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# RELIABILITY ESTIMATION OF A SYSTEM BASED ON THE STRESS-STRENGTH MODEL USING A WEIBULL DISTRIBUTION

K Sruthi and M Kumar

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National Institute of Technology, Calicut
mahesh@nitc.ac.in

**Abstract**

*Reliability analysis is increasingly used in practical applications. The Weibull distribution is versatile, offering increasing, decreasing, bathtub, and unimodal hazard rates. Its flexibility enables a more accurate representation of real-world data, leading to reliability expressions that are both more realistic and more reliable. In this paper, we analyze the reliability of a single component system using a stress strength model, where both the system strength and stress follow a Weibull distribution. We extend the analysis to situations where a single-component system is exposed to two independent stresses, with the strength also following a Weibull distribution to estimate reliability. Furthermore, we estimate the reliability of a stress-strength model for a single component exposed to n stresses. Our analysis includes a series system with two independent components under a common stress, using Weibull distributions for stress and exponential distributions for strength. Furthermore, we study the reliability of a series system with n components under common stress, where the stress follows a Weibull distribution. Numerical computations are performed to demonstrate the results. Finally, a comparative study is conducted to evaluate the reliability expression derived in our work using the Weibull distribution against the existing methods.*

**Keywords:** Reliability estimation, Stress-strength model, Weibull distribution, Exponential distribution

## 1. Introduction

Reliability is a crucial concept in various fields such as engineering, statistics, manufacturing, and more. Najwan et al. [1] estimated various techniques for estimating the Stress-Strength reliability when both stress and strength random variables have the unit Gompertz distribution based on the ranked set sampling (RSS) and simple random sampling (SRS) covered. The methods that have been suggested include the maximum likelihood, least squares, weighted least squares, maximum product spacing, Cramer von Mises, Anderson Darling, and right tail Anderson Darling methods.

Zhang et al. [2] studied the reliability estimation of the multicomponent stress-strength model involving one stress and two correlated strength components of a parallel system. By introducing new latent variables into the model, the likelihood function is simplified to make statistical inference available.

Amer et al. [3] estimated the reliability when the strength X and the stress Y are independent variables following the exponentiated Pareto distribution. The maximum likelihood estimators of R are computed using SRS, RSS, and median ranked set sampling (MRSS). Based on MRSS, the reliability estimate is considered in four different cases.



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Ehsan et al. [4] investigated the engineering properties of waste foundry sand (WFS) with a focus on the shear strength parameters. Comparisons were also made with shear strength properties of recycled glass (RG) as a control material as well as natural sands.

Manal et al. [5] discussed the statistical inference of $R = P(X < Y < Z)$ for a component that has a strength that is independent of the opposite lower and upper bound stresses, when the stresses and strength follow an exponentiated exponential distribution (EED). They assume that both stress and strength random variables are independent, having EED with a common scale parameter. Using the generalized progressive hybrid censoring design, various point and interval estimators are obtained for the reliability model R in view of the ML and Bayesian approaches.

Hassan et al. [6] obtained the reliability of a system when the strength and stress variables are independent Weibull distribution with different scale parameters. The reliability in multi-component stress-strength (MSS) is estimated by using the maximum likelihood and Bayesian methods of estimation when the samples are drawn from strength and stress distributions and their measurements are in terms of the upper record values.

Al-Mutairi et al. [7] studied several point and interval estimation procedures of the stress strength parameter of the Lindley distribution. They have obtained the Uniformly Minimum-Variance Unbiased Estimator (UMVUE) of the stress-strength parameter, however, the exact or asymptotic distribution of it is very difficult to obtain. They have derived the maximum likelihood estimator (MLE) of R and its asymptotic distribution.

Maurya et al. [8] obtained likelihood and Bayesian methods-based estimates of MSS reliability when $\beta$ is known or unknown. They used the Lindly method and Metropolis-Hastings (MH) procedure to evaluate Bayes estimates under informative and noninformative prior distributions. They also obtained UMVUE and exact Bayes estimates of MSS reliability when $\beta$ is known. They observed using Monte Carlo simulations that the Bayes method has an advantage over the maximum likelihood method in case prior distribution is proper.

Mohamed et al. [9] proposed Poisson Modified Weibull Distribution (PMW) distribution and some statistical properties of the proposed distribution including explicit formulas for the density and hazard functions, moments, and quantiles are provided. The usefulness of the proposed distribution is verified by fitting different data sets. The results indicate that PMW is more flexible than other models.

Akram [10] obtained different estimates of the stress-strength parameter, under the hybrid progressive censored scheme, at the time that stress and strength were considered as two independent Kumaraswamy random variables. Also, they consider the existence and uniqueness of the MLE and construct the asymptotic and highest posterior density (HPD) intervals for R.

Alessandro and Asmerilda [11] revised the most popular techniques, highlighting their strengths and weaknesses, and empirically investigated their performance through a comparative study applied to a well-known engineering problem, formulated as a stress-strength model, with the aim of weighing up their feasibility and accuracy in recovering the value of the reliability parameter, also with reference to the number of discrete points.

Amal et al. [12] estimated the stress-strength reliability when the strength X and stress Y are independent but not identically distributed random variables from the generalized inverted exponential distribution. The stress-strength reliability estimators are considered based on different sampling schemes.

Manal and Ehab [13] estimated the reliability R when the observed data are progressive first failure censored coming from Kumaraswamy distribution (KuD). For comparing the results obtained using various methods, they have obtained the mean-squared errors (MSEs) for the reliability R. In the case of Bayesian estimation, looking at the limitation of Lindley estimation to point and interval estimation, they perform the approximation using the Markov chain Monte Carlo (MCMC) technique.

Mayank et al. [14] studied the multicomponent stress-strength reliability for two parameter unit-Gompertz distribution when both stress and strength variates follow the same distribution under progress Type II censoring. The classical and Bayesian approaches have been used to obtain the reliability of the system when the common scale parameter $\mu$ is known and unknown.

Lai et al. [15] reviewed the properties of the basic Weibull distribution and listed the various extensions.Hallinan and Arthur [16]encourages practitioners to use the Weibull distribution by furnishing historical background and parallel development of the essential formulas. Shakhatreh et al. [17] addressed the estimation of differential entropy of the Weibull distribution based on different non-informative prior distributions. They derived non-informative priors using formal rules, such as Jeffreys prior and maximal data information prior based on Fisher information and entropy, respectively. They also developed reference prior and probability matching prior for the differential entropy. Makalic et al. [18] derived a new bias adjusted maximum likelihood estimate for the shape parameter of the Weibull distribution with complete data and type I censored data. The proposed estimate of the shape parameter is significantly less biased and more efficient than the corresponding maximum likelihood estimate, while being simple to compute using existing maximum likelihood software procedures.

Kizilaslan and Nadar [19] studied the multicomponent system with $k$ independent and identical strength components, where each component is exposed to a common random stress when the underlying distributions are Weibull. In our study, we consider both single-component and multi-component systems. The main difference is that we examine cases with two stresses and $n$ stresses. Our study includes the reliability estimation of a single component system, considering two cases, namely, (i) case of two stresses and (ii) case of $n$ stresses. After that, we analyze two-component cases and later extend our study to multi-component systems. In their work, they consider a single stress, whereas in our study, we examine multiple stresses. It is to be observed that the notion of $n$ stresses is the general one. User can set the value of $n$ depends upon the factors which affect the system reliability. This set up is most general and useful to reliability practitioners in a best possible way to estimate system reliability. This is the major contribution of our work compared to existing research. This broader framework allows for a more comprehensive reliability assessment compared to the previous work in the literature.

The rest of this paper is organized as follows. Section 2 provides basic definitions. Section 3 deals with the estimation of the reliability of a system in a stress-strength model, in which the stress and strength of the system follow a Weibull distribution. Section 4 deals with the reliability estimation of a system in a stress-strength model with two independent stresses. Section 5 deals with the reliability estimation of a system in a stress-strength model with $n$ independent stress. Section 6 deals with the reliability estimation of a series system in a stress-strength model with two independent components under common stress. Section 7 deals with the reliability estimation of a series system in a stress-strength model with $n$ independent components under common stress. In Section 8, a numerical illustration of the results obtained is provided. In Section 9, we discuss a comparison study of the reliability expression obtained in our work using the Weibull distribution with the existing method. Finally, the conclusions are presented in Section 10.

## 2. Some basic definitions

In this section, we present some important definitions which will be useful in the following sections to understand the results obtained.

**Definition 1.** (see[20]) Let $V$ denote the strength random variable of the system and $W$ denote the stress random variable. If $V$ and $W$ are independent with respective distribution functions $G$ and $F$, then the traditional stress-strength reliability can be estimated as

$$R = P\{W < V\} = \iint_{w<v} dF(w)dG(v). \tag{1}$$

**Definition 2.** (see [15]) The cumulative distribution function (c.d.f.) of a Weibull random variable, $T$, is given by

$$F(x) = \left(1 - e^{-(x/\lambda)^k}\right), \quad x > 0, k > 0, \lambda > 0 \tag{2}$$

where $k$ is the shape parameter and $\lambda$ is the scale parameter. The probability density function (p.d.f.) of the cumulative distribution function (2) is

$$f(x) = \frac{k}{\lambda}\left[\frac{x}{\lambda}\right]^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k}. \tag{3}$$

## 3. Reliability estimation of a system in a stress-strength model using a Weibull distribution

Reliability estimation has become more popular nowadays. Therefore, it is important to derive more reliable expressions for reliability. To do this, it is necessary to use a distribution that accurately represents realistic data. It has been observed that the Weibull distribution is more flexible and can better represent realistic data. In this section, we derive the reliability estimation using the Weibull distribution. Consider a stress-strength model. The Weibull distribution can represent more realistic data, so we use it for this purpose. Let the random variable $X$ represent the strength of the system following the Weibull distribution given by

$$f(x) = \frac{k_1}{\lambda_1}\left[\frac{x}{\lambda_1}\right]^{k_1-1} e^{-\left(\frac{x}{\lambda_1}\right)^{k_1}}, x > 0 \tag{4}$$

where $k_1 > 0$ is the shape parameter and $\lambda_1 > 0$ is the scale parameter. The random variable $Y$ represents the stress of the system following the Weibull distribution given by

$$g(y) = \frac{k_2}{\lambda_2}\left[\frac{y}{\lambda_2}\right]^{k_2-1} e^{-\left(\frac{y}{\lambda_2}\right)^{k_2}}, y > 0 \tag{5}$$

where and $k_2 > 0$ is the shape parameter and $\lambda_2 > 0$ is the scale parameter. Then, the stress-strength reliability $R$ of the system is given by

$$\begin{aligned} R &= P[Y < X] \\ &= \iint_{y<x} f(x) g(y)\, dxdy \\ &= \iint_{y<x} \frac{k_1}{\lambda_1}\left[\frac{x}{\lambda_1}\right]^{k_1-1} e^{-\left(\frac{x}{\lambda_1}\right)^{k_1}} \frac{k_2}{\lambda_2}\left[\frac{y}{\lambda_2}\right]^{k_2-1} e^{-\left(\frac{y}{\lambda_2}\right)^{k_2}} dxdy. \end{aligned} \tag{6}$$

Hence the stress-strength reliability $R$ of the system is given by

$$R = \frac{k_1 k_2}{\lambda_1 \lambda_2} \iint_{y<x} \left[\frac{x}{\lambda_1}\right]^{k_1-1} \left[\frac{y}{\lambda_2}\right]^{k_2-1} e^{-\left(\frac{x}{\lambda_1}\right)^{k_1}} e^{-\left(\frac{y}{\lambda_2}\right)^{k_2}} dxdy \tag{7}$$

Parameters can be estimated using Maximum Likelihood Estimation as follows. Given a sample $\{x_1, x_2, \ldots, x_n\}$, the likelihood function for the two-parameter Weibull distribution is given by

$$L(\lambda, k) = \prod_{i=1}^{n} \frac{k}{\lambda}\left(\frac{x_i}{\lambda}\right)^{k-1} e^{-(x_i/\lambda)^k}$$

Taking the natural log gives the log-likelihood function:

$$\ell(\lambda, k) = n \ln k - nk \ln \lambda + (k-1)\sum_{i=1}^{n} \ln x_i - \sum_{i=1}^{n}\left(\frac{x_i}{\lambda}\right)^k$$

By solving the partial derivatives $\frac{\partial \ell}{\partial \lambda} = 0$ and $\frac{\partial \ell}{\partial k} = 0$, one can obtain estimates for $\lambda$ and $k$. We observe that the closed form solution does not exist for the estimates of the parameters. The values can be estimated using numerical techniques such as Newton Raphson iteration method.

### 3.1. Confidence Interval for $R$

In this section, we obtain confidence intervals for $R$. Let the random variable $X$ represent the strength of the system following the Weibull distribution given by

$$f(x) = \frac{k_1}{\lambda_1}\left[\frac{x}{\lambda_1}\right]^{k_1-1} e^{-\left(\frac{x}{\lambda_1}\right)^{k_1}}, x > 0 \tag{8}$$

where $k_1 > 0$ is the shape parameter and $\lambda_1 > 0$ is the scale parameter. The random variable $Y$ represents the stress of the system following the Weibull distribution given by

$$g(y) = \frac{k_2}{\lambda_2}\left[\frac{y}{\lambda_2}\right]^{k_2-1} e^{-\left(\frac{y}{\lambda_2}\right)^{k_2}}, y > 0 \tag{9}$$

where and $k_2 > 0$ is the shape parameter and $\lambda_2 > 0$ is the scale parameter.
If $\hat{R} = g(\theta)$, where $\theta$ is a vector of parameter estimates (e.g. $\hat{k}_1, \hat{\lambda}_1, \hat{k}_2, \hat{\lambda}_2$ ), then the variance of $\hat{R}$ is approximately.

$$\text{Var}(\hat{R}) \approx \nabla g(\theta)^T \cdot \text{Cov}(\hat{\theta}) \cdot \nabla g(\theta),$$

where $\nabla g(\theta)$ is the gradient (vector of partial derivatives of $R$ with respect to each parameter). $\text{Cov}(\hat{\theta})$ is the covariance matrix of the parameter estimates.
So the standard error becomes

$$SE(\hat{R}) = \sqrt{\text{Var}(\hat{R})}$$

Then the confidence interval is

$$\hat{R} \pm z_{\alpha/2} \cdot SE(\hat{R})$$

Suppose $\theta = \begin{bmatrix} \hat{k}_1 \\ \hat{\lambda}_2 \\ \hat{k}_2 \\ \hat{\lambda}_2 \end{bmatrix}$ and $\nabla g(\theta) = \begin{bmatrix} \frac{\partial \hat{R}}{\partial k_1} \\ \frac{\partial \hat{R}}{\partial \lambda_1} \\ \frac{\partial \hat{R}}{\partial k_2} \\ \frac{\partial \hat{R}}{\partial \lambda_2} \end{bmatrix}$

Then $\text{Var}(\hat{R}) = \sum_{i=1}^{4}\sum_{j=1}^{4} \frac{\partial \hat{R}}{\partial \theta_i} \cdot \frac{\partial \hat{R}}{\partial \theta_j} \cdot \text{Cov}\left(\hat{\theta}_i, \hat{\theta}_j\right)$

Since there is no closed-form expression for R, it is not possible to compute closed form expressions for the partial derivatives $\frac{\partial R}{\partial k_1}, \frac{\partial R}{\partial \lambda_1}$, etc. Hence it is not possible to proceed for obtaining the true confidence intervals. However, We can use numerical differentiation for performing calculations and one can obtain numerical confidence intervals for the parameters.

## 4. Reliability estimation of a system in a stress-strength model with two independent stresses

In this section, we study the reliability estimation of a system in a stress-strength model with two independent stresses. Consider a system described by the stress-strength model consisting of a component with strength $X$, where the strength follows a Weibull distribution. The probability density function is given by

$$f(x) = \frac{k}{\lambda}\left[\frac{x}{\lambda}\right]^{k-1} e^{-\left(\frac{x}{\lambda}\right)^{k}}, x > 0 \tag{10}$$

where $k > 0$ is the shape parameter and $\lambda > 0$ is the scale parameter. The system has two independent stress $Y_1$ and $Y_2$, respectively, with distributions $g_1(y_1) = \lambda_1 e^{-\lambda_1 y_1}$ and $g_2(y_2) = \lambda_2 e^{-\lambda_2 y_2}$. Then reliability of the system is given by

$$
\begin{aligned}
R &= P[max(Y_1, Y_2) < X] \\
&= \iiint_{\max(y_1,y_2)<x} g_1(y_1)\, g_2(y_2) f(x) dy_1 dy_2 dx \\
&= \int_0^\infty f(x) \left[ \int_0^x \int_0^x g_1(y_1)\, g_2(y_2) dy_1 dy_2 \right] dx \\
&= \int_0^\infty \frac{k}{\lambda} \left[\frac{x}{\lambda}\right]^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} \left[ \int_0^x \int_0^x \lambda_1 e^{-\lambda_1 y_1} \lambda_2 e^{-\lambda_2 y_2} dy_1 dy_2 \right] dx \\
&= \int_0^\infty \frac{k}{\lambda} \left[\frac{x}{\lambda}\right]^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} \lambda_1 \lambda_2 \int_0^x e^{-\lambda_2 y_2} \left[\frac{e^{-\lambda_1 y_1}}{-\lambda_1}\right]_0^x dy_2 \\
&= \int_0^\infty \frac{k}{\lambda} \left[\frac{x}{\lambda}\right]^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} \frac{\lambda_1 \lambda_2}{\lambda_1 \lambda_2} \left[e^{-\lambda_1 x} - 1\right] \left[e^{-\lambda_2 x} - 1\right]
\end{aligned} \tag{11}
$$

Reliability of the system is given by

$$
= \frac{k}{\lambda} \int_0^\infty \left[\frac{x}{\lambda}\right]^{k-1} [e^{-\lambda_1 x} - 1][e^{-\lambda_2 x} - 1] e^{-\left(\frac{x}{\lambda}\right)^k} dx \tag{12}
$$

## 5. Reliability estimation of a system in a stress-strength model with $n$ independent stresses

In this section, we consider the reliability estimation of a system in a stress-strength model with $n$ independent stresses. Consider a system described by the stress-strength model, consisting of a component with strength $X$, where the distribution function is given by

$$
f(x) = \frac{k}{\lambda} \left[\frac{x}{\lambda}\right]^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k}, x > 0 \tag{13}
$$

where $k > 0$ is the shape parameter and $\lambda > 0$ is the scale parameter. The system has n independent stresses $Y_1$, $Y_2$, $Y_3$, ...,$Y_n$ with distributions $g_1(y_1) = \lambda_1 e^{-\lambda_1 y_1}$, $g_2(y_2) = \lambda_2 e^{-\lambda_2 y_2}$, $g_3(y_3) = \lambda_3 e^{-\lambda_3 y_3}$, ..., $g_n(y_n) = \lambda_n e^{-\lambda_n y_n}$ respectively.
Then reliability of the system is given by

$$
\begin{aligned}
R &= P[max\,(Y_1, Y_2, Y_3, ..., Y_n) < X] \\
&= \iiint ... \int_{\max(y_1,y_2,y_3,...,y_n)<x} g_1(y_1) g_2(y_2) g_3(y_3) ... g_n(y_n) f(x) dy_1 dy_2 dy_3 ... dy_n dx \\
&= \int_0^\infty f(x) \left[ \int_0^x \int_0^x \int_0^x ... \int_0^x g_1(y_1) g_2(y_2) g_3(y_3) ... g_n(y_n) dy_1 dy_2 dy_3 ... dy_n \right] dx \\
&= \int_0^\infty \frac{k}{\lambda} \left[\frac{x}{\lambda}\right]^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} \left[ \int_0^x \int_0^x \int_0^x ... \int_0^x \lambda_1 e^{-\lambda_1 y_1} \lambda_2 e^{-\lambda_2 y_2} \lambda_3 e^{-\lambda_3 y_3} ... \lambda_n e^{-\lambda_n y_n} dy_1 dy_2 dy_3 ... dy_n \right] dx \\
&= \int_0^\infty \frac{k}{\lambda} \left[\frac{x}{\lambda}\right]^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} \left[ (e^{-\lambda_1 x} - 1)(e^{-\lambda_2 x} - 1)(e^{-\lambda_3 x} - 1) ... (e^{-\lambda_n x} - 1) \right] dx
\end{aligned}
$$

Reliability of the system is given by

$$
= \frac{k}{\lambda} \int_0^\infty \left[\frac{x}{\lambda}\right]^{k-1} [e^{-\lambda_1 x} - 1][e^{-\lambda_2 x} - 1][e^{-\lambda_3 x} - 1] ... [e^{-\lambda_n x} - 1] e^{-\left(\frac{x}{\lambda}\right)^k} dx \tag{14}
$$

## 6. Reliability estimation of a series system in a stress-strength model with two independent components under common stress

Consider a series system of the stress-strength model with two independent components $C_1$ and $C_2$. Let the random variable $X_1$ and $X_2$ represent the strengths of the components with the probability density function $f_1(x_1) = \lambda_1 e^{-\lambda_1 x_1}$ and $f_2(x_2) = \lambda_2 e^{-\lambda_2 x_2}$. The random variable $Y$

represents the stress of the system with the probability density function $g(y) = \frac{k}{\lambda}\left[\frac{y}{\lambda}\right]^{k-1} e^{-\left(\frac{y}{\lambda}\right)^k}$.
Then, the stress-strength reliability $R$ of the system is given by

$$R = P\left[Y < min\left(X_1, X_2\right)\right]$$
$$= \iiint_{y<min(x_1,x_2)} f\left(x_1\right) f\left(x_2\right) g(y) dx_1 \cdot dx_2 dy$$
$$= \int_0^\infty \int_y^\infty \int_y^\infty f\left(x_1\right) f\left(x_2\right) g(y) dx_1 \cdot dx_2 dy$$
$$= \int_0^\infty g(y) \left[\int_y^\infty \int_y^\infty f\left(x_1\right) f\left(x_2\right) dx_1 \cdot dx_2\right] dy$$
$$= \int_0^\infty \frac{k}{\lambda}\left[\frac{y}{\lambda}\right]^{k-1} e^{-\left(\frac{y}{\lambda}\right)^k} \left[\int_y^\infty \int_y^\infty [\lambda_1 e^{-\lambda_1 x_1}][\lambda_2 e^{-\lambda_2 x_2}] dx_1 \cdot dx_2\right] dy$$
$$= \int_0^\infty \frac{k}{\lambda}\left[\frac{y}{\lambda}\right]^{k-1} e^{-\left(\frac{y}{\lambda}\right)^k} [e^{-(\lambda_1+\lambda_2)y}] dy$$

Reliability of the system is given by

$$= \frac{k}{\lambda}\int_0^\infty \left[\frac{y}{\lambda}\right]^{k-1} [e^{-(\lambda_1+\lambda_2)y}] e^{-\left(\frac{y}{\lambda}\right)^k} dy \tag{15}$$

## 7. Reliability estimation of a series system in a stress-strength model with $n$ independent components under common stress

In this section, we study the reliability estimation of a series system in a stress-strength model with $n$ independent components under a common stress. Consider a system represented by the stress-strength model, consisting of $n$ components $C_1$, $C_2$, $C_3$, ..., $C_n$. Let $X_1$, $X_2$, $X_3$, ..., $X_n$ denote the strength random variables, with the following distribution functions $f_1(x_1) = \lambda_1 e^{-\lambda_1 x_1}$,$f_2(x_2) = \lambda_2 e^{-\lambda_2 x_2}$,$f_3(x_3) = \lambda_3 e^{-\lambda_3 x_3}$,...,$f_n(x_n) = \lambda_n e^{-\lambda_n x_n}$ respectively. Let the random variable $Y$ represent the common stress of system with distribution $g(y)$ given by

$$g(y) = \frac{k}{\lambda}\left[\frac{y}{\lambda}\right]^{k-1} e^{-\left(\frac{y}{\lambda}\right)^k}, y > 0 \tag{16}$$

where $k > 0$ is the shape parameter and $\lambda > 0$ is the scale parameter. Then reliability of the system is given by

$$R = P[Y < min(X_1, X_2, X_3, ..., X_n)]$$
$$= \iiint ... \int_{\min(x_1,x_2,x_3,...,x_n)>y} f_1(x_1) f_2(x_2) f_3(x_3) ... f_n(x_n) g(y) dx_1 dx_2 dx_3 ... dx_n dy$$
$$= \int_0^\infty g(y) \left[\int_y^\infty \int_y^\infty \int_y^\infty ... \int_y^\infty f_1(x_1) f_2(x_2) f_3(x_3) ... f_n(x_n) dx_1 dx_2 dx_3 ... dx_n\right] dy$$
$$= \int_0^\infty \frac{k}{\lambda}\left[\frac{y}{\lambda}\right]^{k-1} e^{-\left(\frac{y}{\lambda}\right)^k} e^{-(\lambda_1+\lambda_2+\lambda_3+...+\lambda_n)y} dy$$

Reliability of the system is given by

$$= \frac{k}{\lambda}\int_0^\infty \left[\frac{y}{\lambda}\right]^{k-1} e^{-(\lambda_1+\lambda_2+\lambda_3+...+\lambda_n)y} e^{-\left(\frac{y}{\lambda}\right)^k} dy \tag{17}$$

## 8. Numerical Results

In this section, we present some numerical results to illustrate the reliability expression obtained in previous sections. Stress-strength reliability applications are crucial in engineering and industrial fields where the performance and durability of products and systems need to be assessed under conditions of varying stress and strength. For pressurized systems (e.g., gas pipelines, tanks, reactors), it is vital to assess both the stress (pressure) and strength (material resistance) to avoid

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rupture or leaks. This application is especially critical in industries like oil and gas, where catastrophic failure could have severe consequences. Therefore, it is important to derive more reliable expressions for reliability. To do this, it is necessary to use a distribution that accurately represents realistic data. Thus, it is essential to use a more realistic distribution to represent this data. The Weibull distribution can exhibit increasing, decreasing, bathtub, and unimodal hazard rates. It has been observed that the Weibull distribution is more flexible and can better represent realistic data. In this section, we illustrate how various choices of parameter values can influence reliability. First, we consider the reliability expression obtained in Section 3. Now, we present some numerical results to illustrate the reliability of the system using equation (7) for various choices of the parameters, namely $\lambda_1, \lambda_2, k_1$, and $k_2$ in equation (4) and (5), which are part of the integral in (7). Table 1 illustrates the results obtained in Section 3.

**Table 1:** *Reliability estimation of a system in the stress-strength model using Weibull distribution*

| $k_1$ | $k_2$ | $\lambda_1$ | $\lambda_2$ | Reliability |
|---|---|---|---|---|
| 12.30 | 0.7 | 11.6 | 1.2 | 0.9908 |
| 14.02 | 1.1 | 14.7 | 1.8 | 0.9999 |
| 11.02 | 2.2 | 12.8 | 3.5 | 0.9999 |
| 13.02 | 1.2 | 14.8 | 2.5 | 0.9995 |
| 14.80 | 0.9 | 13.8 | 1.5 | 0.9991 |
| 12.80 | 0.8 | 12.8 | 1.7 | 0.9917 |

Next, we consider the reliability expression obtained in Section 4. Now, we present some numerical results to illustrate the reliability of the system using equation (12) for various choices of the parameters, namely $\lambda, k, \lambda_1$, and $\lambda_2$ in Section 4. The Table 2 illustrates the results obtained in Section 4.

**Table 2:** *Reliability estimation of a system in the stress-strength model with two independent stresses*

| $\lambda$ | $k$ | $\lambda_1$ | $\lambda_2$ | Reliability |
|---|---|---|---|---|
| 7.2 | 6.6 | 11.96 | 1.73 | 0.9999 |
| 8.3 | 8.7 | 12.26 | 1.14 | 0.9997 |
| 7.3 | 6.7 | 11.26 | 1.24 | 0.9992 |
| 9.3 | 8.8 | 12.12 | 1.23 | 0.9999 |
| 7.7 | 8.6 | 12.22 | 1.17 | 0.9993 |
| 9.4 | 8.6 | 12.22 | 1.22 | 0.9999 |

Next, we consider the reliability expression obtained in Section 5. Now, we present some numerical results to illustrate the reliability of the system via equation (14) for various choices of the parameters, namely, $\lambda, k, \lambda_1$, $\lambda_2$, and $\lambda_3$ in equations in Section(5). We take $n = 3$. Table 3 illustrates the results obtained in Section 5.

**Table 3:** *Reliability estimation of a system in the stress-strength model when $n = 3$*

| $\lambda$ | $k$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | Reliability |
|---|---|---|---|---|---|
| 6.8 | 7.7 | 1.93 | 2.37 | 3.83 | 0.9999 |
| 5.3 | 7.8 | 2.15 | 3.24 | 4.58 | 0.9999 |
| 6.2 | 8.7 | 1.87 | 3.17 | 1.98 | 0.9999 |
| 5.8 | 7.9 | 2.15 | 1.24 | 3.58 | 0.9978 |
| 2.7 | 7.2 | 3.15 | 3.24 | 1.58 | 0.9750 |
| 4.7 | 8.2 | 4.15 | 1.23 | 2.55 | 0.9938 |

Next, we consider the reliability expression obtained in Section 6. Now, we present some numerical results to illustrate the reliability of the system via equation (15) for various choices of the parameters, namely, $\lambda_1$, $\lambda_2$, $k$, and $\lambda$ in the equation in Section 6, which are part of the integral in (15). The Table 4 illustrates the results obtained in Section 6.

**Table 4:** *Reliability estimation of a series system in the stress-strength model with two components*

| $\lambda_1$ | $\lambda_2$ | $k$ | $\lambda$ | Reliability |
|---|---|---|---|---|
| 0.22 | 0.13 | 13 | 0.01 | 0.9966 |
| 0.45 | 0.12 | 22 | 0.07 | 0.9618 |
| 0.32 | 0.53 | 15 | 0.12 | 0.9062 |
| 0.47 | 0.18 | 31 | 0.06 | 0.9624 |
| 0.51 | 0.22 | 21 | 0.07 | 0.9514 |
| 0.39 | 0.32 | 22 | 0.08 | 0.9461 |

Next, we consider the reliability expression obtained in Section 7. Now, we present some numerical results to illustrate the reliability of the system via equation (17) for various choices of the parameters, namely, $\lambda_1$, $\lambda_2$, $k$, and $\lambda$ in equation in Section 7, which are part of the integral in (17). The Table 5 illustrates the results obtained in Section 7.

**Table 5:** *Reliability estimation of a series system in the stress-strength model with three components*

| $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $k$ | $\lambda$ | Reliability |
|---|---|---|---|---|---|
| 0.11 | 0.12 | 0.20 | 29 | 0.12 | 0.9506 |
| 0.18 | 0.13 | 0.19 | 24 | 0.01 | 0.9951 |
| 0.23 | 0.17 | 0.26 | 32 | 0.11 | 0.9311 |
| 0.02 | 0.07 | 0.06 | 38 | 0.13 | 0.9810 |
| 0.01 | 0.02 | 0.03 | 38 | 0.23 | 0.9865 |
| 0.11 | 0.01 | 0.01 | 32 | 0.18 | 0.9773 |

It has been noted that selecting appropriate parameters for the Weibull distribution can result in high levels of reliability. Thus, employing the Weibull distribution can enhance the overall reliability of a system. Specifically, with proper parameter selection, the Weibull distribution can reach a reliability of 0.99, which can enhance system reliability by up to 99%. Hence by correctly utilizing the Weibull distribution, the overall reliability of the system can be enhanced. Thus, employing this distribution can significantly improve system reliability.

## 9. Comparison of results

In this section, we compare the reliability expression obtained in our work using the Weibull distribution with the existing reliability expression. Simulated data is used to estimate the reliability and compute the difference. All computations are performed using MATLAB R2020a. The study involves generating a dataset consisting of 100 observations for simplicity. This sample number can be increased based upon the rquirements. The parameters are estimated using MATLAB built in wblfit function. Suppose the random variables $X$ and $Y$ represent the strength and stress of the system respectively, with the following probability density functions for $x > 0$ and $y > 0$ are

$$f_X(x) = \frac{1}{\lambda_1} e^{-\frac{x}{\lambda_1}} \quad \text{and} \quad f_Y(y) = \frac{1}{\lambda_2} e^{-\frac{y}{\lambda_2}}$$

The reliability $R^*$ (see, [21]) is given by

$$R^* = \frac{\lambda_1}{\lambda_1 + \lambda_2}$$

We examine the difference between the reliability expression $R$ (7) and the existing standard reliability expression $R^*$ using the exponential distribution. The difference is computed and presented in Table 6.

**Table 6:** *Difference in reliability estimation of a system in the stress-strength model using the Weibull distribution and standard reliability estimation using the exponential distribution*

| $k_1$ | $k_2$ | $\lambda_1$ | $\lambda_2$ | R | $R^*$ | $R - R^*$ |
|---|---|---|---|---|---|---|
| 0.3816 | 0.7655 | 0.7952 | 0.1869 | 0.6209 | 0.1903 | 0.4306 |
| 0.9157 | 0.7922 | 0.9595 | 0.6557 | 0.5900 | 0.4060 | 0.1840 |
| 0.88852 | 0.9133 | 0.7962 | 0.0987 | 0.8649 | 0.1103 | 0.7545 |
| 0.2619 | 0.3354 | 0.6797 | 0.1366 | 0.5978 | 0.1673 | 0.4305 |
| 0.3342 | 0.6987 | 0.1978 | 0.0305 | 0.6393 | 0.1337 | 0.5056 |
| 0.7127 | 0.5005 | 0.4711 | 0.0596 | 0.7962 | 0.1123 | 0.6838 |

It is observed from Table 6 that the maximum difference in reliability is about 75%, whereas the minimum difference is about 18%. Therefore, using the Weibull distribution results in a significant difference in evaluating reliability.

## 10. Conclusions

In this study, we explored a stress-strength model where both stress and strength are described using a Weibull distribution. We began by evaluating the reliability of a single-component system within a stress-strength framework utilizing the Weibull distribution. We then expanded the analysis to consider a single-component system exposed to two independent stresses, estimating the system reliability. Additionally, we extended the model to a system subjected to $n$ independent stresses. We also derived reliability expressions for a series system with two components under common stress, where the stress is represented by a Weibull distribution. Moreover, we assessed the reliability of a series system with $n$ components, all subjected to common stress, modeled using the Weibull distribution. Our findings indicate that choosing appropriate parameters for the Weibull distribution can greatly improve reliability. Specifically, employing the Weibull distribution can enhance system reliability by as much as 75% over traditional reliability assessments. With the correct parameters, the Weibull distribution can achieve reliability levels as high as 0.99, potentially boosting reliability by up to 99%. Proper application of the Weibull distribution can significantly improve overall system reliability. The Weibull distribution's flexibility, allowing it to model increasing, decreasing, bathtub-shaped, and unimodal hazard rates, makes it a powerful tool for accurately representing real-world data. Furthermore, the results of this study can be extended to other forms of distributions, such as the modified Weibull distribution.

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