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ANALYSIS OF A SINGLE-SERVER QUEUE WITH ENCOURAGED ARRIVALS, IMPATIENT CUSTOMERS AND DYNAMIC SERVICE SWITCHING

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Abstract

This study examines a single-server queueing system that features encouraged arrivals, customer impatience and a self-switching service mechanism. In this model, the arrival rate increases with the number of customers present in the system, specifically modeled as $\lambda(1+\eta)$, where λ is a positive constant and η represents the current number of customers. The server initially operates at a service rate μ_1 as long as the system size is below a predefined threshold K. Once the number of customers reaches K, the server switches to a faster service rate μ_2. Customers become impatient after entering the queue and may abandon the system at a rate γ. Still, this impatience applies only while the server is operating at the slower service rate μ_1. No reneging occurs once the server switches to the faster rate μ_2. The steady-state probability distribution of the system is derived and performance metrics such as the expected waiting time and the average number of customers in the system are evaluated. Numerical illustrations are also provided to demonstrate the impact of encouraged arrivals and server switching behavior.

Keywords: Encouraged arrival, Customer impatience, Server switch random, Single server queue

1. Introduction

Queueing models play a vital role in various real-world applications such as telecommunications, traffic control, healthcare, banking, business operations, post offices and amusement parks. These models are essential tools for managing congestion and enhancing service quality. In particular, queueing systems involving encouraged arrivals and impatient customers are highly relevant in practical scenarios. Encouraged arrivals refer to situations where potential customers are more likely to enter the queue based on certain system conditions, such as shorter queue lengths or the perception of faster service. Unlike discouraged arrivals, encouraged arrivals increase when customers perceive the system to be efficient or less congested. These behaviors may arise due to visible queue size, shorter expected wait times, or prior experiences. Modeling encouraged arrivals is important because they influence arrival patterns and can significantly impact the accuracy and effectiveness of queueing system analysis.
Impatient customers, on the other hand, are those unwilling to wait for extended periods and may leave the queue before receiving service. Impatience may be triggered by frustration, urgency, uncertainty about waiting times, or other personal constraints. These customers are typically modeled as leaving the system if not served within a certain time threshold. Including customer impatience in queueing models is crucial, as it directly affects performance metrics such as system throughput, average waiting time and service levels.
Another important component of modern service systems is the self-switching server, which has the ability to change its service rate based on the system state. For instance, a server may

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switch to a faster service rate when the number of customers exceeds a certain threshold, thus helping to control congestion and reduce wait times. This dynamic adjustment improves system responsiveness and overall efficiency.

Building upon these concepts, we propose a Markovian single-server queueing model that integrates encouraged arrivals, customer impatience and a self-switching server mechanism. In this model, the arrival rate is positively influenced by the number of customers in the system, while impatience leads customers to abandon the queue if service is delayed. The server switches between service rates depending on queue length thresholds. The steady-state probability distribution of the system is derived and performance measures are evaluated to analyze system behavior.

The following describes the way the article is structured: Section 2 provides the review as the literature. Section 3 generates the queueing model. Section 4 presents the governing equations of the model. In Section 5, special examples for the queuing models are defined and the governing equations are solved constantly to obtain the stationary probability. Section 6 derives measures of effectiveness. The seventh part is a list of the numerical illustrations. The final portion contains the conclusion.

2. Literature review

The concept of encouraged arrivals has gained increasing attention in queueing theory as researchers strive to model customer behaviors that deviate from classical assumptions. Traditional models typically assume a constant arrival rate, often based on a Poisson process, where customers arrive independently of the system state. However, this assumption fails to capture situations where the arrival rate is influenced by the system's perceived efficiency or service conditions. Encouraged arrival behavior, where the arrival rate increases as the queue becomes more appealing or as waiting times decrease, was introduced to address this gap. Several studies have explored variations of arrival behavior depending on the system state. Notably, models incorporating state-dependent arrival rates have been developed, where the arrival rate is a function of the number of customers in the system. In contrast to discouraged arrivals or balking, where customers avoid long queues, encouraged arrivals occur when customers are more inclined to join shorter queues or when the server is operating more efficiently.

The valuable insights into how customer impatience affects system performance is provided in [1] . Its mathematical rigor and practical relevance make it an important work in the field of queueing theory, especially for industries where customer abandonment plays a significant role in service quality and efficiency. The queueing systems where customer arrivals are affected by external factors are studied in [2]. An $M/M/1/N$ queueing system with encouraged arrivals, feedback, balking and retained reneged customers, focusing on minimizing waiting time and improving system performance is studied in [3]. These studies collectively enhance the understanding of reverse balking phenomena across various queueing models, offering valuable frameworks for optimizing service efficiency and customer satisfaction in various applications. A comprehensive stochastic model addressing complex customer behaviors in queueing systems and developed a multi-server, finite capacity Markovian queueing model that incorporates encouraged arrivals, reverse reneging, feedback customers and retention strategies for reneged customers is considered in [4]. An analysis of M/M/1 queueing system with Coxian-2 service, encouraged arrivals and balking behavior, incorporating server vacations and an economic cost model is presented in [11]. Their study extends existing literature by integrating behavioral arrival patterns with phase-type service and cost optimization to evaluate system performance. A server that operates in distinct phases and customers may leave the queue if their impatience threshold is exceeded, representing a more realistic customer behavior in many service systems is introduced in [5]. It investigates how customer impatience and the server's differentiated phases affect the key performance metrics such as waiting times, queue lengths and system utilization. The balking that occurs when customers decide not to join the queue if it appears too lengthy, reneging takes place when customers leave the queue because of extended waiting periods and the addition of servers helps reduce system delay is provided in [6]. The performance analysis of queueing systems affected by scheduled maintenance and temporary server shutdowns is studied in [7] . Using Markovian processes, matrix-analytic techniques and Laplace-Stieltjes transforms, the authors develop a probabilistic model to evaluate key performance metrics, including queue length, waiting time and server utilization. The study highlights the trade-offs between system availability and maintenance scheduling, emphasizing that while maintenance minimizes unexpected failures, excessive downtime can lead to inefficiencies. The system is based on the traditional M/M/1 queue, but it introduces a server that operates in three distinct modes, each affecting its service rate. The focus of the study is to examine the transient behavior of the system, especially how it performs during non-steady state conditions is analyzed in [8]. This is crucial for understanding how the system behaves during periods of fluctuating traffic or when the system is initialized. The paper derives essential performance metrics during the transient phase, providing deeper insights into the system's behavior before it reaches a steady state. A finite Markovian heterogeneous queueing model with encouraged arrivals and focused on optimizing key performance measures such as queue length and system utilization. Their study provides practical insights into managing finite-capacity service systems under dynamic arrival behaviors is developed in [12] . Various queueing theory models and methods that address the behavior of customers who may abandon the system if their wait becomes too long is examined in [9]. Impatience plays a crucial role in many service systems, such as call centers, retail, healthcare and telecommunications, where customers tolerance for waiting is limited. The paper compiles existing research on queueing systems involving impatient customers, providing insights into different strategies for managing impatience and enhancing system performance. The queueing theory to analyze how encouraged arrivals can be leveraged to maximize system size in finite-capacity models is applied in [13]. Their study highlights the impact of arrival incentives on improving system utilization and overall throughput within constrained environments.The server experiences working breakdowns, which cause interruptions in the service and the customers exhibit impatience, with the possibility of them leaving the system if their wait is too long, though they may return later is provided in [10]. This combination of breakdowns and impatient customers is common in manufacturing systems, call centers and other service-oriented industries.

In many practical queueing systems, customer encouragement and impatience significantly influence system performance. When waiting times become long, it is natural for the server to respond by working faster, even if the queue size remains large. Various extensions of customer impatience have been explored in single-server queues. Importantly, customer impatience can have a highly negative impact, leading to the loss of potential customers, an issue critical to many business operations. Considering these effects, this paper investigates the concept of a self-switching server, which dynamically adjusts its service rate in response to queue conditions and customer behavior.

2.1. Definitions

In queueing theory, Poisson processes, exponential distributions and Laplace transforms are fundamental tools used to model and analyze systems involving customer arrivals, service mechanisms and overall system dynamics. These concepts provide the mathematical foundation for deriving performance measures and understanding system behavior under various operational conditions.

2.1.1 Poisson process

The Poisson process is a mathematical model used to describe random events occurring over time, most commonly applied to model arrivals in queueing systems. In this context, it assumes that the number of arrivals in a given time interval follows a Poisson distribution and the inter-arrival times are exponentially distributed, reflecting the memoryless and random nature of customer arrivals.

2.1.2 Exponential distribution

The exponential distribution models the time between successive events in a Poisson process and is widely used in queueing theory to represent service times or waiting times. Its memoryless property makes it ideal for modeling systems where the probability of service completion is independent of how long a customer has already waited. Both the arrival rate and service rate, often assumed to follow exponential distributions, are crucial for evaluating performance metrics such as average wait time, queue length and system utilization.

2.1.3 Laplace transforms

The Laplace transform is a mathematical technique that converts functions from the time domain to the complex frequency domain, making it easier to analyze differential equations commonly found in queueing models. In queueing theory, it is particularly useful for deriving probability generating functions and transform solutions, which help compute key performance measures such as average queue length, waiting time and system utilization.

3. Model description

Customers enter the queuing system with encouragement, following a Poisson process with rate $\lambda(1+\eta)$. The system has a single server that operates in two modes: normal and fast. When the number of customers reaches a threshold K, the server switches from normal mode (with service rate μ_1) to fast mode (with higher service rate μ_2, where $\mu_2 > \mu_1$). Service times in both modes follow exponential distributions with means $\frac{1}{\mu_1}$ and $\frac{1}{\mu_2}$, respectively. During normal service, customers may become impatient and leave the system at an exponential rate γ. However, once the fast mode is activated, no further customer abandonment occurs due to impatience.

3.1. Notations

1. $\mathscr{P}(A)$: Probability of event A.
2. $\pi_n = \lim_{t\to\infty} p_n(t)$: Steady state probability of n customers in the system.
3. $\int_0^t f(u)g(t-u)\,\mathrm{d}u$: Convolution of $f(t)$ and $g(t)$ in Laplace Transform.
4. $\int_0^\infty e^{-st} f(t)\,\mathrm{d}t = f^*(s)$: Laplace transform of $f(t)$.

4. Governing equations

Let $\{X(t); t > 0\}$ be the number of customers present in the system at time t, we denote the probability that there are n customers in the system at time $t.P_n(t) = \Pr\{X(t) = n\}, n = 0, 1, 2, \ldots$

Below differential-difference equations are used to govern the system:
By applying probabilistic laws, we have

$$p(0,t) = e^{-\lambda(1+\eta)t} + \mu_1 p(1,t) e^{-\lambda(1+\eta)t} \tag{1}$$

$$p(n,t) = \left[\frac{\lambda(1+\eta)}{n} p(n-1,t) + (\mu_1 + n\gamma)\, p(n+1,t)\right] e^{-((\lambda(1+\eta)+\mu_1+(n-1)\gamma)t}, \quad n = 1, 2, \ldots, K-2 \tag{2}$$

$$p(K-1,t) = \left[\frac{\lambda(1+\eta)}{K-1} p(K-2,t) + \mu_2 p(K,t)\right] e^{-\left(\frac{\lambda(1+\eta)}{K}+\mu_1+(K-2)\gamma\right)t} \tag{3}$$

$$p(n,t) = \left[\frac{\lambda(1+\eta)}{n} p(n-1,t) + \mu_2 p(n+1,t)\right] e^{-\left(\frac{\lambda(1+\eta)}{K+1}+\mu_2\right)t}, n \geq K \tag{4}$$

Taking the Laplace transform of Equations 1-4, we get

$$(s+\lambda(1+\eta))p^*(0,s) = 1+\mu_1 p^*(1,s) \tag{5}$$

$$(s+(\lambda(1+\eta)+\mu_1+(n-1)\gamma)\,p^*(n,s) = \frac{\lambda(1+\eta)}{n}p^*(n-1,s)+(\mu_1+n\gamma)\,p^*(n+1,s),$$
$$n=1,2,\ldots,K-2 \tag{6}$$

$$\left(s+\frac{\lambda(1+\eta)}{K}+\mu_1+(K-2)\gamma\right)p^*(K-1,s) = \frac{\lambda(1+\eta)}{K-1}p^*(K-2,s)+\mu_2 p^*(K,s) \tag{7}$$

$$(s+(\lambda(1+\eta)+\mu_2)\,p^*(n,s) = \frac{\lambda(1+\eta)}{n}p^*(n-1,s)+\mu_2 p^*(n+1,s), n\geq K \tag{8}$$

5. Steady state probabilities

Taking $\lim_{t\to\infty} p_n(t)=\pi_n$ and therefore $\frac{\mathrm{d}p_n}{\mathrm{d}t}=0$ as $t\to\infty$, hence we get balance equations,

$$\lambda(1+\eta)\pi_0 = \mu_1\pi_1 \tag{9}$$

$$((\lambda(1+\eta)+\mu_1+(n-1)\gamma)\,\pi_n = \frac{\lambda(1+\eta)}{n}\pi_{n-1}+(\mu_1+n\gamma)\,\pi_{n+1}, 1\leq n\leq K-2 \tag{10}$$

$$\left(\frac{\lambda(1+\eta)}{K}+\mu_1+(K-2)\gamma\right)\pi_{K-1} = \frac{\lambda(1+\eta)}{K-1}\pi_{K-2}+\mu_2\pi_K \tag{11}$$

$$((\lambda(1+\eta)+\mu_2)\,\pi_n = \frac{\lambda(1+\eta)}{n}\pi_{n-1}+\mu_2\pi_{n+1}, n\geq K \tag{12}$$

By an iterative procedure, equations from 9-12, yield

$$\pi_n = \begin{cases} \Pi_{r=1}^{n}\left[\frac{\lambda(1+\eta)}{r(\mu_1+(r-1)\gamma)}\right]\pi_0, & 1\leq n\leq K-1 \\ \left[\left(\frac{\lambda(1+\eta)}{\mu_2}\right)^{n-K+1}\frac{1}{(K)_{n-K+1}}\right]\Pi_{r=1}^{K-1}\left[\frac{\lambda(1+\eta)}{r(\mu_1+(r-1)\gamma)}\right]\pi_0, & n\geq K \end{cases} \tag{13}$$

where $(K)_i = K(K+1)(K+2)\cdots(K+i-1)$.
By using the total probability law, we get

$$\pi_0 = \left[1+\sum_{n=1}^{K-1}\prod_{r=1}^{n}\left(\frac{\lambda(1+\eta)}{r\,(\mu_1+(r-1)\gamma)}\right)\right.$$
$$\left.+\sum_{n=K}^{\infty}\left(\left(\frac{\lambda(1+\eta)}{\mu_2}\right)^{n-K+1}\frac{1}{(K)_{n-K+1}}\right)\prod_{r=1}^{K-1}\left(\frac{\lambda(1+\eta)}{r\,(\mu_1+(r-1)\gamma)}\right)\right]^{-1} \tag{14}$$

Hence, the probabilities for the system's steady states for each state are established explicitly.

5.1. Special case

1. When $\gamma=0$, this model reduces encouraged Arrivals and Self-Regulatory Servers in the Markovian Queueing System

$$\pi_n = \begin{cases} \frac{1}{n!}r_1^n\pi_0, & 1\leq n\leq K-1 \\ \frac{1}{n!}r_1^{K-1}r_2^{n-K+1}\pi_0, & n\geq K \end{cases} \tag{15}$$

$$\pi_0 = \left[1+\sum_{n=1}^{K-1}\frac{1}{n!}r_1^n+\left(\frac{r_1}{r_2}\right)^{K-1}\left\{e^{r_2}-\sum_{n=0}^{K-1}\frac{r_2^n}{n!}\right\}\right]^{-1} \tag{16}$$

where $r_1 = \frac{\lambda(1+\eta)}{\mu_1}$ and $r_2 = \frac{\lambda(1+\eta)}{\mu_2}$.
2. When there is no discouragement, no impatience and no server switch, then Equations (15) and (16) becomes

$$\pi_n = \left(\frac{\lambda(1+\eta)}{\mu}\right)^n \pi_0$$

And $\pi_0 = 1 - \frac{\lambda(1+\eta)}{\mu}$.
Hence, proved that the model will be reduced to a simple M/M/1/∞ queueing model.

6. Stationary measures of effectiveness

6.1. Expected number of customers in the system

The average number of customers in the system is denoted by L_s. In an $M/M/1$ queue, this measure helps evaluate overall system performance. Understanding L_s allows us to assess whether the server is operating efficiently or if the system is becoming congested.

$$L_s = \sum_{n=0}^{\infty} n\pi_n = \sum_{n=1}^{K-1} n\pi_n + \sum_{n=K}^{\infty} n\pi_n$$

$$L_s = \Bigg[\sum_{n=1}^{K-1} n \prod_{r=1}^{n} \left(\frac{\lambda(1+\eta)}{r\,(\mu_1 + (r-1)\gamma)}\right)\pi_0 + \sum_{n=K}^{\infty} n \left(\left(\frac{\lambda(1+\eta)}{\mu_2}\right)^{n-K+1} \frac{1}{(K)_{n-K+1}}\right) \prod_{r=1}^{K-1} \left(\frac{\lambda(1+\eta)}{r\,(\mu_1 + (r-1)\gamma)}\right)\pi_0\Bigg] \tag{17}$$

6.2. Expected number of customers in the queue

Let the average number of customers in the queue be denoted by L_q. This metric provides valuable insights into the system's efficiency. A high value of L_q indicates potential system overload, resulting in longer wait times and increased customer dissatisfaction.

$$L_q = L_s\,(1-\pi_0)$$

$$L_q = \Bigg[\sum_{n=1}^{K-1} n \prod_{r=1}^{n} \left(\frac{\lambda(1+\eta)}{r\,(\mu_1 + (r-1)\gamma)}\right)\pi_0 + \sum_{n=K}^{\infty} n \left(\left(\frac{\lambda(1+\eta)}{\mu_2}\right)^{n-K+1} \frac{1}{(K)_{n-K+1}}\right) \prod_{r=1}^{K-1} \left(\frac{\lambda(1+\eta)}{r\,(\mu_1 + (r-1)\gamma)}\right)\pi_0\Bigg] - 1 + \pi_0 \tag{18}$$

6.3. Average impatience rate

The expected number of customers who leave the system due to impatience is denoted by $E[I]$. This metric quantifies the likelihood of customers abandoning the queue before receiving service, particularly in scenarios where prolonged waiting times influence customer behavior.

$$E[I] = \gamma \sum_{n=1}^{K-1} n\pi_n = \gamma \sum_{n=1}^{K-1} n\Pi_{r=1}^{n} \left[\frac{\lambda(1+\eta)}{r\,(\mu_1 + (r-1)\gamma)}\right]\pi_0 \tag{19}$$

7. Numerical illustration

7.1. Mean count of the customers arriving in the system and queue (L_s & L_q)

Take the values of parameters are $\lambda(1+\eta) = 16.5; \mu_1 = 20; \mu_2 = 30; K = 20$ and varying π_n from 0.1 to 1.0, determining the expected count of customers arriving in the system and queue for state probabilities. Table 1 has the values and Figure 1 depicts the variation:

Table 1: *Mean count of the customers arriving in the system and queue (L_s & L_q)*

π_n	L_s	π_n	L_q
0.1	1.4785	0.6	1.1860
0.2	1.6813	0.7	1.2283
0.3	1.7236	0.8	1.2331
0.4	1.7284	0.9	1.2334
0.5	1.7287	1.0	1.2335
0.6	1.7287	0.6	1.2335
0.7	1.7287	0.7	1.2335
0.8	1.7287	0.8	1.2335
0.9	1.7287	0.9	1.2335
1.0	1.7287	1.0	1.2335

Table 1 and demonstrate that as the state probability π_n increases from 0.1 to 1.0, the expected number of customers in the system L_s increases initially and gradually stabilizes around 1.7287 from $\pi_n = 0.5$ onward. Similarly, the expected number of customers in the queue L_q increases and then levels off around 1.2335 from $\pi_n = 0.6$ onward. This behavior indicates that beyond a certain arrival probability, the system reaches a saturation point where additional increases in π_n do not significantly affect system congestion. This suggests efficient server utilization and improved queue stability under higher arrival intensities.

7.2. Mean Count of Customers in the System Against γ

Assume $\lambda(1+\eta) = 16.5$, $\mu_1 = 20$, $\mu_2 = 30$, $K = 20$ and γ varying from 1 to 10. The system's mean customer count is calculated using steady-state probabilities. The table 2 presents the results.

Table 2: *Mean count of customer arrivals in the system L_s*

γ	L_s	γ	L_s
1	1.82490	6	1.74812
2	1.80444	7	1.73792
3	1.78724	8	1.72872
4	1.77245	9	1.72040
5	1.75954	10	1.71280

7.3. Mean Count of Customers in the System against γ

Assume $\lambda(1+\eta) = 16.5$, $\mu_1 = 20$, $\mu_2 = 30$ and $K = 20$, with γ varying from 1 to 10. The mean number of customers in the system, L_s, is presented in Table 3

Table 3: *Mean count of customer arrivals in the system L_s*

γ	L_s	γ	L_s
1	1.8249	6	1.74812
2	1.80444	7	1.73792
3	1.78724	8	1.72872
4	1.77245	9	1.7204
5	1.75954	10	1.7128

Table 3 and show that as γ increases from 1 to 10, the average number of customers in the system L_s gradually decreases from 1.8249 to 1.7128. This trend indicates improved system performance and reduced congestion with higher γ values. However, the rate of decrease slows down at higher values of γ, suggesting diminishing returns beyond a certain point. An optimal range of $\gamma = 6$ to 8 appears to balance efficiency and stability, offering effective queue management without excessive resource adjustment.

8. Sensitivity Analysis

We have investigated the behavior of the customers who are expected to be in the queue and the system based on different switch point values K through the sensitivity analysis presented in Table 4. From the table, it is evident that an increase in the switch point value K results in encouraged arrivals and impatient customers in a single-server system. The self-switching server tends to behave like a single server model with encouraged arrivals and impatient customers, without switching the server mode.

Table 4: *Sensitivity Analysis*

K	π_0	L_s	L_q
2	0.6068	1.7267	1.2335
3	0.6028	1.7392	1.2419
4	0.6044	1.7312	1.2355
5	0.6047	1.7290	1.2337
6	0.6047	1.7288	1.2335
7	0.6047	1.7287	1.2335
8	0.6047	1.7287	1.2335

From Table 4, we observe that when $K \geq 5$, the steady-state probability π_0 stabilizes at 0.6047. Similarly, for $K \geq 6$, the average number of customers in the system L_s remains nearly constant at approximately 1.7287 and the average queue length L_q stabilizes at around 1.2335. These results indicate that as the switch point K increases, its influence on system performance metrics becomes negligible beyond a certain threshold, suggesting diminishing sensitivity to further increases in K.

9. Conclusions

This paper presents a comprehensive analysis of a Markovian queueing system with a single self-switching server, subject to encouraged arrivals, customer impatience and server switching dynamics. The study examines the complex interactions among various system behaviors, including a customer arrival rate that is inversely proportional to the queue length, customer impatience leading to abandonment and the switching of server modes from a normal service rate to a faster one when the queue reaches a predefined threshold. The findings demonstrate that the incorporation of encouraged arrivals, customer impatience and a self-switching server significantly affects system performance metrics such as waiting times, queue lengths and the probability of customer abandonment. Notably, transitioning to a faster service mode once the queue exceeds a threshold improves service delivery and reduces the negative impact of customer impatience.

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