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CONSTRUCTION OF EXPONENTIAL LIFETIME BASED ATTRIBUTE CONTROL CHART: A BAYESIAN APPROACH

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Abstract

Statistical Process Control (SPC) is a quality control method that employs statistical methods to investigate, monitor, and improve a process. A control chart is a tool for monitoring process performance that employs visual indicators to detect unusual deviations due to assignable causes. This chart compares the values of a quality characteristic to its corresponding control limits. In the quality control process, the control chart is frequently constructed while ignoring parameter uncertainty. The detection of changes in the parameter(s) within the probability distribution of one or more process-related variables is an important aspect of monitoring. Estimating the parameters is essential since it may affect the control chart's long-term performance in in-control or out-of-control conditions. This article introduces a new attribute control chart utilising a Bayesian approach founded on the Exponential lifetime distribution and the Hybrid censoring technique. A Bayesian framework will be utilised to compute the control chart parameters and the average run length. The parameters for the control chart are determined across various value combinations, and the performance of the newly developed control chart is assessed using the Average Run Length (ARL). Numerical examples are provided to elucidate the proposed control chart, and simulated data are employed to demonstrate its potential applications.

Keywords: Attribute Control Chart, Exponential Distribution, Inverse Gamma Distribution, Predictive Distribution, Average Run Length

1. Introduction

Nowadays, the manufacturing industry faces significant challenges due to process variation, which often results in defects and increased costs. During continuous production, it is essential to determine whether the process is in control and to identify the presence of any variation. Such variation can arise from common (chance) causes or assignable causes. Quality control involves monitoring, measuring, and adjusting the product's performance to meet predefined standards or specifications. One of the most widely used and effective quality control tools is the control chart, a graphical representation of collected manufacturing data. A control chart typically includes a central line (CL) representing the average, an upper control limit (UCL), and a lower control limit (LCL).

To monitor production processes under varying conditions, numerous control charting techniques have been developed. These charts are designed to suit specific characteristics of the quality attribute being analysed. Broadly, control charts fall into two categories based on the type

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of data: variable or attribute. Variable control charts analyse data measured on a continuous scale with precise values, while attribute control charts classify items as conforming or non-conforming.

Lifetime is considered an important quality characteristic for certain products, and life tests are conducted to monitor their manufacturing process. Based on the results of these tests, a product can be classified as conforming or non-conforming. However, life testing often requires a lengthy duration, making the use of censoring techniques essential. Common censoring methods include Type-I censoring, Type-II censoring, and hybrid censoring. In hybrid censoring, the life test ends at the earlier of two events: the specified test time t or the occurrence of the $(UCL+1)^{th}$ failure. If the number of failures observed by time t falls between the lower control limit (LCL) and upper control limit (UCL), the production process is considered in control; otherwise, it is deemed out of control. Additionally, if a product fails before the test termination time t, it is classified as non-conforming.

The attribute control chart, such as the np control chart, is founded on the fraction of non-conforming items, which is determined by the assumption that the quality characteristic follows a normal distribution. In reality, the distribution of quality characteristics may not conform to normality. In such instances, the use of the current control chart could mislead industrial engineers and increase non-conforming items. A variety of studies on the development of attribute control charts using different lifetime distributions can be found in the literature, including work on the Weibull [1–3], Pareto [4, 5], Birnbaum–Saunders [6], Burr X and XII, Inverse Gaussian, and Exponential [7], Lognormal [8], Exponentiated Half Logistic [9], Dagum [10], Weibull–Pareto [11], Log-Logistic [12], Generalized Log-Logistic [13], Generalized Exponential [14], Inverse Weibull [15], Length-Biased Weighted Lomax [16], Rayleigh [17], Half Normal and Half Exponential Power [18], Exponentiated Inverse Kumaraswamy [19], Lindley [20], Generalized Rayleigh [21], Exponentiated Exponential [22], Inverse Kumaraswamy [23], Exponential–Poisson [24], and Exponential–Rayleigh [25].

Traditional attribute control charts assume that the average number of defectives remains constant over time when the process is in control. However, charts that rely solely on sample variance, such as the np-chart, can produce numerous false alarms if the actual number of defectives fluctuates, even when the process is stable. These false alarms occur because the control limits are typically calculated based only on sampling variation, while the observed number of nonconforming items reflects both sampling and inherent process variability. As sample sizes increase, the impact of process variation becomes more pronounced. Consequently, the actual number of nonconforming items is expected to vary from sample to sample according to an underlying probability distribution. The Bayesian methodology is widely used to handle uncertainty related to the parameter of interest. Regarding attributes, the pioneer work in a Bayesian process control chart was introduced by [26]. Numerous researchers, including [27–36], have enriched the literature and have engaged in developing a variable control chart for various lifetime distributions under a Bayesian approach.

The exponential distribution is a type of continuous distribution that is frequently employed in reliability engineering and estimation to characterise the lifetime failure of a specific product or component of a machine (or system) during a specified time interval, and it is extensively used in electrical products. In the realm of reliability engineering, the exponential distribution is used to model components that maintain a constant failure rate, such as electronic components, because of its mathematical simplicity and memoryless property. Among the various statistical distributions, the exponential distribution is a good fit for most skewed data. One of the reasons for selecting the exponential distribution for the distribution of a time characteristic is when the defect rate in a process is notably low. Literature reviews reveal that there has been no investigation into control charts that include a life test for a non-normal distribution, such as an Exponential distribution with Hybrid censoring under the Bayesian approach.

This paper presents a Bayesian attribute control chart designed using the predictive lifetime distribution. Specifically, it assumes lifetimes follow an Exponential distribution, while the process parameter is assigned an Inverse Gamma prior. A hybrid censoring scheme is utilised to handle incomplete observations. To construct the control chart, an appropriate control coefficient is determined based on the predictive distribution. The chart's performance is assessed using Average Run Length (ARL) under both in-control and out-of-control conditions. The design aims to detect shifts in the process mean, ensuring effective monitoring. Simulation studies are conducted to demonstrate the chart's effectiveness and practical application. Section 2 details the Bayesian design methodology, including the derivation of the control coefficient and the algorithmic procedure for chart development tailored to a specific performance threshold. Section 3 illustrates the proposed chart through a real-world simulation study, demonstrating its operation in practical conditions and validating its sensitivity to process shifts. The last section presents a summary of the overall findings.

2. Construction of a Bayesian Attribute control chart

Let $'T'$ represent the lifetime of the products generated in a manufacturing process. It is assumed that $'T'$ adheres to an Exponential distribution characterised by the parameter θ, defined for $t > 0$ by its density function $g_{T|\{\theta\}}$:

$$g_{T|\{\theta\}}(t) = \frac{1}{\theta}e^{-\frac{t}{\theta}}; t > 0, \theta > 0 \tag{1}$$

The distribution function $G_{T|\{\theta\}}$ of $T|\{\theta\}$ is given by:

$$G_{T|\{\theta\}}(t) = 1 - e^{-\frac{t}{\theta}}; t > 0, \theta > 0 \tag{2}$$

An appropriate prior distribution for $'\theta'$ can be identified based on the insights obtained from the mean lifetime of products in prior lots. It is assumed that the prior distribution of $'\theta'$ is characterised by an inverted gamma distribution, which is recognised as a natural conjugate prior for sampling from an exponential distribution, thus providing mathematical simplicity. The probability density function of the prior distribution of $'\theta'$ is presented as follows:

$$h_{\Theta|\{\alpha,\beta\}}(\theta) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}}\left(\frac{1}{\theta}\right)^{(\alpha+1)} e^{-\frac{1}{\beta\theta}}; \theta > 0, \alpha, \beta > 0 \tag{3}$$

The revised lifetime of the product is characterised by the predictive distribution of "T," with its density function derivable from the prior distribution (3) and the sampling distribution (1) as follows:

$$f_{T|\{\alpha,\beta\}}(t) = \frac{\alpha\beta}{(1+\beta t)^{(\alpha+1)}}; t > 0, \alpha > 1, \beta > 0 \tag{4}$$

Henceforth, the previously mentioned predictive distribution shall be known as the "Exponential-Inverted Gamma" distribution, with the distribution function of $T|\{\alpha, \beta\}$ being expressed as:

$$F_{T|\{\alpha,\beta\}}(t) = 1 - \frac{1}{(1+\beta t)^{(\alpha)}}; t > 0, \alpha > 1, \beta > 0 \tag{5}$$

The average (mean) lifetime of the product, as defined by the Exponential-Inverted Gamma distribution, is expressed as follows:

$$\mu = \frac{1}{\beta(\alpha-1)} \tag{6}$$

When the process is in control, the target mean life of the product is indicated by μ_0, and the time for the experiment t_0 can be formulated in terms of the mean of the in-control process, such that $t_0 = a\mu_0$ (with "a" denoting the experiment termination ratio). Let p denote the probability of an item failing before the experiment time t_0, which can be described as follows:

$$p = 1 - \frac{1}{\left[1 + \left(\frac{1}{\mu(\alpha-1)}\right)(a\mu_0)\right]^{\alpha}} \tag{7}$$

The process is considered to be in control when there is no variation between the process mean and the target mean, that is, $\mu = \mu_0$. Let p_0 represent the failure probability of an item when the process is in control, which is determined by

$$p_0 = 1 - \frac{1}{\left[1 + \left(\frac{a}{(\alpha - 1)}\right)\right]^{\alpha}} \tag{8}$$

If the process mean is considered to be shifted to $\mu_1 = f\mu_0$, where f is identified as a shift constant, the failure probability of an item when the process is out of control, denoted as p_1, is formulated as

$$p_1 = 1 - \frac{1}{\left[1 + \left(\frac{a}{f(\alpha - 1)}\right)\right]^{\alpha}} \tag{9}$$

Generally, for the provided values of α, a and f, the failure probabilities are computed by employing equations (8) and (9).

We formulate a new np control chart aimed at assessing the lifetime of products that are distributed according to the Exponential-Inverted Gamma distribution, based on the count of products in each subgroup. The subsequent section outlines the operational procedure for the designed np control chart:

Step 1: Randomly select a sample of n items from the subgroup of the production process and subject these sample items to a life test for a specified duration of time t_0.
Step 2: Monitor the number of items that fail, which will be referred to as "d".
Step 3: Terminate the life test either when the time t_0 is reached or if d exceeds the upper control limit (UCL) before time t_0, whichever happens first.
Step 4: If d is less than the lower control limit (LCL) or greater than the UCL, then the production process is to be regarded as out-of-control. Otherwise, it should be regarded as in control.

The proposed control chart is regarded as an np control chart since it displays the number of failed items in a subgroup of a predetermined sample size against the subgroup. The count of failed items in the life test follows a binomial (n, p_0) distribution when the production process is in a in-control state. The probability of an item failing before the test ends at time t_0 is denoted as p_0. Thus, the following equations are applied to determine the control limits for the proposed control chart:

$$UCL = np_0 + k\sqrt{np_0(1 - p_0)} \tag{10}$$

$$LCL = max[0, np_0 - k\sqrt{np_0(1 - p_0)}] \tag{11}$$

Here, the term "k" denotes the coefficient for control limits, and the failure probability p_0 of the product before the time t_0 is computed through equation (8).

The probability that the process is recognised as being in control is indicated by P^0_{in} and is provided as

$$P^0_{in} = P(LCL \leq d \leq UCL|p_0)$$

$$P^0_{in} = \sum_{d=[LCL]+1}^{[UCL]} \binom{n}{d} p_0^d (1 - p_0)^{n-d} \tag{12}$$

In general, the average run length is used to measure the performance of the control chart. The in-control ARL of the recommended control chart, indicated as ARL_0, is defined by

$$ARL_0 = \frac{1}{1 - \left[\sum_{d=[LCL]+1}^{[UCL]} \binom{n}{d} p_0^d (1 - p_0)^{n-d}\right]} \tag{13}$$

The probability that the process is considered to be in control upon shifting to μ_1 is now determined by

$$P_{in}^1 = P(LCL \leq d \leq UCL|p_1)$$

$$P_{in}^1 = \sum_{d=[LCL]+1}^{[UCL]} \binom{n}{d} p_1^d (1-p_1)^{n-d} \tag{14}$$

The Average Run Length (ARL) for the shifted process, which is considered out-of-control, is referred to as ARL_1 and is determined by:

$$ARL_1 = \frac{1}{1 - \left[\sum_{d=[LCL]+1}^{[UCL]} \binom{n}{d} p_1^d (1-p_1)^{n-d}\right]} \tag{15}$$

To analyse the effectiveness of the proposed control chart in detecting shifts in the process, we determine the optimal parameters LCL, UCL, a, and k for the specified in-control ARL values. For this purpose, we examined four cases of in-control ARL: 200, 300, 400, and 500, with a fixed sample size of n = 20, 25, 30, and 35, considering the parameters α = 2 and 3. The optimal parameters are chosen to ensure that the in-control ARL is as close as possible to the specified ARL values, while the out-of-control $ARLs$ are reported for various values of the shift constant (0.10 (0.10) 2.0). The optimal parameters along with their corresponding out-of-control $ARLs$ are illustrated in Tables 1 and 2.

The following algorithm is applicable for determining the optimal parameters necessary for the design of the proposed control chart, considering various combinations of the specified in-control Average Run Length (ARL) and sample size.

Step 1: Indicate the values of ARL, referred to as r_0, and the sample size n.

Step 2: Set a = 0.0001 and k = 0.0001.

Step 3: Determine the failure probability of the product (i.e., p_0) when the process is in control by employing equation (8) and also calculate the control chart parameters (LCL and UCL) by substituting the values of n, a and k in equation (10) and (11).

Step 4: Calculate the in-control ARL by using equation (13) and compare it with r_0. Continue this process for various combinations of (a and k) until getting an in-control ARL is very close to the specified ARL, r_0.

Step 5: When such an ARL is found, the corresponding parameters (LCL, UCL, a, k) will be the optimal parameters required. Then calculate the out-of-control ARL (i.e., ARL_1) using these optimal parameters in equation (15).

3. Application of Proposed Control Chart

3.1. Real-Life Application

This section includes a design example intended to illustrate a real-life application of the proposed control chart within the Bayesian approach. Imagine a manufacturer who seeks to enhance the quality of its product. It is known that the failure time of the product is modelled by the Exponential distribution, with the parameter following the Inverted Gamma distribution. The target mean life of the product is established at 1000 hours, with the parameter α = 2. A sample of size n = 25 will be taken from each subgroup and will be subjected to a censored life test. The target in-control Average Run Length (ARL), r_0, is 200. As indicated in Table 1, the constants are a = 0.5081, k = 2.7022, LCL = 7, and UCL = 21. Thus, the control chart that the manufacturer will create is designed as follows:

Step 1: Select a sample consisting of 25 products from each subgroup and carry out a life test for a duration of 508 hours. Record the total number of failed products during the life test, denoting this figure as "d".

Table 1: *ARL when the process parameter two-sided shift*

	$\alpha = 2$															
	$n = 20$				$n = 25$				$n = 30$				$n = 35$			
Parameter	$r_0 = 200$	$r_0 = 300$	$r_0 = 400$	$r_0 = 500$	$r_0 = 200$	$r_0 = 300$	$r_0 = 400$	$r_0 = 500$	$r_0 = 200$	$r_0 = 300$	$r_0 = 400$	$r_0 = 500$	$r_0 = 200$	$r_0 = 300$	$r_0 = 400$	$r_0 = 500$
LCL	5	5	5	5	7	7	7	7	9	9	9	9	12	11	11	11
UCL	18	18	19	19	21	22	22	23	24	25	26	26	29	29	29	29
a	0.5093	0.5392	0.5530	0.5680	0.5081	0.5256	0.5447	0.553	0.5122	0.5194	0.5287	0.5416	0.5351	0.5038	0.5197	0.534
k	2.8929	2.7967	3.0830	3.0202	2.7022	2.9257	2.8944	3.1932	2.7166	2.8054	3.0766	3.0043	2.8566	3.0701	2.9526	3.1511
$Shift(f)$	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
0.10	1.11	1.09	1.61	1.57	1.00	1.03	1.02	1.13	1.00	1.00	1.01	1.01	1.00	1.00	1.00	1.00
0.20	1.92	1.77	4.32	4.07	1.15	1.42	1.37	2.15	1.03	1.08	1.23	1.21	1.04	1.06	1.05	1.04
0.30	4.59	3.92	14.02	12.73	1.85	2.94	2.68	5.99	1.28	1.60	2.30	2.18	1.42	1.56	1.48	1.42
0.40	12.73	10.08	48.22	42.30	3.89	7.78	6.72	20.02	2.13	3.29	6.02	5.44	2.76	3.42	3.05	2.78
0.50	37.19	27.57	165.63	140.90	9.62	23.29	19.12	71.80	4.46	8.38	18.87	16.37	7.06	9.81	8.24	7.14
0.60	109.03	76.40	550.00	456.60	25.76	73.15	57.41	260.74	10.79	24.10	64.62	54.03	21.19	32.74	26.04	21.50
0.70	299.27	205.92	1575.27	1335.35	70.86	228.67	173.70	906.33	28.26	73.23	227.32	184.50	69.14	117.46	89.11	70.53
0.80	549.09	460.85	2042.15	2158.01	181.69	588.84	475.12	2057.24	75.65	217.40	708.60	589.02	219.54	401.02	303.95	236.02
0.90	408.32	524.59	967.18	1197.14	290.87	626.00	698.87	1305.05	173.45	428.39	900.90	969.28	375.86	641.01	655.34	604.38
1.00	200.14	300.44	400.18	500.41	200.43	300.16	399.99	500.14	200.04	300.22	400.11	499.99	200.08	300.21	400.35	499.77
1.10	101.01	151.60	186.60	229.78	101.48	136.12	182.56	210.77	114.43	136.72	163.43	204.18	83.57	119.28	160.57	209.22
1.20	56.33	82.16	98.31	119.02	53.69	69.07	90.58	102.25	58.33	65.93	76.48	93.68	39.48	54.65	71.52	91.29
1.30	34.42	48.68	57.25	68.24	31.22	39.11	50.06	55.80	32.18	35.68	40.70	48.87	21.33	28.77	36.59	45.54
1.40	22.67	31.16	36.15	42.49	19.77	24.24	30.35	33.49	19.50	21.36	24.04	28.36	12.87	16.95	21.00	25.55
1.50	15.88	21.27	24.38	28.29	13.44	16.17	19.84	21.71	12.80	13.89	15.45	17.94	8.49	10.93	13.23	15.77
1.60	11.69	15.30	17.35	19.91	9.68	11.45	13.80	14.99	8.98	9.66	10.64	12.18	6.01	7.58	8.99	10.52
1.70	8.97	11.50	12.99	14.67	7.31	8.52	10.11	10.90	6.65	7.104	7.75	8.76	4.51	5.58	6.50	7.48
1.80	7.13	8.97	9.99	11.24	5.74	6.61	7.73	8.28	5.15	5.45	5.91	6.61	3.55	4.31	4.94	5.61
1.90	5.83	7.21	7.97	8.89	4.67	5.30	6.12	6.52	4.14	4.37	4.69	5.19	2.91	3.47	3.92	4.39
2.00	4.89	5.95	6.53	7.23	3.90	4.38	5.00	5.29	3.44	3.61	3.85	4.21	2.46	2.89	3.22	3.57

Table 2: *ARL when the process parameter two-sided shift*

	$\alpha = 3$															
	$n = 20$				$n = 25$				$n = 30$				$n = 35$			
Parameter	$r_0 = 200$	$r_0 = 300$	$r_0 = 400$	$r_0 = 500$	$r_0 = 200$	$r_0 = 300$	$r_0 = 400$	$r_0 = 500$	$r_0 = 200$	$r_0 = 300$	$r_0 = 400$	$r_0 = 500$	$r_0 = 200$	$r_0 = 300$	$r_0 = 400$	$r_0 = 500$
LCL	4	4	4	4	6	6	5	5	8	7	7	7	9	10	9	8
UCL	17	17	18	18	20	21	20	21	24	23	24	24	26	28	27	26
a	0.5347	0.5687	0.5791	0.5951	0.5554	0.572	0.5197	0.5232	0.5504	0.5198	0.5260	0.5403	0.5017	0.5705	0.5360	0.504
k	2.8292	2.7897	3.0599	2.9869	2.6771	2.9465	2.8612	3.1801	2.9111	2.7663	3.0639	3.2353	2.8349	3.0429	3.0408	3.0291
$Shift(f)$	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
0.10	1.01	1.01	1.05	1.04	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.20	1.26	1.19	1.67	1.60	1.03	1.10	1.05	1.16	1.02	1.01	1.03	1.03	1.00	1.01	1.00	1.00
0.30	2.41	2.09	4.18	3.84	1.36	1.78	1.51	2.19	1.33	1.20	1.42	1.36	1.11	1.23	1.15	1.11
0.40	6.14	4.83	13.15	11.56	2.54	4.16	3.16	6.07	2.59	2.06	3.05	2.76	1.74	2.28	1.95	1.72
0.50	17.84	12.88	44.90	38.00	6.02	12.04	8.37	20.55	6.87	4.84	8.86	7.61	3.95	6.10	4.76	3.87
0.60	53.99	36.26	154.69	126.88	16.20	38.48	24.87	75.02	21.57	13.64	30.17	24.68	11.40	20.41	14.68	11.05
0.70	159.42	102.27	506.23	409.12	46.11	126.41	77.03	275.40	73.03	42.20	109.65	86.19	37.42	76.28	51.20	36.03
0.80	368.96	262.95	1153.49	1038.50	127.55	375.51	230.80	862.69	234.03	132.49	382.29	298.47	126.22	285.40	185.16	124.03
0.90	374.40	421.31	896.95	1045.13	247.66	553.39	487.30	1071.90	377.30	322.92	723.07	700.57	287.48	583.60	501.74	381.47
1.00	199.98	300.39	400.25	500.12	200.34	299.98	400.33	499.91	200.03	300.44	400.23	500.34	200.11	300.29	399.82	499.87
1.10	101.52	157.36	185.96	230.42	103.53	135.57	198.96	216.42	87.28	146.10	167.13	213.80	87.50	116.45	165.51	244.90
1.20	56.50	85.16	97.32	118.45	54.28	67.95	100.24	106.12	42.96	70.80	78.37	98.19	41.74	52.23	73.34	108.71
1.30	34.51	50.24	56.52	67.62	31.31	38.19	55.78	58.54	23.99	38.33	41.85	51.25	22.72	27.17	37.40	54.36
1.40	22.77	32.10	35.70	42.07	19.74	23.60	34.02	35.52	14.85	22.98	24.85	29.82	13.81	15.93	21.48	30.54
1.50	16.00	21.90	24.14	28.06	13.40	15.74	22.38	23.27	9.97	14.98	16.07	18.94	9.16	10.26	13.56	18.88
1.60	11.82	15.77	17.25	19.81	9.65	11.16	15.65	16.22	7.16	10.45	11.13	12.91	6.52	7.13	9.24	12.60
1.70	9.11	11.88	12.90	14.66	7.30	8.33	11.52	11.90	5.41	7.70	8.15	9.32	4.92	5.27	6.70	8.96
1.80	7.27	9.28	10.02	11.27	5.75	6.48	8.84	9.10	4.28	5.94	6.25	7.06	3.88	4.09	5.11	6.71
1.90	5.97	7.48	8.03	8.96	4.68	5.22	7.02	7.22	3.51	4.75	4.98	5.56	3.18	3.31	4.07	5.24
2.00	5.02	6.19	6.61	7.32	3.92	4.33	5.74	5.89	2.96	3.93	4.10	4.53	2.69	2.77	3.35	4.24

Table 3: *Simulated Data*

	Sample Size																				
Sample No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	D
1	333	513	611	40	267	231	368	689	624	13	593	427	2620	717	24	2114	163	363	370	172	12
2	182	1073	272	112	442	136	663	3133	208	3625	774	293	3351	415	24	732	786	862	86	134	11
3	329	383	3569	3706	253	146	695	499	27	99	220	30	378	3816	569	11	429	3024	319	3824	13
4	152	1752	423	332	3574	876	282	428	100	1872	4016	1277	3130	1171	110	348	201	736	279	61	11
5	231	1200	856	2530	21	92	515	2592	525	426	204	334	797	9	44	683	313	1475	18	1233	10
6	211	601	5735	508	595	1254	566	1122	576	28	165	16	7	887	133	9	1276	899	1516	388	9
7	3690	801	216	257	605	94	1429	496	2041	183	3953	283	91	420	180	296	585	532	1215	2854	10
8	463	355	23	2305	85	16	322	211	750	3171	129	878	2603	219	3120	565	149	262	1115	268	12
9	433	313	1914	46	101	1165	134	177	1135	4554	426	10	68	241	453	639	140	742	164	19	14
10	147	2855	80	163	143	293	559	1998	85	209	3726	163	169	350	6672	1869	286	151	979	101	13
11	418	499	510	901	92	1508	206	130	79	145	154	3	401	454	338	260	43	496	24	35	17
12	540	96	310	1017	115	18	656	1599	1128	386	601	511	523	69	32	1829	140	1059	52	1185	9
13	71	195	2270	1600	689	25	1825	1012	619	35	1106	596	263	31	186	477	787	300	359	639	10
14	220	35	26	466	459	490	17	143	372	33	1558	197	3131	91	245	94	994	32	206	1562	16
15	92	4065	49	3228	130	1593	2244	507	984	27	552	393	39	2413	484	676	15	61	393	935	11
16	8	24	48	151	108	526	4025	100	12	158	590	262	49	21	11	82	5	6	226	336	17
17	74	10	94	46	268	127	72	33	56	14	6	191	75	26	57	49	697	33	261	348	19
18	8	123	119	957	271	212	358	20	69	42	50	54	11	14	93	135	598	188	6	717	17
19	233	120	7	19	113	123	414	41	345	409	385	15	96	104	80	4	718	19	42	72	19
20	189	24	3	991	337	96	130	24	510	2	186	69	137	50	19	141	50	103	173	49	18
21	246	58	82	212	106	79	172	264	7	508	81	177	233	193	61	69	3	131	53	133	20
22	49	130	297	129	298	140	38	113	35	56	8	1349	121	28	97	83	13	42	165	40	19
23	164	890	63	336	364	48	46	9	516	27	142	19	22	66	553	238	69	17	254	134	17
24	487	291	504	29	100	113	122	69	2474	22	193	43	33	68	425	75	32	48	368	22	19
25	319	23	121	36	14	667	5	259	19	139	54	174	333	118	62	124	219	195	288	160	19
26	199	5	86	88	142	501	224	685	249	43	25	32	48	21	38	6	234	151	28	646	18
27	341	108	54	373	183	20	141	107	87	118	17	7	5	535	130	11	38	129	53	444	19
28	76	135	3	336	446	9	18	1077	60	86	168	50	2692	41	4	138	11	34	524	15	17
29	1086	91	20	98	133	78	113	57	145	98	10	14	832	21	94	68	123	404	159	19	18
30	70	137	40	268	9	10	25	3	18	32	13	231	28	14	38	198	101	510	20	90	19

Step 2: Terminate the life test either when 508 hours have been completed or if the number of failed products surpasses twenty before reaching 508 hours, depending on which happens first.

Step 3: If d > 21 or d < 7, declare the process to be out of control. If $7 \leq d \leq 21$, declare the process to be in control.

3.2. Simulation Study: Case 1

In this section, the application of the proposed control chart is demonstrated using simulated data. The data is generated from an Exponential distribution while the process is in control, with parameters that follow an Inverted Gamma distribution, specifically with $\alpha = 2$ and a target average lifetime of 1000 hours. We consider a random sample of size $n = 20$ for each batch. The first fifteen samples are produced from the in-control process, and the next fifteen samples are from a shifted process with $f = 0.20$, resulting in 30 sample batches, as illustrated in Table 3. Then, according to Table 1, when $n = 20$ and $ARL_0 = 200$, we have constants $k = 2.8929$, $a = 0.5093$, $UCL = 18$, and $LCL = 5$. The number of products with a lifetime below 509 hours is noted ("D") and is also shown in Table 3. The values of the nonconforming items are depicted with two control limits ($LCL = 5$ and $UCL = 18$) in Figure 1.

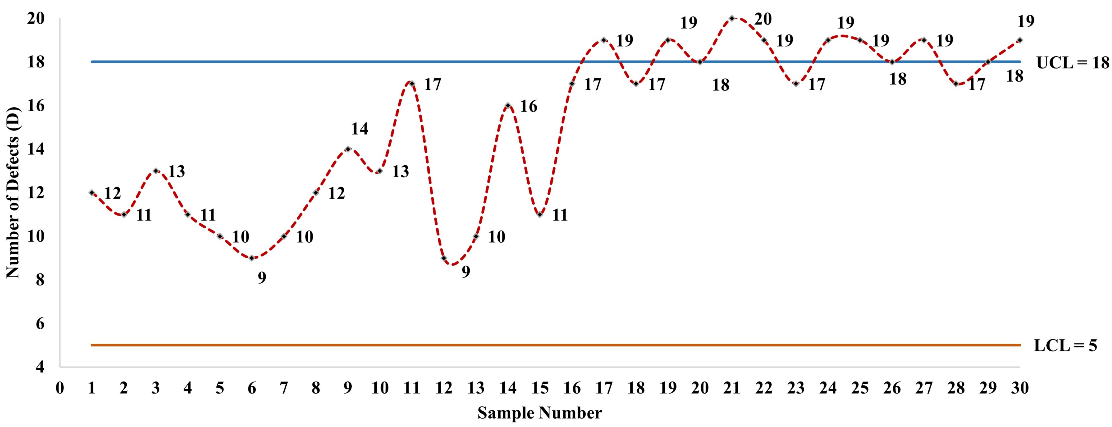

Figure 1: *Control Chart for Simulation Data of Case 1*

From Figure 1, it can be seen that the proposed control chart identifies a shift at the 17^{th} sample (the 2^{nd} observation following the shift), with a recorded ARL of 1.92. Consequently, the developed control chart effectively recognises the shift in the manufacturing process. Therefore, it can be concluded that the proposed control chart will promptly detect any shifts in the process.

3.3. Simulation Study: Case 2

The data is generated from an Exponential distribution while the process is maintained in an in-control state, with parameters that conform to an Inverted Gamma distribution with the parameter α set to 3 and a target average lifetime of 2000 hours. A random sample of size $n = 35$ is utilized for each batch, where the first ten samples are taken from the in-control process, and the next ten samples are from a shifted process with $f = 0.50$, leading to a total of 20 sample batches. From Table 2, with $n = 35$ and $ARL_0 = 400$, the constants are defined as $k = 3.0408$, $a = 0.5360$, $UCL = 27$, and $LCL = 9$. The number of products with a lifetime less than 1072 hours is classified as nonconforming items and is denoted as 'D', with the following values: 18, 24, 23, 22, 17, 12, 17, 15, 21, 18, 25, 24, 26, 25, 29, 25, 24, 23, 23, 27. The values of nonconforming items are plotted against two control limits ($LCL = 9$ and $UCL = 27$), as illustrated in Figure 2.

As shown in Figure 2, the proposed control chart detects a shift at the 15^{th} sample, specifically on the fifth observation after the shift, with an Average Run Length (ARL) of 4.76. This demon-

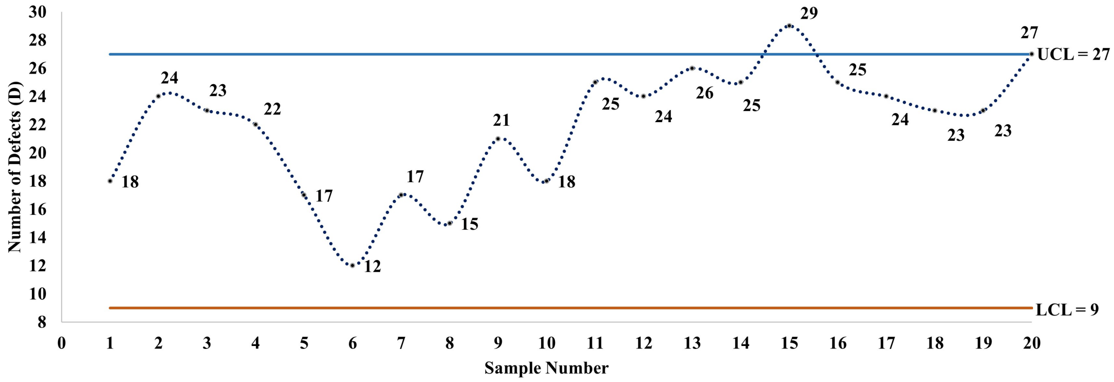

Figure 2: *Control Chart for Simulation Data of Case 2*

strates the chart's effectiveness in identifying shifts in the manufacturing process under standard conditions. Therefore, it can be concluded that the proposed chart is capable of detecting process shifts more quickly.

4. Conclusion

In this paper, we present a methodology for a new Bayesian attribute control chart designed to monitor the lifetime of products. Life tests are conducted under a hybrid censoring scheme, assuming an Exponential distribution for the product lifetime and an Inverse-Gamma distribution as the prior. The proposed control chart aims to ensure that the mean lifetime meets the desired quality standards. This highly adaptable control chart is suitable for monitoring product lifetimes in industrial settings. Supporting tables for practical application are provided and illustrated using simulated data generated in R from the Exponential-Inverse-Gamma model. The performance of the control chart is evaluated in terms of Average Run Lengths (ARLs) for various shift constants (f). Notably, employing a hybrid censoring scheme in life testing reduces both the time and cost involved in sampling inspections. The developed attribute control chart also offers potential for extension to other statistical distributions in future research.

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