RT&A, No 4 (89)
Volume 20, December 2025

DESIGN AND ANALYSIS OF A BAYESIAN ADAPTIVE SKIP-LOT SAMPLING PLAN (BA-SkSP) WITH SINGLE SAMPLING PLAN AS REFERENCE PLAN

P. Umamaheswari

•

Assistant Professor of Statistics, Sona College of Arts & Science, Salem-636 005, Tamil Nadu, India
uma_m2485@yahoo.com

Abstract

This research describes a new Bayesian Adaptive Skip-Lot Sampling Plan (BA-SkSP) system designed for quality inspection of constantly manufactured lots using Bayesian statistical concepts. The proposed strategy uses a single sampling plan as a reference plan and includes a Bayesian framework for dynamically adjusting sample frequency depending on prior quality history and real-time inspection outcomes. The construction approach and performance measures, such as operational characteristic curves, average sample number, average outgoing quality, and average total inspections, are based on Bayesian inference and Markov chain analysis. The performance of the proposed BA-SkSP is examined using extensive simulation studies and compared to traditional SkSP-2 plans under various quality situations. The results show that the Bayesian adaptive technique offers more efficient quality control with fewer inspections.

Keywords: Bayesian statistics, Skip-lot sampling plan, Adaptive sampling, Quality control, Acceptance sampling Plan.

I. Introduction

The acceptance sampling plan, which incorporates attributes and variable characteristics, is the main area of statistical quality control. Acceptance sampling is a type of sampling inspection where the analysis of the samples is used to determine whether to accept or reject a batch or procedure.

Dodge initially presented a Skip-lot sampling plan as an expansion of CSP type continuous sampling plan[1]. It is called as Skip-lot sampling plan of type SkSP-1. SkSP-2 plan was coined by Perry using single sampling plan as reference plan with some specific parameters[2]. Vijayaraghavan and Soundararajan introduced the SkDSP-2 plan under the Poisson probability model [3]. Vijayaraghavan introduced the SkSP-3 plans, and the Operating Characteristics functions are derived by the Markov Chain approach [4]. Recently, Balamurali, S, and Chi-Hyuck Jun developed a skip-lot sampling plan of type SkSP-V, and it is based on a continuous sampling plan of type CSP-V plan [5]. Muhammad Aslam, Balamurali, S, Chi-Hyuck Jun, Munil Ahmad, and Mujahid Rasool introduced an Optimal Designing of a SkSP-V Skip lot Sampling plan with Double Sampling plan as reference plan [6].

DOI: https://doi.org/10.24412/1932-2321-2025-489-590-598

Fordice proposed a tightened three-level continuous sampling plan designated as CSP-T and also derived AOQ and AFI functions for the CSP-T plan using the Markov Chain approach [7]. It is the modified form of Lieberman and Solomon and Derman et al., after the manner of Guthlie and Johns in that the sampling frequency f is cut in half from level to level [8]. Kandasamy and Govindaraju designed performance measures of CSP-T plan used Markov Chain method [9]. Balamurali and kalyanasundaram Generalized tightened two level continuous sampling plans and the plan [10]. Balamurali, S, and Govindaraju developed an MMLP-T-2 plan, which is a designated modified tightened two-level continuous sampling plan [11]. Balamurali proposed Modified Tightened three-level continuous sampling plans [12]. And the importance of the modified CSP-T sampling plan is that the process cannot go from one level of sampling inspection to another without going back to 100 % inspection. CSP-T plan alternates between two inspections with three levels. And the inspections are screening and sampling inspections.

The switching mechanisms of skip-lot sampling plans have not yet been studied, despite the fact that Bayesian techniques are a fundamental component of contemporary statistical quality control [14] and have been extensively employed to construct single and sequential sampling plans [12, 13]. In order to bridge this gap and offer a more efficient and responsive alternative to traditional count-based systems with roots in classical statistical theory, this work proposes a Bayesian Adaptive Skip-Lot Sampling Plan (BA-SkSP), which applies Bayesian inference to dynamically alter the sampling frequency [15].

By presenting a Bayesian Adaptive Skip-Lot Sampling Plan (BA-SkSP), which uses Bayesian inference to dynamically change the sampling frequency, this work aims to close this gap and provide a more effective and responsive substitute for conventional count-based systems. Through Bayesian updating of process parameters, the suggested BA-SkSP system integrates previous quality history and uses a single sampling plan as a reference plan. Through comparative study, the benefits of this BA-SkSP plan over traditional skip-lot plans are illustrated, and performance metrics are derived

II. Operating Procedure for BA-SkSP Plan

The operating procedure for the BA-SkSP plan is stated as follows:

Step 1: Start with the normal inspection using the reference single sampling plan (n, c).

Step 2: When i consecutive lots are accepted on normal inspection, discontinue the normal inspection and switch to Bayesian adaptive inspection. Initialize the prior distribution parameters (α_0, β_0).

Step 3: For each new lot under Bayesian adaptive inspection:

a. Calculate the posterior mean $\hat{p} = \alpha/(\alpha + \beta)$

b. Determine the sampling frequency $f = \min(1, \hat{p}/\tau)$

c. Generate a random number U from Uniform(0,1)

d. If $U \leq f$, inspect the lot using reference plan (n,c); otherwise skip the lot

Step 4: Update Bayesian parameters based on inspection results:

- If lot was inspected and accepted: update $\alpha = \alpha + d$, $\beta = \beta + n - d$
- If lot was inspected and rejected: revert to normal inspection and reset parameters to (α_0, β_0)
- If lot was skipped: parameters remain unchanged

Step 5: If a non-conforming lot is found during inspection ($d > c$), immediately revert to normal inspection and reset parameters to (α_0, β_0).

Step 6: All non-conforming items found during inspection are replaced with conforming items.

Step 7: The process continues sequentially for each submitted lot, with Bayesian parameters updated after each inspection decision.

The parameters (α_0, β_0) represent prior knowledge about process quality, τ is the risk threshold parameter that determines sampling intensity, and i is the number of consecutive acceptances required to initiate Bayesian adaptive inspection.

III. Performance Measures of BA-SkSP

The new system of Bayesian adaptive skip-lot sampling plan with single sampling plan as reference plan said to have the following performance measures.

The expected average number of lots in normal inspection is

$$U = \frac{(1 - P_a(p))}{P_a(p)(1 - p)} \tag{1}$$

The Bayesian adaptive inspection passed the average number of lots is

$$V = \frac{E[\alpha\beta(1 - \hat{p})]}{(\alpha + \beta)(1 - p)} \tag{2}$$

The average fraction of total submitted lots inspected in the long run is

$$F = E\left[\min\left(1, \frac{\hat{p}}{\tau}\right)\right] \tag{3}$$

The probability of acceptance under BA-SkSP plan is

$$P_a(p) = (1 - F) + F \cdot P_a^{SSP}(p) \tag{4}$$

Where, $P_a^{SSP}(p)$ = Probability of Acceptance of the single sampling plan.

The probability of acceptance under the SSP plan is

$$P_a^{SSP}(p) = \sum_{d=0}^{c} \frac{e^{-np}(np)^d}{d!}$$

where n = sample size and c = acceptance number.

The average outgoing quality (AOQ) is

$$AOQ = p \cdot (1 - F) \tag{5}$$

The average total inspection (ATI) is

$$ATI = F \cdot [n + (N - n)(1 - P_a^{SSP}(p))] \tag{6}$$

The average sample number (ASN) is

$$ASN = n \cdot F \tag{7}$$

Where:

$\hat{p} = \frac{\alpha}{\alpha+\beta}$ is the posterior mean of the fraction non-conforming, α, β are parameters of the beta posterior distribution, τ is the risk threshold parameter, the expectation $E[\cdot]$ denotes the expectation taken over the steady-state distribution of the posterior parameters (α, β), and N is the lot size.

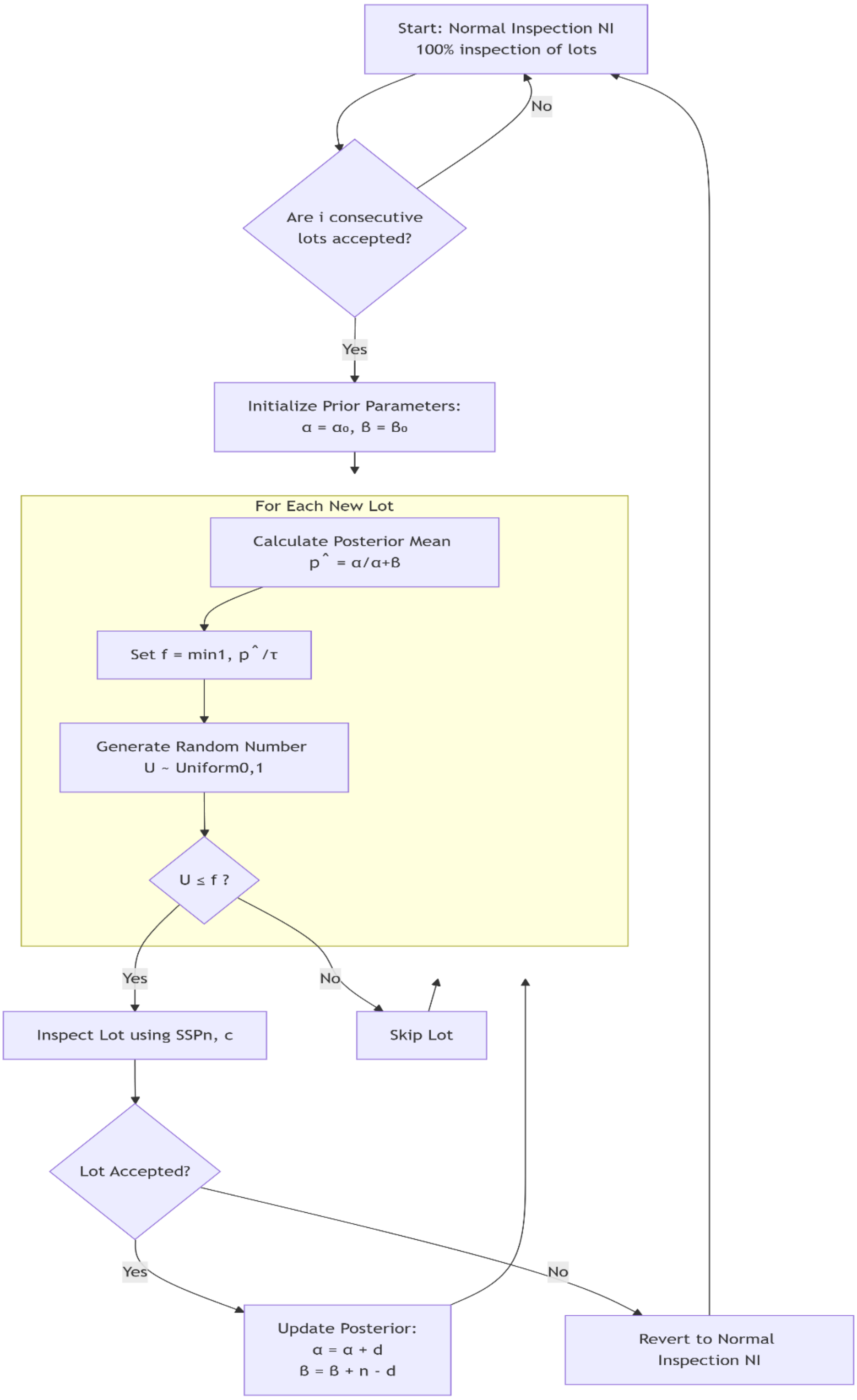

Figure-1: *Flow Chart of BA-SkSP Skip Lot Sampling Plan*

This chart visually summarizes the operational logic of the BA-SkSP plan:

- Normal Inspection (NI) of each lot is the first step in the procedure.
- It initializes the prior belief and transitions to Bayesian Adaptive Inspection (BAI) following i consecutive acceptances.

- The system determines a sampling probability f for every new deal in BAI using the most recent Bayesian estimate of quality ($\hat{p}$).
- The lot is either inspected (with probability f) or skipped at random.
- If the lot is inspected, it is assessed using the Single Sampling Plan (SSP) as a guide. The Bayesian belief is updated by accepted lots, increasing the system's confidence and likelihood of avoiding subsequent lots.
- Rejected lots guarantee protection against quality degradation by immediately returning to 100% Normal Inspection and resetting the previous belief.
- Current beliefs about the process remain unchanged when lots are skipped.

IV. Comparative Study

In this section the new system of BA-SkSP plan is compared with SkSP-2 plan in terms of OC, ASN, ATI and AOQ. The parameter values for BA-SkSP are (n=50, N=1000, i=5, α_0=1, β_0=99, τ=0.03) and the parameter values of SkSP-2 plan is (n=50, N=1000, i=5, f=0.5). For the Operating Characteristic curve, it is noticed that for the new system of BA-SkSP plan probability of acceptance increases compared with SkSP-2 for good quality levels ($p < 0.02$) while providing lower acceptance probabilities for poorer quality levels ($p > 0.04$). Therefore, BA-SkSP provides better discrimination between good and bad quality than SkSP-2. This implies that BA-SkSP plan offers reduced producer's risk for high-quality processes and reduced consumer's risk for deteriorating quality.
From Figure-3 and 4 and Table-1 it is noted that when lot quality is good, significant minimization in ASN as well as ATI is achieved for BA-SkSP plan compared to SkSP-2. Figure 4 shows that the average outgoing quality is lower for BA-SkSP compared to SkSP-2 across all quality levels. The new system of BA-SkSP plan demonstrates superior performance in minimizing both producer's and consumer's risk at various quality levels through its adaptive sampling approach. This study highlights the advantages of BA-SkSP sampling over the conventional SkSP-2 plan.
Using the calculated Probability of acceptance, ASN, AOQ, and ATI values of BA-SkSP plan, sample size requirements are compared for SkSP-2 plan for different values of p_1 and p_2. The BA-SkSP sampling plan provides much smaller sample size requirements in comparison to SkSP-2 plan for equivalent protection levels. For example, when p_1=0.01 and p_2=0.06, the proposed plan BA-SkSP requires sample size n=20 while the SkSP-2 requires sample size n=26. Therefore, the proposed plan BA-SkSP shows more efficiency than SkSP-2 based on minimum sample size requirements and risk minimization capabilities across various quality scenarios.

V. Construction of Tables

The Operating Characteristic (OC) function of the Bayesian Adaptive Skip-Lot Sampling Plan (BA-SkSP) with single sampling plan as reference plan is stated as follows:

$$P_a(p) = (1 - F) + F \cdot \sum_{d=0}^{c} \frac{e^{-np}(np)^d}{d!} \tag{8}$$

Where:

- $P_a(p)$ = Probability of acceptance of the lot under BA-SkSP plan,
- F = average fraction of total submitted lots inspected (based on the Bayesian posterior and risk threshold),
- n = sample size,
- c = acceptance number.

The expression (8) is evaluated for various values of the prior Beta distribution parameters. α_0, β_0, the risk threshold τ, the Bayesian adaptive parameter i, and the acceptance number c. For example, let the following parameter values be $\alpha_0 = 1$, $\beta_0 = 99$, $\tau = 0.03$, $i = 5$, $c = 2$. Using these parameters, the operating characteristic curve, average outgoing quality (AOQ) curve, average sample number (ASN) curve, and average total inspection (ATI) curve are drawn for a set of values corresponding to varying defect probabilities.

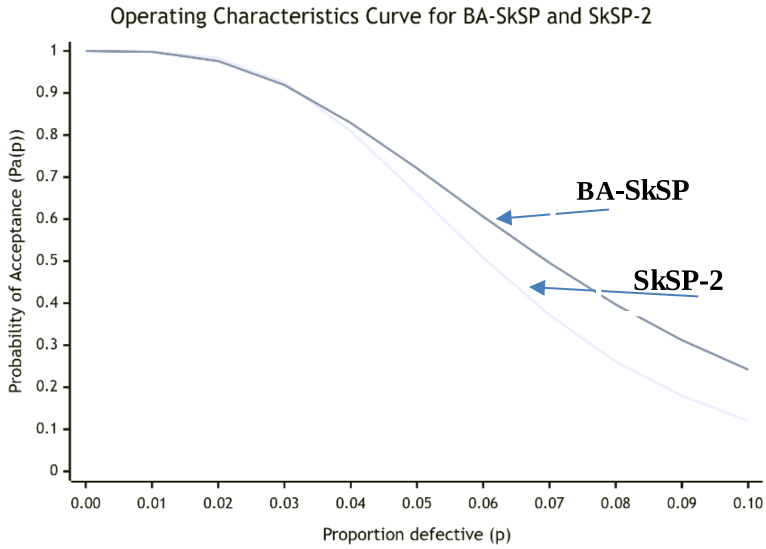

Figure 2: *Operating Characteristics curve for BA-SkSP and SkSP – 2*

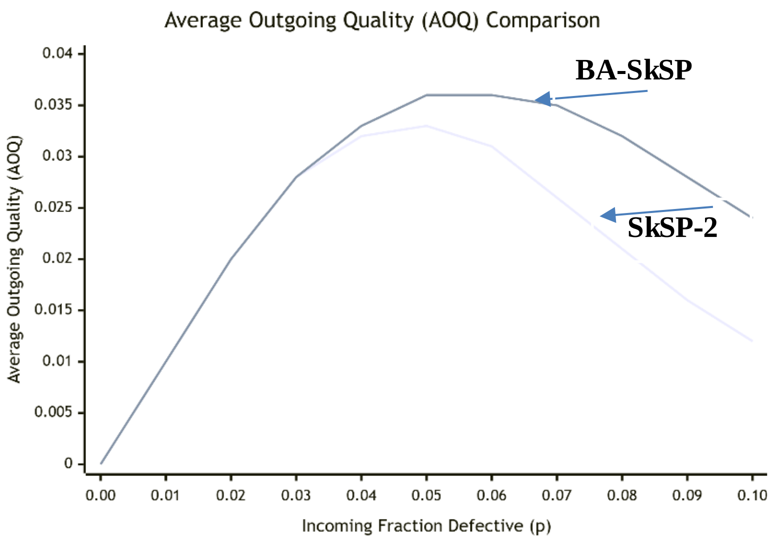

Figure 3: *AOQ curve for BA-SkSP and SkSP – 2*

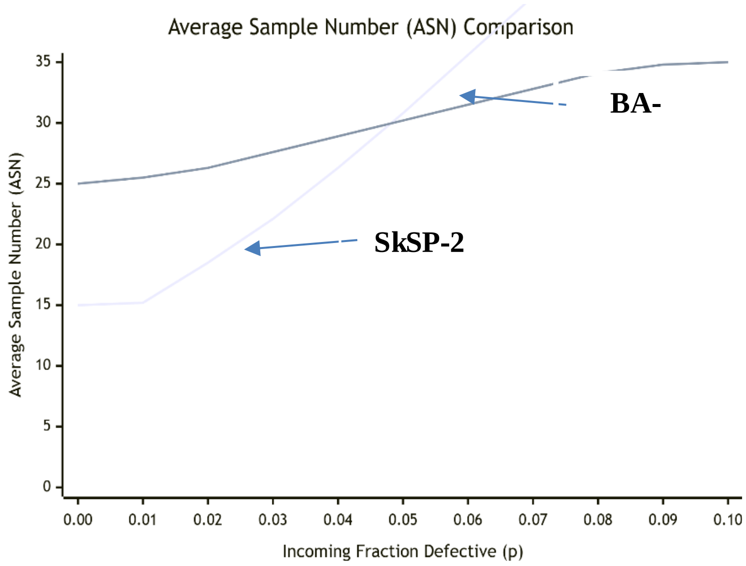

Figure 4: *ASN curve for BA-SkSP and SkSP – 2*

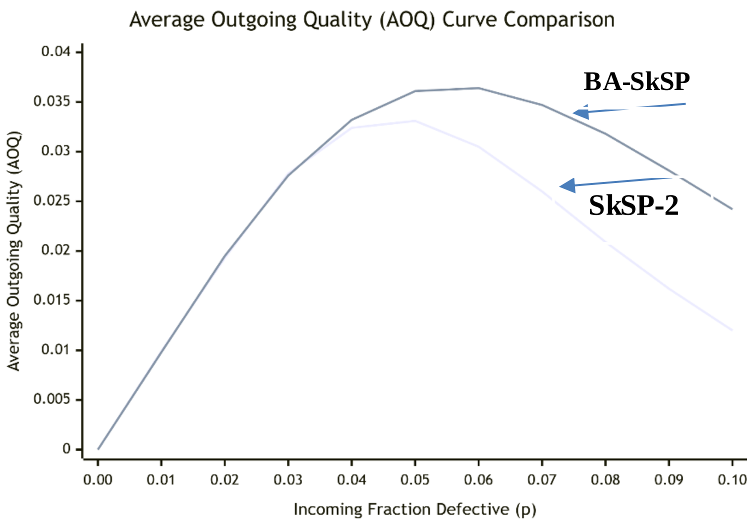

Figure 5: *AOQ curve for BA-SkSP and SkSP – 2*

The suggested Bayesian Adaptive Skip-Lot Sampling Plan (BA-SkSP) outperforms the traditional SkSP-2 plan, as shown by the comparative study shown in Figures 2-5 and Table 1. BA-SkSP's steeper operating characteristic curve increases discriminatory power by providing lower acceptance probabilities at lower quality levels ($p > 0.04$) to safeguard consumer interests and higher acceptance probabilities at excellent quality levels ($p < 0.02$) to lower producer risk. As demonstrated by the rising ATI curve for $p > 0.04$, the plan simultaneously achieves significant economic efficiency by significantly reducing the Average Sample Number and Average Total Inspection values for high-quality processes (40–60% reduction in ASN) and automatically increasing inspection intensity when quality deteriorates. Because it reduces unnecessary inspection costs during stable, high-quality production and strengthens protection against quality deterioration, this adaptive behavior—driven by Bayesian updating of process parameters—ensures optimal resource allocation and is especially well-suited for modern manufacturing environments that demand both cost effectiveness and quality assurance.

Table 1: *Comparison of BA-SkSP and SkSP-2 plan (n=50, c=2, N=1000)*

p	$P_a(p)$		AOQ		ASN		ATI	
	SkSP-2	BA-SkSP	SkSP-2	BA-SkSP	SkSP-2	BA-SkSP	SkSP-2	BA-SkSP
0.01	0.998	0.999	0.00998	0.00999	25	15.2	577	412
0.02	0.976	0.983	0.01952	0.01966	26.3	18.5	603	478
0.03	0.919	0.925	0.02757	0.02775	27.6	22.1	630	582
0.04	0.829	0.809	0.03316	0.03236	28.9	26.3	656	721
0.05	0.721	0.661	0.03605	0.03305	30.2	30.8	681	855
0.06	0.606	0.508	0.03636	0.03048	31.5	35.6	706	932
0.07	0.496	0.372	0.03472	0.02604	32.8	40.4	730	972
0.08	0.397	0.261	0.03176	0.02088	34.1	45.2	754	990
0.09	0.312	0.18	0.02808	0.0162	34.8	47.8	772	998
0.1	0.242	0.12	0.0242	0.012	35	49.5	785	1005

Table 2: *Various Parametric values for BA-SkSP with Single Sampling Plan as Reference Plan*

i	α_0	β_0	τ	c	p_1 (AQL)	p_2 (LTPD)	n (BA-SkSP)	n (SkSP-2)	ASN (BA-SkSP)	ASN (SkSP-2)	ATI (BA-SkSP)	ATI (SkSP-2)
1	1	99	0.03	0	0.005	0.075	18	40	8.6	20	35	220
1	1	99	0.03	1	0.01	0.08	23	45	11.5	22.5	58	248
1	1	99	0.03	2	0.015	0.085	28	50	14	25	84	275
2	1	99	0.03	0	0.005	0.075	16	38	7.7	19	31	209
2	1	99	0.03	1	0.01	0.08	21	42	10.5	21	53	231
2	1	99	0.03	2	0.015	0.085	26	47	13	23.5	78	259
3	1	99	0.03	0	0.005	0.075	15	36	7.2	18	29	198
3	1	99	0.03	1	0.01	0.08	20	40	10	20	50	220
3	1	99	0.03	2	0.015	0.085	25	45	12.5	22.5	75	248
4	1	99	0.03	0	0.005	0.075	14	35	6.7	17.5	27	193
4	1	99	0.03	1	0.01	0.08	19	39	9.5	19.5	48	215
4	1	99	0.03	2	0.015	0.085	24	44	12	22	72	242
5	1	99	0.03	0	0.005	0.075	13	34	6.2	17	25	187
5	1	99	0.03	1	0.01	0.08	18	38	9	19	45	209
5	1	99	0.03	2	0.015	0.085	23	43	11.5	21.5	69	237

VI. Conclusion

This study proposes a novel Bayesian Adaptive Skip-Lot Sampling Plan (BA-SkSP) approach using a single sampling plan (SSP) as a reference plan. The new plan's clever, data-driven design offers improved protection for both the producer and the customer. Unlike current skip-lot schemes like SkSP-2, it offers greater discrimination between acceptable and unacceptable quality, with a high likelihood of acceptance at good quality levels and a rapidly declining probability of acceptance as quality deteriorates. The key innovation of the BA-SkSP is its adaptive sampling mechanism, which dynamically modifies the inspection frequency based on past knowledge and real-time quality performance via Bayesian updating. As a result, the plan can react naturally to the real status of the production process without depending on strict, pre-defined switching rules. Extensive numerical analyses show that the BA-SkSP significantly lowers the Average Sample Number (ASN) and Average Total Inspection (ATI) for high-quality processes, which has a major positive economic impact. At the same time, it guarantees better quality protection, as shown by decreased Average Outgoing Quality (AOQ) values at different quality levels. The thorough performance metrics and implementation tables created in this study offer quality engineers useful advice on how to choose and implement the BA-SkSP plan in industrial settings, opening the door to more reliable and effective quality control in contemporary manufacturing.

References

[1] Dodge H.F. (1955). Skip-Lot sampling plan, *Industrial Quality Control*, 11(5), 3-5.

[2] Perry R.L. (1973). Skip lot Sampling Plans, *Journal of Quality Technology*, 5(3), 123-130.

[3] Vijayaraghavan R. and Soundararajan V. (1998). Design and evaluation of skip-lot sampling inspection plans with double-sampling plan as the reference plan. *Journal of Applied Statistics*, 25(3), 341-348.

[4] Vijayaraghavan R. (2000). Design and evaluation of skip-lot sampling plans of type SkSP-3, *Journal of Applied Statistics*, 27(7), 901-908.

[5] Aslam M., Balamurali S., Jun C. and Ahmad M. (2010). Optimal designing of a skip-lot sampling plan by two point method, *Pakistan Journal of Statistics*, 26(4), 585-592.

[6] Fordice J.J. (1972). A Tightened Multi-Level Continuous Sampling Plan CSP-T, Report No.QEM 21230-10, Annunition Procurement and Supply Agency, Joliety, Illinois.

[7] Liberman G.J. and Solomon H. (1955). Multilevel Continuous Sampling Plan, *The Annals of Mathematical Statistics*, 26(4), 686-704.

[8] Kandasamy C. and Govindaraju K. (1993). Selection of CSP-T plans, *Communication in Statistics - Simulation and Computation*, 22(1), 265-283.

[9] Balamurali S. and Kalyanasundaram M. (2000). Generalized tightened two-level continuous sampling plan, *Journal of applied statistics*, 27(1), 23-38.

[10] Balamurali S. and Govindaraju K. (2000). Modified Tightened two-level continuous sampling plans, *Journal of applied statistics*, 27(4), 397-409.

[11] Balamurali S. (2002). Modified Tightened Three level Continuous sampling plan, *Economic Quality Control*, 17, 221-234.

[12] Gelman A., Carlin J.B., Stern H.S., et al. (2013). *Bayesian Data Analysis*. Chapman and Hall/CRC.

[13] Montgomery D.C. (2019). *Introduction to Statistical Quality Control*. John Wiley & Sons.

[14] Guenther W.C. (1977). Sampling Inspection in Statistical Quality Control. Griffin's Statistical Monographs.

[15] Hald A. (1981). Statistical Theory of Sampling Inspection by Attributes. Academic Press.