DESIGNING AND EVALUATION OF BAYESIAN SKIP LOT SAMPLING PLAN - 2 WITH DOUBLE SAMPLING PLANS AS THE REFERENCE PLAN UNDER GAMMA ZERO-INFLATED POISSON DISTRIBUTION

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Abstract

Sampling plans are an effective statistical method for assessing and deciding whether to approve or decline a lot based on quality inspection. Skip-lot sampling plans focus on inspecting only a fraction of the submitted lots, significantly reducing costs by minimizing time and effort. Skip-lot sampling is intended for immediate use, and it has been demonstrated that the quality of the submitted lots is consistently high. This article presents designing of Bayesian Skip Lot Sampling Plan – 2 (BSkSP – 2) with Double Sampling Plan (DSP) as the reference plan based on Gamma Zero-inflated Poisson (GZIP) distribution. The Zero-inflated Poisson (ZIP) distribution is suitable for count data with a high frequency of zeros, especially when non-conformities are rare. The Gamma distribution serves as a conjugate prior to the ZIP distribution. The Operating Characteristic (OC) function of skip lot sampling plan is derived and numerical illustrations are provided to illustrate the proposed sampling plan.

Keywords: Acceptance sampling, Sampling inspection by attributes; Double sampling plan; Prior distribution; Zero-inflated Poisson distribution; Operating characteristics function.

I. Introduction

Statistical quality control (SQC) is a systematic approach for monitoring and controlling product quality using statistical techniques. It helps industries to maintain consistency, reduce variability, and improve efficiency in manufacturing processes. Sampling inspection examines a subset of items from a production batch is examined to decide whether to accept or reject the entire lot. This method is used when 100% inspection is impractical due to high costs, time constraints, or product destructiveness. Sampling inspection helps maintain product quality while minimizing inspection efforts.

An acceptance sampling plan determines whether a lot should be accepted or rejected based on the number of nonconformities items found in a sample. A single sampling plan (SSP) involves selecting a single sample from a lot and making an acceptance or rejection decision based on the number of nonconformities found. A double sampling plan (DSP) allows for a second sample if the

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first sample results are inconclusive. DSP provide more flexibility than SSP and help reduce unnecessary rejections or inspections.

Bayesian methodology incorporates prior knowledge about the process into decision-making. In Bayesian acceptance sampling, prior information is combined with observed sample data to update the probability of lot acceptance. Quality variations can occur both within and between lots. While traditional methods often focus on within-lot variation, real-world scenarios demand consideration of both types. This approach allows for more informed decisions, especially when dealing with variable-quality processes.

Technological advancements have significantly improved process monitoring and product quality, resulting in frequent instances of zero nonconformities. However, inherent variations still occur, making the Zero-inflated Poisson (ZIP) distribution a suitable distribution. This statistical approach integrates a Poisson component with an additional zero-inflation factor, making it particularly effective in scenarios where nonconformities are rare but occasional deviations exist.

Loganathan and Shalini [9, 10] pioneered ZIP Single Sampling Plans (SSPs), while Vijayaraghavan et al. [23] explored Bayesian methodologies for Gamma-Poisson SSPs. Additionally, Shalini et al. [19] and Palanisamy and Latha [11, 12] worked on the refinement of GZIP SSPs.

The design of ZIP-based Double Sampling Plans (DSPs) has been examined by Shalini and Sheik [18], Pramote and Wimonmas [15], and Wimonmas and Pramote [24]. The integration of Bayesian principles into DSP design has also been investigated by Vijayaraghavan and Sakthivel [22], Balamurali et al. [3], and Suresh and Usha [21], who explored their impact on sampling efficiency and risk optimization. More recently, Priyadharshini and Shalini [16] developed GZIP DSP**s**.

Dodge [4] pioneered the concept of continuous sampling and established the mathematical foundation and operational rules for the first continuous sampling plan (CSP-1). Building on this, Dodge [5] introduced the idea of skip-lot sampling, applying the principles of CSP-1 to a continuous series of lots. This led to the development of the first skip-lot sampling plan, known as SkSP-1, which is widely used for bulk materials or products manufactured in consecutive lots. Compared to a standard single acceptance sampling plan, SkSP-1 requires fewer units for inspection, effectively reducing inspection costs.

Subsequent research by Dodge and Perry [6] and Perry [7] expanded upon this concept, leading to the introduction of the SkSP-2 plan, which employs a reference plan for skip-lot sampling. Under this framework, certain lots are exempted from inspection when the overall product quality is consistently high. Skip-lot sampling is particularly advantageous in industries such as chemical and physical processing, where inspection costs are substantial and product quality remains stable.

Parker and Kessler [13] refined the SkSP-2 approach, proposing advanced skip lot sampling plans. More recently, Aslam et al. [1] formulated design parameter tables for SkSP-2, using a single sampling plan as a reference. Further, Balamurali and Subramani [4] determined optimal design parameters that satisfy both producer and consumer risk under the binomial model. Recently, Kaviyarasu and Sivakumar [8] developed a Bayesian skip-lot sampling plan (BSkSP-2) based on a zero-inflated Poisson distribution with a gamma prior.

According to the literature, there has been no research conducted on developing GZIP BSkSP - 2 with DSP as the reference plan. This study aims to address by focusing on the determination of GZIP BSkSP - 2 with DSP. Section 2 presents the BSkSP - 2 with DSP under GZIP distribution. Section 3 focuses on designing GZIP BShSP - 2 with DSP, and Section 4 provides a numerical illustration. Finally, Section 5 concludes the findings.

II. SkSP - 2 with DSP as the reference plan under GZIP Distribution

A DSP is structured around five specific parameters: n_1, n_2, c_1, c_2 and c_3, where $c_1 < c_2$ and $c_2 \leq c_3$ (the third acceptance number). When c_2 is taken in equal to c_3 (i.e., $c_2 = c_3$), the DSP is described through its parameters n_1, n_2, c_1 and c_2, which represent the sizes of the first and second samples and the first and second acceptance numbers, respectively (Duncan [7] and Stephen [20]).

Let d_1 and d_2 denote the number of nonconformities in the first sample and second sample and $D = d_1 + d_2$ represent the combined nonconformities from both samples.

Schilling and Neubauer [17] outlined the steps involved in double sampling plan and Priyadharshini and Shalini [16] derived the Operating Characteristic (OC) function of GZIP DSPs.

A skip lot sampling plan of BSkSP - 2 is studied using the method of attributes with a double sampling plan as the reference plan. In this context, the Bayesian DSP with zero acceptance number under the conditions of GZIP distribution is considered as a reference plan.

In contrast to a normal inspection procedure, which requires every lot to be examined, a skip-lot plan reduces inspection by evaluating only a fraction of the lots when historical data indicate consistently good quality. The use of a zero acceptance number ensures stricter quality control, while the Bayesian approach incorporates prior knowledge about the nonconformities rate to make more informed sampling decisions.

To maintain quality assurance, BSkSP - 2 includes specific switching rules based on recent records of lot acceptances and rejections, which determine when to continue reduced inspection and when to revert to full inspection, thereby balancing cost efficiency with consumer protection.

Operating Procedure of SkSP-2

The SkSP-2 plan, as described by Perry [14], follows these operational steps:

Step 1. Initiate normal inspection, where in all lots undergo assessment based on the reference plan. Lots are examined sequentially in accordance with production or submission order.
Step 2. If i consecutive lots are accepted under the normal inspection then switch to skipping inspection procedure.
Step 3. During skipping inspection, only a specified fraction f of the lots is randomly inspected. This continues until a lot fails inspection.
Step 4. If a lot is rejected during skipping inspection, revert to normal inspection as outlined in Step 1.
Step 5. Monitor each rejected lot and correct or replace all defective units obtained.

SkSP-2 is defined by two parameters associated with its reference sampling plan: f and i. In general, $0 < f < 1$, and i is a positive integer. When $f = 1$, the skip-lot sampling plan effectively reverts to the reference plan, eliminating the skipping mechanism.

The performance of SkSP – 2 with DSP as the reference plan is assessed through its OC function.

$$P_a = \frac{(1-f)P_a(p)^i + fP_a(p)}{(1-f)P_a(p)^i + f} \tag{1}$$

III. Designing BSkSP – 2 with DSP under GZIP Distribution

GZIP BSkSPs - 2 with DSPs are designed by determining the optimum parameters n_1, n_2, c_1, c_2, i and f based on the prescribed points $(p_1, 1\text{-}\alpha)$ and (p_2, β) on the OC curve so that the determined GZIP BSkSPs - 2 with DSPs provide adequate protection to both producer and consumer.

The plan must satisfy the following requirements:

(i) $P_a(p_1) \geq 1- \alpha$

(ii) $P_a(p_2) \leq \beta$

The values of the plan parameters n_1, n_2, c_1, c_2, i and f can be derived for each set of φ, m, p_1, α, p_2 and β by applying the unity value approach. The values np_1 and np_2 satisfying respectively equations *(i)* and *(ii)* are termed as unity values. The plan parameters can be arranged in tables for different combinations of $(p_1, \alpha, p_2, \beta)$. The use of an operating ratio $R = \frac{np_2}{np_1}$, reduces the number of tables.

The plan parameters are determined for specific sets of values of φ, m, p_1, α, p_2 and β under the condition of GZIP distribution. The unity values are computed for various combinations of (φ, m, $P_a(p)$) by solving the OC function of GZIP BSkSP – 2 with DSP for each combination of c_1, c_2,$i$and f with $n_1 = n_2 = n$. The parameter values consider are as follows; m = 10 and 100, φ = 0.01 and 005, i = 2, 3, 4 and 5, f = 2/3, 1/2 and 1/4 and for $Pa\ (p)$ are 0.99, 0.90, 0.50, 0.20 and 0.10. The specific values used for these parameters are presented in Table 1. The corresponding operating ratio values calculated for $\alpha = 0.05$ and $\beta = 0.10$ are listed in Table 2. Based on specified strength values $(p_1, \alpha, p_2, \beta)$ together with φ and m, these tables enable the determination of plan parameters using the unity value approach described by Schilling and Neubauer [17].

IV. Numerical Illustration

This section describes the process for choosing GZIP BSkSP - 2 with DSP for a specified strength, along with numerical examples.

Illustration 1

For instance, to determine the plan parameter for proposed GZIP BSkSP- 2 with DSP as the reference plan with values of φ = 0.01, m = 10, c_1 = 2, c_2 = 4 and plan strength p_1 = 0.008, α = 0.05, p_2 = 0.10 and β = 0.10; compute the operating ratio $R = p_2/p_1$ = 0.05/0.008 = 6.25. Choose f and i and the unity values np_1 corresponding to the nearest operating ratio (Table 2). Here the values are obtained as f = 2/3, i = 3, andnp_1 = 1.0439. The sample size $n = np_1/p_1$ = 130.44 ~ 130. Hence the plan parameters of the BSkSP-2 with DSP for given values of c_1 = 2, c_2 = 4, φ = 0.01, m = 10, i = 3 and f = 2/3 are n = 130.

Illustration 2

Assuming φ = 0.05, m = 100, c_1 = 2, c_2 = 4 and strength p_1= 0.008, α = 0.05, p_2 = 0.05 and β = 0.10. The operating ratio R corresponding to these specifications is computed as 6.25. Choose f and i and the unity value np_1 corresponding to the nearest R (Table 2). Based on the R value, the corresponding values of f and i are chosen as 2/3 and 5 respectively. The sample size is then calculated as n=np_1/p_1= 134.375, which is approximated to 134. Since, the optimum BSkSP - 2 with DSP under GZIP is given as (134; 2, 4; 2/3, 4).

The corresponding BSkSP - 2 with DSP as the reference plan under ZIP for the same specified strength is (136; 2, 4; 2/3, 4). This comparison indicates that the sample size for GZIP BSkSP - 2 with DSP are lesser than ZIP BSkSP - 2 with DSP.

***Table 1:** Unity values for GZIP BSkSP-2 with DSP when $n_1 = n_2 = n$ for $\varphi = 0.05$, $\alpha = 0.05$, $\beta = 0.10$*

m	*φ*	*f*	*i*	Probability of Acceptance ($P_a(p)$)				
				0.99	0.95	0.50	0.20	0.10
10	0.01	2/3	2	0.5101	1.0547	3.4916	5.4205	6.8829
			3	0.5091	1.0439	3.3677	5.2999	6.7962
			4	0.5081	1.0341	3.3042	5.2738	6.7870
			5	0.5071	1.0251	3.2708	5.2684	6.7861
		1/2	2	0.5747	1.1950	3.6821	5.5480	6.9698
			3	0.5724	1.1720	3.4712	5.3306	6.8061
			4	0.5701	1.1520	3.3619	5.2805	6.7880
			5	0.5679	1.1342	3.3025	5.2698	6.7862
		1/4	2	0.7726	1.5927	4.1692	5.9193	7.2513
			3	0.7633	1.5217	3.7488	5.4356	6.8441
			4	0.7546	1.4650	3.5286	5.3056	6.7921
			5	0.7465	1.4181	3.4020	5.2752	6.7866
	0.05	2/3	2	0.5187	1.0754	3.6345	5.9219	8.2431
			3	0.5177	1.0643	3.4992	5.7636	8.0720
			4	0.5166	1.0542	3.4301	5.7297	8.0543
			5	0.5156	1.0450	3.3939	5.7227	8.0525
		1/2	2	0.5846	1.2191	3.8442	6.0927	8.4217
			3	0.5822	1.1956	3.6121	5.8035	8.0913
			4	0.5799	1.1750	3.4927	5.7383	8.0563
			5	0.5777	1.1568	3.4283	5.7245	8.0527
		1/4	2	0.7867	1.6279	4.3925	6.6147	9.0618
			3	0.7772	1.5548	3.9183	5.9418	8.1652
			4	0.7683	1.4965	3.6750	5.7709	8.0637
			5	0.7600	1.4482	3.5365	5.7314	8.0531
100	0.01	2/3	2	0.5377	1.0846	3.3352	4.8541	5.8840
			3	0.5366	1.0739	3.2302	4.7649	5.8254
			4	0.5356	1.0641	3.1760	4.7455	5.8192
			5	0.5346	1.0552	3.1474	4.7415	5.8186
		1/2	2	0.6037	1.2230	3.4947	4.9476	5.9425
			3	0.6013	1.2004	3.3180	4.7876	5.8322
			4	0.5990	1.1806	3.2252	4.7504	5.8199
			5	0.5968	1.1631	3.1746	4.7425	5.8186
		1/4	2	0.8037	1.6116	3.8925	5.2158	6.1299
			3	0.7943	1.5427	3.5500	4.8651	5.8578
			4	0.7856	1.4875	3.3664	4.7691	5.8227
			5	0.7774	1.4417	3.2593	4.7465	5.8189
	0.05	2/3	2	0.5465	1.1050	3.4577	5.2304	6.8004
			3	0.5455	1.0941	3.3439	5.1159	6.6909
			4	0.5444	1.0841	3.2853	5.0912	6.6795
			5	0.5434	1.0750	3.2545	5.0861	6.6784
		1/2	2	0.6138	1.2467	3.6319	5.3528	6.9138
			3	0.6114	1.2236	3.4389	5.1448	6.7033
			4	0.6090	1.2033	3.3384	5.0974	6.6808
			5	0.6067	1.1854	3.2838	5.0874	6.6785

m	φ	f	i	Probability of Acceptance ($P_a(p)$)				
				0.99	0.95	0.50	0.20	0.10
100	0.05	1/4	2	0.8179	1.6456	4.0753	5.7195	7.3127
			3	0.8083	1.5748	3.6929	5.2447	6.7509
			4	0.7994	1.5181	3.4916	5.1212	6.6858
			5	0.7911	1.4711	3.3754	5.0924	6.6790

Table 2: *Operating ratio for GZIP BSkSP-2 with DSP when $n_1 = n_2 = n$ for $\varphi = 0.05$, $\alpha = 0.05$, $\beta = 0.10$*

m	φ	f	i	Probability of Acceptance ($Pa(p)$)		Operating Ratio
				0.95	0.1	$\alpha = 0.05$, $\beta = 0.10$
10	0.01	2/3	2	1.0547	6.8829	6.5259
			3	1.0439	6.7962	6.5104
			4	1.0341	6.7870	6.5632
			5	1.0251	6.7861	6.6199
		1/2	2	1.1950	6.9698	5.8325
			3	1.1720	6.8061	5.8073
			4	1.1520	6.7880	5.8924
			5	1.1342	6.7862	5.9833
		1/4	2	1.5927	7.2513	4.5528
			3	1.5217	6.8441	4.4977
			4	1.4650	6.7921	4.6363
			5	1.4181	6.7866	4.7857
	0.05	2/3	2	1.0754	8.2431	7.6652
			3	1.0643	8.0720	7.5843
			4	1.0542	8.0543	7.6402
			5	1.0450	8.0525	7.7057
		1/2	2	1.2191	8.4217	6.9081
			3	1.1956	8.0913	6.7676
			4	1.1750	8.0563	6.8564
			5	1.1568	8.0527	6.9612
10	0.05	1/4	2	1.6279	9.0618	5.5666
			3	1.5548	8.1652	5.2516
			4	1.4965	8.0637	5.3884
			5	1.4482	8.0531	5.5608
100	0.01	2/3	2	1.0846	5.8840	5.4250
			3	1.0739	5.8254	5.4245
			4	1.0641	5.8192	5.4687
			5	1.0552	5.8186	5.5142
		1/2	2	1.2230	5.9425	4.8590
			3	1.2004	5.8322	4.8586
			4	1.1806	5.8199	4.9296
			5	1.1631	5.8186	5.0027
		1/4	2	1.6116	6.1299	3.8036
			3	1.5427	5.8578	3.7971
			4	1.4875	5.8227	3.9144
			5	1.4417	5.8189	4.0361

m	*φ*	*f*	*i*	Probability of Acceptance (*Pa(p)*)		Operating Ratio
				0.95	0.1	α = 0.05, β = 0.10
100	0.05	2/3	2	1.1050	6.8004	6.1542
			3	1.0941	6.6909	6.1154
			4	1.0841	6.6795	6.1613
			5	1.0750	6.6784	6.2125
		1/2	2	1.2467	6.9138	5.5457
			3	1.2236	6.7033	5.4783
			4	1.2033	6.6808	5.5521
			5	1.1854	6.6785	5.6340
		1/4	2	1.6456	7.3127	4.4438
			3	1.5748	6.7509	4.2868
			4	1.5181	6.6858	4.4041
			5	1.4711	6.6790	4.5401

V. Conclusion

Acceptance sampling is particularly useful when it is impractical to inspect every unit in a production lot. Traditional single and double sampling plans are effective but can become costly when product quality is consistently high. To address this, Skip-lot Sampling Plans (SkSP) reduce inspection by checking only a fraction of lots while still protecting consumers. The integration of Bayesian methods enables the incorporation of prior knowledge with observed data, leading to more reliable decision-making, which is particularly important when nonconformity levels are low and consistent product quality is observed. This study enhances quality control in production by utilizing a BSkSP-2 with DSP as the reference plan under GZIP distribution, for which the OC function has been derived. The plan achieves reduced inspection effort, smaller sample sizes compared to ZIP distribution, and balanced protection of both producer's and consumer's risks. Thus, the BSkSP-2 with DSP under GZIP represents an effective and reliable approach to modern industrial quality control.

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