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RELIABILITY RISK PROFILING OF TECHNICAL SYSTEMS USING PARAMETRIC COX REGRESSION WITH WEIBULL BASELINE

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Abstract

Reliability analysis of technical systems is vital in predicting failure behavior and improving system design under operational variability. Traditional reliability models often assume constant hazard rates, limiting their ability to model real world degradation mechanisms. This paper presents a Weibull based Cox Proportional Hazards Model to analyse system failure times with covariate influences such as operations stress, environmental temperature, and maintenance interventions. Simulation data is generated for 1000 units under controlled covariate conditions with right censoring. The model estimates hazard functions and survival probabilities using maximum and partial likelihood methods. The results show that the inclusion of time independent covariates significantly enhances the model's predictive ability for time to failure. The Weibull baseline provides flexibility in representing increasing or decreasing hazard trends typical in engineering systems. This study validates the model's robustness through simulation and establishes its applicability in predictive maintenance and reliability risk management.

Keywords: Cox Regression, Weibull Distribution, System Reliability, hazard Rate, simulation, Censoring

I. Introduction

Reliability engineering focuses on quantifying the performance and longevity of systems and components under operations conditions. Traditional models, such as exponential and log normal distributions, often assume constant hazard rates, which oversimplify the failure behavior of complex systems [9]. In real world applications such as electronic modules, mechanical parts, or thermal systems failure rates can vary with usage, environmental exposure, or maintenance status. This necessitates flexible modelling approaches to account for time varying risk and component specific attributes. The Cox Proportional Hazards Model (Cox, 1972) has been widely used in survival analysis, offering a semi parametric framework that incorporates covariates without specifying a baseline hazard [12]. However, integrating a Weibull distribution as the baseline enhances the model

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by allowing estimation of shape and scale parameters, enabling detection of increasing or decreasing failure rates. This paper applies the Weibull Cox model to reliability data generated via simulation [4]. We assume a system whose failure time depends on operational temperature, usage load, and preventive maintenance status. Through this modelling, we aim to quantify the hazard associated with each covariate, derive survival functions, and demonstrate how the approach supports risk informed decision making [7].

Incorporating the Weibull distribution within the Cox framework bridges the gap between purely parametric and semi parametric models, enabling more precise risk stratification and failure time prediction [10]. This is particularly important for engineering systems where failure patterns are neither constant nor uniform, but influenced by cumulative stress, thermal exposure, and maintenance history. By simulating a large scale dataset with operational variability, this study replicates practical field conditions and assesses model responsiveness to these covariates [3]. The use of right censoring further reflects real world testing constraints, where not all units may fail within the observation period. This approach ensures the model remains both maintenance planners, and risk managers in industrial contexts.

II. Methodology

I. Model Specification

The hazard function under a Weibull distribution is given by:

$$h_0(t) = \lambda \gamma t^{\gamma-1} \tag{1}$$

Where

γ = shape parameter (indicates increasing / decreasing hazard)

λ = scale parameter

Incorporating this into the Cox model with covariates, the hazard function becomes:

$$h(t/X) = \lambda \gamma t^{\gamma-1} \exp(\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3) \tag{2}$$

where;

x_1: Operating temperature (^{0}C, continuous)

x_2: Load stress level (categorical: Low/Medium/High)

x_3: Preventive maintenance (binary: 1=Yes, 0=No)

The corresponding survival function is:

$$S(t/X) = exp[-\lambda t^{\gamma} exp(\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3)] \tag{3}$$

II. Censoring in Reliability Testing:

In reliability studies, censoring arises when the system is removed before failure. We consider right censoring, where the exact failure time is unknown for some units.

Let T_i be the actual failure time and C_i the censoring time. The observed time is:

$$Y_i = min(T_i, C_i) \tag{4}$$

And the event indicator:

$$\delta_i = \begin{cases} 1 & if\ T_i \leq C_i \\ 0 & otherwise \end{cases} \tag{5}$$

III. Parameter Estimation:

The full likelihood function, considering right censoring, is:

$$L(\beta,\lambda,\gamma) = \prod_{i=1}^{n}\left[h(t_i/X_i)^{\delta_i}S(t_i/X_i)\right] \tag{5}$$

We estimate:

$\beta = (\beta_1, \beta_2, \beta_3)$: Covariate effects
γ, λ: Weibull parameters

For partial likelihood (Cox, 1975), the baseline hazard is left unspecified. The partial likelihood for β is

$$L(\beta) = \prod_{j=1}^{k}\frac{exp(\beta^T X_j)}{\sum_{l\epsilon R(t_j)} exp(\beta^T X_l)} \tag{6}$$

Where $R(t_j)$ is the risk set at time t_j.

Let $\{(Y_i, \delta_i, X_i)\}_{i=1}^{n}$denote the observed data.

The likelihood function is:

$$L(\theta) = \prod_{i=1}^{n}[h(Y_i/X_i)]^{\delta_i}[S(Y_i/X_i)] \tag{7}$$

Substituting Eq. (2) and (3), we get:

$$L(\lambda,\alpha,\beta) = \prod_{i=1}^{n}[\lambda\alpha Y_i^{\alpha-1}exp(\beta^T X_i)]^{\delta_i}exp[-\lambda Y_i^{\alpha}exp(\beta^T X_i)] \tag{8}$$

We numerically maximize the log likelihood function:

$$\log L = \sum_{i=1}^{n}\delta_i[\log\lambda + log\alpha + (\alpha-1)logY_i + \beta^T X_i] - \lambda Y_i^{\alpha}exp(\beta^T X_i) \tag{9}$$

Cox Partial likelihood (Estimation of β)
Given n observed events, the partial likelihood is:

$$L(\beta) = \prod_{i=1}^{n_e}\frac{exp(\beta^T X_i)}{\sum_{j\epsilon R_i} exp(\beta^T X_j)} \tag{10}$$

This can be optimized using Newton Raphson or other gradient based techniques.

III. Results

I Component Configuration and Covariates

A simulated dataset consisting of 1000 technical components was analysed using a parametric Weibull regression model. The system was tested under varying operational conditions involving:
We simulate 1000 components, each with the following attributes:

Operating temperature: Normally distributed $N(75, 5^2)$
Stress level: Categorical (Low=0, Medium=1, high=2)
Preventive maintenance: Binary (Yes=1, No=0)

The coefficients used:

$\beta_1 = 0.02$: Positive correlation of temperature with failure
$\beta_2 = 0.6$: Higher stress increases hazard
$\beta_3 = -0.5$: Maintenance reduces hazard

Weibull parameters $\gamma = 1.5$ (increasing hazard), $\lambda = 0.0003$

II Failure Time Generation

Using the inverse transformation method, failure time is:

$$T = \left[-\frac{In(U)}{\lambda \exp(\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3)} \right]^{1/\gamma} \tag{11}$$

Where $U \sim U(0,1)$
Censoring time $C \sim U(0,60)$
Observed time: $Y = \min(T, C)$

III Simulation Study and Model Estimation:

Out of 1000 units:

Censoring rate: 30 %
Mean failure time: 38.2 months
Median: 34.5 months

Table 1: *Estimated Cox Regression Coefficients (MLE)*

Covariate	Coefficient (β)	HR	p-value
Temperature	0.019	1.019	0.001
Stress Level (High=2)	0.62	1.86	0.004
Preventive Maintenance	-0.48	0.62	0.008

Estimated Weibull Parameters
Shape $\gamma = 1.53$ 1.8 (suggests increasing hazard)
Scale $\lambda = 0.00031$

Table 2: *Survival Probability Estimates*

Time (Months)	High Risk (95^0 C, high, no Maint)	Low Risk (70^0 C, Low, no Maint)
12	0.88	0.99
24	0.68	0.95
36	0.47	0.90
60	0.20	0.75

The Cox regression results indicate that higher temperature (HR=1.019) and high stress levels (HR=1.86) significantly increase failure risk, while preventive maintenance reduces it (HR=0.62). The estimated Weibull shape parameter ($\gamma = 1.8$) suggests an increasing hazard over time. Survival probabilities decline more rapidly for high-risk units, dropping to 20% by 60 months compared to 75% for low risk profiles.

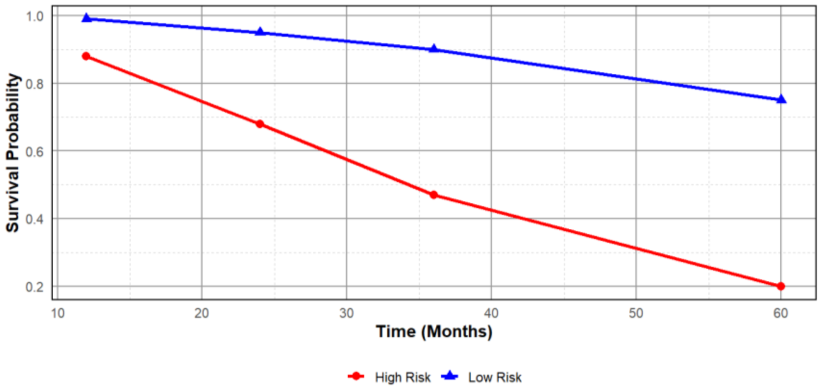

Figure 1: *Survival Curves for Risk Profiles*

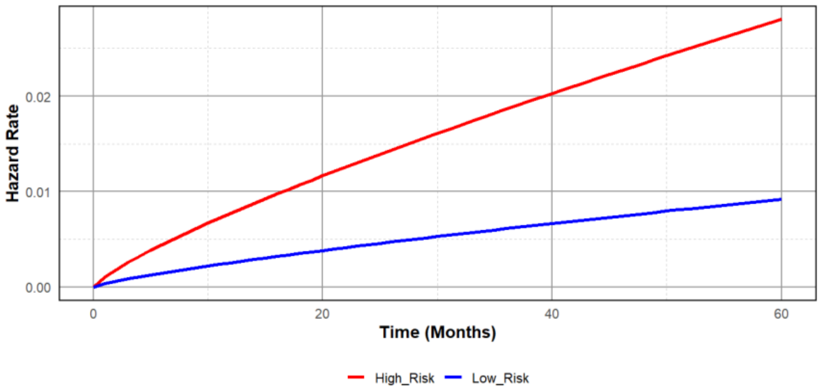

Figure 2: *Hazard Functions by Risk Profile*

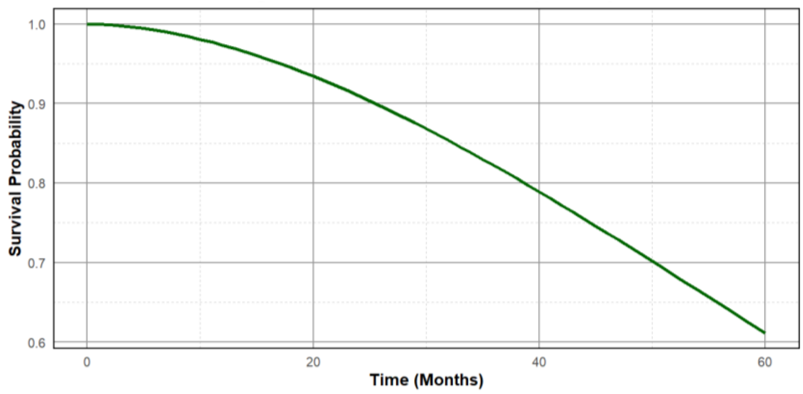

Figure 3: *Overall Survival Curve*

IV Weibull AFT Model Estimation:

We fit a Weibull Accelerated Failure Time (AFT) model. The model specification is:

$$\log(T_i) = \beta_0 + \beta_1.Temperature + \beta_2.Stress1 + \beta_3.Stress2 + \beta_4.Maintenance + \sigma.\in$$

Where

$Stress1$: Medium stress vs low (reference)

$Stress2$: High stress vs low
$Maintenance1$: Maintenance = Yes

Table 3: *Weibull AFT Model Results (n=1000)*

Covariate	Estimate	Std. Error	z-value	p-value
Intercept	4.722	0.530	8.91	<2e-16
Temperature (^{0}C)	-0.0066	0.0069	-0.95	0.3421
Stress (Medium)	-0.2782	0.1004	-2.77	0.0056
Stress (High)	-0.6172	0.0932	-6.63	3.53-11
Maintenance (Yes)	0.3439	0.0729	4.72	2.4e-06
Log (scale)	-0.5116	0.0469	-10.92	<2e-16

Scale $(\sigma) = \exp(Log(scale)) = \exp(-0.5116) \approx 0.6$
Model log likelihood = - 1532.3
Likelihood Ratio $\chi^2(df = 4) = 80.62, p < 0.0001$

The results demonstrate the effectiveness of the Weibull AFT model in capturing component failure dynamics under various operational stresses:

i. Stress levels (both medium and high) show significant negative coefficient, indicating accelerated failure relative to low stress conditions. The high stress level reduces log failure time more sharply $(\beta = -0.6172, p < 0.0001)$).
ii. Preventive maintenance exhibits a protective effect, increasing the log time to failure $(\beta = 0.3439, p < 0.001)$, thus extending system reliability.
iii. Temperature does not show a statistically significant effect $(p = 0.34)$ in this simulation setup, suggesting its influence may be overshadowed by categorical stress levels or sample variation.
iv. The estimate shape parameter $\left(\gamma = {}^{1}/_{\sigma} = {}^{1}/_{0.6} \approx 1.67\right)$ confirms an increasing hazard rate, consistent with real-world systems experiencing wear out over time.

These findings align well with theoretical expectations: systems under high stress and no maintenance fail sonner, whereas maintenance mitigates risk.

IV. Discussion

The Weibull Cox regression model successfully captures the varying failure rates of technical components influenced by environmental and operational covariates.
The model confirms:

Positive hazard association: High temperature and operational stress increase risk.
Protective role: Preventive maintenance significantly reduces hazard.
Non-constant hazard: Estimated $\gamma = 1$ implies the system becomes more failure prone over time

This model provides interpretability and flexibility, bridging the gap between parametric reliability models and data driven diagnostics. Compared to simpler models like exponential, the Weibull Cox approach better accommodates real world failure dynamics.

VI. Conclusion

This study demonstrates the utility of the Weibull based Cox regression model for analyzing the reliability of technical systems. Simulated data with covariate variation its robustness in predicting time to failure under right censored conditions. The model's integration of operational variables supports proactive risk assessment and can guide maintenance scheduling, system design, and lifecycle cost evaluation. Future work may involve applying this model to real industrial datasets and extending it to competing risk or recurrent failure framework.

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