V)}\right) - M \left(\frac{{r_V}^a - {r_V}^b + [(\beta_r + \psi) + (b_0^r - (\beta_r + \psi))]({r_V}^b - {r_V}^{a-1})}{{r_V}^b(1 - r_V)}\right) \qquad (18)$$

Hence the steady state queue size probabilities of described model are expressed in terms of Q_0^v are becomes

$$Q_n^v = {r_V}^n Q_0^v \qquad n \geq 0 \tag{19}$$

$$P_n^r = (L{r_R}^n + M\,{r_V}^n)\,Q_0^v \qquad n \geq 0 \tag{20}$$

$$R_n^i = \left[L\left(\frac{1-r_R-\beta_r}{1-r_R}\right)\left(\frac{{r_R}^{a-1}-{r_R}^n}{1-{r_R}^b}\right) + (M+1)\left(\left(\frac{1-r_V-\beta_v}{1-r_V}\right)\left(\frac{{r_V}^{a-1}-{r_V}^n}{1-{r_V}^b}\right)\right) + \frac{{r_V}^{a-1}-{r_V}^n}{{r_V}^b(1-r_V)} + \frac{{r_V}^b-{r_V}^a}{c_0^v\ {r_V}^b\ (1-r_V)} \right] Q_0^v$$

$$0 \leq n \leq a-1 \tag{21}$$

where;

$$L = \left(\frac{{r_R}^b(1-r_R)}{({r_R}^a-{r_R}^b)+[(\beta_r+\psi)+((b_0^r-(\beta_r+\psi))]({r_R}^b-{r_R}^{a-1})}\right)\left[\frac{d_0^{ia}}{c_0^v}\left(\frac{{r_V}^a-{r_V}^b}{{r_V}^b(1-r_V)}\right)+\beta_v\left(\frac{{r_V}^{a-1}-{r_V}^b}{{r_V}^b(1-r_V)}\right) - M\left(\frac{({r_V}^a-{r_V}^b)+[(\beta_r+\psi)+(b_0^r-(\beta_r+\psi))]({r_V}^b-{r_V}^{a-1})}{{r_V}^b(1-r_V)}\right)\right]$$

$$M = \frac{\sum_{k=0}^{\infty} {r_V}^{kb}\ d_k^{ia}}{\beta_v+\varphi} = \left(\frac{1}{\beta_v+\varphi}\right)\left[\frac{\lambda\eta}{\left(\frac{\mu_r}{\lambda+\mu}-\frac{\mu_v}{\lambda+\mu_v+\eta}\right)}\left(\frac{\mu_r}{\lambda+\mu_r-{r_v}^b\mu_r}-\frac{\mu_v}{\lambda+\mu_r+\eta-{r_v}^b\mu_v}\right)\right]$$

$$d_0^{ia} = \lambda\eta\ ;\ ;\ c_0^v = \frac{\lambda}{\lambda+\eta+\mu_v};\ b_0^r = \frac{\lambda}{\lambda+\beta_v}$$

By using normalizing conditions, the value of Q_0^v is calculated.

(i.e.) $\sum_{n=0}^{\infty} P_n^r + \sum_{n=0}^{\infty} Q_n^v + \sum_{n=0}^{\infty} R_n^i = 1$

Hence, $$(Q_0^v)^{-1} = L\left(h(r)\right) + (R+1)h(r_v) + \frac{1}{{r_V}^b(1-r_V)}\left(\frac{{r_V}^b-{r_V}^a}{c_0^v} + ({r_V}^{a-1}-{r_V}^b)\right) \tag{22}$$

where $h(x) = \left[\frac{1}{1-x^b}\left(\frac{1-x-\beta_r}{1-x}\right)\right]\left(\frac{x^a-x^b}{1-x}+x^{a-1}\right)$

Thus, the steady state queue size probabilities for the general arrival bulk service with types break down are given by

$$Q_n^v = {r_V}^n\ Q_0^v \qquad n \geq 0$$

$$P_n^r = (L{r_R}^n + M\,{r_V}^n)\,Q_0^v$$

$$R_n^i = \left[L\left(\frac{1-r_R-\beta_r+\psi}{1-r_R}\right)\left(\frac{{r_R}^{a-1}-{r_R}^n}{1-{r_R}^b}\right) + (M+1)\left(\left(\frac{1-r_V-\beta_v+\varphi}{1-r_V}\right)\left(\frac{{r_V}^{a-1}-{r_V}^n}{1-{r_V}^b}\right)\right) + \frac{{r_V}^{a-1}-{r_V}^n}{{r_V}^b(1-r_V)} + \frac{{r_V}^b-{r_V}^a}{c_0^v{r_V}^b(1-r_V)}\right] Q_0^v$$

$$0 \leq n \leq a-1$$

where Q_0^v is determined using equation (17).

VI. Performance Measures

I. Mean Queue Length

The mean queue length L_q of the model is as follows:

$$L_q = \sum_{n=0}^{\infty} nP_n^r + \sum_{n=0}^{\infty} nQ_n^v + \sum_{n=0}^{a-1} nR_n^i$$

By substituting P_n^r, Q_n^v *and* R_n^i it is termed that

$$L_q = \left[L(\xi(r_R)) + (M+1)(\xi(r_V)) + \frac{a(a-1)}{2}\left(\frac{{r_V}^{a-1}-{r_V}^b}{{r_V}^b(1-r_V)} + \frac{{r_V}^b-{r_V}^a}{c_0^v{r_V}^b(1-r_V)}\right)\right] Q_0^v \tag{23}$$

where $\xi(x) = \frac{x}{(1-x)^2} + \frac{1-x-\beta_r+\psi}{(1-x)(1-x^b)}\left(\frac{a(a-1)x^{a-1}}{2} + \frac{ax^a(1-x)-x(1-x^a)}{(1-x)^2}\right)$ and $c_0^v = \frac{\lambda}{\lambda+\eta+\mu_v}$

II. Probability Measures

Let P^v , P^b , P^i denote the probability that the server is vacation, busy, and idle respectively. Then we have,

$$P^v = \sum_{n=0}^{\infty} r_V{}^n \; Q_0^v = \left(\frac{1}{1-r_V}\right) Q_0^v$$

$$P^b = \sum_{n=0}^{\infty} \left(L\, r_R{}^n + M\, r_V{}^n\right) Q_0^v = \left(\frac{L}{1-r_R} + \frac{M}{1-r_V}\right) Q_0^v$$

$$P^i = \sum_{n=0}^{a-1} R_n^i$$

$$= \Bigg(L\left(\frac{1-r_R-\beta_r+\psi}{1-r_R}\right)\left(\frac{r_R{}^{a-1}}{1-r_R{}^b}\right)+(M+1)\left(\frac{1-r_V-\beta_v+\varphi}{1-r_V}\right)\left(\frac{r_V{}^{a-1}}{1-r_V{}^b}\right)+\frac{r_V{}^{a-1}-r_V{}^n}{r_V{}^b\,(1-r_V)}+\frac{r_V{}^b-r_V{}^a}{c_0^v\; r_v{}^b\,(1-r_V)}-$$

$$L\left(\frac{1-r_R-\beta_r+\psi}{(1-r_R)(1-r_R{}^b)}\right)\left(\frac{1-r_R{}^a}{1-r_R}\right)-(M+1)\left(\frac{1-r_V-\beta_v+\varphi}{(1-r_V)(1-r_V{}^b)}\right)\left(\frac{1-r_V{}^a}{1-r_V}\right)\Bigg)\; Q_0^v$$

VII. Sensitivity Analysis

Steady state probability values are computed based on the derived equations. These probabilities are systematically calculated and tabulated to provide a clear view of the systems behavior under the varying parameters. Additionally, graphical representations have been developed to visually illustrate the trends and variation's in probability values.

Based on the different values by considering $\lambda = 0.3$, $\mu_r = 2$, $\mu_v = 0.6$ and η = 0. 05 and by varying batch size of "a" from 5 to 10, φ assumes the values from 0.02 to 0.05, ψ ranges from 0.1 to 0.5 probability values of the prescribed model is tabulated and graphically represented in Table 1 to 6 and Figure 1.

Table 1: *System size probabilities w.r.t φ and batch size 5*

φ	a	b	Ψ	Pv	Pb	Pi
			0.1	0.3542	0.3875	0.3483
			0.2	0.3389	0.4139	0.3362
0.02	5	12	0.3	0.3334	0.4235	0.3211
			0.4	0.3305	0.4285	0.3205
			0.5	0.3287	0.4315	0.3217
			0.1	0.3928	0.3507	0.3429
			0.2	0.3769	0.3761	0.333
0.03	5	12	0.3	0.371	0.3855	0.3294
			0.4	0.368	0.3903	0.3275
			0.5	0.3662	0.3932	0.3264
			0.1	0.4155	0.3619	0.3158
			0.2	0.3992	0.3876	0.3126
0.04	5	12	0.3	0.3933	0.389	0.3113
			0.4	0.3902	0.4019	0.2185
			0.5	0.3883	0.4049	0.2105
			0.1	0.4304	0.3238	0.3412
			0.2	0.414	0.3483	0.3315
0.05	5	12	0.3	0.4079	0.3573	0.3279
			0.4	0.4048	0.362	0.3261
			0.5	0.4029	0.3649	0.3205

Table 2: *System size probabilities w.r.t φ and batch size 6*

φ	a	b	Ψ	Pv	Pb	Pi
			0.1	0.3969	0.3619	0.3412
			0.2	0.3809	0.3876	0.3315
0.02	6	12	0.3	0.375	0.3970	0.3279
			0.4	0.372	0.4019	0.3261
			0.5	0.3701	0.4049	0.3205
			0.1	0.4371	0.3378	0.3252
			0.2	0.4206	0.3628	0.3167
0.03	6	12	0.3	0.4145	0.3720	0.3135
			0.4	0.4113	0.3767	0.3119
			0.5	0.4094	0.3797	0.3109
			0.1	0.4437	0.3483	0.308
			0.2	0.4375	0.3573	0.3051
0.04	6	12	0.3	0.4343	0.3620	0.3036
			0.4	0.4324	0.3649	0.3027
			0.5	0.4756	0.3147	0.3098
			0.1	0.4526	0.3478	0.2996
			0.2	0.4494	0.3524	0.2982
0.05	6	12	0.3	0.4474	0.3552	0.2974
			0.4	0.4371	0.3378	0.3252
			0.5	0.4206	0.3628	0.3167

Table 3: *System size probabilities w.r.t φ and batch size 7*

φ	a	b	Ψ	Pv	Pb	Pi
			0.1	0.4343	0.3394	0.3263
			0.2	0.4178	0.3645	0.3177
0.02	7	12	0.3	0.4118	0.3737	0.3145
			0.4	0.4086	0.3785	0.3129
			0.5	0.4067	0.3814	0.3119
			0.1	0.4753	0.3148	0.3099
			0.2	0.4585	0.3390	0.3025
0.03	7	12	0.3	0.4523	0.3479	0.2997
			0.4	0.4491	0.3526	0.2983
			0.5	0.4471	0.3554	0.2975
			0.1	0.482	0.3243	0.2937
			0.2	0.4757	0.3331	0.2912
0.04	7	12	0.3	0.4725	0.3376	0.2899
			0.4	0.4705	0.3404	0.2891
			0.5	0.5141	0.2916	0.2944
			0.1	0.491	0.3234	0.2856
			0.2	0.4877	0.3278	0.2844
0.05	7	12	0.3	0.4858	0.3306	0.2837
			0.4	0.4343	0.3394	0.3263
			0.5	0.4178	0.3645	0.3177

Table 4: *System size probabilities w.r.t φ and batch size 8*

φ	a	b	Ψ	Pv	Pb	Pi
			0.1	0.4674	0.3196	0.313
			0.2	0.4506	0.3440	0.3054
0.02	8	12	0.3	0.4445	0.3529	0.3026
			0.4	0.4413	0.3576	0.3011
			0.5	0.4393	0.3604	0.3002
			0.1	0.5087	0.2948	0.2965
			0.2	0.4918	0.3182	0.29
0.03	8	12	0.3	0.4856	0.3268	0.2876
			0.4	0.4823	0.3313	0.2864
			0.5	0.4803	0.3341	0.2856
			0.1	0.5153	0.3034	0.2812
			0.2	0.5091	0.3119	0.279
0.04	8	12	0.3	0.5059	0.3163	0.2779
			0.4	0.5039	0.3189	0.2772
			0.5	0.5473	0.2716	0.2811
			0.1	0.5243	0.3022	0.2735
			0.2	0.5211	0.3065	0.2724
0.05	8	12	0.3	0.5191	0.3091	0.2717
			0.4	0.4674	0.3196	0.313
			0.5	0.4506	0.3440	0.3054

Table 5: *System size probabilities w.r.t φ and batch size 9*

φ	a	b	Ψ	Pv	Pb	Pi
			0.1	0.4968	0.3019	0.3013
			0.2	0.4799	0.3256	0.2945
0.02	9	12	0.3	0.4737	0.3343	0.2919
			0.4	0.4705	0.3389	0.2906
			0.5	0.4685	0.3417	0.2898
			0.1	0.538	0.2772	0.2848
			0.2	0.5212	0.2998	0.279
0.03	9	12	0.3	0.515	0.3081	0.2769
			0.4	0.5118	0.3125	0.2758
			0.5	0.5098	0.3151	0.2751
			0.1	0.5613	0.2632	0.2755
			0.2	0.5447	0.2851	0.2703
0.04	9	12	0.3	0.5385	0.2932	0.2683
			0.4	0.5352	0.2974	0.2673
			0.5	0.5333	0.3000	0.2667
			0.1	0.5763	0.2542	0.2695
			0.2	0.5598	0.2756	0.2646
0.05	9	12	0.3	0.5536	0.2836	0.2628
			0.4	0.5504	0.2877	0.2619
			0.5	0.5484	0.2903	0.2613

Table 6: *System size probabilities w.r.t φ and batch size 10*

φ	a	b	Ψ	Pv	Pb	Pi
			0.1	0.5326	0.3059	0.2577
			0.2	0.5300	0.3085	0.2571
0.02	10	12	0.3	0.5274	0.3111	0.2565
			0.4	0.5248	0.3137	0.2559
			0.5	0.4982	0.3248	0.2872
			0.1	0.5066	0.3319	0.2517
			0.2	0.5040	0.3345	0.2511
0.03	10	12	0.3	0.5014	0.3371	0.2505
			0.4	0.4988	0.3397	0.2499
			0.5	0.4962	0.3423	0.2493
			0.1	0.4806	0.3579	0.2457
			0.2	0.4780	0.3605	0.2451
0.04	10	12	0.3	0.4754	0.3631	0.2445
			0.4	0.4728	0.3657	0.2439
			0.5	0.4702	0.3683	0.2433
			0.1	0.4546	0.3839	0.2397
			0.2	0.4520	0.3865	0.2391
0.05	10	12	0.3	0.4494	0.3891	0.2385
			0.4	0.4468	0.3917	0.2379
			0.5	0.4442	0.3943	0.2373

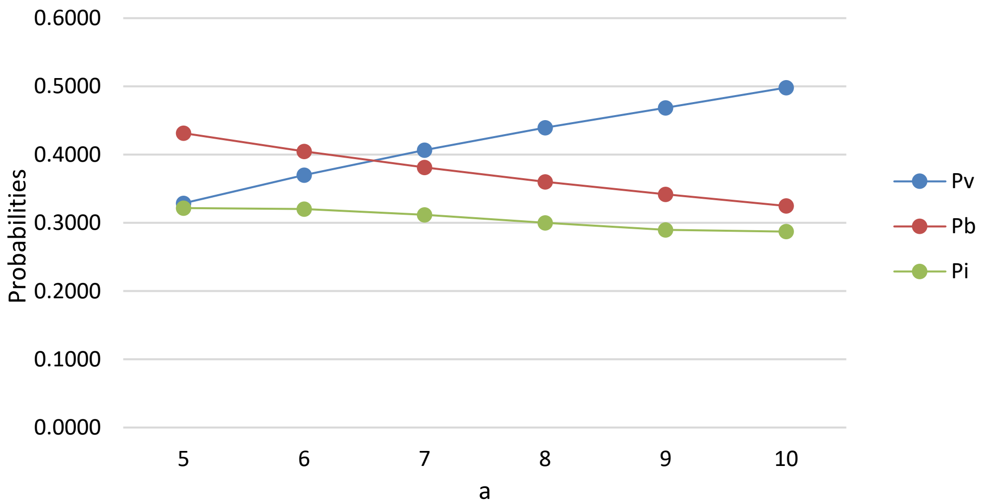

Figure 1: *System size probabilities w.r.t batch size 'a' where* P_v , P_b *and* P_i *are probabilities during working vacation Regular busy period and idle*

The graphical comparison states that as repair efficiency improves and batch size increases, the server transitions from idle and busy states into a more sustained working vacation mode. This shift helps to balance service continuity and repair flexibility, there by influencing overall system performance and expected cost. These finding emphasize the importance of tuning the system parameters to achieve optimal state probabilities for efficient queuing operations.

I. Expected cost

The expected cost (EC) for the prescribed model is formulated by combining various cost components associated with system behavior along with compulsory and extend repair for batch size of a= 5,6,7,8,9,10. As the total expected cost includes costs due to the customer waiting, server idle, during regular busy, working vacation, system breakdown, and repair rate during working vacation and regular busy periods. The expected cost is expressed as:

$$EC = C^{v}\ P^{v} + \ C^{b}\, P^{b} + C^{i}\ P^{i} + \ C^{BV}\, \beta_{v} + \ C^{BR}\, \beta_{r} + \ C^{CR}\ \ \psi^{b} + \ C^{EWV}\ \ \varphi^{v} + \ C^{LQ}\, L_{q}$$

where: C^{v}: Cost per unit time during working vacation period, P^{v}: Probability that the system is in working vacation, C^{b}: Cost incurred during Regular busy period, P^{b}: Probability that the system is in Regular busy period, C^{i}: Cost during the server is in idle P^{i}: Probability that the server is idle, C^{BV}: Breakdown cost during working vacation, β_{v}: Breakdown probabilities during working vacation C^{BR}: Breakdown cost during regular busy, β_{r}: Breakdown probabilities during regular busy, C^{CR}: Cost of compulsory repair during regular busy period, , ψ^{b}: probability of compulsory repair, C^{EWV}: Cost of extended repair during working vacation period, φ^{v}: probability of extended repair, C^{LQ}: Cost per customer waiting in the queue L_{q} (i.e.) Expected queue length.

By letting $\lambda = 0.3$, $\mu_{R} = 2$, $\mu_{v} = 0.6$ and η = 0. 05 and by varying batch size of "a" from 5 to 10, $C^{v} = 5$, $C^{b} = 15$, $C^{i} = 5$, $C^{BV} = 200$, $C^{BR} = 250$, $C^{CR} = 175$, $C^{EWV} = 150$, $C^{LQ} = 25$, and for various values of ψ *and* φ the expected cost is calculated and tabulated in Table 7.

Table 7: *Expected cost for GI/M(a,b)/1/MWV with respect compulsory repair ψ and extended repair φ*

Φ	A	B	ψ	EC
			0.1	1658.861
			0.2	1554.603
0.02	5	12	0.3	1542.931
			0.4	1549.826
			0.5	1536.346
			0.1	1343.860
			0.2	1337.118
0.03	6	12	0.3	1334.684
			0.4	1331.417
			0.5	1328.898
			0.1	1250.039
			0.2	1254.183
0.04	7	12	0.3	1258.806
			0.4	1262.754
			0.5	1266.862
			0.1	1368.284
			0.2	1372.559
0.05	8	12	0.3	1376.164
			0.4	1381.260
			0.5	1386.657
			0.1	1387.569
			0.2	1398.217
0.06	9	12	0.3	1408.865
			0.4	1419.513
			0.5	1430.161
			0.1	1452.369
			0.2	1463.017
0.07	10	12	0.3	1471.665
			0.4	1484.313
			0.5	1490.961

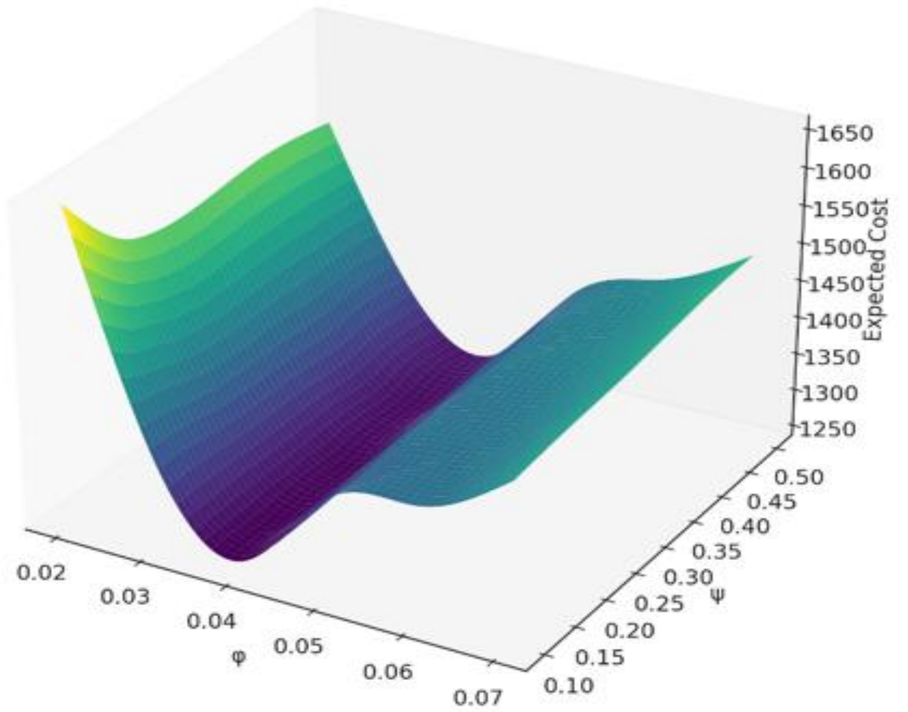

Figure 2: *Expected cost for GI/M(a,b)/1/MWV with respect to compulsory repair ψ and extended repair φ*

From the tabulated values and from the Figure 2, it is observed that the expected cost initially decreases as the repair rate increases. this is due to faster recovery from the breakdowns, leading to reduced waiting time s and improve server availability. However, beyond a certain point, further increases in the repair rate leads to a rise in the expected cost. this happens since very high repair rates along with larger batch sizes can make the system more complicated to manage and increase the overall cost. As both the repair rate and batch size increase simultaneously the system handles more customers per cycle which can introduce new delays or recourse strain. Therefore, the expected cost shows nonlinear U – shaped behavior, indicating that there exists an optimal range of repair rate and batch size where the system performs most cost effectively. Hence from φ is 0.04, EC equals 1250.039 we can conclude that the expected cost increases as the repair rate increases. This means that once the repair rate goes beyond 0.04, making repairs faster does not reduce the cost—instead, it causes the cost to go up. so, φ = 0.04 seems to be the optimal point, beyond which further increasing the repair rate becomes less efficient and costlier.

VIII. Conclusion

The analysis of the prescribed queuing model incorporating multiple working vacations, compulsory and extended repairs, and batch service reveal important trends in both system behavior and cost performance. The study shows that as key parameters such as repair rate and batch size vary, the systems operational states shift significantly. The probability of the working vacation state increases while the probabilities of the regular busy and idle states decrease indicating that the server remains more engaged in partial service modes under higher efficiency. In terms of cost analysis, the expected cost initially decreases with increase repair rate due to the improved service reliability and reduced downtime. However, beyond a certain threshold further increase in repair rate and batch sizes results in raising expected costs, highlighting the tradeoff between service speed and operational complexity. This nonlinear behavior underlines the need to identify an optimal range of parameter values that minimizes system cost while maintaining performance. Hence this study results in understanding how the system behaves under different operating conditions, providing the guidance for designing and managing efficient queuing systems. In future the study can be extended to reduce the expected cost for the increase of repair rate, batch size and balking so as to maintain the service mechanism.

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