ASSESSMENT OF CRUDE PROBABILITIES OF FAILURE FOR TWO-COMPONENT SERIES SYSTEM SHOCK MODEL

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Abstract

A Series system with two components having a single risk on each component is same as a single component with two independent risks acting on it. The third risk may be a catastrophic risk to whole system. A two-component series system shock model with two kinds of shocks namely, damage shock and catastrophic shock, is studied from competing risks perspective. The crude probabilities of different modes of system failure are derived. Information from life testing experiment is used in assessment of crude probabilities for system failure. Results are validated through simulation studies.

Keywords: Shock Model, Series System, Damage and Catastrophic Shock, Crude Probability for Failure, Life Testing Experiment.

I. Introduction

The arrangement of subassemblies of a system plays a crucial role in determining system reliability. While parallel setups offer redundancy, series configurations are often essential despite their vulnerability to single-point failures. Applications such as fire alarm systems, rocket launch sequences, water purification processes, and power distribution rely on series systems, where failure of any component like generators or transformers can disrupt the entire operation. Thus, the system functions only if all components operate effectively.

Often, system failure arises from one risk out of several competing risks. Identifying the dominant failure mode aids in implementing effective preventive or corrective actions to improve system reliability and hence, assessment of crude probabilities of system failure is crucial. Crude probability is defined in [4] as the probability of failure due to a specific risk in the presence of all other risks provides insights into the most probable causes of failure. A realistic and effective framework for modeling reliability of systems operating within random environments is through shock models. The definition of a shock model deals with the inter-arrival times of shocks, the damage induced by each shock, system failure criteria and their relationships.

The life distribution properties of a device under a homogeneous Poisson shock model were studied by [9]. The results obtained by [9] are extended to a shock model governed by a non-homogeneous Poisson process in [1]. A measure of component importance in coherent systems and fault trees has been introduced in [3]. Shock models with non-stationary pure birth process has

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been discussed in [2]. The properties of survival function of a pure birth shock model are demonstrated in [12]. The IFRA property of life distribution of non-homogeneous Poisson shock model has been established in [17]. The distribution, mean, variance and asymptotic behavior of the system failure time for a class of cumulative shock models have been analyzed in [20].

A semi-Markov shock model with additive damages and control limit replacement policies was examined in [16]. A non-homogeneous Poisson shock model leading to multivariate distribution has been studied in [18]. A two-dimensional random walk has been used to explain cumulative shock models in [10]. The MLE and UMVUE of the reliability function for a two-component parallel system have been derived in [13]. A shock model reliability using a renewal point process has been studied in [19]. A detailed account of competing risks theory has been provided in [6]. In [7] NBU and NWU ageing notions within competing risks have been introduced. The survival under two types of shocks has been modeled in [14]. The crude probabilities of failure for such models have been examined in [15]. Analytic expressions for hazard rates, failure densities, cause-specific probabilities, and survival functions in a bivariate shock setting have been obtained in [8].

Understanding not only when but how a system fails is crucial. In [5] and [11] the concept of crude probability from competing risks theory has been provided and it offers insights into failure causes - key for maintenance, design, and root-cause analysis. However, while the concept is well-studied in general reliability contexts, its application to shock-driven series systems remains relatively unexplored.

This study aims to assess the crude probabilities of failure in a two-component (components A and B) series system subjected to an external shock process. Each component is characterized by a fixed threshold, and the system fails when either component A or component B experiences a damage shock exceeding its respective threshold or when the system endures the catastrophic shock.

The model considered in this study is explained in Section 2. The explicit expressions for the crude probabilities of system failure due to different causes are also derived in the same section. Life testing experiment is described in Section 3. Simulation study is presented in Section 4. The results that contribute to a deeper understanding of failure attribution in shock models are outlined in Section 5.

II. Crude Probabilities of System Failure

A Poisson process with intensity $\lambda, (\lambda > 0)$ is considered for a randomly occurring sequence of shocks to a two-component series system. The shocks are of two types namely damage shocks and catastrophic (fatal) shocks with probabilities $p, (0 < p < 1)$ and $(1 - p)$ repsectively. Let X_i and Y_i denote amounts of damages to components A and B respectively due to damage shocks. Assuming X_i, Y_i $i = 1, 2, \ldots$ are exponential random variables (r.vs) with respective parameters θ_1 and θ_2 and u_1, u_2 are the thresholds of components A and B respectively. The damages are non-accumulating and the component fails whenever a damage exceeds its threshold and failure of a component leads to system failure (series system). The system also fails when it experiences a catastrophic shock. Thus, the system failure can be associated with one of the following three cases.

i) Failure of component A due to damage (X) exceeding its threshold (u_1).
ii) Failure of component B due to damage (Y) exceeding its threshold (u_2).
iii) Failure of the system when it experiences a catastrophic shock.

In this scenario, it is not feasible to study the system failure with an isolated cause (of the 3 causes). Hence, each cause of system failure is studied in the presence of other two causes which are referred to as risks and the corresponding probabilities of failure are referred to as crude probability of failure in the presence of the other.

Denoting Q_{tAd} as the crude probability of system failure during $(0, t)$ due to failure of component A because of damage exceeding its threshold even when the system has risks of component B failure and experiencing a catastrophic shock and is given by

$$Q_{tAd} = \int_0^t \sum_{k=1}^{\infty} \frac{e^{-p\lambda t_1}(p\lambda t_1)^k}{k!} \left(1 - e^{-u_1\theta_1}\right)^{k-1} e^{-u_1\theta_1} e^{-(1-p)\lambda t_1} \left(1 - e^{-u_2\theta_2}\right)^k dt_1 \quad (1)$$

Expression (1) is obtained in the light of following probability considerations:

- Component A has failed at $t_1 \epsilon (0, t)$.
- During $(0, t_1)$ the system has experienced k number of damage shocks $(k = 1, 2, \ldots)$.
- All the damages due to $(k - 1)$ shocks are less than u_1(component A), the damage due to k^{th} shock is greater than u_1.
- Component B has lived through all the k damage shocks.
- The system has not experienced any catastrophic shock during $(0, t_1)$.

Q_{tAd} is further simplified as

$$Q_{tAd} = \frac{e^{-u_1\theta_1}}{(1-e^{-u_1\theta_1})} \left\{ \frac{1-e^{-\left(\lambda - p\lambda\left(1-e^{-u_1\theta_1}\right)\left(1-e^{-u_2\theta_2}\right)\right)t}}{\lambda - p\lambda(1-e^{-u_1\theta_1})(1-e^{-u_2\theta_2})} - \frac{(1-e^{-\lambda t})}{\lambda} \right\} \quad (2)$$

On the similar grounds, the crude probability of system failure due to failure of component B is obtained as

$$Q_{tBd} = \int_0^t \sum_{k=1}^{\infty} \frac{e^{-p\lambda t_1}(p\lambda t_1)^k}{k!} \left(1 - e^{-u_1\theta_1}\right)^k e^{-(1-p)\lambda t_1} \left(1 - e^{-u_2\theta_2}\right)^{k-1} e^{-u_2\theta_2}\, dt_1 \quad (3)$$

$$Q_{tBd} = \frac{e^{-u_2\theta_2}}{(1-e^{-u_2\theta_2})} \left\{ \frac{1-e^{-\left(\lambda - p\lambda\left(1-e^{-u_1\theta_1}\right)\left(1-e^{-u_2\theta_2}\right)\right)t}}{\lambda - p\lambda(1-e^{-u_1\theta_1})(1-e^{-u_2\theta_2})} - \frac{(1-e^{-\lambda t})}{\lambda} \right\} \quad (4)$$

Q_{tf}, the crude probability of system failure by enduring a catastrophic shock at $t_1 \epsilon (0, t)$ when both the components A and B surviving through all previous damage shocks is given by

$$Q_{tf} = \int_0^t \sum_{k=0}^{\infty} \frac{e^{-p\lambda t_1}(p\lambda t_1)^k}{k!} \left(1 - e^{-u_1\theta_1}\right)^k (1-p)\lambda e^{-(1-p)\lambda t_1} \left(1 - e^{-u_2\theta_2}\right)^k dt_1 \quad (5)$$

Expression (5) is obtained in the light of following probability considerations:

- System has failed at $t_1 \epsilon (0, t)$.
- During $(0, t_1)$ the system has experienced k number of damage shocks $(k = 0, 1, 2, \ldots)$.
- Components A and B both have lived through all the k damage shocks.
- The system has experienced catastrophic shock at $t_1 \epsilon (0, t)$.

Q_{tf} is further simplified as,

$$Q_{tf} = (1-p) \left\{ \frac{1-e^{-\left(\lambda - p\lambda\left(1-e^{-u_1\theta_1}\right)\left(1-e^{-u_2\theta_2}\right)\right)t}}{1-p(1-e^{-u_1\theta_1})(1-e^{-u_2\theta_2})} \right\} \quad (6)$$

It can be noted that Q_{tAd} , Q_{tBd} and Q_{tf} are all non-decreasing functions of t.

Table 1 and 2 provide the values of Q_{tAd} , Q_{tBd} and Q_{tf} for given values of λ, θ_1 and θ_2, at different combinations of p, u_1 and u_2, and for various values of t.

Table 1: *Theoretical computed crude probabilities of failure*
$p = 0.7, \lambda = 0.2, \theta_1 = 0.6, \theta_2 = 0.8$

t	$u_1 = 1.2, u_2 = 1.6$			$u_1 = 1.8, u_2 = 2.4$			$u_1 = 2.4, u_2 = 3.2$		
	Q_{tAd}	Q_{tBd}	Q_{tf}	Q_{tAd}	Q_{tBd}	Q_{tf}	Q_{tAd}	Q_{tBd}	Q_{tf}
1.0	0.0219	0.0089	0.0558	0.0182	0.0061	0.0565	0.0139	0.0037	0.0571
1.5	0.0466	0.0189	0.0807	0.0389	0.0130	0.0823	0.0297	0.0080	0.0835
2.0	0.0783	0.0318	0.1039	0.0657	0.0220	0.1066	0.0502	0.0136	0.1086
2.5	0.1157	0.0470	0.1254	0.0975	0.0326	0.1294	0.0748	0.0202	0.1325
3.0	0.1576	0.0640	0.1453	0.1335	0.0446	0.1509	0.1027	0.0277	0.1552
3.5	0.2030	0.0824	0.1639	0.1727	0.0577	0.1712	0.1332	0.0360	0.1768
4.0	0.2511	0.1020	0.1811	0.2145	0.0716	0.1902	0.1660	0.0448	0.1973
4.5	0.3011	0.1223	0.1971	0.2582	0.0863	0.2082	0.2005	0.0541	0.2168
5.0	0.3523	0.1431	0.2119	0.3034	0.1014	0.2250	0.2364	0.0638	0.2353
5.5	0.4042	0.1641	0.2257	0.3496	0.1168	0.2409	0.2732	0.0737	0.2529

Table 2: *Theoretical computed crude probabilities of failure*
$p = 0.6, \lambda = 0.2, \theta_1 = 0.6, \theta_2 = 0.8$

t	$u_1 = 1.2, u_2 = 1.6$			$u_1 = 1.8, u_2 = 2.4$			$u_1 = 2.4, u_2 = 3.2$		
	Q_{tAd}	Q_{tBd}	Q_{tf}	Q_{tAd}	Q_{tBd}	Q_{tf}	Q_{tAd}	Q_{tBd}	Q_{tf}
1.0	0.0187	0.0076	0.0741	0.0156	0.0052	0.0749	0.0118	0.0032	0.0756
1.5	0.0398	0.0162	0.1070	0.0332	0.0111	0.1088	0.0253	0.0068	0.1102
2.0	0.0668	0.0271	0.1375	0.0559	0.0187	0.1406	0.0427	0.0115	0.1429
2.5	0.0986	0.0400	0.1657	0.0828	0.0277	0.1703	0.0634	0.0171	0.1737
3.0	0.1341	0.0545	0.1918	0.1131	0.0378	0.1981	0.0868	0.0234	0.2028
3.5	0.1726	0.0701	0.2159	0.1461	0.0488	0.2241	0.1124	0.0303	0.2303
4.0	0.2132	0.0866	0.2383	0.1811	0.0605	0.2484	0.1397	0.0377	0.2563
4.5	0.2553	0.1037	0.2589	0.2177	0.0727	0.2712	0.1683	0.0454	0.2807
5.0	0.2984	0.1212	0.2780	0.2554	0.0853	0.2926	0.1980	0.0534	0.3039
5.5	0.3420	0.1389	0.2957	0.2937	0.0981	0.3125	0.2283	0.0616	0.3257

From Table 1 and Table 2 the crude probabilities of failure exhibit a non-decreasing trend overtime. When the probability of damage shock p is reduced, the associated probabilities of failure from such shock also decline. At the same time, the crude probability of failure due to fatal shock Q_{tf} rises since the chance of encountering a fatal event, quantified by $(1 - p)$ increase as p decreases. As threshold of the components increase the crude probabilities of failure due to damage shock decrease, reflecting the increased resistance of components of damaging shocks.

III. Life Testing Experiment

Suppose n two-component series systems whose modes of failure are described by Q_{tAd} , Q_{tBd} and Q_{tf} derived as in Section 2 are subjected to sequence of shocks. Let r_1 systems fail due to failure of component A, r_2 systems fail due to failure of component B and r_3 $(= n - r_1 - r_2)$ systems fail due to experiencing catastrophic shock. Let n_i be the shock at which the i^{th} system has failed, t_{ij} be the time epoch at which i^{th} system has experienced j^{th} shock $j = 1, 2, \dots, n_i$, $i = 1, 2, \dots, n$. $(t_{ij} - t_{i(j-1)})$

are exponential r.vs with parameter $p\lambda$ for a damage shocks and $(1-p)\lambda$ for a catastrophic shock. It is assumed that the damage shock at which a component fails is not observable but it is known to exceed its threshold and the damages due to catastrophic shocks are also not observed.

Hence, the joint distribution of the random variables r_1, n_i, t_{i1}, $t_{i2}, \dots, t_{in_i}$, x_{i1}, $x_{i2}, \dots, x_{i(n_i-1)}$, $y_{i1}, y_{i2}, \dots, y_{in_i}$ for the r_1 systems that fail due to failure of component A because of the damage shock is given by

$$\prod_{i=1}^{r_1}(p\lambda)^{n_i}e^{-p\lambda t_{n_i}}\,\theta_1^{\,n_i-1}e^{-\theta_1\sum_{j=1}^{n_i-1}x_{ij}}e^{-\theta_1 u_1}\theta_2^{\,n_i}e^{-\theta_2\sum_{j=1}^{n_i}y_{ij}} \tag{7}$$

The joint distribution of all the random variables involved for the r_2 systems that have failed due to component B failure by damage shocks is given by

$$\prod_{i=1}^{r_2}(p\lambda)^{n_i}e^{-p\lambda t_{n_i}}\,\theta_1^{\,n_i}e^{-\theta_1\sum_{j=1}^{n_i}x_{ij}}\theta_2^{\,n_i-1}e^{-\theta_2\sum_{j=1}^{n_i-1}y_{ij}}e^{-\theta_2 u_2} \tag{8}$$

Similarly, for r_3 systems that have failed due to experiencing fatal shocks, the joint distribution of all the underlying random variables is given by

$$\prod_{i=1}^{r_3}(p\lambda)^{n_i-1}e^{-p\lambda t_{n_i-1}}\;\theta_1^{\,n_i-1}e^{-\theta_1\sum_{j=1}^{n_i-1}x_{ij}}\;\theta_2^{\,n_i-1}e^{-\theta_2\sum_{j=1}^{n_i-1}y_{ij}}\;(1-p)\lambda e^{-(1-p)\lambda\left(t_{n_i}-t_{n_i-1}\right)} \tag{9}$$

combining (7), (8) and (9) the joint distribution of all the random variables observed in this life testing experiment is obtained as

$$f=\frac{n!}{r_1!\,r_2!\,r_3!}(p\lambda)^{(n_.-r_3)}e^{-p\lambda t_.}\,\theta_1^{\,(n_.-r_1-r_3)}e^{-\theta_1 x_.}\,\theta_2^{\,(n_.-r_2-r_3)}e^{-\theta_2 y_.}\left((1-p)\lambda\right)^{r_3}e^{-\lambda t_{..}} \tag{10}$$

with, $n_. = \sum_{i=1}^{r_1}n_i+\sum_{i=1}^{r_2}n_i+\sum_{i=1}^{r_3}n_i$, $\quad t_. = \sum_{i=1}^{r_1}t_{n_i}+\sum_{i=1}^{r_2}t_{n_i}+\sum_{i=1}^{r_3}t_{(n_i-1)}-t_{..}$

$x_. = \sum_{i=}^{r_1}\sum_{j=1}^{n_i-1}x_{ij}+\sum_{i=}^{r_2}\sum_{j=1}^{n_i}x_{ij}+\sum_{i=}^{r_3}\sum_{j=1}^{n_i-1}x_{ij}+r_1u_1$

$y_. = \sum_{i=}^{r_1}\sum_{j=1}^{n_i}y_{ij}+\sum_{i=}^{r_2}\sum_{j=1}^{n_i-1}y_{ij}+\sum_{i=}^{r_3}\sum_{j=1}^{n_i-1}y_{ij}+r_2u_2$

$t_{..} = \sum_{i=1}^{r_3}(t_{n_i}-t_{(n_i-1)})$

Treating Equation (10) as function of parameters, equating the partial derivatives with respect to θ_1, θ_2, p and λ of natural logarithm of (10) to 0, the maximum likelihood estimators (MLEs) $\hat{\theta}_1, \hat{\theta}_2, \hat{p}, \hat{\lambda}$ of $\theta_1, \theta_2, p, \lambda$ respectively are obtained as below:

$$\frac{\partial logf}{\partial\theta_1}=0,\;\Rightarrow\hat{\theta}_1=\frac{n_.-r_1-r_3}{x_.}$$

$$\frac{\partial logf}{\partial\theta_2}=0,\;\Rightarrow\hat{\theta}_2=\frac{n_.-r_2-r_3}{y_.}$$

$$\frac{\partial\log f}{\partial\lambda}=0,\;\Rightarrow\frac{n_.-r_3}{\lambda}-pt_.+\frac{r_3}{\lambda}-t_{..}=0 \tag{12}$$

$$\frac{\partial\log f}{\partial p}=0,\;\Rightarrow\frac{n_.-r_3}{p}-\lambda t_.-\frac{r_3}{(1-p)}=0 \tag{13}$$

Solving equations (12) and (13) simultaneously, the MLEs of λ and p are obtained as

$$\hat{\lambda}=\frac{n_.}{\hat{p}t_.+t_{..}},\qquad\hat{p}=\frac{(n_.-r_3)t_{..}}{n_.t_{..}+r_3t_.}$$

Using the invariance property of MLE, the MLEs $\hat{Q}_{tAd}, \hat{Q}_{tBd}$ and $\hat{Q}_{tf}$ of three crude probabilities are obtained by plugging in the MLE of parameters in the expressions for Q_{tAd}, Q_{tBd} and Q_{tf}.

III. Simulation study

Given the values of $p = p_o, \lambda = \lambda_0, \theta_1 = \theta_{1o}, \theta_2 = \theta_{2o}, u_1 = u_{1o}, u_2 = u_{2o}$ for i^{th} system, the random variables X_{ij}, Y_{ij}, t_{ij} ; $i = 1, 2, 3, \ldots, n$; $j = 1, 2, 3, \ldots, n_i$ are generated as follows

Step 1: At the beginning, we have to judge whether the i^{th} system has failed due to damage shock or catastrophic shock. To do this, the following algorithm is used

i) Generate a standard uniform random number U_o.
ii) If $0 < U_o < p_o$, the system has failed due to damage shock, else the system has failed due to catastrophic shock.

Step 2: Depending on the results of Step 1 proceed to Step 3 if the system has failed due to damage shock or to Step 4, if the system has failed due to catastrophic shock.

Step 3: Initialize $r_1 = 0, r_2 = 0, n_i = 0$. For $p = p_o, \lambda = \lambda_o, \theta_1 = \theta_{1o}, \theta_2 = \theta_{2o}$ the exponential random variables X_{i1} with parameter θ_{1o} and Y_{i1} with parameter θ_{2o} are generated.

i) If $X_{i1} > u_{1o}$ and $Y_{i1} > u_{2o}$, this condition is against model assumption so the current iteration is discarded and a new set of random variables are generated by initializing $n_i = 0$.
ii) The process of generation of X_{ij}, Y_{ij} and incrementation of n_i by 1 is continued until system fails due to either failure of component A or B.
iii) r_1 is incremented by 1 if system fails due to component A else r_2 is incremented by 1.
iv) n_i number of exponential random variables with parameter $p_o\lambda_o$ are generated to compute t_{in_i} here, $i = 1, 2, \ldots, r_1$ or r_2.

Step 4: Initialize $r_3 = 0, n_i = 0$. For $p = p_o, \lambda = \lambda_o, \theta_1 = \theta_{1o}, \theta_2 = \theta_{2o}$ the exponential random variables X_{i1} with parameter θ_{1o} and Y_{i1} with parameter θ_{2o} are generated.

(i) and (ii) of Step 3 are repeated until n_i is noted. r_3 is incremented by 1, $n_i - 1$ number of exponential random variables with parameter $p_o\lambda_o$ are generated to compute $t_{i(n_i-1)}$ here, $i = 1, 2, \ldots, r_3$. One inter-arrival time $\left(t_{in_i} - t_{i(n_i-1)}\right)$ having exponential distribution with parameter $(1 - p_o)\lambda_o$ is generated.

Steps 1 to 4 are repeated for $n = 25, 45, 75$. The statistics $n_{.}$, $t_{.}$, $x_{.}$, $y_{.}$, $t_{..}$ are computed, using which $\hat{Q}_{tAd}, \hat{Q}_{tBd}$ and $\hat{Q}_{tf}$ are obtained for $t = 1\ (0.5)\ 5.5$.

The absolute biases are computed for each of the estimated $\hat{Q}_{tAd}, \hat{Q}_{tBd}$ and $\hat{Q}_{tf}$, and the respective mean absolute biases are computed for 10,000 (m) simulations as

$$M_{\hat{Q}_{tAd}} = \frac{\sum_{j=1}^{m} |Q_{tAd} - \hat{Q}_{tAd_j}|}{m} \ ; \ M_{\hat{Q}_{tBd}} = \frac{\sum_{j=1}^{m} |Q_{tBd} - \hat{Q}_{tBd_j}|}{m} \ ; M_{\hat{Q}_{tf}} = \frac{\sum_{j=1}^{m} |Q_{tf} - \hat{Q}_{tf_j}|}{m}$$

Table 3: *Mean absolute biases of estimated crude probabilities of failure*

t	$p = 0.7, \lambda = 0.2, \theta_1 = 0.6, \theta_2 = 0.8, u_1 = 1.2, u_2 = 1.6$								
	Q_{tAd}			Q_{tBd}			Q_{tf}		
	$n = 25$	$n = 45$	$n = 75$	$n = 25$	$n = 45$	$n = 75$	$n = 25$	$n = 45$	$n = 75$
1.0	0.0050	0.0040	0.0034	0.0028	0.0024	0.0022	0.0205	0.0131	0.0102
1.5	0.0105	0.0085	0.0071	0.0060	0.0051	0.0047	0.0289	0.0188	0.0146
2.0	0.0175	0.0141	0.0118	0.0101	0.0085	0.0079	0.0365	0.0239	0.0187
2.5	0.0257	0.0206	0.0172	0.0149	0.0126	0.0116	0.0432	0.0287	0.0225
3.0	0.0348	0.0277	0.0231	0.0203	0.0170	0.0156	0.0492	0.0330	0.0259
3.5	0.0446	0.0354	0.0294	0.0261	0.0219	0.0200	0.0546	0.0369	0.0290
4.0	0.0549	0.0435	0.0360	0.0324	0.0270	0.0246	0.0594	0.0405	0.0319
4.5	0.0657	0.0518	0.0427	0.0388	0.0323	0.0294	0.0637	0.0437	0.0346
5.0	0.0766	0.0602	0.0494	0.0454	0.0377	0.0342	0.0676	0.0467	0.0370
5.5	0.0877	0.0687	0.0561	0.0521	0.0431	0.0390	0.0710	0.0494	0.0393

Table 4: *Mean absolute biases of estimated crude probabilities of failure*

t	$p = 0.7, \lambda = 0.2, \theta_1 = 0.6, \theta_2 = 0.8, u_1 = 1.8, u_2 = 2.4$								
	Q_{tAd}			Q_{tBd}			Q_{tf}		
	$n = 25$	$n = 45$	$n = 75$	$n = 25$	$n = 45$	$n = 75$	$n = 25$	$n = 45$	$n = 75$
1.0	0.0041	0.0036	0.0034	0.0018	0.0015	0.0014	0.0209	0.0137	0.0101
1.5	0.0088	0.0078	0.0073	0.0038	0.0031	0.0029	0.0296	0.0196	0.0145
2.0	0.0149	0.0131	0.0123	0.0065	0.0053	0.0049	0.0374	0.0250	0.0185
2.5	0.0221	0.0193	0.0182	0.0096	0.0079	0.0072	0.0443	0.0300	0.0223
3.0	0.0302	0.0265	0.0249	0.0131	0.0107	0.0098	0.0506	0.0345	0.0257
3.5	0.0391	0.0341	0.0320	0.0169	0.0139	0.0126	0.0561	0.0386	0.0288
4.0	0.0486	0.0423	0.0396	0.0210	0.0172	0.0156	0.0611	0.0423	0.0317
4.5	0.0585	0.0507	0.0474	0.0254	0.0208	0.0188	0.0656	0.0457	0.0343
5.0	0.0689	0.0595	0.0555	0.0299	0.0244	0.0220	0.0696	0.0488	0.0368
5.5	0.0795	0.0685	0.0637	0.0345	0.0281	0.0252	0.0732	0.0516	0.0390

IV. Results

Table 3 and Table 4 presents mean absolute biases of the estimated crude probabilities of system failure Q_{tAd}, Q_{tBd} and Q_{tf} for varying time (t) and number of systems subject to test (n) values. Across all time points the mean absolute biases of crude probabilities of failure decrease as n increase this shows the consistency of the MLE.

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Conflict of Interest

The authors declare that there is no conflict of interest.

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