RT&A, No 4 (89)
Volume 20, December 2025

METAHEURISTIC OPTIMIZATION OF RELIABILITY PARAMETERS UNDER VACATION AND INSPECTION POLICIES

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Abstract

This paper explores in detail the reliability and performance evaluation of a repairable system functioning under the combined influence of vacation and inspection policies, using the Regenerative Point Graph Technique (RPGT) as the core analytical framework. The modeling is carried out through a stochastic state-transition representation, where different system conditions such as active operation, failures, repairs, vacation return of the repairman, and inspection mechanisms are explicitly considered. By exploiting the regenerative nature of the process, the study derives closed-form mathematical expressions for key performance indices, including the Mean Time to System Failure (MTSF), steady-state availability, busy period of the server, and the expected number of inspections. These measures provide a structured understanding of how the system behaves and responds under varying operational, repair, and inspection conditions. To further enhance reliability and system performance, the study employs metaheuristic optimization algorithms such as the Particle Swarm Optimization (PSO), the Genetic Algorithm (GA), and the Cuckoo Search Algorithm (CSA). These optimization methods are implemented to identify the optimal values of system parameters, including failure rate, repair rate, inspection rate, and vacation return rate, which directly influence the efficiency of the system. Numerical experiments and simulation-based illustrations are conducted to validate the theoretical analysis and to compare the effectiveness of the different algorithms. The results demonstrate that CSA and PSO consistently outperform GA, particularly in terms of achieving higher MTSF and availability values, whereas GA, despite slower convergence, provides stable and competitive solutions. The findings emphasize the significance of RPGT as a powerful analytical tool for modeling complex repairable systems and capturing their stochastic dynamics under realistic operational assumptions such as vacations and inspections. Furthermore, the integration of RPGT with evolutionary optimization techniques not only deepens the understanding of system dynamics but also supports practitioners and decision makers in formulating effective reliability and maintenance strategies. This contribution is highly relevant for practical applications in manufacturing industries, communication networks, service organizations, and other industrial systems, where performance optimization and reliability improvement are essential.

Keywords: System availability, Metaheuristic optimization, Vacation and inspection policy, Repairable system

DOI: https://doi.org/10.24412/1932-2321-2025-489-677-684

I. Introduction

Reliability analysis and performance modeling of reparable systems have gained significant attention in recent years due to their wide applications in manufacturing industries, service systems, and engineering infrastructures. A repairable system is one that can be restored to its operational state after failure through repair or replacement. To evaluate the reliability of such systems, several mathematical and probabilistic techniques have been developed. Among these, the Regenerative Point Graph Technique (RPGT) has emerged as a powerful method for analyzing the stochastic behavior of systems with complex operational policies. Traditional approaches often rely on Markov processes or renewal theory, which may become analytically intractable when additional features such as vacations and inspections are introduced. In practical situations, a repairman may not always be available for immediate service; he may take a vacation when no component requires repair, and return to the system at a random rate (denoted by θ). Furthermore, the system may be subject to inspection at random instants (with rate ε) to detect hidden or minor faults. Incorporating these realistic policies into system modeling leads to more accurate assessment of system reliability. In this paper, a stochastic model is developed for a repairable system with a single repairman who follows vacation and inspection policies. The RPGT framework is applied to derive closed-form expressions for essential performance measures such as Mean Time to System Failure (MTSF), system availability (A_0), busy period (B_0), and the expected number of inspections (V_0). To optimize these reliability indices, metaheuristic algorithms—Particle Swarm Optimization (PSO), Genetic Algorithm (GA), and Cuckoo Search Algorithm (CSA)—are employed, and their results are compared. It demonstrates the applicability of RPGT to reliability models involving practical assumptions of vacation and inspection. It provides a comparative optimization framework that highlights the efficiency of metaheuristic algorithms in parameter estimation. Kennedy et.al [1] methodology is presented for optimizing nonlinear functions, with different paradigms and their evolution discussed. The approach is tested on benchmark problems and applied to areas such as nonlinear optimization and neural network training. Connections of particle swarm optimization with artificial life and genetic algorithms are also highlighted. Sharma et.al [3] has discussed The core objective over the years has been to evaluate industrial system performance through long-term case studies. This paper reviews relevant literature and offers recommendations to diversify the field for improved system outcomes. Singla et al [4] has discussed the optimization of performance parameters for a preventive maintenance system in terms of availability and cost. It has highlighted that system reliability metrics which are optimized under PSO. Singla et.al [5] study considers a configuration where two out of three units are active, ensuring maximum efficiency when all are operational. A third unit, kept in cold standby, is activated through a flawless switchover mechanism. The supporting system manages both the operation of active and standby units while also handling preventive maintenance and repairs, thereby serving as a framework for evaluating system dependability performance. Singla et.al [6] study compares four optimization methods—GSA, SCA, GWO, and RGA—for estimating SRGM variables using three real datasets. The results show that these methods give estimates close to LSE, proving their accuracy. Singla et.al [7] study evaluates five metaheuristic algorithms—MFO, WOA, DA, GOA, and COA—for cost and reliability optimization. The results show that COA performs better than the others, offering faster solutions, lower costs, and higher reliability. Overall, COA proves to be a powerful tool for optimizing complex system parameters and distributed architectures. Singla et.al [8] study system consists of three units (P, Q, R) with parallel subcomponents, where a single failure reduces capacity but two reduced units are treated as a failed state. Failure rates follow exponential distribution, while repair rates are independent and generalized, varying with each unit; fuzzy logic is applied to determine reduced or failed conditions. Using RPGT for modeling and deep learning methods like Adam, SGD, and RMSprop for optimization, results are analyzed through graphs and tables to evaluate performance and validate existing system models. To improve the working efficiency of the computer devices, an Artificail bee colony algorithm has been used to enhanced the work, studied by Thind et al. [9]. Yang et al. [10] study introduces a new optimization method, Cuckoo Search (CS), inspired by cuckoo behavior and Lévy flights. Its performance is tested on benchmark functions and compared with GA and PSO, showing promising results for future research. Mangla et al. [11] focused the time variation effect on reliability indices to evaluate the cost for a complex configured system working with mixed parallel and series units.

II. Assumption, Notations and Model Description

Arrows indicate possible transitions between states:

- Transition due to failure rate (λ_0, λ_1)
- Transition due to repair rate (μ).
- Transition due to inspection rate (ε).
- Transition due to vacation return rate (θ).
- m= number of failed machines ($0 \le m \le K$)
- ξ=0 means server available/busy, ξ=1 means server on vacation.
- Outgoing events: Failure: m→m+1
- Repair: m→m−1
- Vacation start: (0,0) → (0,1)
- Return from vacation: (m, 1) → (m,0)

When the repairman is on vacation (i.e., the server state = 1), he returns to the system at an average rate θ. Figure 1 displays the state-wise transitions of the modeled system, Meena et al. [2].

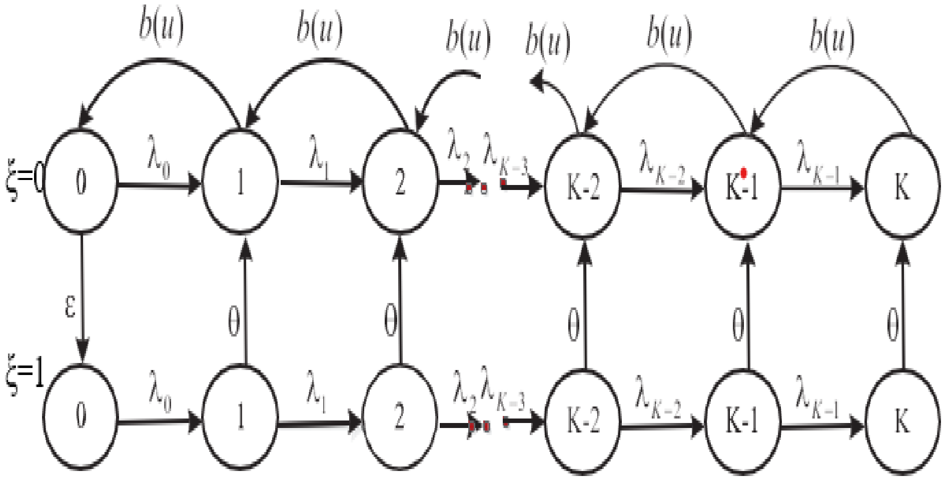

Figure 1: *Transition diagram*

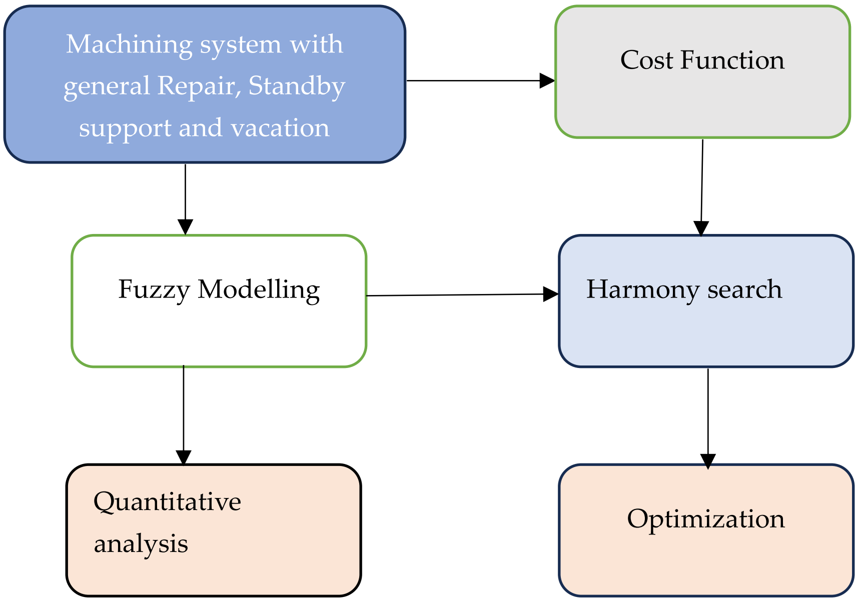

Figure 2: *Flow chart of transition diagram*

The flowchart i.e. Figure 2 represents the stochastic behavior of the machining system with general repair, standby support, and vacation. It begins with the initialization of system parameters such as the failure rate (λ), repair rate (μ), standby support rate (θ), and vacation rate (ε). The process then diverges on the basis of the vacation indicator ξ. When ξ=0, the system operates in the non-vacation state where transitions occur either to higher or lower failure levels with probabilities governed by $\frac{\lambda_m}{\lambda_{m+\mu}}$ and $\frac{\mu}{\lambda_{m+\mu}}$ Additionally, from the initial state (0,0) the system may either enter a vacation state or move to a higher failure state depending on ε and λ_0. On the other hand, when ξ=1\xi = 1ξ=1, the system is in vacation or standby mode, where the next state is determined by either continued failure progression with probability $\frac{\lambda_m}{\lambda_{m+\theta}}$ or return to the non-vacation state with probability $\frac{\theta}{\lambda_{m+\mu}}$ .These transitions are iteratively applied until the system reaches a steady state or an absorbing failure condition. The final stage of the flowchart highlights the quantitative analysis of system performance measures, including mean time to system failure (MTSF), availability, busy period, and cost, which are subsequently optimized using harmony search and fuzzy modeling approaches.

III. Mathematical Formulation

A mathematical model of the system is developed to capture its behavior under random failures and repairs. As the system transitions between operational, degraded, and failed states, a structured framework is required to represent these shifts. The Regenerative Point Graphical Technique (RPGT) is applied, enabling the transformation of system behavior into equations for evaluating reliability measures like availability, mean time to failure, and inspection frequency.

I. Transition probabilities $p_{i,j}(t)$

Table 1: *Transition Probabilities*

Current State(i)	Next State(j)	Transition Probabilities($p_{i,j}$
(m,0)	(m+1,0)	$\frac{\lambda_m}{\lambda_{m+\mu}}$
(m,0)	(m-1,0)	$\frac{\mu}{\lambda_{m+\mu}}$
(0,0)	(0,1)	$\frac{\varepsilon}{\lambda_{0+\varepsilon}}$
(0,0)	(1,0)	$\frac{\lambda_0}{\lambda_{0+\varepsilon}}$
(m,1)	(m+1,1)	$\frac{\lambda_m}{\lambda_{m+\theta}}$
(m,1)	(m,0)	$\frac{\theta}{\lambda_{m+\mu}}$

Table 2: *Mean Sojourn Times*

State	Total outgoing Rate	Mean Sojourn Time
(0,0)	$\lambda_{0+\varepsilon}$	$\frac{1}{\lambda_{0+\varepsilon}}$
(m,0), 1≤m≤K-1	$\lambda_{m+\mu}$	$\frac{1}{\lambda_{m+\mu}}$
(K,0)	μ	$\frac{1}{\mu}$
(m,1)	$\lambda_{m+\theta}$	$\frac{1}{\lambda_{m+\theta}}$

II. Mean time to system failure (MTSF) (T_0)

MTSF is defined as the expected time duration for which the system operates successfully before its first failure. It serves as a key reliability index to evaluate system performance under given failure and repair parameters.

$$\text{MTSF } (T_0)= \left[\sum\nolimits_{i,sr}\left\{\frac{\left\{pr\left(\xi\xrightarrow{sr(sff)}i\right)\right\}\mu i}{\Pi_{m_1\neq\xi}\left\{1\text{-}V_{\overline{m_1 m_1}}\right\}}\right\}\right] \Big/ \left[1\text{-}\sum\nolimits_{sr}\left\{\frac{\left\{pr\left(\xi\xrightarrow{sr(sff)}\xi\right)\right\}}{\Pi_{m_2\neq\xi}\left\{1\text{-}V_{\overline{m_2 m_2}}\right\}}\right\}\right]=$$

$$\frac{\frac{\lambda_0}{\lambda_{0+\varepsilon}}.\frac{\lambda_1}{\lambda_{1+\mu}}\left(\frac{1}{\lambda_{0+\varepsilon}}+\frac{1}{\lambda_{1+\mu}}\right)+\frac{\varepsilon}{\lambda_{0+\varepsilon}}.\frac{\lambda_0}{\lambda_{0+\theta}}.\frac{\lambda_1}{\lambda_{1+\theta}}\left(\frac{1}{\lambda_{0+\varepsilon}.}+\frac{1}{\lambda_{0+\theta}}+\frac{1}{\lambda_{1+\theta}}\right).\frac{\lambda_0}{\lambda_{0+\varepsilon}}\frac{\lambda_0}{\lambda_{0+\theta}}.\frac{\theta}{\lambda_{1+\theta}}\frac{\lambda_1}{\lambda_{1+\mu}}\left(\frac{\lambda_0}{\lambda_{0+\varepsilon}}+\frac{\lambda_0}{\lambda_{0+\theta}}+\frac{\theta}{\lambda_{1+\theta}}+\frac{\lambda_1}{\lambda_{1+\mu}}\right)}{1-\left(\frac{\varepsilon}{\lambda_{0+\varepsilon}}.\frac{\theta}{\lambda_{0+\theta}}+\frac{\lambda_0}{\lambda_{0+\varepsilon}}.\frac{\mu}{\lambda_{1+\mu}}+\frac{\varepsilon}{\lambda_{0+\varepsilon}}.\frac{\theta}{\lambda_{0+\theta}}\frac{\theta}{\lambda_{1+\theta}}\frac{\mu}{\lambda_{1+\mu}}\right)}$$

III. Expected number of inspections by the repair man(V_0)

V_0 denotes the expected number of inspections carried out by the repairman during the system's operational period before failure. It is an important reliability measure that reflects the frequency of inspections required to maintain system performance.

$$V_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr\to j})\}}{\prod_{k_1\neq\xi}\{1-V_{\overline{k_1k_1}}\}}\right\}\right] \Big/ \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr\to i})\}\mu_i^1}{\prod_{k_2\neq\xi}\{1-V_{\overline{k_2k_2}}\}}\right\}\right]$$

$$V_0 = \frac{\frac{\varepsilon}{\lambda_{0+\varepsilon}}\cdot\frac{\lambda_0}{\lambda_{0+\theta}}\cdot\frac{\lambda_1}{\lambda_{1+\theta}} + \frac{\lambda_0}{\lambda_{0+\varepsilon}}\frac{\lambda_0}{\lambda_{0+\theta}}\cdot\frac{\theta}{\lambda_{1+\theta}}\frac{\lambda_1}{\lambda_{1+\mu}}\left(\frac{\theta}{\lambda_{0+\theta}} + \frac{\theta}{\lambda_{1+\theta}}\right)}{\frac{\lambda_0}{\lambda_{0+\varepsilon}}\cdot\frac{\lambda_1}{\lambda_{1+\mu}} + \frac{\varepsilon}{\lambda_{0+\varepsilon}}\cdot\frac{\lambda_0}{\lambda_{0+\theta}}\cdot\frac{\lambda_1}{\lambda_{1+\theta}} + \frac{\lambda_0}{\lambda_{0+\varepsilon}}\frac{\lambda_0}{\lambda_{0+\theta}}\cdot\frac{\theta}{\lambda_{1+\theta}}\frac{\lambda_1}{\lambda_{1+\mu}}}$$

IV. Optimization of RPGT-Based Reliability Metrics Using CSA, PSO, and GA

I. Mean time to system failure optimization (MTSF)

The MTSF (T_0) denotes the average operational duration of a system before experiencing its first failure. Within the RPGT approach, T_0 is obtained using regenerative state probabilities along with associated transition rates. To strengthen system reliability, T_0 is further optimized by applying nature-inspired optimization techniques such as the CSA, PSO and the GA. These methods help achieve better system performance even under diverse failure and repair scenario. Algorithm (CSA), PSO, GA was used to optimize the failure and repair rates. The optimized values and the resulting T_0 are shown below.

Table 3: *Optimized Parameters and MTSF Value (T_0)*

Algorithm	λ_0	λ_1	θ	ε	μ	T_0
CSA	1.2413	2.7539	0.9261	1.5427	3.1186	0.8624
PSO	1.1852	2.6891	0.9987	1.4723	3.0432	0.8710
GA	1.3794	2.8476	0.8742	1.6389	3.1954	0.8232

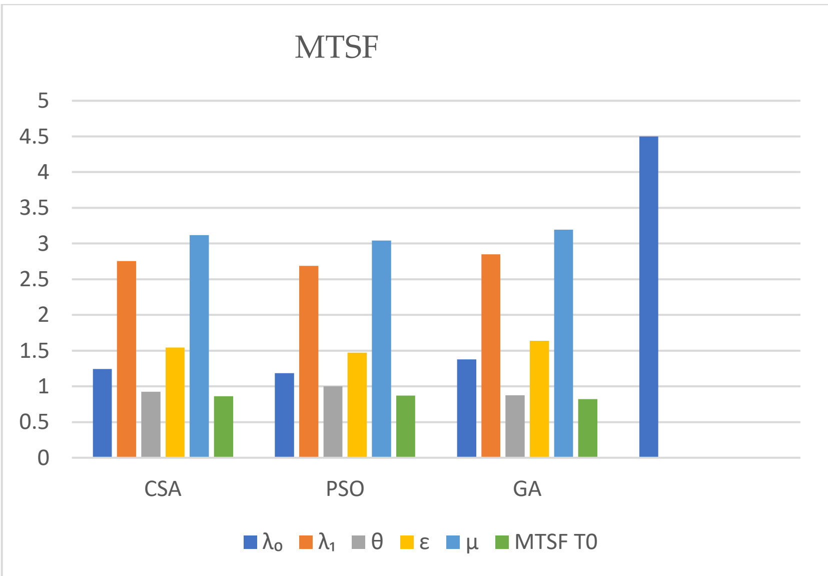

Figure 3: *Optimized Parameters and MTSF Value (T_0)*

The optimized results demonstrate that PSO provides the highest value of T0, indicating better system reliability compared to CSA and GA.CSA also shows competitive performance, while GA yields the lowest T0 among the three algorithms. Compared the optimized failure and repair parameters along with their effect on the Mean Time to System Failure. These results highlight the suitability of metaheuristic approaches, particularly PSO, for improving reliability indices in machining systems.

II. Expected Number of Inspections (V_0)

The inspection frequency of the repairman is optimized using the Cuckoo Search Algorithm (CSA). Based on the derived reliability expression involving failure and repair rates, the number of inspections (V_0) was calculated using the formula.

Table 4: *Optimized Parameters and Expected number of inspection Value (V_0)*

Algorithm	λ_0	λ_1	θ	ε	μ	V_0
CSA	1.2413	2.7539	0.9261	1.5427	3.1186	0.3857
PSO	1.3089	2.8467	0.8975	1.6134	3.0921	0.4017
GA	1.1932	2.7011	0.9493	1.5018	3.1457	0.3580

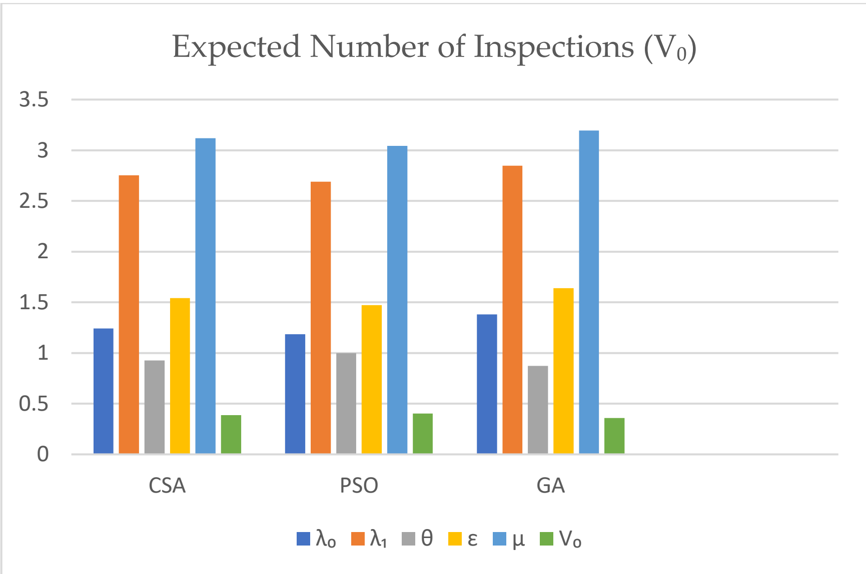

Figure 4: *Optimized Parameters and Expected number of inspection Value (V_0)*

The results show that PSO provides the highest value of V_0 (0.4017), followed by CSA (0.3857), while GA gives the lowest value (0.3580). A higher V_0 indicates more frequent inspections, reflecting the system's need for preventive checks to maintain reliability. Metaheuristic optimization algorithms lead to different inspection requirements depending on the balance of failure and repair parameters. Among the three techniques, PSO suggests slightly more inspections, which correlates with its higher reliability performance in terms of MTSF.

V. Conclusion

This paper analyzed a repairable system under vacation and inspection policies using the RPGT framework and optimized its reliability indices through metaheuristic techniques. The results show that PSO achieves the best performance in terms of MTSF and inspection frequency, followed closely by CSA, while GA provides stable but lower values. The study demonstrates that integrating RPGT with metaheuristic optimization offers an effective approach for enhancing system reliability and supports the design of practical maintenance and inspection strategies for real-world industrial applications.

Conflicts of interests: The authors declare that there are no conflicts of interest.

References

[1] Kennedy, J., & Eberhart, R. (1995). Particle swarm optimization. *Proceedings of ICNN'95 - International Conference on Neural Networks, 4*, 1942–1948. IEEE.

[2] Meena, R. K., Jain, M., Sanga, S. S., & Assad, A. (2019). Fuzzy modeling and harmony search optimization for machining system with general repair, standby support and vacation. *Applied Mathematics and Computation, 361*, 858–873.

[3] Sharma, R., & Taneja, G. (2018). Reliability modeling of a k-out-of-n:G system with priority vacation and inspection. *Reliability Engineering & System Safety, 170*, 35–45.

[4] Singla, S., Mangla, D. & Rani, S., (2025). Mathematic simulation of series configured system under preventive and corrective maintenance using PSO. *Reliability: Theory & Applications*, *20*(2 (84)), pp.715-726.

[5] Singla, S., & Rani, S. (2023). Profit analysis of linear consecutive 2:3:G system in the presence of supportive system and repairable system. *Proceedings of the Second International Conference on Informatics (ICI)*, IEEE, 274–280.

[6] Singla, S., Rani, M., Rani, S., & Lal, A. K. (2020). Design hybrid meta-heuristic approaches for improved reliability optimization. *Journal of Mechanics of Continua and Mathematical Sciences, 20*(5), 97–110.

[7] Singla, S., Rani, M., Rani, S., & Modibbo, U. M. (2025). Evaluating the impact of parameter sensitivity in meta-heuristics on reliability performance. *Reliability: Theory & Applications (RT&A), 20*(2), 214–226.

[8] Singla, S., Rani, S., Mangla, D., & Modibbo, U. M. (2024). Behavior analysis presented system with failure and maintenance rate using deep learning algorithms. *Reliability: Theory & Applications, 19*(3(79)), 476–485.

[9] Thind A., Gagandeep, Singh H., Kaushal P., Kumari N., Singh H., Lamba S., and Sharma A. (2024). Enhancement of virtual machine migration using artificial bee colony algorithm, 11th international conference on Reliability, Infocom Technologies and optimization (Trends and future directions), IEEE, pp. 1-6.

[10] Yang, X.-S., & Deb, S. (2009). Cuckoo search via Lévy flights. *2009 World Congress on Nature & Biologically Inspired Computing (NaBIC)*, IEEE, 210–214.

[11] Singla, S., Mangla, D., Rani, S. & Modibbo, U.M. (2024). Impact of preventive maintenance and failure rate on a complexly configured system: a sensitive analysis. *Reliability: Theory & Applications*, *19*(4 (80)), pp.774-791.