AN ATTRIBUTE CONTROL CHART BASED ON GENERALIZED EXPONENTIAL-POISSON DISTRIBUTION UNDER HYBRID CENSORING

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Abstract

This paper presents a novel attribute control chart utilizing a Generalized Exponential- Poisson (GEP) distribution to detect the effect of indeterminacy in manufacturing processes. The proposed chart demonstrates enhanced performance compared to traditional GEP-based control charts, particularly in uncertain environments, by effectively monitoring shifts in process performance and exhibiting lower out-of-control average run lengths (ARL). A case study from the automobile industry further highlights its practical applicability. In addition, the paper explores a hybrid censoring scheme that integrates multiple censoring techniques to address incomplete data in survival and reliability analysis. This approach enhances the accuracy of lifetime estimates, especially in cases where data may be incomplete due to time or resource constraints. A comparative analysis of the proposed chart against traditional control charts underscores its superior performance in real-world applications, especially in situations with uncertain process distributions. The charts parameters are carefully optimized to ensure that the ARL for the in-control process closely aligns with a predefined target. The paper demonstrates the charts efficiency through numerical examples and simulation studies, thereby validating its performance and highlighting its practical advantages.

Keywords: Generalized Exponential-Poisson distribution, Hybrid censoring scheme, Control Chart, Average Run Length.

I. Introduction

A control chart is a graphical tool used in manufacturing industries to monitor and evaluate quality characteristics of production processes. Its primary function is to assess process stability by detecting deviations from specified tolerance limits, thereby ensuring consistency in production. Control charts visualize data in the sequence it occurs, making them an effective tool for tracking variations and identifying potential issues in real time. Over the years, various types of control chart techniques have been developed to suit different industrial applications. Every control chart includes at least two control limits: Lower Control Limit (LCL) and Upper Control Limit (UCL). A process is considered in control when the control statistic remains within these limits. Control charts are generally classified into two types:

- Attribute Control Charts – Used to classify products as conforming or non-conforming.
- Variable Control Charts – Applied when data is collected through direct measurement.

For some products, lifetime is a critical quality characteristic, necessitating life tests to assess the manufacturing process. Products are classified as conforming or non-conforming based on test

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results. However, such life testing is can be time-consuming making censoring techniques essential for efficient data analysis. Common censoring methods include: Type-I Censoring, Type-II Censoring, Hybrid Censoring. In Hybrid censoring a life test terminated when the earliest of two conditions occurs: either the test reaches predetermined time t or when the $(UCL+1)^{th}$ failure occurs. If the number of observed failures at time t falls within the specified control limits LCL and UCL range, the process is deemed to be considered in control otherwise it is considered out of control. Attribute control charts, such as the np control chart are frequently used in this context often assuming a normal distribution for quality characteristics. However, in practice quality characteristics may not always follow a normal distribution. This misalignment can lead to incorrect process evaluations, potentially increasing the number of non-conforming products.

Several researchers have studied attribute control charts across various lifetime distributions, however there remains limited research has been conducted on control charts incorporating life tests for non-normal distributions, particularly the Generalized Exponential-Poisson Distribution under Hybrid censoring. The **Generalized Exponential-Poisson Distribution**, first introduced by Gupta and Kundu [10] has been recognized for its excellent fit to specific lifetime data, often outperforming traditional models such as Weibull, gamma, lognormal, and Generalized Rayleigh distributions. In recent years, mixture models have gained significant attention in fields such as biology, genetics, medicine, engineering, economics, and social sciences. A mixture distribution is constructed by combining two or more component distributions typically using mixing parameters such as weights, which must be non-negative and sum to one. These models are especially valuable for accurately representing heterogeneous populations more offering greater flexibility and precision than single distribution model.

This study proposes an attribute control chart based on the Generalized Exponential-Poisson Distribution under hybrid censoring scheme. The control chart coefficient is derived and the chart performance is evaluated using the Average Run Length (ARL) criterion. The proposed control chart is designed to sensitive to changes in the process scale parameter median and is validated through simulated data. The structure of this paper is as follows: section 2 presents a comprehensive Review of Literature, section 3 introduces the Generalized Exponential-Poisson Distribution and details the control charts design. section 4 presents a simulation study demonstrating the control charts applicability. section 5 provides a comprehensive summary of the findings.

II. Review of Literature

Granting numerous researchers have explored attribute control charts for lifetime distributions, there is still a lack of studies on control charts for non-normal distributions like the Generalized Exponential-Poisson distribution under Hybrid censoring, despite the increasing interest in generalized models for reliability and life testing. Adeoti and Ogundipe [2] introduced a control chart for the Generalized Exponential distribution under a time-truncated life test, providing a structure methodology for quality monitoring in reliability engineering. However, its practical applicability could be enhanced through further comparisons with existing control chart methods. Aslam and Jun [4] proposed attribute control charts for the Weibull distribution under truncated life tests, enhancing quality control in reliability engineering, though further comparisons with other lifetime distributions could improve applicability. Baklizi and Ghannam [6] proposed an attribute control chart for the inverse Weibull distribution under truncated life tests, enhancing reliability engineering. However, further comparisons with existing methods could improve its applicability. Gunasekaran [10] proposed an attribute control chart based on the Exponentiated Exponential distribution under accelerated life testing with hybrid censoring, contributing to quality monitoring in reliability engineering, though further comparisons with traditional methods could further enhance its applicability. Kavitha and Gunasekaran [13] introduced a novel process monitoring approach using Exponentiated Exponential Distribution, with proposed enhancements including title refinement, abstract clarification, deeper explanation, detailed mathematical framework, comparative simulations, practical applications, and suggestions for future research.

Ristic and Nadarajah [17] introduced a novel lifetime distribution that addresses the limitations of traditional models, offering flexibility for real-world data. They also suggested further improvements through comparisons, simulations, and parameter estimation methods. Hussain et al., [21] offers a comprehensive approach to enhancing attribute control charts for popular distributions. However, their method could benefit from further simulation studies, real-world data comparisons, and more detailed mathematical analysis. Rao et al., [25] introduced an attribute control chart for the Dagum distribution, designed for truncated life tests, offering a robust method for quality control and reliability monitoring, though further validation and comparisons with traditional methods would enhance its practical application. Marshadi et al., [3] introduced a time-truncated control chart for Weibull distribution under uncertainty, offering a robust method for monitoring processes in reliability testing, though further validation and comparison with traditional methods would enhance its applicability. Aslam et al., [5] introduced a control chart method for time-truncated life tests based on the Pareto distribution of the second kind, aimed at improving process monitoring in reliability testing. Balamurali and Jeyadurga [7] presented an attribute np control chart for monitoring mean life, utilizing multiple deferred state sampling in the context of truncated life tests. Jayadurga et al., [12] proposed an attribute np control chart for process monitoring using repetitive group sampling in truncated life test. Srinivasa Rao [23] introduced a control chart method based on the Exponentiated Half Logistic Distribution for monitoring time-truncated life data in reliability testing. Srinivasa Rao and Edwin Paul [25] presented a control chart method based on the Log Logistic distribution for monitoring time-truncated life data in reliability testing. Shaheen et al., [22] proposed a control chart method utilizing repetitive sampling to monitor lognormal process variations in quality control.

Muhammad and Cordeiro [18] offered an in-depth review of compounding distributions, presented new generalized classes that expand the modeling capabilities for complex data in fields like reliability theory, survival analysis, and actuarial science. The paper's strengths lie in its comprehensive survey, the introduction of new distribution classes, and its broad applicability, while improvements could include empirical validation and a discussion on computational challenges in practical use. Coskun Kus [15] presented an innovative lifetime distribution, broadening the scope of statistical models used in reliability and survival analysis. The study thoroughly explores the distribution's mathematical properties, such as its probability density function, cumulative distribution function, and moments, demonstrating its versatility in modelling diverse lifetime data. By offering practical applications and effective parameter estimation techniques, the paper underscores the distribution potential for real-world reliability studies. Adamidis and Loukas [1] introduced a new lifetime distribution with a decreasing failure rate (DFR), relevant for modeling systems, particularly in electronics, where the failure rate decreases over time. Daghestani et al., [8] explored a mixture model combining the Lindley and Weibull distributions to enhance the modeling of real-life data with diverse characteristics. Aldosari et al., [16] presented an innovative method for process monitoring by incorporating multiple dependent state repetitive sampling into attribute control charts, improving the detection of process shifts and variations. Nadarajah and Kotz [19] introduced exponentiated distributions, extending traditional models to better handle skewed data and offering enhanced flexibility for various statistical applications. Nassar and Eissa [20] introduced an extension of the Weibull distribution, incorporating an exponentiation parameter to handle a broader range of lifetime data, particularly with complex failure rates, including both increasing and decreasing hazard rates. This generalization enhances the flexibility of the Weibull distribution, making it useful for diverse fields like reliability engineering, survival analysis, and medical research.

Recently Kavitha and Gunasekaran [14] proposed a novel attribute np control chart specifically designed to track the average lifetime of products within a hybrid audit framework, assuming that the lifetime follows a mixed exponential-Rayleigh distribution. Based on statistical process control principles, this study demonstrated the chart's ability to sensitively detect changes in process performance, with a specific focus on product longevity. The performance of the control chart is assessed using the average run length (ARL) criterion, which ensures that the ARL under control is as close as possible to the nominal target. The authors perform extensive parameter estimation under various scenarios, and verify the practical suitability of the chart with simulated data. The proposed method not only reduces the time and cost associated with the model by using a hybrid audit approach, but also provides a flexible and efficient solution for lifetime monitoring in quality control.

Gokila and Sheikh Abdullah [9] introduced a novel attribute np control chart that aims to track the average lifetime of products within a hybrid censoring framework, under the assumption that product lifetimes follow an exponential-poison distribution a compound model that is well suited for processes with early-life failures. Acknowledged the inherent limitations of conventional life testing, particularly in terms of time and cost, this study uses hybrid censoring techniques and compound distributions to improve the practicality and robustness of the monitoring process

Gupta and Kundu [10] presented a flexible framework that extends the traditional exponential distribution, offering improved versatility for modeling lifetime data. The paper provides a detailed exploration of the distribution properties, parameter estimation, and statistical inference, making valuable contributions to reliability engineering and survival analysis. The study highlights the advantages of the Generalized Exponential-Poisson distribution for specific types of lifetime data, demonstrating its effectiveness compared to other common lifetime distributions. Additionally, the paper introduces an attribute control chart based on the Generalized Exponential-Poisson distribution within a hybrid censoring Scheme, establishing the chart's coefficient and evaluating its performance using the Average Run Length. The design of the control chart is adapted to account for changes in the median of the process scale parameter, and its applicability is demonstrated through simulated data.

III. Design of the control chart

Let t be a random variable with Generalized Exponential-Poisson distribution with parameters λ,β and α, i.e.,$t \sim GEP(\lambda,\beta,\alpha)$ and the probability density function is given by

$$f(t) = \frac{\alpha\beta\lambda\{1-e^{-\lambda+\lambda\exp(-\beta t)}\}^{\alpha-1}.e^{-\lambda-\beta t+\lambda\exp(-\beta t)}}{(1-e^{-\lambda})^{\alpha}}, t, \lambda, \beta, \alpha > 0 \tag{1}$$

and the cumulative density function of the model can be obtained as

$$F(t) = \left(\frac{1-e^{-\lambda+\lambda\exp(-\beta t)}}{1-e^{-\lambda}}\right)^{\alpha}, t, \lambda, \beta, \alpha > 0 \tag{2}$$

The lifetime of the units follows Generalized Exponential – Poisson Distribution with a median life defined as

$$m_0 = \left(-\frac{1}{\beta}\right) log\left[1 + \frac{1}{\lambda} log\left[1 - (2^{-1})^{\frac{1}{\alpha}}.\left(1 - e^{-\lambda}\right)\right]\right] \tag{3}$$

The truncation time, denoted as t_0, along with failure rate, is introduced to reduce testing in life experiments. The number of failures with lifetimes ≤ t_0, is recorded where the truncation time is predefined based on the desired median life or testing constraints

$$t_0 = am_0 \tag{4}$$

where t_0 is the truncated time, a is the truncation coefficient and m_0 is the desired median life of the product. The probability of failure before t_0 is given by

$$p = \left[\frac{1-e^{-\lambda+\lambda e^{-\beta t_0}}}{1-e^{-\lambda}}\right]^{\alpha}, t, \lambda, \beta, \alpha > 0 \tag{5}$$

using equation (3), the probability of failure of a product when the process is in control is

$$p_0 = \left[\frac{1-e^{-\lambda+\lambda\left\{1+\frac{1}{\lambda}log\left[1-2^{\left(-\frac{1}{\alpha}\right)}\left(1-e^{-\lambda}\right)\right]\right\}^{a}}}{1-e^{-\lambda}}\right]^{\alpha}, \lambda, \alpha > 0 \tag{6}$$

when the median shifts to m_1, the probability of failure of a product becomes
Let $m_1 = fm_0$, where f is the shift coefficient, then

$$p_1 = \left[\frac{1-e^{-\lambda+\lambda\left\{1+\frac{1}{\lambda}log\left[1-2^{\left(-\frac{1}{\alpha}\right)}\left(1-e^{-\lambda}\right)\right]\right\}^{\frac{a}{f}}}}{1-e^{-\lambda}}\right]^{\alpha}, \lambda, \alpha > 0 \tag{7}$$

Based on the number of products in each subgroup, we propose the following *np* control chart for Generalized Exponential-Poisson under Hybrid censoring scheme.

Algorithm

Step 1: Randomly select a set of n products from the production process.
Step 2: Conduct a life test on the selected products, using t_0 as the test termination time, and record the number of failures observed during the test, denoted as D.
Step 3: Terminate the life test either when the truncated time t_0 is reached or the number of failures exceeds the upper control limit (UCL) whichever occurs first.
Step 4: If D exceeds the UCL or falls below the lower control limit (LCL), declare the process out of control. If D falls within the range of LCL ≤ D ≤ UCL, declare the process in control.

This method involves monitoring the number of failures denoted as D within each sample. Any unit that fails before the predetermined truncation time t_0 is considered defective or nonconforming. For a sample of size n, the number of defective units D is shown using a Binomial distribution, where the probability of failure before t_0 is p. The probability mass function of the Binomial distribution is as follows:

$$p(D = x) = \binom{n}{x} p^{x}(1-p)^{n-x}, x = 0,1,2,3\ldots \tag{8}$$

The Lower and Upper control limits are given by

$$LCL = max(0, np_0 - k\sqrt{np_0(1-p_o)}) \tag{9}$$

$$UCL = np_0 + k\sqrt{np_0(1-p_o)} \tag{10}$$

Where n is the sample size and k is the coefficient of the control limits and p_0 is the probability of nonconforming when the process is in control. The probability of maintaining that the process is in control when it is truly in control is obtained using the binomial probability given in equation (8)

$$p_{in}^{0} = p(LCL \leq D \leq UCL|p_0) = \sum_{d=LCL+1}^{UCL}\binom{n}{d} p_0^{d}(1-p_0)^{n-d} \tag{11}$$

The probability of stating that the process is in control when the median life of the product has shifted to m_1 is given by

$$p_{in}^{1} = p(LCL \leq D \leq UCL|\, p_1) = \sum_{d=LCL+1}^{UCL} \binom{n}{d} p_1^{d}(1-p_1)^{n-d} \quad (12)$$

It follows from the values p_{in}^{0} and p_{in}^{1} given in Equation (11) and (12) that the in-control ARL is given by

$$ARL_0 = \frac{1}{1-p_{in}^{o}} \quad (13)$$

and the out of control ARL are given by

$$ARL_1 = \frac{1}{1-p_{in}^{1}} \quad (14)$$

To construct the performance evaluation tables for the proposed control chart, based on the Generalized Exponential Poisson Distribution under a hybrid censoring scheme, the following algorithm is applied.

Step 1: Specification of ARL and sample size

Define the desired in control average run length (ARL), denoted as r_0 , and select an appropriate sample size, *n.*

Step 2: Determination of control chart parameters

For each combination of r_0 , n and the scale parameter λ, determine the control chart parameters. Compute the truncation coefficient a, such that the average run length calculated using equation (13)

Step 3: Evaluation of ARL under process shift

Introduce a shift constant *f*, which defines the new median $m_1 = fm_0$ using the values obtained from step 2, compute the out of control ARL, based on equation (14).

The results of various values of r_0, n and λ are presented in Tables 1-4. Notably, the tables demonstrate that the out- of control ARL decreases as the shift constant f becomes smaller, indicating that the chart is more sensitive to larger shifts in the median life of the product.

Table 1: *The Values of ARLs when λ = 1, α=0.5*

λ = 0.5, α=0.5									
n	**20**			**25**			**30**		
r0	**260**	**320**	**420**	**260**	**320**	**420**	**260**	**320**	**420**
a	0.445	0.801	0.536	0.305	0.342	0.35	0.387	0.562	0.329
k	3.5	2.8881	3.5	3.5	3.4695	3.5	2.894	2.9937	3.2095
LCL	0	0	0	0	0	0	0	1	0
UCL	11	12	12	12	12	13	13	15	13
Shift(f)	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
0.10	1.15	1.03	1.24	1.26	1.16	1.30	1.02	1.01	1.05
0.20	2.80	1.74	3.95	4.01	3.06	4.72	1.59	1.45	2.05
0.30	7.75	3.98	13.34	13.20	9.10	17.70	3.40	3.13	5.19
0.40	19.42	9.13	38.84	37.29	24.25	55.71	7.52	7.32	12.80
0.50	43.31	19.65	97.37	90.20	57.02	147.87	15.85	16.61	29.08
0.60	86.27	39.42	210.04	183.26	118.30	324.50	31.43	35.58	60.71
0.70	150.67	73.97	371.11	291.24	210.47	536.10	58.67	71.41	116.74
0.80	220.91	130.13	494.89	341.93	302.89	615.24	103.20	132.99	204.68
0.90	262.86	213.28	497.20	315.88	342.08	537.63	170.29	223.12	317.81
1.00	260.30	319.73	420.40	260.13	319.81	420.42	259.75	319.50	420.30

Table 2: *The Values of ARLs when λ = 1, α=1*

				λ = 1, α=1					
N	**20**			**25**			**30**		
r_0	**260**	**320**	**420**	**260**	**320**	**420**	**260**	**320**	**420**
A	0.637	0.751	0.836	0.52	0.968	0.564	0.571	0.9	0.757
K	2.878	3.0605	2.9802	2.8864	3.0967	2.9883	2.9303	3.1105	2.9931
LCL	1	2	2	1	5	1	3	6	4
UCL	13	15	15	14	20	15	18	22	20
Shift(f)	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
0.10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.20	1.02	1.05	1.02	1.00	1.02	1.01	1.00	1.00	1.00
0.30	1.30	1.65	1.37	1.16	1.35	1.20	1.15	1.09	1.07
0.40	2.33	3.86	2.64	1.89	2.86	2.09	1.97	1.72	1.58
0.50	5.15	10.89	6.34	3.99	8.11	4.78	4.76	3.91	3.31
0.60	12.42	32.62	16.66	9.54	26.28	12.38	13.70	11.10	8.61
0.70	30.59	96.82	44.79	23.89	88.72	33.47	42.16	35.11	24.95
0.80	73.65	256.59	117.80	59.68	280.72	90.10	126.32	113.71	75.02
0.90	160.76	420.13	274.81	139.83	510.03	228.58	275.53	300.58	214.82
1.00	260.23	319.82	419.53	259.98	320.88	421.30	260.19	320.49	419.64

Table 3: ***The Values of ARLs when λ = 1, α=0. 2***

				λ = 1, α = 0.2					
N	**20**			**25**			**30**		
r_0	**260**	**320**	**420**	**260**	**320**	**420**	**260**	**320**	**420**
A	0.145	0.782	0.546	0.821	0.898	0.628	0.625	0.704	0.719
K	2.9012	3.1212	2.9811	3.0122	3.0962	3.1747	2.9966	3.1105	3.1311
LCL	1	3	2	5	5	4	6	6	6
UCL	13	16	15	20	20	19	22	22	23
Shift(f)	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
0.10	9.27	5.46	4.78	5.37	4.50	4.62	3.23	2.66	4.16
0.20	31.05	22.82	17.26	27.77	21.92	20.54	13.88	10.42	21.78
0.30	67.20	60.09	41.30	86.91	66.97	58.20	40.54	29.19	71.79
0.40	116.29	123.35	79.65	202.62	155.88	128.17	92.90	65.62	179.24
0.50	171.84	211.36	133.77	366.15	293.71	235.84	176.89	125.99	361.94
0.60	222.65	305.46	201.91	492.40	439.67	366.23	277.83	209.73	577.92
0.70	257.97	371.35	276.92	497.12	509.21	472.82	349.07	297.50	695.82
0.80	273.52	387.36	346.20	420.11	477.70	510.08	355.24	352.79	654.21
0.90	272.21	362.85	396.24	331.90	399.31	480.71	314.22	355.47	536.39
1.00	260.09	320.25	419.63	259.89	320.45	419.51	260.45	320.49	419.99

Table 4: *The Values of ARLs when λ = 2, α=1*

				λ = 2, α=1					
N		**20**			**25**			**30**	
r_0	**260**	**320**	**420**	**260**	**320**	**420**	**260**	**320**	**420**
A	0.161	0.508	0.331	0.375	0.13	0.17	0.654	0.546	0.3
K	3.2501	3.1896	2.9882	2.995	3.4668	3.0557	2.9855	3.1626	2.9936
LCL	0	5	2	5	0	0	12	10	4
UCL	11	19	15	20	12	13	27	26	20
Shift(f)	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
0.10	1.00	1.10	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.20	1.08	2.38	1.04	1.05	1.06	1.02	1.03	1.02	1.00
0.30	1.73	7.87	1.48	1.59	1.65	1.29	1.36	1.34	1.10
0.40	3.83	9.93	2.95	3.69	3.79	2.36	2.62	2.65	1.69
0.50	9.65	17.04	7.07	10.92	10.25	5.38	6.54	7.02	3.58
0.60	24.91	46.65	18.30	35.97	28.73	13.44	19.05	21.93	9.25
0.70	62.34	86.97	48.28	121.42	78.19	34.28	59.99	74.17	26.38
0.80	140.76	204.02	124.45	360.12	188.89	86.08	189.56	249.41	77.85
0.90	240.85	293.37	283.09	490.02	322.52	204.88	403.91	529.71	218.35
1.00	260.36	319.90	420.37	260.37	320.41	419.94	259.81	320.33	419.78

I. Illustration 1

Consider the production scenario where the lifetimes of the manufactured products follow a Generalized Exponential-Poisson distribution with parameters $\lambda = 1$ and $\alpha = 1$. The following conditions and parameters are assumed for monitoring the quality of the process. The initial lifetime (m_0) is 1000 hours, the Average Run Length (ARL_0) is 260, and the sample size (n) is 20. The control chart parameters obtained from Table 2 are $k = 2.878$, $a = 0.637$, $LCL = 1$, and $UCL = 13$. Based on this information, the following steps were taken to establish the control chart:

Step 1: **Sample Selection**

Select a random sample of 20 products from each subgroup and subject them to the life test.

Step 2: **Life testing process**

During the test, monitor the number of failures and it is denoted as D and track the elapsed time.

Step 3: Test Termination Criteria

Terminate the test once the first failure occurs or the specified time period expires, whichever happens first.

Step 4: Process Evaluation

If the number of failures (D) is between 1 and 13 (inclusive), the process is declared in control. If D exceeds 13 or falls below 1, the process is considered out of control.

Interpretation

This control chart allows for timely detection of shifts in the products median lifetime by observing the number of failures before a predetermined truncation point. The use of hybrid censoring combined with the Generalized Exponential-Poisson work enhances detection capability especially under right censored data scenarios.

II. Illustration 2

Assume the product lifetimes follow a Generalized Exponential-Poisson distribution with shape parameters $\alpha = 1$ and Scale parameter $\lambda = 2$. The monitoring setup is based on the following specifications: The product values $m_0 = 1000$ hours, $ARL = 420$, and $n = 30$, the control chart parameters from Table 4 are $k = 2.9936$, $a = 0.3$, $LCL = 4$, and $UCL = 20$.

The control chart is set up as follows:

Step 1: Sample Selection

From each subgroup, select a sample of 30 products and subject them to a life test.

Step 2: Life test execution

During the life testing, observe the number of failed items and it is denoted as D, which corresponds to the minimum number of failures or the maximum time reached, whichever occurs first.

Step 3: Test monitoring

Monitor the number of failures and the elapsed time during the test.

Step 4: Process control decision

If the number of failures D is between 4 and 20, or the maximum time is reached before D exceeds these limits, the process is in control. If the number of failures D falls outside the range of 4 to 20, or the maximum time is reached prematurely, the process is deemed to be out of control.

Interpretation:

This illustration demonstrates the effectiveness of the proposed hybrid censoring scheme-based control chart in detecting changes in the lifetime distribution of products. The chart ensures balanced sensitivity and specificity by incorporating both failures counts and censoring time, allowing for robust decision making even under right censored data conditions.

IV. Simulation Study

This section demonstrates the practical application of the developed control chart using simulated data. The simulation data was performed under the assumption that product lifetimes follows Generalized Exponential-Poisson distribution with the following a median lifetime (m_0) of 1000 hours, and the shape parameters $\lambda = 1$ and $\alpha = 1$. The sample size (n) was set to 20, and the specified $ARL\ (r_0)$ was 420. For $m_0 = 1000$ hours and $f = 1$, the process is considered in control. These in-control parameters were used to construct the first 15 observations, each with a subgroup size of 20. Next, assuming the median of the Exponentiated Exponential-Poisson distribution shifts, we set the shift constant f to 0.5. The experiment was conducted at t = 836 hours. The number of products having the lifetime below 836 hours is noted ("D") and the Values are : 6,11,10,5,10,7,13,8,10,5,9,7,9,8,4,16,16,17,13,10,15,10,11,16,12,13,14,11,12,12. The control chart was constructed using the following control limits, derived from the earlier parameters: Lower Control Limit (LCL): 2 Upper Control Limit (UCL): 15. These limits were applied uniformly across all subgroups. Figure 1 displays the plotted values of D against subgroup indices, along with the LCL and UCL. Subgroups where D falls outside the control limits are indicative of an out-of-control process, thereby validating the sensitivity of the proposed control chart to shifts in product median lifetime.

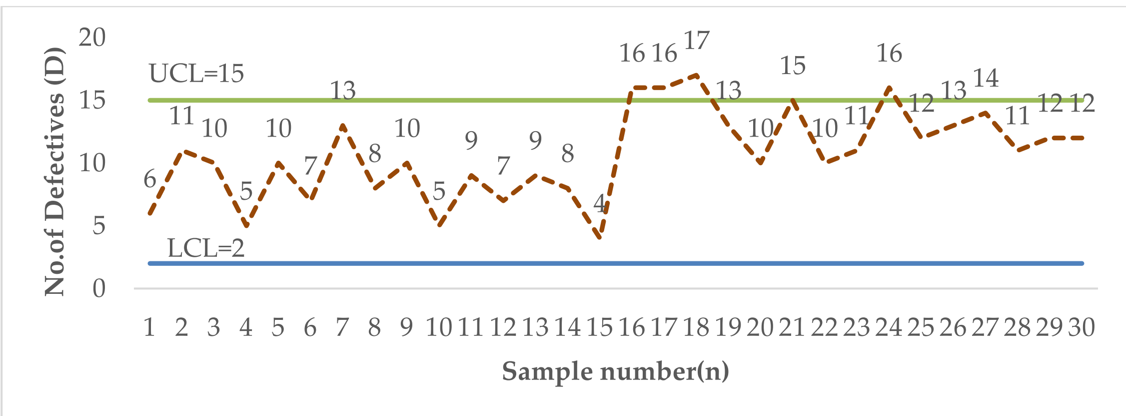

Figure 1: *Control Chart for Simulated Data*

V. Conclusion

This article presents an advanced attribute control chart that utilizes the Generalized Exponential-Poisson distribution under a Hybrid censoring Scheme designed to ensure the median lifetime of a product serves as the quality standard. The newly developed control chart offers significant flexibility, making it highly suitable for monitoring the longevity of high quality products across various industries. To demonstrate its application, the article includes tables tailored for industrial settings, with simulated data generated using R software based on the Generalized Exponential-Poisson distribution. The effectiveness of the control chart is assessed through the calculation of Average Run Lengths *(ARLs)* under varying shift constants *(f)*, providing insights into its performance. A notable advantage of employing the Hybrid censoring scheme in life testing is the substantial reduction in both time and costs associated with sampling inspection. Moreover, the control chart work is designed to be extensible, holding potential for adaptation to a variety of other statistical distributions in ongoing and future research activities.

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