PERFORMANCE METRICS OF M/M(A,B)/(2,1)/RV QUEUING SYSTEM WITH RENEGING AND RETENTION OF CUSTOMERS

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Abstract

In general, the quality of service management ensures the quality of the business outcome. Customer behavior trivially has an impact on it. To obtain a potential output, it is necessary to speed up the customer service request. Adequate among of losses happen when the customers withdraw the service request. Thus, it is vital to implement a significant strategy based on the number of request in queue. By modeling the service system mathematically in concern with the customers behavior, it is elegant to make a strategy for estimating demands. This paper aims to interpret a system, M/M(a,b)/(2,1)/RV providing service in batches with the aid of two servers and encounters the measures of performance of the system under the possibility of servers vacation and customers behavior of reneging and retention. A server withdraws the system whenever he is not available with an accessible batch to start his service. This behavior of the server can be termed as vacation. The vacation policy used in this paper involves the repeated mode of vacation of the servers. Customers entering the system experience three states of availability of the servers. Thus, for the proposed model the equations and solutions for the steady state is derived and computed theoretically, and the results are verified through numerical illustration.

Keywords: General bulk service rule, Markovian process, Poisson arrival, Reneging, Repeated vacations

1. Introduction

The sustainable growth in all fields of this real world urges everyone to undergo a systematic progressing of a work. In that sense, lining up to fulfill once need also becomes one among this progressing. Now a days researchers focus on modeling the practical scenarios into a computational frame work and introduce them into the literature. In the similar way, the study on queues and its behavior is known as queuing theory was introduced into the mathematical literature by Agner K. Erlang, the Danish mathematician. Queuing prevails almost everywhere, even from the grocery shops to digital markets. The mathematical approach on queuing provides solution in the way of optimizing a work in fields like manufacturing systems, business management, traffic flow, computer systems, and so on. To optimize a work, it is important to have efficient servers, service time and service quality that means a good quality service in a good period of time. Long-term waiting for a work may anticipates many clients and it stimulates them to withdraw from the queue of waiting. This paper aims to interpret this impatience behavior of customers waiting in the queue. Reneging of customers will ultimately create a greater impact on the system. A credential amount of loss may happen.

On the other hand while obtaining service, there may be situations where a customer may

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experience the unavailability of the server, since he may left the system for a random interval of time either for some other job or to take rest. This period of unavailability of the server is called vacation. So far, the research on queuing theory, studying queuing models with vacation policy, has also become a crucial area of study for many researchers.

A methodological overview of the queuing system with two vacation models was done by B.T Doshi [6]. Divya and Indhira [5] surveyed on these vacation models as it is an important existing real-life situation that has been encountered in our daily life routine and industrial management. Similarly, models with working vacations was surveyed by Fiems [8]. S. Sindhu et al. [19]has also worked on queuing models with vacation policies, which was found with arrival and service variants.

There may also exist some situations where the servers undergo random vacation from the system multiple times in order to avoid being idle, which means not having an accessible batch size in the system. This behavior of the server is called repeated vacation (RV). At first, Palaniammal [16] worked on the queuing model, M/M(a,b)/(2,1) in this perspective. Later, Baskar and Palaniammal [2] both again worked on this model, M/M(a,b)/(2,1) with repeated and delayed vacation taken by the servers. Maragathasundari et al. [14] made a study on non Markovian queuing system and analyzed it with two categories over vacation period. Motivated in analyzing health care service facilities Sundararaman et al. [21] estimated the performance metrics of the system that allow customers to join the usual and retrial waiting line are estimated using SVT technique. It is also noticeable that the servers are on working vacations.

In order to minimize customer waiting time during the unavailability of the servers, it is preferable if the system has a few more servers to grant service so that customers are served at a moment. In view of assisting the system continuously irrespective of breakdown, repair, and vacation, employing additional server helps to assist the system effectively. Srinivas R. Chakravarthy and Kulshrestha [3] discussed a queuing system from this point of view. By the way, in order to save energy and to improve the quality of service in virtual machines Ma et al. [13] proposed an energy saving measure by combining synchronous and asynchronous vacation and modeled them into a Markov stochastic model of two dimension with many servers. Queuing systems with cost optimization was done by Afanasyev [1] considering the behavior of the system during vacation period.

The concept of impatient attitude of customers was analysed by Dbeis and Sahili [4] for the Markovian queuing system with single channel. Customers encouraged arrival into the system may result in the reneging of customers from the queue. Som et al. [20] practically valid this scenario for a system with single server. Economic analysis was done by Pan et al. [17] for the customers alternative level of impatient behavior in a single server system. Khan and Paramasivam [9] analysed the quality control policy for finite limit Markovian queuing system with to reduce the waiting time and maintain the reneged customers not to join the system again. Lidiya and K.J.R. Mary [12] interpreted the queuing model consisting of one server and analysed the act of reneging of customers with working vacation policy. Kumar [11] addressed a queuing system with some probable perspectives, including customers reneging with a threshold policy. With the aid of the metrics evaluated optimality values were also interpreted. Fayama et al. [7] evaluated the performance of a queuing model with two servers with finite capacity under the scenario of reneging. Seenivasan et al. [18] studied a queuing model under the circumstances of system failure and reneging of customers. Heterogeneous service system involving two servers with customers reverse balking and reneging was studied by Tamuli et al. [22] Since queuing is an integral part of life in many fields, the health care department faces this mainly. Patients renege the health care facilities when they are dissatisfied but the probability of reneging varies with different time marks thus a correlated reneging was proposed and studied by Kuaban et al. [10] for a multi-server queuing model.

Previously, Mary K.J.R and Jenifer Princy studied the queuing model M/M(a,b)/(2,1) [15] under the perspective of an encouraged arrival of customers and breakdown of the servers. With an aid of this survey the model M/M(a,b)/ (2,1)/RV under customer impatient behavior reneging is proposed and the steady state equations and solutions are derived in the following sections. A numerical illustration is carried out to verify the validity of the outcomes.

2. Mathematical Model Description

This model involves two servers where the arrival of customers to the system follows the Poisson distribution with the parameter λ. The service rate follows an exponential distribution with parameter μ. The service is done in batches according to the General Bulk Service Rule introduced by Neuts. The service is provided if there are a minimum 'a' number of customers in the queue. If the number of customers is more than the maximum limit 'b' only the first b customers are allowed to take the service.

The servers are in idle state if there is not even a minimum number of customers in a queue. In this case, a server leaves the system for a random period called vacation. This follows an exponential distribution with parameter θ. On returning from the vacation, if the server finds less than 'a' customers in the queue while the other server is busy or idle, the server leaves the system for another random period called repeated vacation. The server will continue the same activity until he finds minimum 'a' number of customers to start his service. This particular queuing system always be available with at least one server either in the idle or busy state.

Meanwhile, long waiting in the queue to get a service anticipates clients and it stimulates them to withdraw from the queue without fulfilling their need. The customers may renege whenever it takes time to reach the minimum batch size or at the time of servers' vacation. These customers renege at the rate of ζ. Let $p^{'}$ be the probability that the customers may renege and $q^{'} = 1 - p'$ be the probability that the reneged customers are retained in the system.

In the state space $(j, n), j = 0, 1, 2; n \geq 0$, the queue is studied as a Markov process where $n \geq 0$ denotes the number of customers waiting in the queue and j denotes the level of the server.

1. $(0, n)$ - represents that one server is idle and the other server is on vacation, $0 \leq n \leq a - 1$.
2. $(1, n)$ - represents that one server is busy and the other server is on vacation, $n \geq 0$.
3. $(2, n)$ - represents that both the servers are busy, $n \geq 0$.

Defining $P_{jn}(t) = \text{Prb}\ \{$ At time t, the system is in the state $(j,\ n) j = 0, 1, 2; n \geq 0\}$ and considering that the steady state probabilities are $P_{0n} = \lim_{t\to\infty} p_{0n}(t)$,$P_{1n} = \lim_{t\to\infty} p_{1n}(t)$ and $P_{2n} = \lim_{t\to\infty} p_{2n}(t)$.

The steady state equations are

$$\lambda p_{00} = \mu p_{10} \tag{1}$$

$$\lambda p_{0n} = \lambda p_{0n-1} + (\mu + n\zeta p^{'}) p_{1n} \quad (1 \leq n \leq a-1) \tag{2}$$

$$(\lambda + \mu) p_{10} = \lambda p_{0a-1} + 2\mu p_{20} + (\mu + n\zeta p^{'}) \sum_{n=a}^{b} p_{1n} \tag{3}$$

$$(\lambda + \mu + n\zeta p^{'}) p_{1n} = \lambda p_{1n-1} + (2\mu + 2n\zeta p^{'}) p_{2n} + (\mu + (n+b)\,\zeta p^{'}) p_{1n+b} \quad (1 \leq n \leq a-1) \tag{4}$$

$$(\lambda + \mu + n\zeta p^{'} + \theta) p_{1n} = \lambda p_{1n-1} + (\mu + (n+b)\,\zeta p^{'}) p_{1n+b} + \mu p_{1n+b} \quad (n \geq a) \tag{5}$$

$$(\lambda + 2\mu) p_{20} = \theta \sum_{l=a}^{b} p_{1n} + (2\mu + 2n\zeta p^{'}) \sum_{n=a}^{b} p_{2n} \tag{6}$$

$$(\lambda + 2\mu + 2n\zeta p^{'}) p_{2n} = \lambda p_{2n-1} + \theta p_{1n+b} + (2\mu + 2(n+b)\zeta p^{'}) p_{2n+b} \quad (n \geq 1) \tag{7}$$

3. Steady State Solutions

Let E be the forward shifting operator defined by $E(p_{1\,n}) = p_{1\,n+1}$. Equation 5 implies $[(\mu + n\zeta p' + b\zeta p')E^{b+1} - (\mu + n\zeta p' + b\zeta p')E + \lambda]p_{1\,n} = 0. \quad (n \geq 1)$.

The corresponding characteristic equation is as follows.

$$(\mu + n\zeta p' + b\zeta p')Z^{b+1} - (\mu + n\zeta p' + b\zeta p')Z + \lambda = 0. \tag{8}$$

Then by Rouche's Theorem., it has only one real root inside the circle $|z| = 1$ when $\rho = \dfrac{\lambda + \theta}{b(\mu + n\zeta p')} < 1$. Let r_0 be the root of the above characteristic equation with $|r_0| < 1$. Therefore, the homogeneous difference equation has solution of the form.

$$p_{1\,n} = A_1 r_0^n \quad (n \geq a-1)$$

so we get

$$p_{1\,n} = r_0^{n-a+1} p_{1\,a-1} \quad (n \geq a) \tag{9}$$

Using equation 7 we get

$$[2(\mu + n\zeta p' + b\zeta p')E^{b+1} - (\lambda + 2\mu + 2n\zeta p')E + \lambda]p_{2\,n} = -\theta p_{1\,n+b+1} \quad (n \geq 1)$$

The corresponding characteristic equation is

$$2(\mu + n\zeta p' + b\zeta p')Z^{b+1} - (\lambda + 2(\mu + n\zeta p'))Z + \lambda = 0 \tag{10}$$

If r_1 is the root of the above characteristic equation with $|r_1| < 1$ which exists when $\rho_1 = \left(\dfrac{\lambda}{2b(\mu + n\zeta p')}\right) < 1$. This non - homogeneous difference equation (7) has the solution

$$p_{2\,n} = (A_1 r_1^n + K r_0^n) p_{1\,a-1} \quad (n \geq 0) \tag{11}$$

where A_1 is constant and $K = \dfrac{\left(-\theta r_0^{b-a+2}\right)}{(\lambda + 2\theta)\, r_0 - \lambda}$

Using equation 4 and substituting for $p_{2\,n+1}$ and $p_{1\,n+b+1}$ we have after simplification.

$$p_{1\,n} = [A_2 R^n + B_1(r_0) r_0^n + B_2(r_1) r_1^n]\, p_{1\,a-1} \quad (0 \leq n \leq a-1) \tag{12}$$

where $R = \dfrac{\lambda}{\lambda + \mu + n\zeta p'}$, $\quad B_1(r_0) = \dfrac{2(\mu + n\zeta p')K r_0 + (\mu + (n+b)\zeta p') r_0^{b-a+2}}{\theta[(\lambda + \mu + n\zeta p') r_0 - \lambda]}$ and $B_2(r_1) = \dfrac{2(\mu + n\zeta p') A_1 r_1}{(\lambda + \mu + n\zeta p') r_1 - \lambda}$

Adding equation 2 over $k = 1$ to n and substituting for $p_{1\,k}$ from equation 12 and simplifying, we get

$$p_{0\,n} = \left\{ \frac{\mu}{\lambda} \left[A_2 \frac{1 - R^{n+1}}{1 - R} + B_1(r_0) \frac{1 - r_0^{n+1}}{1 - r_0} + B_2(r_1) \frac{1 - r_1^{n+1}}{1 - r_1} \right] + \frac{\zeta p'}{\lambda} \left[A_2 H(R) + B_1(r_0) H(r_0) + B_2(r_1) H(r_1) \right] \right\} p_{1\,a-1} \quad (0 \leq n \leq a-1) \tag{13}$$

where $H(u) = \left(\dfrac{u(1 - u^{n+1}) - (n+1)u^{n+1}(1-u)}{(1-u)^2} \right)$

Using equation 9 and 11 in 6 then we obtain

$$A_1 = \frac{\theta \left(r_0 - r_0^{b-a+2} + 2K(E(r_0) - \mu) - \lambda K \right)}{\lambda - 2(E(r_1) - \mu)} \tag{14}$$

where $E(u) = \left(\frac{u^a - u^{b+1}}{1-u}\right)(\mu + a\zeta p^{'}) + \zeta p^{'} \left(\frac{(1-u^{b-a+1})u^{a+1} - (b-a+1)(1-u)u^{b+1}}{(1-u)^2}\right)$
Further, In equation 12 substituting $n = a-1$ we get the value of A_1 as

$$A_2 = \frac{1}{R^{a-1}}\left[1 - B_1(r_0)r_0^{a-1} - B_2(r_1)r_1^{a-1}\right] \tag{15}$$

The value of $p_{1\,a-1}$ is obtained using the normalizing condition.

$$\sum_{n=0}^{\infty} p_{2\,n} + \sum_{n=a}^{\infty} p_{1\,n} + \sum_{n=0}^{a-1} (p_{0\,n} + p_{1\,n}) = 1 \tag{16}$$

substituting for $p_{2\,n}$, $p_{1\,n}$ and $p_{0\,n}$ and simplifying we get,

$$\begin{aligned} p_{1\,a-1}^{-1} = A_2\big(S(R) + J(R)\big) + B_1(r_0)\big(S(r_0) + J(r_0)\big) + B_2(r_1)\big(S(r_1) + J(r_1)\big) \\ + \frac{A_1}{1-r_1} + \frac{K}{1-r_0} + \frac{r_0}{1-r_0} \end{aligned} \tag{17}$$

where $S(u) = \frac{1-u^a}{1-u} + \frac{\mu}{\lambda}\left(\frac{a}{1-u} - \frac{u}{1-u}\frac{1-u^a}{1-u}\right)$
$J(u) = \frac{\zeta p^{'}}{\lambda}\left\{\frac{(a+1)(u^{a+1} - u^2) + (a-1)(u - u^{a+2})}{(1-u)^3}\right\}$ and the values of $R, B_1(r_0), B_2(r_1)$ are obtained from equation 12.

4. Analysis of the Measures of Performance

In this section, we have computed the measures of performance of the system M/M (a,b)/ (2,1)/RV under customers impatient attitude of reneging.

1. **Expected Queue Length**

   The expected queue length is given by

$$L_q = \sum_{n=1}^{\infty} np_{2\,n} + \sum_{n=a}^{\infty} np_{1\,n} + \sum_{n=1}^{a-1} n\,(p_{0\,n} + p_{1\,n})$$

   Using equations (10) to (16) and simplifying, we have

$$\begin{aligned} L_q = \Bigg[& A_2(U(R) + V(R)) + B_1(r_0)(U(r_0) + V(r_0)) + B_2(r_1)(U(r_1) + V(r_1)) \\ & + \frac{ar_0}{1-r_0} + \frac{r_0^2}{(1-r_0)^2} + \frac{A_1 r_1}{(1-r_1)^2} + \frac{Kr_0}{(1-r_0)^2}\Bigg] p_{1\,a-1} \end{aligned}$$

   where,
   $U(u) = \left(\frac{1-u^a - au^{a-1}(1-u)}{(1-u)^2}\right)\left(u - \frac{u^2\mu}{\lambda(1-u)}\right) + \frac{\mu a(a-1)}{2\lambda(1-u)}$ and
   $V(u) = \frac{\zeta p^{'}}{\lambda}\left[\frac{a(a-1)u}{2(1-u)^2} + \frac{u^2(3u^{a+1} - u - 2)}{(1-u)^4} + \frac{au^{a-1} - (a+1)(u^2 - u^{a+2})}{(1-u)^3}\right]$

2. **Expected Waiting Time,** W_q

$$W_q = \frac{L_q}{\lambda}$$

3. **Expected Rate of Reneging,** R_g

$$R_g = l\zeta p^{'} L_q$$

4. Let $P_{2\,B}$ denote the probability that both the servers are busy then

$$P_{2\,B} = \sum_{n=0}^{\infty} p_{2\,n} = \left(\frac{A_1}{1-r_1} + \frac{K}{1-r_0} \right) p_{1\,a-1}$$

5. Let $P_{1\,B}$ denote the probability that one server is busy and the other server is on vacation then

$$P_{1\,B} = \sum_{n=0}^{\infty} p_{1\,n} = \left(A_2 \frac{1-R^a}{1-R} + B_1(r_0)\frac{1-r_0^a}{1-r_0} + B_2(r_1)\frac{1-r_1^a}{1-r_1} + \frac{r_0}{1-r_0} \right) p_{1\,a-1}$$

6. Let $P_{0\,B}$ denote the probability that one server is idle and one server is on vacation then

$$P_{0\,B} = \sum_{n=0}^{a-1} p_{0\,n} = \Big[A_2\big(G(R)+F(R)\big) + B_1(r_0)\big(G(r_0)+F(r_0)\big) + B_2(r_1)\big(G(r_1)+F(r_1)\big) \Big] p_{1\,a-1}$$

where $F(u) = \frac{\zeta p'}{\lambda}\left[\frac{au}{(1-u)^2} - \frac{u^2(1-u^a)}{(1-u)^3} + \frac{u(u^{a+1}+(a+1)(1-u)u^a-1}{(1-u)^3} \right]$ and $G(u) = \frac{\mu}{\lambda}\left(\frac{a}{1-y} + \frac{u(1-u^a)}{(1-u)^2} \right)$ and the values of $R, B_1(r_0), B_2(r_1)$ are obtained from equation (12).

5. Numerical Analysis

In practice, many manufacturing industries involve production in batches. However, it is notable to manage the manufacturing system effectively. Those sectors must be aware of the orders received for production and the quantity they need to deliver during the recommended period of time, as it decides the economic outcome of the business. Reneging of orders also a catastrophe happens when the orders are not served on time.

Thus, considering sample values, the performance measures are verified in this section in order to validate the findings.

For a batch (a, b) of size with lower limit, $a = 3$ and upper limit $b = 15$ service is granted with a service rate, $\mu = 2$. The following Table 1, shows the expected queue size of the proposed queuing system and the probability measures of the system with respect to the probability of reneging and retention of customers in correspondence to the arrival rate $\lambda = 12$, reneging rate, $\zeta = 0.04$ and vacation rate $\theta = 0.1$.

Table 1: *L_q, P_{0B}, P_{1B}, P_{2B} with respect to p' and q'*

p'	q'	L_q	P_{0B}	P_{1B}	P_{2B}
0	1	5.8615	0.1062	0.8756	0.02072
0.1	0.9	5.71201	0.1069	0.8751	0.02039
0.2	0.8	5.5686	0.10762	0.8746	0.02005
0.3	0.7	5.4301	0.10834	0.8741	0.01973
0.4	0.6	5.2915	0.10912	0.8736	0.01938
0.5	0.5	5.1575	0.1099	0.87301	0.01904
0.6	0.4	5.0280	0.1107	0.8725	0.01869
0.7	0.3	4.9029	0.1115	0.8719	0.0184
0.8	0.2	4.7823	0.1123	0.8714	0.01804
0.9	0.1	4.6656	0.1131	0.8708	0.0177
1	0	4.5485	0.1139	0.87021	0.0174

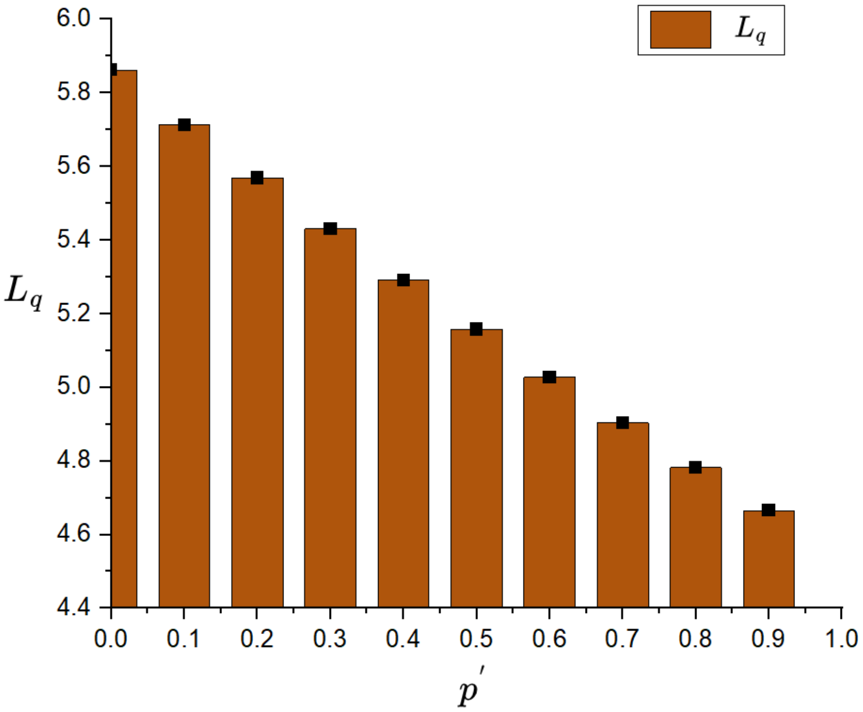

Figure 1: *Probability of reneging Vs Expected length of the queue*

The graphical representation in Figure 1 shows that the expected queue length decreases with an increase in the probability of reneging p'.

Table 2: *L_q, W_q, R_g with respect to ζ*

ζ	L_q	W_q	R_g
0.01	4.6407	0.3867	0.4455
0.02	4.5573	0.3798	0.8749
0.03	4.5263	0.3772	1.3036
0.04	4.4675	0.3723	1.7155
0.05	4.3543	0.3629	2.0900
0.06	4.1916	0.3493	2.4143
0.07	3.9992	0.3333	2.6875
0.08	3.7973	0.3164	2.9163
0.09	3.6229	0.3019	3.1302
0.1	3.4494	0.2875	3.3114

The increase in reneging directly impacts the length of the queue. Customer withdrawal from the queue reduces the congestion in the waiting line, it seems that there is a reduction in the length of the waiting queue. Meanwhile, the waiting time in the queue also decreases. The above Table 2. shows the sample result for different values of ζ. The length of the queue decreases from 4.4607 to 3.4494 and the waiting time in the queue also decreases from 0.3867 to 0.2875.

It is efficient to estimate the expected rate of reneging, that is, the number of customers renege during a period of time, which is also estimated and tabulated in Table 2. As the reneging rate increases, the expected number of customers reneging from the system also increases. This result validates our findings.

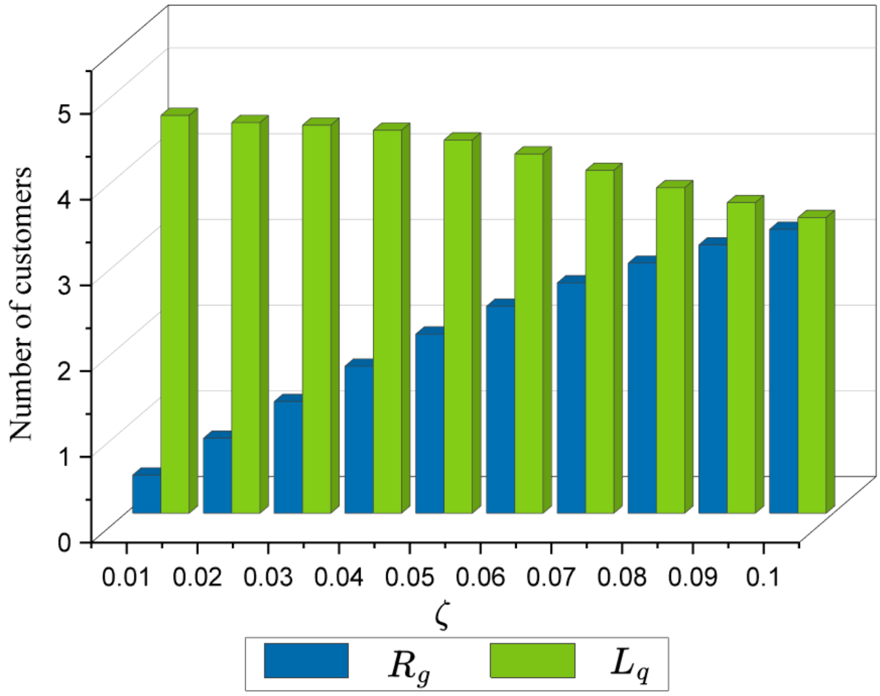

Figure 2: *L_q, R_g with respect to ζ*

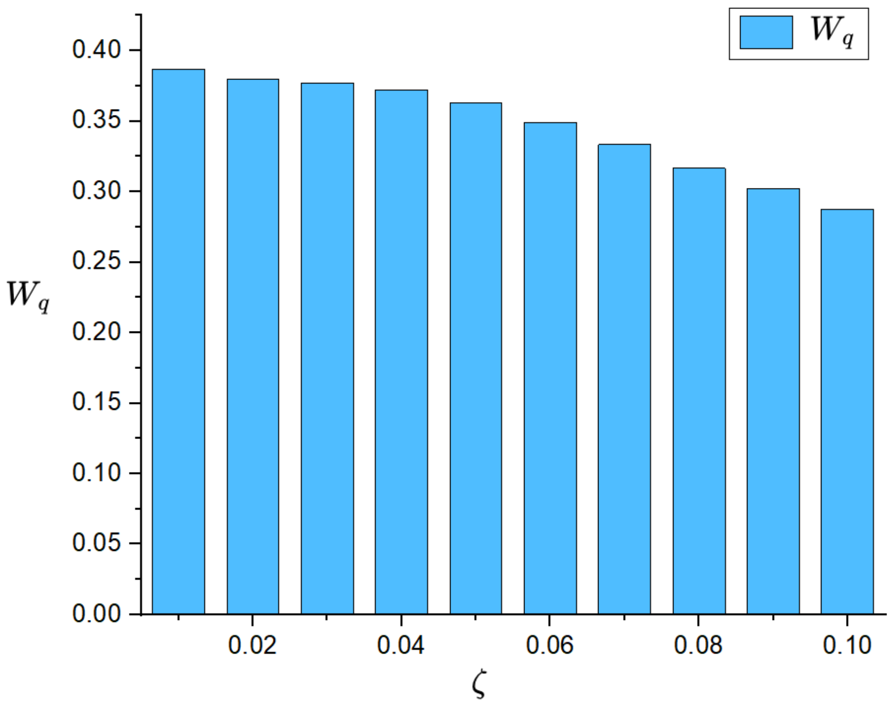

Figure 3: *W_q with respect to ζ*

Figure 2 and Figure 3 are the graphical representation of the tabulated values in Table 2 estimated for the batch size $a = 5$ and $b = 20$ with the probability of reneging of customers as $p' = 0.8$ and the probability of customers entering into the system as $q' = 0.2$.

6. Comparative Analysis

When the system experiences no reneging the value of $\zeta = 0$, this synchronizes with the classical queuing system M/M(a,b)/(2,1)/RV [16]. The following table shows a comparative outcome corresponding to the queue length for a batch of size a=5 and $b = 20$.

Table 3: *Classical queuing system Vs Queuing system with reneging*

	L_q	
λ	M/M(a,b)/(2,1)/RV	M/M(a,b)/(2,1)/RV - Reneging
5	2.4364	2.2961
8	3.3835	3.0448
10	4.2362	3.7393
12	5.2353	4.5574
15	7.0116	6.72298

By comparing the classical system without reneging with the contributed system it is observed from Table 3, there is a considerable decrease in the queue length in our proposed system, which is graphically interpreted in Figure 4.

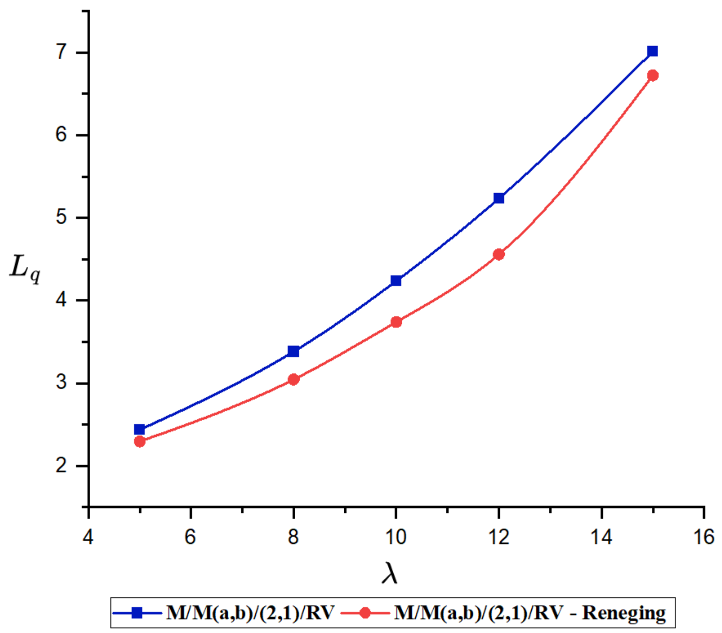

Figure 4: *Comparison between M/M(a, b)/(2,1)/RV and M/M(a, b)/(2,1)/RV with reneging*

7. Results and Discussion

The numerical analysis showcases the effect of our proposed model and the comparative analysis validates the result both theoretically and graphically. With the aid of the finding, it is possible to estimate the length of the queue for the system experiencing reneging of customers. Revoking of arrived customers showcases the inability of the system performance which may affect the system economically. Thus, estimating the length of the queue or else the quantity of products yet to manufacture helps to coordinate the system in accordance to their business outcome.

8. Conclusion

The steady state solutions of the model M/M (a, b)/ (2,1)/RV with the notion of customers behavior of reneging was discussed. Performance measure like average queue length, probability of servers being idle and engaged are formulated and analysed with a numerical example. The proceeded work helps ultimately the manufacturing system in making strategic decisions to coordinate the system and to yield a well outreach among the community. This work can be further investigated from the side decision making invoking the performance measures that has been found.

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