A STATISTICAL ANALYSIS OF FUZZY CONFOUNDING FACTORIAL EXPERIMENTS

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Abstract

The factorial experiment is used to test more than one factor at a time, whereas simple experiments can test only one factor at a time. The technique of reducing the block size by making one or more interaction contrast identical to block contrast is known as confounding. Only the higher-order interactions are confounded in factorial experiments, and the interaction effects have been confounded within the blocks. This article proposes that confounding design is performed with Latin squares with trapezoidal fuzzy numbers (TrFNs). In order to obtain an unbiased estimate of an error from a Latin square, the rows and columns must be rearranged in random order. Finally, we provide numerical example to clarify the discussions.

Keywords: Latin Square Design, Trapezoidal Fuzzy Number, α-cut interval method, Symmetrical Factorial Experiments, Confounding, Partial Confounding.

I. Introduction

Factorial experiments are experiments which include several factors or different sets of treatments at different levels. It is classified into two types: complete and partial confounding. Confounding the same interactions in different replications is complete confounding, and confounding different interactions for different replications is partial confounding. In certain cases, the confounding adapts Latin square designs. For instance, a 2^4factorial involves 16 combinations, which can be arranged in 8×8 Latin square. Treatment combinations can also be arranged in the type of Latin square, since the interactions of no interest are confounded with columns and rows of Latin square.

Sometimes, the day-to-day situations deal with imprecise data. In that situation, fuzzy plays a vital role. Some of the authors scrutinized to the relevant study are: Cotter, S.C., (1974) shows that which component for the sum of squares is confounded with blocks by considering the problem of m^n factorial experiment with block of size m^l. Cotter, S.C., (1975) developed the method of partial confounding which combines confounding and incomplete block design, since some are totally and some are partially confounded. Jalil, M.A., (2012) constructed a simultaneous confounding in p^n (p is prime) factorial experiment confounded with a single factorial effect by

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using a matrix method. In factorial Ahmed, L.A., (2016) compares the result among confounding and fractional replicated design by using 2^4 factorial experiment in RCBD with 4 blocks layout. By using fuzzy environments, Parthiban, S and Gajivaradhan, P (2016) proposed the study of LSD with three factor ANOVA test. Parthiban, S and Gajivaradhan, P (2016) describes the 2^2 factorial experiment using fuzzy environments and compared the result with various tests. Without replications Akra, U.P., (2017) confounded 2^5 factorial experiments for different block sizes like 2, 3 and 8 to minimize the error. When $k > 2$, Eze, F.C., (2019) considers the selection of confounding, as it depends on size, blocks and factors. Sri Devi, P. and Pachamuthu, M., (2022) constructed fuzzy complete confounding for vague data. This article proposes that confounding design is performed with Latin squares using α -cut interval method of TrFNs.

II. Preliminaries

I. Fuzzy Set

A fuzzy set $\tilde{A}$ is defined as the set of ordered pairs and it is denoted $\tilde{A} = \{x, \mu_{\tilde{A}}(x) | x \in X\}$ where, $\mu_{\tilde{A}}(x)$ - membership function of $\tilde{A}$.

II. Trapezoidal Fuzzy Number (TFN)

The TFN is defined as if a fuzzy set$\tilde{A} = (a_1, a_2, a_3, a_4)$, then its membership function is defined as;

$$\mu_{\tilde{A}}(x) = \begin{cases} 0; x < a_1 \text{or} x > a_4 \\ \frac{x - a_1}{a_2 - a_1}; a_1 < x \le a_2 \\ 1; a_2 < x \le a_3 \\ \frac{a_4 - x}{a_4 - a_3}; a_3 < x \le a_4 \end{cases} \quad (1)$$

where, $a_1 \le a_2 \le a_3 \le a_4$. A trapezoidal fuzzy number becomes triangular fuzzy number if it satisfies$a_2 = a_3$. In terms of α −cut interval, TrFN is defined as

$$\tilde{A} = [a_1 + (a_2 - a_1)\alpha, a_4 - (a_4 - a_3)\alpha]; 0 \le \alpha \le 1 (2).$$

III. Methodology

I. A Statistical analysis on Partial Confounding of 2^3Factorial Design in two 4×4 LSD with α −Cut Interval Method

The 2^3 partial confounding factorial experiment can be arranged in blocks of 4 plots in such a way that confounding treatments with 2 degrees of freedom (df). In this 4×4 Latin square, considering rows as blocks with 1 df and considering columns as blocks with 1 df. To get an adequate estimate of error, atleast 2 replicates are needed. It may be completely confounded with 2 df or partially with 4 df or partially with 2 df and completely with 1 df. The 2^3partial confounding factorial experiment is formulated in two 4×4 Latin squares and the linear model is defined as,

$$y_{ijk} = \mu + s_i + r_{ij} + c_{ik} + t_l + e_{ijkl}; i = 1,2; j = 1,2, \dots, m; k, l = 1,2, \dots, n (3)$$

Where y_{ijk} is the observation corresponding to i^{th}square, j^{th}row, k^{th}column, l^{th}treatment. If the sample observations are in the form of TrFNs, then the 2^3partial confounding factorial model is converted into interval 2^3partial confounding factorial model using relation (2), which is rewritten as,

$$\tilde{y}_{ijk} = \left[a_{ijk} + \alpha\left(b_{ijk} - a_{ijk}\right), d_{ijk} - \alpha\left(d_{ijk} - c_{ijk}\right)\right]; i = 1,2; j = 1,2,\ldots,m; k,l = 1,2,\ldots,n \quad (4)$$

Where $\tilde{y}_{ijk}$ is the observation corresponding to i^{th}square, j^{th}row, k^{th}column, l^{th}treatment in the interval observations and split this expression into lower and upper levels as,

$$L = \tilde{y}^{L}_{ijk} = a_{ijk} + \alpha\left(b_{ijk} - a_{ijk}\right) \text{ and } U = \tilde{y}^{U}_{ijk} = d_{ijk} - \alpha\left(d_{ijk} - c_{ijk}\right).$$

Hypothesis: null hypothesis $\tilde{H}_0: \tilde{\mu}_1 = \tilde{\mu}_2 = \ldots = \tilde{\mu}_n$ against alternative hypothesis $\tilde{H}_1: \tilde{\mu}_1 \neq \tilde{\mu}_2 \neq \ldots \neq \tilde{\mu}_n$. The crisp hypothesis can be turned as fuzzy hypothesis for both lower and upper levels as, $\left[\tilde{H}^{L}_0, \tilde{H}^{U}_0\right]: \left[\tilde{\mu}^{L}_1, \tilde{\mu}^{U}_1\right] = \left[\tilde{\mu}^{L}_2, \tilde{\mu}^{U}_2\right] = \ldots = \left[\tilde{\mu}^{L}_n, \tilde{\mu}^{U}_n\right]$ against $\left[\tilde{H}^{L}_1, \tilde{H}^{U}_1\right]: \left[\tilde{\mu}^{L}_1, \tilde{\mu}^{U}_1\right] \neq \left[\tilde{\mu}^{L}_2, \tilde{\mu}^{U}_2\right] \neq \ldots \neq \left[\tilde{\mu}^{L}_n, \tilde{\mu}^{U}_n\right]$. Yates' introduced a computational procedure for calculating main effects and interactions (Yates, 1937) of factorial experiment. The expression (linear contrast) for various main effects and interactions are obtained as (Sri Devi, P., 2022),

$A = [a] - [1] + [ab] - [c] - [b] - [bc] + [ac] + [abc]$; $B = [b] - [1] + [ab] + [bc] - [a] + [abc] - [ac] - [c]$; $C = [ac] + [c] + [abc] + [bc] - [1] - [b] - [a] - [ab]$; $AB = [1] - [a] - [b] + [ab] - [bc] + [abc] + [c] - [ac]$; $AC = [abc] + [ac] - [c] - [bc] + [1] - [a] - [ab] + [b]$; $BC = [ab] + [abc] - [ac] - [c] - [b] + [bc] - [a] - [1]$; $ABC = [a] - [ac] - [bc] + [c] + [b] - [ab] + [abc] - [1]$.

Table 1: *ANOVA table for 2^3 partial confounding factorial design in two 4×4 Latin squares of L.L.M*

Sources of Variation	Degrees of Freedom	Sum of Squares	Mean Sum of Squares	F-Ratio
Square	$(n-7)$	SS^{L}_{Squ}	$M^{L}_{Squ} = \frac{SS^{L}_{Squ}}{(n-7)}$	$F^{L}_{Squ} = \frac{M^{L}_{Squ}}{M^{L}_{E}}$
Row	$(n-2)$	SS^{L}_{Row}	$M^{L}_{Row} = \frac{SS^{L}_{Row}}{(n-2)}$	$F^{L}_{Row} = \frac{M^{L}_{Row}}{M^{L}_{E}}$
Column	$(n-2)$	SS^{L}_{Col}	$M^{L}_{Col} = \frac{SS^{L}_{Col}}{(n-2)}$	$F^{L}_{Col} = \frac{M^{L}_{Col}}{M^{L}_{E}}$
Treatment	$(n-3)$	SS^{L}_{Treat}	$M^{L}_{Treat} = \frac{SS^{L}_{Treat}}{(n-3)}$	$F^{L}_{Treat} = \frac{M^{L}_{Treat}}{M^{L}_{E}}$
Main Effect$A$	1	SS^{L}_{A}	$M^{L}_{A} = \frac{SS^{L}_{A}}{(n-3)}$	$F^{L}_{A} = \frac{M^{L}_{A}}{M^{L}_{E}}$
Main Effect$B$	1	SS^{L}_{B}	$M^{L}_{B} = \frac{SS^{L}_{B}}{(n-3)}$	$F^{L}_{B} = \frac{M^{L}_{B}}{M^{L}_{E}}$
Main Effect$C$	1	SS^{L}_{C}	$M^{L}_{C} = \frac{SS^{L}_{C}}{(n-3)}$	$F^{L}_{C} = \frac{M^{L}_{C}}{M^{L}_{E}}$
Interaction Effect AB	1	SS^{L}_{AB}	$M^{L}_{AB} = \frac{SS^{L}_{AB}}{(n-3)}$	$F^{L}_{AB} = \frac{M^{L}_{AB}}{M^{L}_{E}}$
Interaction Effect AC	1	SS^{L}_{AC}	$M^{L}_{AC} = \frac{SS^{L}_{AC}}{(n-3)}$	$F^{L}_{AC} = \frac{M^{L}_{AC}}{M^{L}_{E}}$
Interaction Effect BC	1	SS^{L}_{BC}	$M^{L}_{BC} = \frac{SS^{L}_{BC}}{(n-3)}$	$F^{L}_{BC} = \frac{M^{L}_{BC}}{M^{L}_{E}}$
Interaction Effect ABC	1	SS^{L}_{ABC}	$M^{L}_{ABC} = \frac{SS^{L}_{ABC}}{(n-3)}$	$F^{L}_{ABC} = \frac{M^{L}_{ABC}}{M^{L}_{E}}$
Error	$(nm-4n+13)$	SS^{L}_{E}	$M^{L}_{E} = \frac{SS^{L}_{E}}{(nm-4n+13)}$	-
Total	$(nm-1)$	SS^{L}_{T}	-	-

The sum of squares for squares, rows, columns, treatments, main effects and interactions are calculated as follows,

Lower Level Model (L.L.M): Let $\tilde{x}^L_{i..} = \sum_{i=1}^{2}[a_{i..} + \alpha(b_{i..} - a_{i..})] = S^L_i$; $\tilde{x}^L_{ij.} = \sum_{i=1}^{2}\sum_{k=1}^{n}[a_{ij.} + \alpha(b_{ij.} - a_{ij.})] = \sum_{i=1}^{2}\sum_{k=1}^{m}[a_{ij.} + \alpha(b_{ij.} - a_{ij.})] + \sum_{i=1}^{2}\sum_{k=m+1}^{n}[a_{ij.} + \alpha(b_{ij.} - a_{ij.})]$. R^L_{ij}; $\bar{x}^L_{i.k} = \sum_{i=1}^{2}\sum_{j=1}^{m}\left[a_{i.k} + \alpha\left(b_{i.k} - a_{i.k}\right)\right] = C^L_{ik}$; $\bar{x}^L_{..k}$ =sum of all $[a_{ijk} + \alpha(b_{ijk} - a_{ijk})]$ receiving l^{th} treatment $= T^L_l$ (since, it is the square of unconfounded effects).

The sum of squares are calculated as follows: $SS^L_{Squ} = \frac{\sum_{i=1}^{2}(S^L_i)^2}{m^2} - \frac{(G^L)^2}{N}$ with $(n-7)$ df; $SS^L_{Row} = \frac{\sum_{i=1}^{2}\sum_{j=1}^{m}[R^L_{ij}]^2}{m} - \frac{[G^L]^2}{N}$ with $(n-2)$ df; $SS^L_{Col} = \frac{\sum_{i=1}^{2}\sum_{k=1}^{n}[C^L_{ik}]^2}{m} - \frac{[G^L]^2}{N}$ with $(n-2)$ df; $SS^L_{Treat} = \frac{\sum_{l=1}^{n}[T^L_l]^2}{N}$ with $(n-3)$ df; $SS^L_T = \sum_{i=1}^{2}\sum_{j=1}^{m}\sum_{k=1}^{n}(x^L_{ijk})^2 - \frac{[G^L]^2}{N}$ with $(nm-1)$ df; $SS^L_A = \frac{[A^2]^L}{N}$; $SS^L_B = \frac{[B^2]^L}{N}$; $SS^L_C = \frac{[C^2]^L}{N}$; $SS^L_{AB} = \frac{[(AB)^2]^L}{N}$; $SS^L_{AC} = \frac{[(AC)^2]^L}{N}$; $SS^L_{BC} = \frac{[(BC)^2]^L}{N}$ and $SS^L_{ABC} = \frac{[(ABC)^2]^L}{N}$ with 1 df. $SS^L_E = SS^L_T - [SS^L_{Square} + SS^L_{Row} + SS^L_{Col} + SS^L_T + SS^L_A + SS^L_B + SS^L_C + SS^L_{Interactions}]$ with $(nm - 4n + 13)$df; where, $N = mn$, $G^L = \sum_{i=1}^{2}\sum_{j=1}^{m}\sum_{k=1}^{n} x^L_{ijk}$, $SS^L_{Interactions} = SS^L_{AB} + SS^L_{AC} + SS^L_{BC} + SS^L_{ABC}$ (omit the effects which is confounded). All these calculated values are presented in the table 1.

Table 2: *ANOVA table for 2^3 partial confounding factorial design in two 4×4 Latin squares of U.L.M*

Sources of Variation	Degrees of Freedom	Sum of Squares	Mean Sum of Squares	F-Ratio
Square	$(n-7)$	SS^U_{Squ}	$M^U_{Squ} = \frac{SS^U_{Squ}}{(n-7)}$	$F^U_{Squ} = \frac{M^U_{Squ}}{M^U_E}$
Row	$(n-2)$	SS^U_{Row}	$M^U_{Row} = \frac{SS^U_{Row}}{(n-2)}$	$F^U_{Row} = \frac{M^U_{Row}}{M^U_E}$
Column	$(n-2)$	SS^U_{Col}	$M^U_{Col} = \frac{SS^U_{Col}}{(n-2)}$	$F^U_{Col} = \frac{M^U_{Col}}{M^U_E}$
Treatment	$(n-3)$	SS^U_{Treat}	$M^U_{Treat} = \frac{SS^U_{Treat}}{(n-3)}$	$F^U_{Treat} = \frac{M^U_{Treat}}{M^U_E}$
Main Effect A	1	SS^U_A	$M^U_A = \frac{SS^U_A}{(n-3)}$	$F^U_A = \frac{M^U_A}{M^U_E}$
Main Effect B	1	SS^U_B	$M^U_B = \frac{SS^U_B}{(n-3)}$	$F^U_B = \frac{M^U_B}{M^U_E}$
Main Effect C	1	SS^U_C	$M^U_C = \frac{SS^U_C}{(n-3)}$	$F^U_C = \frac{M^U_C}{M^U_E}$
Interaction Effect AB	1	SS^U_{AB}	$M^U_{AB} = \frac{SS^U_{AB}}{(n-3)}$	$F^U_{AB} = \frac{M^U_{AB}}{M^U_E}$
Interaction Effect AC	1	SS^U_{AC}	$M^U_{AC} = \frac{SS^U_{AC}}{(n-3)}$	$F^U_{AC} = \frac{M^U_{AC}}{M^U_E}$
Interaction Effect BC	1	SS^U_{BC}	$M^U_{BC} = \frac{SS^U_{BC}}{(n-3)}$	$F^U_{BC} = \frac{M^U_{BC}}{M^U_E}$
Interaction Effect ABC	1	SS^U_{ABC}	$M^U_{ABC} = \frac{SS^U_{ABC}}{(n-3)}$	$F^U_{ABC} = \frac{M^U_{ABC}}{M^U_E}$
Error	$(nm-4n+13)$	SS^U_E	$M^U_E = \frac{SS^U_E}{(nm-4n+13)}$	-
Total	$(nm-1)$	SS^U_T	-	-

The table 2 represents are Upper Level Model (U.L.M): Let $\tilde{x}^{U}_{i..} = \sum_{i=1}^{2}[d_{i..} - \alpha(d_{i..} - c_{i..})] = S^{U}_{i}$; $\bar{x}^{U}_{i..} = \sum_{i=1}^{2}\sum_{k=1}^{n}[d_{ij.} - \alpha(d_{ij.} - c_{ij.})] = \sum_{i=1}^{2}\sum_{k=1}^{m}[d_{i..} - \alpha(d_{i..} - c_{i..})] + \sum_{i=1}^{2}\sum_{k=m+1}^{n}[d_{i..} - \alpha(d_{i..} - c_{i..})] = R^{U}_{ij}$; $\bar{x}^{U}_{i.k} = \sum_{i=1}^{2}\sum_{j=1}^{m}\left[d_{i.k} - \alpha\left(d_{i.k} - c_{i.k}\right)\right] = C^{U}_{ik}$; $\bar{x}^{U}_{..k}$ =sum of all $[d_{ijk} - \alpha(d_{ijk} - c_{ijk})]$ receiving l^{th} treatment $= T^{U}_{l}$ (since, it is the square of unconfounded effects).

The sum of squares are calculated as follows: $SS^{U}_{Squ} = \frac{\sum_{i=1}^{2}(S^{U}_{i})^{2}}{m^{2}} - \frac{(G^{U})^{2}}{N}$ with $(n-7)$ df; $SS^{U}_{Row} = \frac{\sum_{i=1}^{2}\sum_{j=1}^{m}[R^{U}_{i}]^{2}}{m} - \frac{[G^{U}]^{2}}{N}$ with $(n-2)$ df; $SS^{U}_{Col} = \frac{\sum_{i=1}^{2}\sum_{k=1}^{n}[C^{U}_{ik}]^{2}}{m} - \frac{[G^{U}]^{2}}{N}$ with $(n-2)$ df; $SS^{U}_{Treat} = \frac{\sum_{l=1}^{n}[T^{2}_{l}]^{U}}{N}$ with $(n-3)$ df; $SS^{U}_{T} = \sum_{i=1}^{2}\sum_{j=1}^{m}\sum_{k=1}^{n}(x^{U}_{ijk})^{2} - \frac{[G^{U}]^{2}}{N}$ with $(nm-1)$ df; $SS^{U}_{A} = \frac{[A^{2}]^{U}}{N}$; $SS^{U}_{B} = \frac{[B^{2}]^{U}}{N}$; $SS^{U}_{C} = \frac{[C^{2}]^{U}}{N}$; $SS^{U}_{AB} = \frac{[(AB)^{2}]^{U}}{N}$; $SS^{U}_{AC} = \frac{[(AC)^{2}]^{U}}{N}$; $SS^{U}_{BC} = \frac{[(BC)^{2}]^{U}}{N}$ and $SS^{U}_{ABC} = \frac{[(ABC)^{2}]^{U}}{N}$ with 1 df; $SS^{U}_{E} = SS^{U}_{T} - [SS^{U}_{Square} + SS^{U}_{Row} + SS^{U}_{Col} + SS^{U}_{T} + SS^{U}_{A} + SS^{U}_{B} + SS^{U}_{C} + SS^{U}_{Interactions}]$ with $(nm - 4n + 13)$df; where, $N = mn$, $G^{U} = \sum_{i=1}^{2}\sum_{j=1}^{m}\sum_{k=1}^{n} x^{U}_{ijk}$, $SS^{U}_{Interactions} = SS^{U}_{AB} + SS^{U}_{AC} + SS^{U}_{BC} + SS^{U}_{ABC}$ (omit the effects which is confounded). All these values are presented in the ANOVA table (table 2).

Decision Rule: Lower-Level Model: If $F^{L}_{(CalculatedValue)} < F_{t}$, then H^{L}_{0} is accepted. Otherwise, it is rejected. That is, there is no significant difference between treatments. Similarly, rows and columns can also be tested.

Upper-Level Model: If $F^{U}_{(CalculatedValue)} < F_{t}$, then H^{U}_{0} is accepted. Otherwise, it is rejected. That is, there is no significant difference between treatments. Similarly, rows and columns can also be tested.

Table 3: *Conclusion Table*

Null Hypothesis $\tilde{H}_0$		
L.L.M	U.L.M	Conclusion
$H^{L}_{0}$is accepted	H^{U}_{0} is accepted	$\tilde{H}_0$ is accepted
$H^{L}_{0}$is rejected	H^{U}_{0} is accepted	$\tilde{H}_0$ is rejected
H^{L}_{0} is accepted	H^{U}_{0} is rejected	$\tilde{H}_0$ is rejected
H^{L}_{0} is rejected	H^{U}_{0} is rejected	$\tilde{H}_0$ is accepted

IV. Applications

Example: The data given in the table 4 provides the barley grains yield. Three fertilizers used for cultivation are Urea (A), SSP (B) and Muriate of Potash (C). The observations are in the form trapezoidal fuzzy numbers, which are arranged in a superimposed Latinsquares. The problem is to examine whether the treatments differ significantly or not?

Table 4: *The yield of barley grain cultivation*

b (45,47,52, 54)	*a* (58,62,65, 68)	*c* (86,89,93, 97)	*abc* (55,58,61, 63)	*c* (49,52,53, 56)	*ac* (75,79,81, 82)	*ab* (67,69,72, 74)	*b* (75,76,79, 82)
ab (54,57,59, 61)	*bc* (73,77,79, 82)	*ac* (48,51,53, 54)	(1) (65,68,70, 73)	*a* (57,59,61, 63)	*bc* (92,94,95, 96)	(1) (86,89,93, 95)	*abc* (63,65,67, 69)
ac (75,77,79, 81)	(1) (94,95,96, 98)	*ab* (57,59,62, 64)	*bc* (63,67,69, 72)	*abc* (49,52,53, 54)	(1) (77,81,82, 85)	*bc* (46,48,51, 52)	*a* (84,85,88, 90)
c (91,94,95)	*abc* (43,45,47)	*b* (73,75,77)	*a* (67,69,71)	*b* (73,75,79)	*ab* (58,62,65)	*ac* (63,66,67)	*c* (46,49,52)

The conclusion table (Table 3) shows the overall result. For both L.L.M and U.L.M, the partial acceptance of $\widetilde{H}_0$ at the intersection of certain level can be taken as acceptance of $\widetilde{H}_0$.

Null Hypothesis $\widetilde{H}_0$: There is no significant difference between squares, rows, columns and treatments respectively. The converting the trapezoidal fuzzy observations into interval data as shown in table 5.

Lower-Level Model (L.L.M): There is no significant difference between squares, rows, columns and treatments respectively.

Null HypothesisH_0^L: The sum of squares of squares, row, column, treatments, main effects and interactions for L.L.M are calculated as follows: $SS_A^L = \frac{1}{32}(25921)$; $SS_B^L = \frac{1}{32}(18225 + 1080\alpha + 16\alpha^2)$; $SS_C^L = \frac{1}{32}(5329 - 1460\alpha + 100\alpha^2)$; $SS_{AB}^L = \frac{1}{32}(729 - 324\alpha + 36\alpha^2)$; $SS_{AC}^L = \frac{1}{32}(121 + 88\alpha + 16\alpha^2)$; $SS_{Square}^L = \frac{1}{32}(169)$; $SS_{Row}^L = \frac{1}{16}(13907 - 1850\alpha + 70\alpha^2)$; $SS_{Col}^L = \frac{1}{16}(11795 + 382\alpha - 66\alpha^2)$; $SS_{Treat} = \frac{1}{32}(50325 - 616\alpha + 168\alpha^2)$; $SS_T^L = \frac{1}{32}(223239 - 6628\alpha + 924\alpha^2)$; $SS_{Error}^L = \frac{1}{32}(121341 - 3076\alpha + 484\alpha^2)$

Between Squares: $F_{Square}^L = \frac{13(169)}{(121341-3076\alpha+484\alpha^2)} < 1; 0 \le \alpha \le 1$. Here, $F(1,13) = 3.14$and $F_{Square}^L < F(1,13)$.

Between Rows: $F_{Row}^L = \frac{13(13907-1850\alpha+70\alpha^2)}{3(121341-3076\alpha+484\alpha^2)} < 1; 0 \le \alpha \le 1$. Here, $F(6,13) = 2.28$ and $F_{Row}^L < F(6,13)$.

Table 5: *Converting Trapezoidal Observations to Interval Observations*

b (45 + 2α, 54 − 2α)	a (58 + 4α, 68 − 3α)	c (86 + 3α, 97 − 3α)	abc (55 + 3α, 63 − 2α)	c (49 + 4α, 56 − 3α)	ac (75 + 4α, 82 − α)	ab (67 + 2α, 74 − 2α)	b (75 + α, 82 − 3α)
ab (54 + 3α, 61 − 2α)	bc (73 + 4α, 82 − 3α)	ac (48 + 3α, 54 − α)	(1) (65 + 3α, 73 − 3α)	a (57 + 2α, 63 − 2α)	bc (92 + 2α, 96 − α)	(1) (86 + 3α, 95 − 2α)	abc (63 + 2α, 69 − 2α)
ac (75 + 2α, 81 − 2α)	(1) (94 + α, 98 − 2α)	ab (57 + 2α, 64 − 2α)	bc (63 + 4α, 72 − 3α)	abc (49 + 4α, 54 − α)	(1) (77 + 4α, 85 − 3α)	bc (46 + 2α, 52 − α)	a (84 + α, 90 − 2α)
c (91 + 3α, 97 − 2α)	abc (43 + 2α, 49 − 2α)	b (73 + 2α, 79 − 2α)	a (67 + 2α, 74 − 3α)	b (73 + 2α, 81 − 3α)	ab (58 + 4α, 67 − 2α)	ac (63 + 3α, 69 − 2α)	c (46 + 3α, 54 − 2α)

Between Columns: $F_{Col}^L = \frac{13(11795+382\alpha-66\alpha^2)}{3(121341-3076\alpha+484\alpha^2)} < 1; 0 \le \alpha \le 1$. Here, $F(6,13) = 2.28$ and $F_{Col}^L < F(6,13)$.

Between Treatments: $F_{Treat}^L = \frac{13(50325-616\alpha+168\alpha^2)}{5(121341-3076\alpha+484\alpha^2)} > 1; 0 \le \alpha \le 1$. Here, $F(5,13) = 2.35$ and $F_{Treat}^L < F(5,13)$.

Main Effect A: $F_A^L = \frac{13(25921)}{(121341-3076\alpha+484\alpha^2)}$; Here, $F(1,13) = 3.14$ and $F_A^L < F(1,13)$.

Main Effect B: $F_B^L = \frac{13(18225+1080\alpha+16\alpha^2)}{(121341-3076\alpha+484\alpha^2)}$; Here, $F(1,13) = 3.14$ and $F_B^L < F(1,13)$.

Main Effect C: $F_C^L = \frac{13(5329-1460\alpha+100\alpha^2)}{(121341-3076\alpha+484\alpha^2)}$; Here, $F(1,13) = 3.14$ and $F_C^L < F(1,13)$.

Interaction Effect AB: $F_{AB}^L = \frac{13(729-324\alpha+36\alpha^2)}{(121341-3076\alpha+484\alpha^2)}$; Here, $F(1,13) = 3.14$ and $F_{AB}^L < F(1,13)$.

Interaction Effect AC: $F_{AC}^L = \frac{13(121+88\alpha+16\alpha^2)}{(121341-3076\alpha+484\alpha^2)}$; Here, $F(1,13) = 3.14$ and $F_{AC}^L < F(1,13)$.

Table 4.3 indicate that a Inference: There is no significant difference between squares, rows, columns and treatments respectively.

Upper-Level Model (U.L.M): Null Hypothesis H_0^U: There is no significant difference between squares, rows, columns and treatments respectively.

The sum of squares of squares, row, column, treatments, main effects and interactions for U.L.M are calculated as follows

$SS_A^U = \frac{1}{32}(29241 - 2394\alpha + 49\alpha^2)$; $SS_B^U = \frac{1}{32}(18769 - 3288\alpha + 144\alpha^2)$; $SS_C^U = \frac{1}{32}(6561 -$

$1134\alpha + 49\alpha^2)$; $SS^U_{AB} = \frac{1}{32}(529 + 46\alpha + \alpha^2)$; $SS^U_{AC} = \frac{1}{32}(1 - 34\alpha + 289\alpha^2)$; $SS^U_{Square} = \frac{1}{32}(9 + 30\alpha + 25\alpha^2)$; $SS^U_{Row} = \frac{1}{16}(10751 + 388\alpha + 19\alpha^2)$; $SS^U_{Col} = \frac{1}{16}(12119 + 708\alpha + 43\alpha^2)$; $SS^U_{Treat} = \frac{1}{32}(55101 - 6804\alpha + 532\alpha^2)$; $SS^U_T = \frac{1}{32}(220831 - 4106\alpha + 455\alpha^2)$; $SS^U_{Error} = \frac{1}{32}(119981 + 476\alpha - 226\alpha^2)$

Table: 4.3 *Three factor ANOVA table for lower-level model*

S.V.	DF	S.S.	M.S.S.	F-Ratio
Squares	1	$\frac{1}{32}(169)$	$\frac{1}{32}(169)$	$\frac{13(169)}{(121341 - 3076\alpha + 484\alpha^2)} < 1$; $0 \le \alpha \le 1$.
Row	6	$\frac{1}{16}(13907 - 1850\alpha + 70\alpha^2)$	$\frac{1}{16}(13907 - 1850\alpha + 70\alpha^2)$	$\frac{13(13907 - 1850\alpha + 70\alpha^2)}{3(121341 - 3076\alpha + 484\alpha^2)} < 1$; $0 \le \alpha \le 1$.
Column	6	$\frac{1}{16}(11795 + 382\alpha - 66\alpha^2)$	$\frac{1}{16}(11795 + 382\alpha - 66\alpha^2)$	$\frac{13(11795 + 382\alpha - 66\alpha^2)}{3(121341 - 3076\alpha + 484\alpha^2)} < 1$; $0 \le \alpha \le 1$.
Treatment	5	$\frac{1}{32}(50325 - 616\alpha + 168\alpha^2)$	$\frac{1}{32}(50325 - 616\alpha + 168\alpha^2)$	$\frac{13(50325 - 616\alpha + 168\alpha^2)}{5(121341 - 3076\alpha + 484\alpha^2)}$; $0 \le \alpha \le 1$.
Main Effect A	1	$\frac{1}{32}(25921)$	$\frac{1}{32}(25921)$	$\frac{13(25921)}{(121341 - 3076\alpha + 484\alpha^2)}$; $0 \le \alpha \le 1$
Main Effect B	1	$\frac{1}{32}(18225 + 1080\alpha + 16\alpha^2)$	$\frac{1}{32}(18225 + 1080\alpha + 16\alpha^2)$	$\frac{13(18225 + 1080\alpha + 16\alpha^2)}{(121341 - 3076\alpha + 484\alpha^2)}$; $0 \le \alpha \le 1$
Main Effect C	1	$\frac{1}{32}(5329 - 1460\alpha + 100\alpha^2)$	$\frac{1}{32}(5329 - 1460\alpha + 100\alpha^2)$	$\frac{13(5329 - 1460\alpha + 100\alpha^2)}{(121341 - 3076\alpha + 484\alpha^2)} < 1$; $0 \le \alpha \le 1$.
Interaction Effect AB	1	$\frac{1}{32}(729 - 324\alpha + 36\alpha^2)$	$\frac{1}{32}(729 - 324\alpha + 36\alpha^2)$	$\frac{13(729 - 324\alpha + 36\alpha^2)}{(121341 - 3076\alpha + 484\alpha^2)} < 1$; $0 \le \alpha \le 1$
Interaction Effect AC	1	$\frac{1}{32}(121 + 88\alpha + 16\alpha^2)$	$\frac{1}{32}(121 + 88\alpha + 16\alpha^2)$	$\frac{13(121 + 88\alpha + 16\alpha^2)}{(121341 - 3076\alpha + 484\alpha^2)} < 1$; $0 \le \alpha \le 1$
Error	13	$\frac{1}{32}(121341 - 3076\alpha + 484\alpha^2)$	$\frac{1}{13(32)}(121341 - 3076\alpha + 484\alpha^2)$	-
Total	31	$\frac{1}{32}(223239 - 6628\alpha + 924\alpha^2)$	-	-

Between Squares: $F^U_{Square} = \frac{13(9+30\alpha+25\alpha^2)}{(119981+476\alpha-226\alpha^2)} < 1; 0 \le \alpha \le 1$. Here, $F(1,13) = 3.14$ and $F^U_{Square} < F(1,13)$.

Between Rows: $F^U_{Row} = \frac{13(10751+388\alpha+19\alpha^2)}{3(119981+476\alpha-226\alpha^2)} < 1; 0 \leq \alpha \leq 1$. Here, $F(6,13) = 2.28$ and $F^U_{Row} < F(6,13)$.

Between Columns: $F^U_{Col} = \frac{13(12119+708\alpha+43\alpha^2)}{3(119981+478\alpha-226\alpha^2)} < 1; 0 \leq \alpha \leq 1$. Here, $F(6,13) = 2.28$ and $F^U_{Col} < F(6,13)$.

Between Treatments: $F^U_{Treat} = \frac{13(55101-6804\alpha+532\alpha^2)}{5(119981+476\alpha-226\alpha^2)} > 1; 0 \leq \alpha \leq 1$. Here, $F(5,13) = 2.35$ and $F^U_{Treat} < F(5,13)$.

Main Effect A: $F^U_A = \frac{13(29241-2394\alpha+49\alpha^2)}{11981+476\alpha-226\alpha^2}$; Here, $F(1,13) = 3.14$ and $F^U_A > F(1,13)$.

Main Effect B: $F^U_B = \frac{13(18769-3288\alpha+144\alpha^2)}{11981+476\alpha-226\alpha^2}$; Here, $F(1,13) = 3.14$ and $F^U_B > F(1,13)$.

Main Effect C: $F^U_C = \frac{13(6561-1134\alpha+49\alpha^2)}{11981+476\alpha-226\alpha^2}$; Here, $F(1,13) = 3.14$ and $F^U_C > F(1,13)$.

Interaction Effect AB: $F^U_{AB} = \frac{13(529+46\alpha+\alpha^2)}{11981+476\alpha-226\alpha^2} < 1; 0 \leq \alpha \leq 1$. Here, $F(1,13) = 3.14$ and $F^U_{AB} < F(1,13)$.

Interaction Effect AC: $F^U_{AC} = \frac{13(1-34\alpha+289\alpha^2)}{11981+476\alpha-226\alpha^2} < 1; 0 \leq \alpha \leq 1$. Here, $F(1,13) = 3.14$ and $F^U_{AC} < F(1,13)$.

Table: 4.4 *Three factor ANOVA table for upper-level model*

S.V.	D.F.	S.S.	M.S.S.	F-Ratio
Squares	1	$\frac{1}{32}(9+30\alpha+25\alpha^2)$	$\frac{1}{32}(9+30\alpha+25\alpha^2)$	$\frac{13(9+30\alpha+25\alpha^2)}{(119981+476\alpha-226\alpha^2)} < 1; 0 \leq \alpha \leq 1.$
Row	6	$\frac{1}{16}(10751+388\alpha+19\alpha^2)$	$\frac{1}{6(16)}(10751+388\alpha+19\alpha^2)$	$\frac{13(10751+388\alpha+19\alpha^2)}{3(119981+476\alpha-226\alpha^2)} < 1; 0 \leq \alpha \leq 1.$
Column	6	$\frac{1}{16}(12119+708\alpha+43\alpha^2)$	$\frac{1}{6(16)}(12119+708\alpha+43\alpha^2)$	$\frac{13(12119+708\alpha+43\alpha^2)}{3(119981+478\alpha-226\alpha^2)} < 1; 0 \leq \alpha \leq 1.$
Treatment	5	$\frac{1}{32}(55101-6804\alpha+532\alpha^2)$	$\frac{1}{5(32)}(55101-6804\alpha+532\alpha^2)$	$\frac{13(55101-6804\alpha+532\alpha^2)}{5(119981+476\alpha-226\alpha^2)} > 1; 0 \leq \alpha \leq 1.$
Main Effect A	1	$\frac{1}{32}(29241-2394\alpha+49\alpha^2)$	$\frac{1}{32}(29241-2394\alpha+49\alpha^2)$	$\frac{13(29241-2394\alpha+49\alpha^2)}{11981+476\alpha-226\alpha^2}$
Main Effect B	1	$\frac{1}{32}(18769-3288\alpha+144\alpha^2)$	$\frac{1}{32}(18769-3288\alpha+144\alpha^2)$	$\frac{13(18769-3288\alpha+144\alpha^2)}{11981+476\alpha-226\alpha^2}$
Main Effect C	1	$\frac{1}{32}(6561-1134\alpha+49\alpha^2)$	$\frac{1}{32}(6561-1134\alpha+49\alpha^2)$	$\frac{13(6561-1134\alpha+49\alpha^2)}{11981+476\alpha-226\alpha^2}$
Interaction Effect AB	1	$\frac{1}{32}(529+46\alpha+\alpha^2)$	$\frac{1}{32}(529+46\alpha+\alpha^2)$	$\frac{13(529+46\alpha+\alpha^2)}{11981+476\alpha-226\alpha^2} < 1; 0 \leq \alpha \leq 1$
Interaction Effect AC	1	$\frac{1}{32}(1-34\alpha+289\alpha^2)$	$\frac{1}{32}(1-34\alpha+289\alpha^2)$	$\frac{13(1-34\alpha+289\alpha^2)}{11981+476\alpha-226\alpha^2} < 1; 0 \leq \alpha \leq 1$
Error	13	$\frac{1}{32}(119981+476\alpha-226\alpha^2)$	$\frac{1}{13(32)}(119981+476\alpha-226\alpha^2)$	-
Total	31	$\frac{1}{32}(220831-4106\alpha+455\alpha^2)$	-	-

Inference: There is no significant difference between squares, rows, columns and treatments except main effectsA, B, C respectively.

Finally, it is concluded that there is no significant difference between the fertilizers Urea (A), SSP (B) and Muriate of Potash (C).

V. Conclusion

In this article, the new methodology has been constructed for the fuzzy partial confounding factorial experiment. This can be widely used when the observations are vague. This work can further be to fractional factorial designs, asymmetrical factorial experiments and so on.

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