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CONTROL CHARTS WITH SIX SIGMA FOR RANGE UNDER TWO-PARAMETER GAMMA DISTRIBUTION

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Abstract

Quality monitoring of goods and services has been practiced for centuries, whether directly or indirectly. However, the modern approach incorporates quantitative methods rooted in statistical principles for quality control. Statistical quality control (SQC) encompasses a variety of problem-solving techniques used in industrial production, some of which rely on straightforward statistical theories for process control and monitoring. Over time, statistical process control (SPC) has evolved to include a broad range of statistical and optimization methods employed by professionals to enhance quality. In this research paper, we present a comprehensive review and development of a Six Sigma-based control chart for range under the Gamma distribution, examining its various aspects and applications.

Keywords: Gamma distribution, Range, Process capability and Six sigma

I. Introduction

Quality planning is not merely a technical or operational task it is a critical strategic function that plays a pivotal role in shaping the long-term success and sustainability of an organization. Much like the product development plan, financial strategy, marketing roadmap, and human resource management plan, the strategic quality plan forms a foundational pillar of an organization's overall business strategy. Neglecting to implement a well-structured quality planning framework can lead to significant inefficiencies, including wasted time, financial losses, and diminished workforce productivity. More critically, the absence of a strategic focus on quality often results in the proliferation of design flaws, manufacturing errors, product failures in the field, and recurring customer complaints—all of which can seriously damage a company's reputation and market position.

At its core, quality planning is about proactively identifying the expectations and requirements of all stakeholders involved in or impacted by the product or service. This includes not only external customers who ultimately purchase and use the product but also internal customers, such as departments and teams within the organization that depend on one another to deliver consistent value. Understanding these needs in detail is essential for designing processes, products, and services that not only meet but exceed expectations.

By aligning quality goals with the broader objectives of the organization, quality planning

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enables a culture of continuous improvement, fosters cross-functional collaboration, and ensures that quality is embedded into every stage of the business lifecycle from design and production to delivery and customer support. In this way, strategic quality planning becomes a proactive investment that drives operational excellence, enhances customer satisfaction, and positions the organization for sustained competitive advantage.

II. Review of Literature

I have conducted a thorough review of several scholarly works that build upon and are closely related to this paper, examining their methodologies, key findings, and contributions in order to highlight the broader academic context, identify recurring themes, and assess how these studies extend, refine, or challenge the ideas originally presented.

Radhakrishnan and Balamurugan [9] present a paper that addresses the limitations of traditional 3-sigma Shewhart control charts when applied to processes operating under a Six Sigma initiative. The authors propose a new "Six Sigma-based exponentially weighted moving average (EWMA) control chart," designed to be more effective for high-quality processes. They argue that this chart is better at detecting out-of-control conditions in processes with reduced variation, thereby enabling timely corrective actions.

Radhakrishnan and Balamurugan [10] introduce a Six Sigma-based control chart for range, demonstrating that even within Six Sigma practices, processes may still fall out of control due to unexpected variations. The proposed charts provide a more effective tool for monitoring and improving process quality compared to traditional 3-sigma Shewhart charts. Similarly, Radhakrishnan and Balamurugan [11] develop Six Sigma-based control charts for monitoring defects, showing that processes can still deviate despite Six Sigma implementation. The proposed charts and tables serve as practical tools for engineers seeking to improve quality control beyond the capabilities of traditional Shewhart methods.

In another study, Radhakrishnan and Balamurugan [12] present research that highlights the limitations of traditional 3-sigma Shewhart control charts in high-quality processes under Six Sigma. They propose a "Six Sigma-based cumulative-sum (CUSUM) control chart," which is argued to be more effective in detecting process deviations and potentially replacing existing Shewhart charts.

In a related work, Radhakrishnan and Balamurugan [13] propose a Six Sigma-based $\bar{X}$ control chart using standard deviation with varying sample sizes, showing that processes may still fall out of control under Six Sigma initiatives. The suggested charts and tables help engineers detect and correct variations more effectively than conventional Shewhart control charts.

Radhakrishnan and Balamurugan [14] propose a Six Sigma-based control chart for regression, emphasizing that even under Six Sigma initiatives, processes may experience unexpected variations. The suggested control charts allow companies to detect such issues more effectively than traditional Shewhart charts.

Pavithra and Balamurugan [7] present a new statistical process control (SPC) method, introducing a "Six Sigma-based fuzzy mean using range control chart" under a "moderate distribution," leveraging process capability (CP). The authors claim that this new chart is both more accurate and more flexible than traditional methods, addressing key weaknesses of existing control charts.

Finally, Kanneswari and Sivakumaran [4] present a control chart based on Six Sigma using the interquartile range (IQR) to address cases where normality assumptions do not hold. Their study demonstrates that processes remained out of control under all tested methods, emphasizing the ongoing need for quality improvements.

III. Quality control and improvement

It represents a structured and strategic set of activities aimed at ensuring that products and services not only meet predefined requirements but are also continuously enhanced over time. These functions are vital in fostering customer satisfaction, maintaining consistency, reducing waste, and driving operational excellence. Together, they form the backbone of any organization's commitment to delivering value and excellence in a competitive marketplace.

At the heart of both quality control and improvement lies the need to manage and reduce variability a common and often detrimental source of quality issues. Variability in processes, materials, equipment, or human performance can lead to defects, inefficiencies, and inconsistent outcomes. To address this, organizations rely heavily on statistical techniques that provide objective, data-driven insights into how processes behave and where improvements are needed. Among the most widely used tools are:

Statistical Process Control (SPC): This method involves the use of control charts and statistical methods to monitor production and service processes in real time. By identifying patterns, trends, or anomalies, SPC enables early detection of potential problems before they result in defective output.

Design of Experiments (DOE): This approach allows organizations to systematically test and analyse the effects of multiple process variables to determine the optimal conditions for performance. DOE is particularly useful for identifying the root causes of variation and implementing effective changes.

3.1 Control chart techniques

The area between the upper and lower control limits is considered the normal range of variation. As long as the data points fall within this region, the process is deemed to be "in control," implying that it is functioning as expected. However, when one or more data points fall outside of these control limits, it signals that the process may be out of control, likely due to an assignable cause Montgomery[5]. Such an occurrence prompts further investigation to identify and correct the issue before it impacts product quality or process performance.

By regularly monitoring process data using control charts, organizations can detect potential issues early, minimize the risk of defects, and maintain a consistent level of quality in their operations.

Consider the statistic $t = t(x_1, x_2, \ldots x_n)$, function of the sample observations $(x_1, x_2, \ldots x_n)$ $Mean_t = \mu_t$ and $Variance_t = \sigma_t^2$. If the Statistic 't' is normally distributed, then from the fundamental area property of the normal distribution, we have

$$P[\mu_t - 3\sigma_t < t < \mu_t + 3\sigma_t] = 0.9973 \Rightarrow P[|t - \mu_t| < 3\sigma_t] = 0.9973 \Rightarrow P[|t - \mu_t| > 3\sigma_t] = 0.0027 \quad (1)$$

Then the control limits of three sigma are as follows:

$$\text{UCL} = \mu_t + \varphi\sigma_t$$

$$\text{Center Line (CL)} = \mu_t$$

$$\text{LCL} = \mu_t - \varphi\sigma_t$$

The symbol φ represents the distance between the control limits and the centre line of a control chart, measured in units of standard deviation. This distance defines the threshold for identifying whether variations in the process are within acceptable bounds. When the value of φ is set to 3, the chart is referred to as a 3-sigma control chart, meaning that the control limits are placed three standard deviations above and below the process mean. This configuration is widely used because it provides a balance between sensitivity to process changes and the avoidance of false alarms, effectively capturing approximately 99.73% of all expected data points under normal conditions.

Control charts for variables are used to monitor quality characteristics that can be measured on a continuous scale, such as height, weight, volume, length, or width. These characteristics are quantitative in nature, allowing for precise measurement rather than mere classification. When a product or component is inspected, the specific variable of interest is measured and recorded for analysis.

3.2 Concept of six sigma

In organizations that adopt Six Sigma practices, the implementation of these advanced quality techniques often results in such significant process improvement that traditional control charts may no longer be effective, as they may not register any points outside the control limits. Consequently, there arises a need for specialized control charts that can effectively monitor and reflect the performance of processes operating at Six Sigma quality levels. These new charts are essential for accurately evaluating the outcomes of companies that have integrated Six Sigma initiatives into their operations. Radhakrishnan and Balamurugan [8], proposed the development of control charts tailored to Six Sigma initiatives, specifically designed to monitor the proportion of defectives in a process.

The Table 1 illustrates the relationship between sigma quality levels, the corresponding percentage of defect-free output, and the number of defects per million opportunities (PPM). At 3σ quality, a process produces 99.73% good output, which still results in about 66,807 defects per million opportunities. As the sigma level increases, the percentage of quality improves dramatically while defects decrease sharply. For example, at 4σ quality, defects are reduced to about 6,210 PPM (roughly one-tenth of 3σ). At 5σ, the defect rate drops further to only 233 PPM, representing near-perfect performance. Finally, 6σ quality reaches 99.99966% defect-free output, allowing only 3.4 defects per million opportunities.

Table 1: *Levels of sigma and defect rate*

Quality Level	%Quality	Defective PPM*
3σ	99.73	66807
4σ	99.9937	6210
3σ	99.73	66807
4σ	99.9937	6210

*Parts per Million (PPM)

3.3 Process Capability (CP)

The Cp index is a straightforward and commonly applied metric that compares the allowable spread of the specification limits to the actual spread of the process variation. Specifically, it is calculated by taking the difference between the Upper Specification Limit (USL) and the Lower

Specification Limit (LSL) which defines the range of acceptable values and dividing this by six times the process standard deviation (6σ). Mathematically, the CP index is expressed as: $C_p = \frac{USL - LSL}{6\sigma}$.

Here, σ (sigma) is the estimated standard deviation of the process, which reflects the inherent variability in the system. A higher Cp value indicates that the process variation is well within the specification limits, suggesting that the process is capable of consistently meeting product quality standards. However, it is important to note that Cp assumes the processis centred; if it is not, a different index such as Cpk should be used to account for process shift.

3.4 Construction of control charts under lifetime distributions

The development of control charts specifically designed for lifetime distributions plays a crucial role in accurately tracking and managing the reliability and performance of products and systems throughout their operational lifespan. Conventional control charts are typically based on the assumption that the underlying data follows a normal distribution. However, this assumption does not hold true in many practical scenarios, particularly when dealing with lifetime or reliability data, which often conform to non-normal distributions. By tailoring control charts to these particular lifetime distributions, organizations can significantly improve the effectiveness of their quality control efforts. These customized charts offer greater sensitivity to changes in process behaviour and provide more precise insights into potential failures or deviations, thereby supporting more informed decision-making and fostering enhanced product reliability.
Processes with a constant hazard rate where the interval between subsequent occurrences or failures stays constant throughout time are specifically monitored by Shewhart-type [16], control charts that are adapted to the Gamma distribution. Because it assumes a constant failure rate, the Gamma distribution is frequently used in survival analysis and reliability engineering. This makes it especially well-suited for estimating the lifespans of specific systems and components. The primary parameter of the distribution, which is closely related to the mean time between failures (MTBF), is what these control charts are intended to identify as it changes. Shewhart-type charts based on the Gamma distribution are a useful and practical tool for assessing process reliability and assisting in the early detection of possible quality problems or failures in maintenance and production environments by identifying deviations from the expected process behaviour. The following are the conditions for application

- When the primary goal is to directly analyze the time intervals between successive events or occurrences within a process.
- When there is a requirement to implement conventional Six Sigma control charts as part of a broader strategy for quality enhancement and process optimization.
- When the nature of the process demands a more gradual, steady response to variations or shifts, ensuring stability and minimizing overreactions to minor fluctuations.

IV. Methods and materials

A random variable 'X' is said to follow a two-parameter gamma distribution, which is represented as:

$$X \sim Gamma(k, \theta) \tag{2}$$

Where, k>0 is the shape parameter. The shape parameter 'k' determines the skewness of the gamma distribution:

k<1: Highly skewed right, with a peak near zero, k=1: Exponential distribution (memoryless property) and k>1: Distribution becomes more symmetric as 'k' increases.The scale parameter 'θ' determines the spread. θ>0 is the scale parameter.

The probability density function (PDF) of two-parameter Gamma distribution Ross[15], is given by:

$$f\,(x;k,\theta)=\frac{x^{k-1}e^{-x/\theta}}{\theta^{k}\Gamma k},\,x>0, and\; k,\theta>0 \tag{3}$$

Where, Γk is the gamma function, defined as:

$$\Gamma k=\int_0^{\infty} t^{k-1}\;e^{-t}dt, k>0 \tag{4}$$

which generalizes the factorial function, such that $\Gamma n=(n-1)!$ for integer values of 'n'.By definition:

$$E(x)\;=\;\int_0^{\infty} x\; f(x;k,\theta)\,dx \tag{5}$$

Substituting the gamma probability density function:

$$E[x]=\int_0^{\infty} x\;\frac{x^{k-1}e^{-x/\theta}}{\theta^{k}\Gamma k}\,dx \tag{6}$$

Using the substitution u = $x/\theta \Rightarrow du=dx/\theta, we\,get:$

$$E(x)=\frac{\theta^{k+1}}{\theta^{k}\Gamma k}\int_0^{\infty} u^{k}e^{-u}\;du \tag{7}$$

Since:

$$\int_0^{\infty} u^{k}e^{-u}du=\Gamma(k+1)\;=k\Gamma k \tag{8}$$

we obtain:

$$\text{E(x) = k}\theta \tag{9}$$

The second moment is:

$$E(x)\;=\;\int_0^{\infty} x^{2}\;f(x;k,\theta)\,dx \tag{10}$$

$$E[x]=\int_0^{\infty} x^{2}\;\frac{x^{k-1}e^{-x/\theta}}{\theta^{k}\Gamma k}\,dx \tag{11}$$

Using the property of the gamma function , Casella and Berger, [2]:

$$E(x)=\theta^2\, k(k+1) \tag{12}$$

The variance is given by:

$$V(x)=E(x^2)-[E(x)]^2=k\theta^2 \tag{13}$$

In the two-parameter gamma distribution, we typically define the distribution in terms of its shape parameter 'k' and scale parameter 'θ'. However, an alternative parametrization often used in statistical modelling expresses the gamma distribution in terms of its mean 'μ' and standard deviation 'σ' (Johnson et.al., [3]. From $E[X] = k\theta$, we solve for 'θ':

In the two-parameter gamma distribution, the distribution is conventionally characterized using two fundamental parameters: the shape parameter, denoted as 'k', and the scale parameter, represented by 'θ'. However, in various statistical modelling applications, an alternative parameterization is frequently employed, where the gamma distribution is expressed in terms of its mean (μ) and standard deviation (σ). This approach provides a more intuitive understanding of the distribution's properties, particularly in applied settings. As noted by Johnson et al. [3],from this alternative representation, we can derive an expression for the scale parameter 'θ':

$$\theta=\frac{\mu}{k} \tag{14}$$

Thus, the equivalence in terms of 'μ' and 'σ' is:

$$k=\frac{\mu^2}{\sigma^2}\ and\ \ \theta=\frac{\sigma^2}{\mu} \tag{15}$$

For a given tolerance level (TL) and a specified process capability (CP), the standard deviation 'σ' (termed as $\eta_{6\sigma}$for exponential distribution) often referred to as the process variation is determined using a predefined mathematical relationship $CP=\frac{Tolerance\ Level}{6\sigma}$.This calculation is performed for various combinations of tolerance levels and process capability values to assess the consistency and precision of the manufacturing process by Pavithra and Balamurugan, [6]. By analysing these values, organizations can better understand the extent of variation within the process and ensure that it remains within acceptable limits to meet quality and performance requirements.

To determine the process standard deviation $\eta_{6\sigma}$, first, establish the tolerance level (TL) and the desired process capability $C_p = 2$, as outlined by Montgomery [4]. Once the values of $\eta_{6\sigma}$ is obtained, it is applied to the control limit equations for mean as follows:

$$\left(\theta_{6\sigma}^{\bar{x}}+\frac{1}{\lambda_{6\sigma}^{\bar{x}}}\right)\pm\left[\left(\frac{Z_{6\sigma}}{\sqrt{n}}\right)\left(\frac{1}{\lambda_{6\sigma}^{\bar{x}}}\right)\right] \tag{16}$$

4.1 Illustration 1

The example presented by Amitava Mitra [1], is examined in this context. It pertains to a paper manufacturing company that must regulate the thickness of the cardboard sheets it produces to ensure quality control. To achieve this, random samples consisting of five sheets each are periodically selected, and their thickness measurements are recorded. The dataset for 30 such samples is provided in Table 2, offering valuable insights into the variability and consistency of the manufacturing process.

Table 2: *Data (in mm) from thickness of cardboard sheets*

Sample No	Observations (in mm)					Mean	Standard deviation (S)	Range
1	0.52	0.55	0.49	0.51	0.52	0.5180	0.0217	0.06
2	0.53	0.50	0.51	0.51	0.50	0.5100	0.0122	0.03
3	0.52	0.51	0.55	0.50	0.52	0.5200	0.0187	0.05
4	0.42	0.45	0.43	0.42	0.46	0.4360	0.0182	0.04
5	0.48	0.50	0.47	0.51	0.51	0.4940	0.0182	0.04
6	0.49	0.52	0.51	0.48	0.50	0.5000	0.0158	0.04
7	0.54	0.48	0.51	0.48	0.49	0.5000	0.0255	0.06
8	0.43	0.46	0.53	0.48	0.49	0.4780	0.0370	0.10
9	0.51	0.52	0.48	0.53	0.54	0.5160	0.0230	0.06
10	0.53	0.48	0.49	0.51	0.48	0.4980	0.0217	0.05
11	0.51	0.46	0.51	0.55	0.50	0.5060	0.0321	0.09
12	0.50	0.54	0.52	0.51	0.52	0.5180	0.0148	0.04
13	0.46	0.48	0.53	0.51	0.51	0.4980	0.0277	0.07
14	0.52	0.49	0.52	0.47	0.48	0.4960	0.0230	0.05
15	0.49	0.51	0.50	0.51	0.52	0.5060	0.0114	0.03
16	0.50	0.52	0.48	0.53	0.51	0.5080	0.0192	0.05
17	0.45	0.51	0.50	0.52	0.44	0.4840	0.0365	0.08
18	0.52	0.49	0.55	0.52	0.53	0.5220	0.0217	0.06
19	0.56	0.51	0.54	0.56	0.52	0.5380	0.0228	0.05
20	0.49	0.53	0.51	0.52	0.51	0.5120	0.0148	0.04
21	0.52	0.48	0.47	0.50	0.49	0.4920	0.0192	0.05
22	0.55	0.46	0.48	0.53	0.50	0.5040	0.0365	0.09
23	0.51	0.50	0.53	0.50	0.49	0.5060	0.0152	0.04
24	0.50	0.47	0.49	0.52	0.51	0.4980	0.0192	0.05
25	0.48	0.52	0.47	0.46	0.47	0.4800	0.0235	0.06
26	0.52	0.49	0.50	0.48	0.50	0.4980	0.0148	0.04
27	0.46	0.52	0.48	0.51	0.52	0.4980	0.0268	0.06
28	0.49	0.55	0.52	0.50	0.49	0.5100	0.0255	0.06
29	0.52	0.51	0.53	0.47	0.48	0.5020	0.0259	0.06
30	0.50	0.48	0.52	0.49	0.48	0.4940	0.0167	0.04
						$\overline{X} = 0.5013$	$\overline{S} = 0.0220$	$\overline{R} = 0.0570$

The scale parameter $(\theta_{3\sigma-R.\gamma})$ of the two-parameter Gamma distribution is estimated based on the 3-Sigma criterion for the range as follows:

$$(\theta_{3\sigma-R.\gamma}) = \frac{\overline{R}}{\ln(n)} = \frac{0.0203}{\ln(5)} = 0.0340 \tag{17}$$

The analysis of Table 3 and Figure 1 clearly indicates that the process is out of control when assessed using the Six Sigma methodology for the range under two-parameter Gamma distribution. This conclusion is based on the observation that only fifteen samples fall within the established control limits, signalling significant deviations from the expected process stability. Additionally, the control limit interval (CLI) for the Six Sigma-based control chart is found to be 0.0200, which is distinctly different when compared to the Three Sigma control limits for a sample size of n=5. This comparison underscores the heightened sensitivity of the Six Sigma approach, which enables more effective detection of variations, making it a more robust and reliable tool for process monitoring and quality control.

Table3: *3σ and six sigma control limits for range undertwo-parameter Gamma distribution*

S.No	Type of control chart	LCL	CL	UCL	CLI
1	Shewhart 3σ	0	0.0547	0.1851	0.2609
2	Six sigma	0.0446	0.0547	0.0647	0.0200

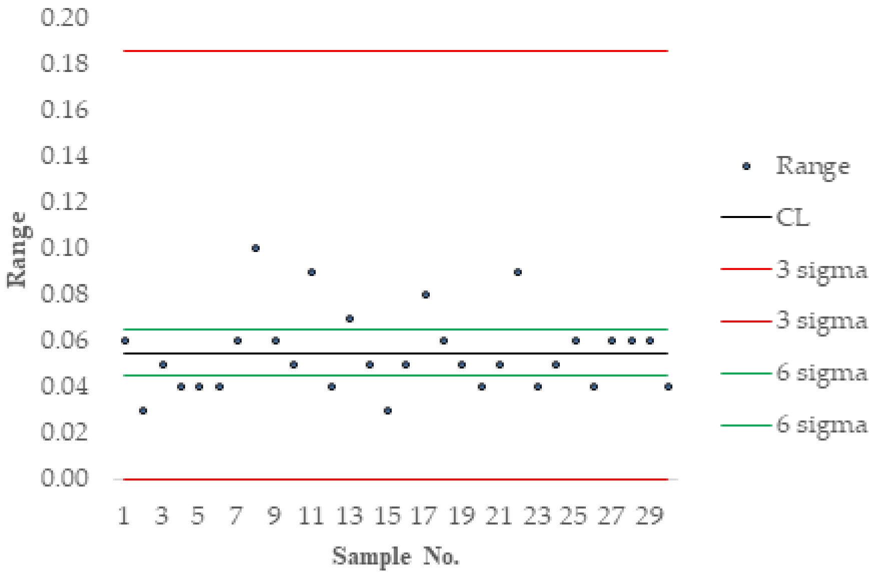

Figure1: *Shewhart 3σ and 6σ control charts for range under two-parameter Gamma distribution*

Then the 3σ control limits for range are

$$[\theta_{3\sigma-R.\gamma} \times \ln(n)] \pm [3\times 1.28 \frac{\overline{R}}{\ln(n)}]$$

$$UCL_{3\sigma-R.\gamma=} [\theta_{3\sigma-R.\gamma} \times \ln(n)] + [3\times 1.28 \frac{\overline{R}}{\ln(n)}]$$

$$=[0.0340\times \ln(5)] + \left[3\times 1.28 \frac{0.0547}{\ln(5)}\right] = 0.0547 + 0.1304 = 0.1851$$

$$CL_{3\sigma-R.\gamma} = [\theta_{3\sigma-R.\gamma} \times \ln(n)] = [0.0340\times \ln(5)] = 0.0547$$

$$LCL_{3\sigma-R.\gamma=} [\theta_{3\sigma-R.\gamma} \times \ln(n)] - [3\times 1.28 \frac{\overline{R}}{\ln(n)}]$$

$$= [0.0340\times \ln(5)] - \left[3\times 1.28 \frac{0.0547}{\ln(5)}\right] = 0.0547 - 0.1304 = -0.0758 \approx 0 \qquad (18)$$

and also,the six sigma-based control limits for range are $\left[\theta_{6\sigma-R.\gamma} \times \ln(n)\right] \pm \left[\frac{z_{6\sigma} \times \tau_{6\sigma.R-\gamma}}{\sqrt{n}}\right]$

$$UCL_{6\sigma-R.\gamma} = \left[\theta_{6\sigma-R.\gamma} \times \ln(n)\right] + \left[\frac{z_{6\sigma} \times \tau_{6\sigma.R-\gamma}}{\sqrt{n}}\right] = [0.0340 \times \ln(5)] \pm \left[\frac{4.381 \times 0.0046}{\sqrt{5}}\right]$$

$$= 0.0547 + 0.0100 = 0.0647$$

$$CL_{6\sigma-R.\gamma} = [\,\theta_{6\sigma-R.\gamma} \times \ln(n)] = [0.0340 \times \ln(5)\,] = 0.0547$$

$$LCL_{6\sigma-R.\gamma} = \left[\theta_{6\sigma-R.\gamma} \times \ln(n)\right] - \left[\frac{z_{6\sigma} \times \tau_{6\sigma.R-\gamma}}{\sqrt{n}}\right] = [0.0340 \times \ln(5)\,] - \left[\frac{4.381 \times 0.0046}{\sqrt{5}}\right]$$

$$= 0.0547 - 0.0100 = 0.0446$$

Furthermore, the analysis presented in Table 4 and Figure 2 demonstrates that the traditional Shewhart 3-Sigma control chart for monitoring the range, when evaluated alongside the average run length (ARL), is not as effective as the proposed Six Sigma control chart under two-parameter Gamma distribution. The Six Sigma approach provides a more stringent and precise method for detecting variations in process performance, making it more sensitive to shifts and deviations. This comparison highlights the superior performance of the Six Sigma-based control chart, reinforcing its suitability for organizations aiming to achieve higher levels of quality control and process consistency.

Table 4: *Average run length (ARL) of 3σ and six sigma control limits for range under two-parameter Gamma distribution*

S.No	Multiple of σ	Shewhart 3σ	Six sigma
1	0.0050	348	129
2	0.0051	347	113
3	0.0052	346	100
4	0.0053	345	88
5	0.0054	344	77
6	0.0055	343	68

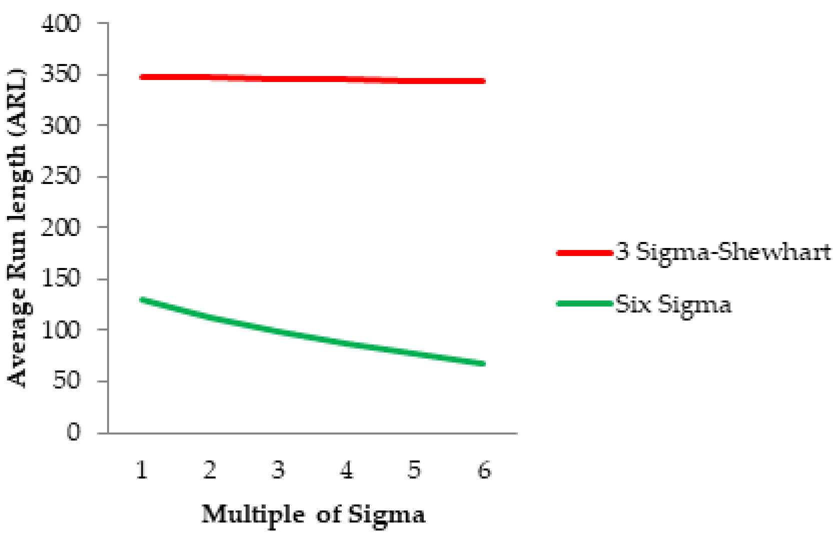

Figure 2: *ARL of Shewhart 3σ and 6σ range control charts under two-parameter Gamma distribution*

Conclusion

In this research paper, we have demonstrated how Six Sigma methodologies can be effectively employed to account for parameter uncertainty in the construction of control charts for the range statistic under the Gamma distribution framework. Unlike control charts developed under the assumption of known process parameters, the presence of parameter uncertainty typically results in wider control limits, reflecting the added variability and ensuring a more realistic representation of process behavior.

Our analysis further illustrates how these control limits, derived from the two-parameter Gamma distribution, can be dynamically evaluated across a range of process parameter values. In particular, we emphasize the utility of expected run length (ERL) calculations as a means of assessing chart performance under varying conditions. The Six Sigma-based control charting approach introduced in this study is grounded in the statistical modeling of observable quantities specifically, the sample range in this context under the two-parameter Gamma distribution.

By systematically identifying and eliminating assignable causes of variation within a production or service process, the proposed methodology offers practical applicability across multiple stages of the operational workflow. This flexibility supports the attainment of management-defined quality objectives while promoting a data-driven strategy for process stability and improvement.

Ultimately, the findings highlight the importance of ongoing quality enhancement initiatives, which may be realized through the refinement of existing procedures or through thoughtful design changes to the product or service. Such continuous improvement efforts not only enhance customer satisfaction but also contribute significantly to operational efficiency by reducing waste, time, and cost.

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