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# FM/FM/1 QUEUE WITH N-POLICY TWO-PHASE,SERVER START-UP,TIME-OUT AND BREAKDOWNS USING L-R METHOD

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**Abstract**

*This study centers on evaluating the performance metrics of the system,specifically emphasizing two key elements:the expected length of the system (L) and the minimum anticipated cost T(N) derived from the optimal strategy analysis of the N-policy L-R(Left-Right),FM/FM/1 two-phase queueing model.This system incorporates essential factors such as server start-up,time-out,breakdown, and repair processes,making the analysis more comprehensive and reflective of real-world operational challenges.The arrival rate and service rate in this model are represented as triangular fuzzy numbers,which provide a more flexible and realistic approach in capturing the uncertainty and variability inherent in these rates.The study employs the L-R method,a technique known for its efficiency and simplicity when compared to the more traditional alpha-cuts method.The L-R method minimizes computational complexity and enhances the ease of differentiation, making it a more convenient approach for handling fuzzy numbers in queueing theory.By applying this method,the study effectively addresses the limitations of previous models and provides a more accurate estimation of the system's performance metrics.To demonstrate the validity and applicability of the proposed model,a numerical example is provided.This example illustrates the practicality of the L-R method in optimizing the system's cost and performance,showcasing how the model can be applied to real-world scenarios.The results of the numerical example confirm the robustness and reliability of the proposed approach,making a strong case for its adoption in queueing system analysis.This study thus offers significant insights and a novel methodological contribution to the field of fuzzy queueing systems.*

**Keywords:** N-policy fuzzy queue,Performance measures,L-R method,Triangular fuzzy numbers.

## 1. Introduction

Queueing theory examines how people,tasks, or information pass through waiting lines.By understanding congestion and its root causes in various situations,it becomes possible to develop services and systems that are more efficient and economical.Typical examples include ration depots,postal services,medical stores, and vaccination clinics.In this paper,we investigate a two-phase N-policy FM/FM/1 fuzzy queuing system that includes several important operational characteristics such as server start-up delay,time-out,breakdown, and repair processes.The system operates under a fuzzy environment,where both arrival and service processes follow flexible Markovian(FM) distributions.These fuzzy parameters allow for better modeling of real-world


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uncertainty and vagueness in service systems.The server does not start operating immediately when the system becomes idle;instead,it waits until a certain number of customers,N,accumulate before starting up.We apply the L-R(Left-Right combination) method to analyze the fuzzy behavior of the queuing system,allowing us to derive approximate measures for performance characteristics such as average queue length,waiting time, and system availability.This approach is useful for systems that cannot be effectively modeled using crisp values due to uncertain operating conditions.The results of this study can be applied to various practical fields including production systems,computer networks, and service industries,helping in better system planning and efficient resource utilization under uncertainty.

Explores an N-policy two-phase $M/E_k/1$ queueing system,addressing server breakdowns and gating mechanisms, and discusses optimal operational strategies [12].Examines an N-policy two-phase $M_X/M/1$ queueing system considering server startup delays and breakdowns,providing insights into optimal control strategies [13].

Fuzzy logic was first introduced [1].Since then,fuzzy queueing systems have been studied extensively by various researchers conducted a fuzzy analysis of a bulk arrival two-phase retrial queue that includes vacation and admission control policies,demonstrating the use of fuzzy set theory in intricate queueing systems [2-3,4,14,16].Similarly,an FM/FM/C queueing model using a fuzzy framework integrated with parametric nonlinear programming,offering techniques relevant to fuzzy queueing analysis [15].

Representing and analyzing fuzzy numbers,especially triangular or trapezoidal fuzzy values,in a systematic and mathematically manageable way [5].Examined a single-server vacation queueing system variant involving uncertain or imprecise parameters [7].The characteristics of N-policy fuzzy queue under vogue data [6].Derive and analyze variance-based performance measures for an FM/FM/1 queueing system using crisp (non-fuzzy) values,with the goal of obtaining more precise and quantitative insights into the behavior of the system [8].Analyzes an N-policy FM/FM/1 vacation queueing system incorporating server start-up and time-out, employing the L-R method for performance evaluation [9].Introduce the L-R method for computing performance measures in fuzzy queueing systems, demonstrating its advantages over traditional alpha-cuts method[10].Analyzed bulk arrival queuing model with flow communication network using L-R method.A significant number of these studies focus on determining system performance metrics using the alpha-cuts technique [11].Here we calculate the expected system length L and the minimum expected cost T(N*) in the fuzzy queue by the L-R method, especially based on the L-R fuzzy arithmetic.

## 2. Methods

### 2.1. Preliminaries

A fuzzy set is a set (which often does not need to be empty) having degrees of membership between 0 and 1.The membership function $\varphi_A(x) : A \to [0,1]$ is the membership function, and $\varphi_A(x)$ is the grading value called as real numbers between 0 and 1 (covered within 0 and 1).

A fuzzy set $A$ is called normal if there exists an element $x \in X$ whose membership value is one,i.e.,$\varphi_A(x) = 1$.

A fuzzy set $A$ is convex if and only if for any $x_1, x_2 \in X$, the membership function of $A$ satisfies the condition:

$$\varphi_A\left(\lambda x_1 + (1-\lambda)x_2\right) \geq \min\left\{\varphi_A(x_1), \varphi_A(x_2)\right\}, \quad 0 \leq \lambda \leq 1.$$

For a triangular fuzzy number $\widehat{A}_{\text{3-tuples}}$, it is represented by $\overline{A}(m,n,p)$,where $m,n,p$ are real numbers, and its membership function $\mu_{\overline{A}}$ is given by:

$$\mu_{\overline{A}}(x) = \begin{cases} \frac{x-a}{b-a}, & a \le x \le b \\ 1, & x = b \\ \frac{c-x}{c-b}, & b \le x \le c \\ 0, & \text{otherwise} \end{cases}$$

The representation of L-R can be recorded as:

$$\overline{A}(m,n,p) = \langle n, n-m, p-m \rangle_{\text{LR}}, \quad \text{for } L(x) = \max(0, 1-x).$$

## 2.2. Triangular Fuzzy Number of L-R Method

Fuzzy number $\overline{A}$ is an L-R fuzzy number only if it has three positive numbers $\mu$,$m > 0$, $n > 0$, and two positive,continuous, and decreasing functions from the actual number $[0,1]$:

$$L(0) = R(0) = 1, \quad L(1) = 0, \quad L(x) > 0, \ \lim_{x\to\infty} L(x) = 0,$$
$$R(1) > 0, \quad \lim_{x\to\infty} R(x) = 0.$$

$$\phi_{\overline{\mu}}(x) = \begin{cases} L\left(\dfrac{\mu - x}{m}\right), & \text{if } x \in [\mu - m, \mu] \\ R\left(\dfrac{x-\mu}{n}\right), & \text{if } x \in [\mu, \mu + n] \\ 0, & \text{otherwise} \end{cases}$$

Fuzzy number $\overline{\mu}$ is the L-R fuzzy number,then $\overline{\mu} = \langle \mu,\ m,\ n \rangle_{\text{LR}}$.$\mu$ is called the average value or modal value of $\overline{\mu}$;$m$ and $n$ are called the left spread and right spread of $\overline{\mu}$.Conventionally, $\overline{\mu} = \langle \mu,\ 0,\ 0 \rangle_{\text{LR}}$ is the ordinary real number $\mu$,called a fuzzy singleton.

$$\text{Supp}(\overline{\mu}) = [\mu - m, \mu] \cup [\mu, \mu + n] = [\mu - m, \mu + n]$$

## 2.3. L-R Fuzzy Arithmetical Operators

If $\overline{\mu} = \langle \mu,\ m,\ n \rangle_{\text{LR}}$ and $\overline{n} = \langle n,\ p,\ q \rangle_{\text{LR}}$ are same fuzzy numbers,then their sum is the same L-R fuzzy number, and their difference is also the same L-R fuzzy number,given by:

$$\overline{\mu} + \overline{n} = \langle \mu + n,\ m + p,\ n + q \rangle_{\text{LR}}$$

$$\overline{\mu} - \overline{n} = \langle \mu - n,\ m + q,\ n + p \rangle_{\text{LR}}$$

The product of L-R fuzzy numbers $\overline{\mu} = \langle \mu,\ m,\ n \rangle_{\text{LR}}$ and $\overline{n} = \langle n,\ p,\ q \rangle_{\text{LR}}$ is given by:

$$\overline{\mu} \cdot \overline{n} = \langle \mu \cdot n,\ \mu p + nm - mp,\ \mu q + nn + nq \rangle_{\text{LR}}$$

The quotient (secant approximation) of L-R fuzzy numbers $\overline{\mu} = \langle \mu,\ m,\ n \rangle_{\text{LR}}$ and $\overline{n} = \langle n,\ p,\ q \rangle_{\text{LR}}$ is given by:

$$\frac{\overline{\mu}}{\overline{n}} = \left\langle \frac{\mu}{n},\ \frac{\mu q}{n(n+q)} + \frac{m}{n} - \frac{mq}{n(n+q)},\ \frac{\mu p}{n(n-p)} + \frac{n}{n} + \frac{np}{n(n-p)} \right\rangle_{\text{LR}}$$

## 3. Model Description

Customer arrivals follow a Poisson process with a fuzzy average arrival rate $\lambda$ and join the phase-1 of batch service.The server served all customers with mean service rate $1/\beta$.Once the batch service is completed,each customer in the batch receives an individual fuzzy service at a mean rate $1/\mu$.During this individual service phase,the server is subject to failure at a rate $\alpha$,but it is immediately repaired at a rate $\beta$,allowing service to resume without delay.

After completing the personal services,the server returns to Phase-1 to serve any waiting customers and proceeds to the second phase accordingly.If no customers are waiting in the batch queue,the server remains idle for a fixed duration $C$,known as the server timeout.If customers arrive during this timeout,the server begins batch service followed by individual service.However,if no arrivals occur within this timeout period,the server goes on vacation.It returns to begin pre-service activities at a mean rate of $1/\theta$ once $N$ customers have accumulated in the batch queue.

Let:
$C_h$ = Holding cost for each customer
$C_o$ = Operational cost of server
$C_m$ = Pre-service cost per cycle
$C_t$ = Timeout cost per cycle
$C_s$ = Setup cost per cycle
$C_b$ = Breakdown cost
$C_r$ = Reward for the server being on vacation

Now we consider N-policy FM/FM/1 two-phase queuing model server start-up and time-out,break down and repair of expected system length $L(N)$ and cost function $T(N)$.

## 4. Performance Measures

### 4.1. The expected system length is

$$\begin{aligned}
L(N) = {} & \frac{N(N-1)}{2} p_{000} + \frac{\lambda(\lambda + N\theta)}{\theta^2} p_{000} + \frac{\lambda}{\beta}\left(\frac{\lambda}{c} + \frac{\lambda + N\theta}{\theta}\right) \\
& + \frac{\lambda\left[\lambda^2\beta\xi + \mu\gamma^2(\lambda+\beta)\right]}{\mu\gamma\beta\left[\mu\gamma - \lambda(\xi+\gamma)\right]} \\
& + \frac{\lambda\gamma\left[2\lambda(\lambda+N\theta) + \theta^2 N(N-1)\right]}{2\theta^2\left[\mu\gamma - \lambda(\xi+\gamma)\right]} p_{000} \\
& + \frac{\lambda\xi}{\gamma^2} \cdot \frac{\lambda\mu\gamma R_1(1) + \beta\theta\gamma \cdot \frac{\lambda}{\theta} p_{000}}{\beta\left[\mu\gamma - \lambda(\xi+\gamma)\right]} \\
& + \frac{\alpha}{\gamma} \cdot \frac{\lambda\left[\lambda^2\beta\alpha + \mu\gamma^2(\lambda+\beta)\right]}{\mu\gamma\beta\left[\mu\gamma - \lambda(\xi+\gamma)\right]} \\
& + \frac{\lambda\gamma\left[2\lambda(\lambda+N\theta) + \theta^2 N(N-1)\right]}{2\theta^2\left[\mu\gamma - \lambda(\xi+\gamma)\right]} p_{000}
\end{aligned}$$

Where:

$$p_{000} = \frac{1 - \frac{\lambda}{\mu}\left(1 + \frac{\lambda}{\gamma}\right)\frac{\lambda}{\beta}}{N + \frac{\lambda}{\theta} + \frac{\lambda}{c}} \quad \text{and} \quad R_1(1) = \frac{\lambda}{\mu}$$

### 4.2. The function of total expected cost is

$$
\begin{aligned}
T(N) = C_h \Bigg[ & \frac{N(N-1)}{2} p_{000} + \frac{\lambda(\lambda+N\theta)}{\theta^2} p_{000} + \frac{\lambda}{\beta}\left(\frac{\lambda}{c} + \frac{\lambda+N\theta}{\theta}\right) + \frac{\lambda\left[\lambda^2\beta\xi + \mu\gamma^2(\lambda+\beta)\right]}{\mu\gamma\beta\left[\mu\gamma-\lambda(\xi+\gamma)\right]} \\
& + \frac{\lambda\gamma\left[2\lambda(\lambda+N\theta)+\theta^2 N(N-1)\right]}{2\theta^2\left[\mu\gamma-\lambda(\xi+\gamma)\right]} p_{000} \\
& + \frac{\lambda\xi}{\gamma^2} \cdot \frac{\lambda\mu\gamma R_1(1) + \beta\theta\gamma \cdot \frac{\lambda}{\theta} p_{000}}{\beta\left[\mu\gamma-\lambda(\alpha+\gamma)\right]} \\
& + \frac{\xi}{\gamma} \cdot \frac{\lambda\left[\lambda^2\beta\xi + \mu\gamma^2(\lambda+\beta)\right]}{\mu\gamma\beta\left[\mu\gamma-\lambda(\xi+\gamma)\right]} \\
& + \frac{\lambda\gamma\left[2\lambda(\lambda+N\theta)+\theta^2 N(N-1)\right]}{2\theta^2\left[\mu\gamma-\lambda(\xi+\gamma)\right]} p_{000} \Bigg] \\
& + C_o\left(\frac{\lambda}{\beta} + \frac{\lambda}{\mu}\right) + C_m\left(\frac{\lambda}{\theta} p_{000}\right) + C_t\left(\frac{\lambda}{c} p_{000}\right) + C_b \cdot \frac{\lambda\xi}{\mu\gamma} \\
& + C_s \cdot \frac{\lambda\left[1 - \frac{\lambda}{\mu}\left(1+\frac{\lambda}{\gamma}\right)\frac{\lambda}{\beta}\right]}{N + \frac{\lambda}{\theta} + \frac{\lambda}{c}} - C_r \cdot (N p_{000})
\end{aligned}
$$

Where:

$$N = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$a = \frac{C_h\mu\gamma\theta C}{2\left[\mu\gamma - \lambda(\xi+\gamma)\right]} \quad , \quad b = \frac{C_h\mu\gamma\lambda(\theta+C)}{\left[\mu\gamma-\lambda(\xi+\gamma)\right]}$$

$$c = \frac{C_h\lambda(2\lambda-\mu\gamma)}{2\left[\mu\gamma-\lambda(\xi+\gamma)\right]} + \lambda\left[C(C_m+C_r) + \theta(C_t+C_r) + C_s\theta C\right]$$

## 5. Numerical analysis and Graphical Presentation

### 5.1. Triangular fuzzy number

We consider an N-policy FM/FM/1 two-phase queuing model with server start-up,time-out,breakdown, and repair,where the arrival rate and service rate are triangular fuzzy numbers represented by:

$$\lambda = [0.4, 0.5, 0.6]; \quad \mu = [2.4, 2.5, 2.6]; \quad \beta = [1, 2, 3]; \quad \theta = [2, 3, 4]$$

Let the cost and system parameters be given as:

$$C = 1, \quad \alpha = 0.1, \quad C_b = 100, \quad C_m = 100, \quad C_t = 30, \quad C_s = 500, \quad C_r = 40, \quad C_h = 5.$$

The interval of confidence at possibility level $\alpha$ is given by:

$$
\begin{aligned}
&\lambda \in [0.4 + 0.1\alpha,\ 0.6 - 0.1\alpha], \\
&\mu \in [2.4 + 0.1\alpha,\ 2.6 - 0.1\alpha], \\
&\beta \in [1 + \alpha,\ 3 - \alpha], \\
&\theta \in [2 + \alpha,\ 4 - \alpha].
\end{aligned}
$$

## 5.2. The Expected System Length Is

Let:

$$\vec{a} = \langle 0.4, 0.5, 0.6\rangle, \quad \vec{b} = \langle 2, 3, 4\rangle, \quad \vec{c} = \langle 1, 2, 3\rangle, \quad \vec{d} = \langle 2.4, 2.5, 2.6\rangle$$

All vector operations (like $\vec{a} \cdot \vec{b}$) are dot products.

$$\begin{aligned}
L(N) = &\frac{9.80(9.80-1)}{2} \cdot p_{000} + \frac{\vec{a} \cdot (\vec{a} + 9.80 \cdot \vec{b})}{\vec{b} \cdot \vec{b}} \cdot p_{000} \\
&+ \frac{\vec{a}}{\vec{c}} \left( \frac{\vec{a}}{c} + \frac{\vec{a} + 9.80 \cdot \vec{b}}{\vec{b}} \right) \\
&+ \frac{\vec{a}\left[(\vec{a} \cdot \vec{a}) \cdot \vec{c} \cdot 0.1 + \vec{d} \cdot 4 \cdot (\vec{a} + \vec{c})\right]}{\vec{d} \cdot 2 \cdot \vec{c}\left(\vec{d} \cdot 2 - \vec{a} \cdot (0.1+2)\right)} \\
&+ \frac{\vec{a} \cdot 2\left[2 \cdot \vec{a} \cdot (\vec{a} + 9.80 \cdot \vec{b}) + (\vec{b} \cdot \vec{b}) \cdot 9.80(9.80-1)\right]}{2 \cdot (\vec{b} \cdot \vec{b})\left(\vec{d} \cdot 2 - \vec{a} \cdot (0.1+2)\right)} \cdot p_{000} \\
&+ \frac{\vec{a} \cdot 0.1}{4} \cdot \frac{\vec{a} \cdot \vec{d} \cdot 2 \cdot R_1(1) + \vec{c} \cdot \vec{b} \cdot 2 \cdot \left(\frac{\vec{a}}{\vec{b}} \cdot p_{000}\right)}{\vec{c} \cdot \left(\vec{d} \cdot 2 - \vec{a} \cdot (0.1+2)\right)} \\
&+ \frac{\alpha}{2} \cdot \frac{\vec{a}\left[(\vec{a} \cdot \vec{a}) \cdot \vec{c} \cdot \alpha + \vec{d} \cdot 4 \cdot (\vec{a} + \vec{c})\right]}{\vec{d} \cdot 2 \cdot \vec{c}\left(\vec{d} \cdot 2 - \vec{a} \cdot (0.1+2)\right)} \\
&+ \frac{\vec{a} \cdot 2\left[2 \cdot \vec{a} \cdot (\vec{a} + 9.80 \cdot \vec{b}) + (\vec{b} \cdot \vec{b}) \cdot 9.80(9.80-1)\right]}{2 \cdot (\vec{b} \cdot \vec{b})\left(\vec{d} \cdot 2 - \vec{a} \cdot (0.1+2)\right)} \cdot p_{000}
\end{aligned}$$

Where

$$p_{000} = \langle 0.042, 0.058, 1.23\rangle, \qquad R_1(1) = \frac{\lambda}{\mu}$$

$$L(N) = \langle 3.12, 3.34, 443.82\rangle$$

## 5.3. The Function of Total Expected Cost

$$\begin{aligned}
T(N) = &C_h \cdot \langle 2.98, 2.35, 161.35\rangle \\
&+ C_o \cdot \left( \frac{\langle 0.4, 0.5, 0.6\rangle}{\langle 1, 2, 3\rangle} + \frac{\langle 0.4, 0.5, 0.6\rangle}{\langle 2.4, 2.5, 2.6\rangle} \right) \\
&+ C_m \cdot \left( \frac{\langle 0.4, 0.5, 0.6\rangle}{\langle 2, 3, 4\rangle} \cdot p_{000} \right) \\
&+ C_t \cdot \left( \frac{\langle 0.4, 0.5, 0.6\rangle}{c} \cdot p_{000} \right) \\
&+ C_b \cdot \left( \frac{\langle 0.4, 0.5, 0.6\rangle \cdot \xi}{\langle 2.4, 2.5, 2.6\rangle \cdot 2} \right) \\
&+ C_s \cdot \left[ \frac{\langle 0.4, 0.5, 0.6\rangle \cdot \left(1 - \frac{\langle 0.4,0.5,0.6\rangle}{\langle 2.4,2.5,2.6\rangle} \cdot \left(1 + \frac{\langle 0.4,0.5,0.6\rangle}{2}\right) \cdot \frac{\langle 0.4,0.5,0.6\rangle}{\langle 1,2,3\rangle}\right)}{9.80 + \frac{\langle 0.4, 0.5, 0.6\rangle}{\langle 2, 3, 4\rangle} + \frac{\langle 0.4, 0.5, 0.6\rangle}{c}} \right] \\
&- C_r \cdot (9.80 \cdot p_{000})
\end{aligned}$$

$$T(N) = \langle 160.35, 1894.33, 3604.8\rangle$$

## 5.4. Graphical Presentation

Triangular fuzzy membership function and results based on $\alpha$-levels. Based on the below graphs, which illustrate a triangular fuzzy membership function defined by the L-R representation method, the corresponding values of $L(N)$ and $T(N)$ have been evaluated. These evaluations consider fuzzy parameters such as arrival rate, service rate, breakdown, and repair rate, to determine the expected system length $L$ and the cost function $T(N)$ within the queue at 11 distinct $\alpha$-levels: 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0.

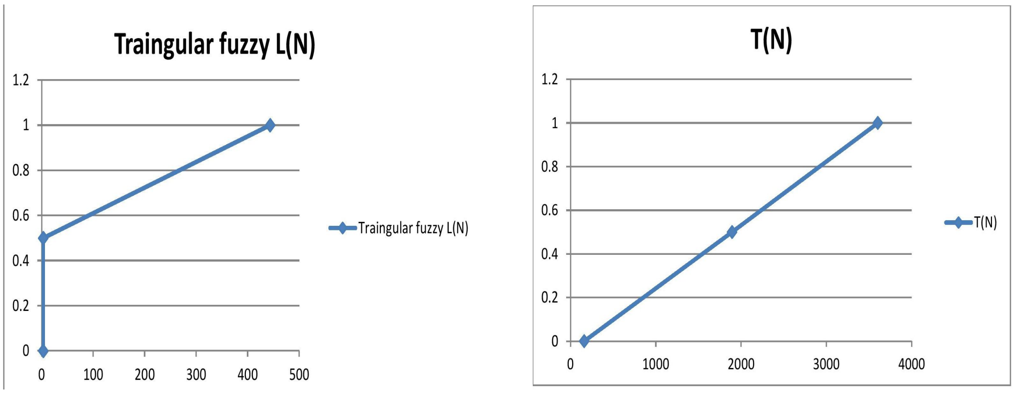


**Figure 1:** *Expected system length $L(N)$ and Cost function $T(N)$*

The L-R values of the fuzzy sets $L$ and $T(N)$, corresponding to different levels of possibility ($\alpha$ values), have been calculated as follows: From Table 1 and Figure 1 the estimated system length $L(N)$ is found to be 3.34, with a left spread of 3.12 and a right spread of 443.82. Similarly, the cost function $T(N)$ is determined to be 160.35, with a left spread of 1894.33 and a right spread of 3604.8.

## 6. Conclusion

In this study,we calculated key performance measures of the system,focusing on the expected system length $L$ and the cost function $T(N)$.The expected system length $L$ provides insight into the average number of units in the system,which reflects system congestion.The cost function $T(N)$ was evaluated to analyze the trade-off between performance and associated operational costs.

Through these calculations,we were able to assess the system's efficiency under varying conditions.The results highlight how changes in system parameters affect both $L$ and $T(N)$.Overall,the analysis aids in optimizing system performance while minimizing costs.

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