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LOG EXPONENTIATED KUMARASWAMY DISTRIBUTION WITH REAL LIFE APPLICATIONS

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Abstract

The generalization of probability distributions plays a crucial role in broadening their applications across various domains. To achieve this, this paper introduces a novel three-parameter generalization of the Kumaraswamy distribution. The new distribution is termed as the Log Exponentiated Kumaraswamy (LEK) distribution. Some essential properties of this distribution are studied and the estimate of parameters are obtained using the maximum likelihood estimation procedure. Also, a simulation study is conducted and two real life data sets demonstrate the proposed distribution's applicability. The performance of the proposed LEK distribution proved to be better than comparative models.

Keywords: Log exponentiated technique, Reverse Hazard rate, Kumaraswamy distribution, Mixture representations

1. Introduction

Probability distributions play an indispensable role in mathematical statistics, as they contribute to understanding and interpreting the underlying patterns in the data with greater accuracy. The diverse data in applied domains often transcend the capabilities of traditional probability distributions, necessitating the need of deriving new distributions. Researchers have introduced new probability distributions designed to accommodate the vast data across applied domains. The process of creating new distributions often involve modifying existing distributions by introducing additional parameters. Numerous researchers have utilized different techniques to make the base distribution more flexible and versatile. Some contributions include MTI Rayleigh distribution introduced by [1] as an extension of the Rayleigh distribution, the MTI inverted exponential distribution proposed by [2], the PNJ power function distribution proposed by [3], the exponentiated power inverse Lomax distribution introduced by [4], Inverse power XLindley distribution with statistical inference and applications to engineering data by [5].

The Kumaraswamy distribution [6], introduced by Poondi Kumaraswamy in 1980, is a continuous probability distribution defined in the interval (0,1). The Kumaraswamy distribution is particularly effective in addressing hydrological challenges and modelling natural phenomena with bounded process values. Its distinctive properties make it an excellent choice for applications that require straightforward and efficient modelling techniques. In recent years, numerous extensions of the Kumaraswamy distribution have been introduced, significantly increasing its versatility and applicability. These advancements include the generalized odd maxwell-kumaraswamy distribution by [7], second order Transmuted Kumaraswamy distribution by [8], Kumaraswamy-Gull Alpha Power Rayleigh distribution introduced by [9], SMPKumaraswamy distribution put forward by [10], Transmuted Exponentiated Kumaraswamy distribution put forward by [11]. In this paper, we introduce a novel three-parameter extension of Kumaraswamy

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distribution with pdf and cdf given in equations 1 and 2 respectively, using a novel approach for constructing distributions introduced by [12] termed as the Log Exponentiated Transformation (LET).

$$g(x;\beta,\lambda) = \beta\lambda x^{\lambda-1}(1-x^{\lambda})^{\beta-1}; \quad 0 < x < 1,\ \beta > 0, \lambda > 0 \tag{1}$$

$$G(x;\beta,\lambda) = 1-(1-x^{\lambda})^{\beta}; \quad 0 < x < 1,\ \beta > 0, \lambda > 0 \tag{2}$$

The cdf and pdf of LET are respectively given by equation 3 and 4

$$F(x) = 1-\log\left(e+\bar{e}(G(x;\beta,\lambda))^{\theta}\right); \quad x \in \mathbb{R} \quad \theta,\beta,\lambda > 0 \tag{3}$$

$$f(x) = \frac{(e-1)\theta g(x;\beta,\lambda)(G(x;\beta,\lambda))^{\theta-1}}{e+\bar{e}\left(G(x;\beta,\lambda)\right)^{\theta}}; \quad x \in \mathbb{R} \quad \theta,\beta,\lambda > 0 \tag{4}$$

where $G(x;\beta,\lambda)$ is cdf of distribution to be generalized.

In this paper a novel three-parameter Kumaraswamy distribution using Log Exponentiated technique is introduced. The research goals of this study are:

- To introduce a new probability distribution with an additional parameter. The new model is more flexible than the baseline distribution and can analyze datasets of different shapes.
- To estimate the model parameters and conduct a comprehensive simulation study to assess the behavior of derived estimators.
- To show the flexibility and applicability of the proposed distribution using two real life datasets from different fields.

The structure of the manuscript is organized as follows: Section 2 introduces Log exponentiated Kumaraswamy distribution along with mixture representations. In Section 3, some statistical properties are discussed. Section 4 addresses the reliability characteristics of the proposed distribution, while Section 5 is devoted to generating functions and parameter estimation method is given in section 6, followed by a detailed simulation study in section 7 and applications in section 8. Finally, conclusion is provided in section 9.

2. Log Exponentiated Kumaraswamy Distribution

Using the Kumaraswamy distribution as baseline distribution for proposed distribution, the Log Exponentiated Kumaraswamy distribution (LEK) cdf function is given by

$$F(x;\lambda,\beta,\theta) = 1-\log\left(e+\bar{e}[1-(1-x^{\lambda})^{\beta}]^{\theta}\right); \quad 0 < x < 1, \lambda,\beta,\theta > 0 \tag{5}$$

The pdf function is given by

$$f(x;\lambda,\beta,\theta) = \frac{(e-1)\beta\lambda\theta x^{\lambda-1}(1-x^{\lambda})^{\beta-1}[1-(1-x^{\lambda})^{\beta}]^{\theta-1}}{e+\bar{e}[1-(1-x^{\lambda})^{\beta}]^{\theta}}; \quad 0 < x < 1, \lambda,\beta,\theta > 0 \tag{6}$$

Figure 1 illustrates graphically different pdf plots of LEK distribution for several parameter combinations. The legend specifies the parameter values for each curve. The shape of the pdf changes with different parameter values showcasing how the distribution behaves under various parameter combinations.

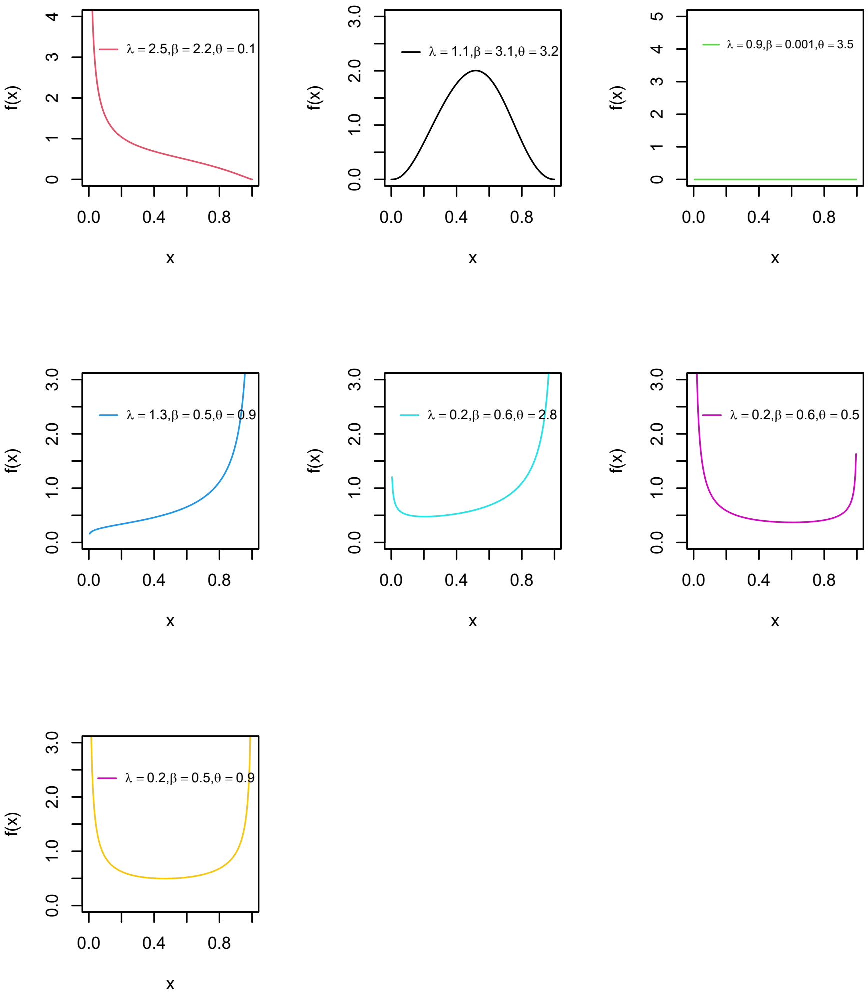

Figure 1: *Probability Density Plots of LEK distribution*

2.1. Mixture Representations

Using the binomial expansion , $(1+z)^{-1} = \sum_{p=0}^{\infty} (-1)^p z^p \quad ; |z| \leq 1$, an alternative representation for the probability density function is given below

$$f(x;\lambda,\beta,\theta) = \frac{(e-1)\beta\lambda\theta x^{\lambda-1}(1-x^{\lambda})^{\beta-1}[1-(1-x^{\lambda})^{\beta}]^{\theta-1}}{e+\bar{e}[1-(1-x^{\lambda})^{\beta}]^{\theta}}; \quad 0<x<1, \beta,\lambda,\theta>0$$

$$= \frac{(e-1)}{e}\beta\lambda\theta x^{\lambda-1}(1-x^{\lambda})^{\beta-1}\left[1-(1-x^{\lambda})^{\beta}\right]^{\theta-1}\left[1+\frac{\bar{e}}{e}\left[1-(1-x^{\lambda})^{\beta}\right]^{\theta}\right]^{-1}$$

3. Statistical Properties of Log Exponentiated Kumaraswamy Distribution

This section deals with the statistical characteristics of the proposed LEK distribution, which presents insights into the behavior and features of the distribution.

3.1. Quantile function

Theorem 1. If X $\sim$ *LEK* (λ,β,θ) distribution, then the quantile function of X is given by

$$x = \left[1 - \left(1 - \left(\frac{e^{1-u} - e}{\bar{e}}\right)^{\frac{1}{\theta}}\right)^{\frac{1}{\beta}}\right]^{\frac{1}{\lambda}} \tag{7}$$

Where u is a uniform random variable, $0 < u < 1$.

Proof. Let $F(x;\lambda,\beta,\theta)$ = u

$$log\left[e + \bar{e}(1-(1-x^{\lambda})^{\beta})^{\theta}\right] = 1 - u$$

$$x^{\lambda} = 1 - \left[1 - \left(\frac{e^{1-u} - e}{\bar{e}}\right)^{\frac{1}{\theta}}\right]^{\frac{1}{\beta}}$$

$$x = \left[1 - \left(1 - \left(\frac{e^{1-u} - e}{\bar{e}}\right)^{\frac{1}{\theta}}\right)^{\frac{1}{\beta}}\right]^{\frac{1}{\lambda}}$$

■

3.2. Moments

The r^{th} ordinary moment of the LEK distribution is obtained as

$$\mu_{r'} = \int_0^1 x^r \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}(-1)^{p+q+1}\binom{(p+1)\theta - 1}{q}\left[(1-x^{\lambda})\right]^{q\beta+\beta-1}\theta\beta\lambda x^{\lambda-1}\,dx$$

$$\mu_{r'} = \int_0^1 \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\delta_{p,q}\theta\beta\lambda x^r x^{\lambda-1}\left[(1-x^{\lambda})^{\beta(1+q)-1}\right]\,dx$$

where $\delta_{p,q} = (-1)^{p+q+1}\binom{(p+1)\theta-1}{q}\left(\frac{\bar{e}}{e}\right)^{p+1}\theta$

$$\mu_{r'} = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\delta_{p,q}\int_0^1 \beta z^{\frac{r}{\lambda}}\left[(1-z)^{\beta(1+q)-1}\right]\,dz$$

$$\mu_{r'} = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\delta_{p,q}\beta B\left[\left(\frac{r}{\lambda}+1\right),\beta(1+q)\right]$$

$B[(\frac{r}{\lambda}+1),\beta(1+q)]$ represents the beta function.

From above moment expression, we can easily derive the mean, variance, moment skewness coefficient and kurtosis coefficient.

Mean: $\mu_{1'} = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\delta_{p,q}\beta B\left[\left(\frac{1}{\lambda}+1\right),\beta(1+q)\right]$

3.3. Incomplete Moment

Let $I_r(t) = \int_0^t x^r f(x;\lambda,\beta,\theta)\,dx$ denotes the r^{th} incomplete moment, then we have

$$I_r(t) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\delta_{p,q}\left[t^{\lambda};(\frac{1}{\lambda}+1),\beta(1+q)\right] \tag{8}$$

Using $I_r(t) = \int_0^t x^r f(x;\lambda,\beta,\theta)\,dx$

$$I_r(t) = \int_0^t x^r \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} \delta_{p,q}\lambda\beta x^{\lambda-1}\left[(1-x^{\lambda})^{\beta-1}\right]\left[(1-x^{\lambda})^{\beta}\right]^q\,dx$$

$$I_r(t) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} \delta_{p,q}\beta B\left[t^{\lambda};\left(\frac{r}{\lambda}+1\right),\beta(1+q)\right]$$

Where $B[z;a,b] = \int_0^z x^{a-1}(1-x)^{b-1}\,dx$ is incomplete beta function.

Remark: For r=1 in equattion (8) we get first incomplete moment.

$$I_1(t) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} \delta_{p,q}\beta B\left[t^{\lambda};\left(\frac{1}{\lambda}+1\right),\beta(1+q)\right]$$

4. Reliability Analysis Of the Log Exponentiated Kumaraswamy Distribution

This section introduces some reliability expressions of the LEK distribution, such as survival function, Reverse hazard function, hazard rate.

4.1. Survival Function

The survival function is a key concept in probability, survival analysis, and reliability theory. It represents the probability that a subject or system survives beyond a certain time without experiencing a specified event (e.g., failure, death).The survival function for the LEK distribution is given as

$$R(x;\lambda,\beta,\theta) = \log\left[e + \bar{e}(1-(1-x^{\lambda})^{\beta})^{\theta}\right]$$

4.2. Reverse Hazard Function

The reverse hazard rate is given as

$$h_r(x;\lambda,\beta,\theta) = \frac{(\frac{e-1}{e})\beta\lambda\theta x^{\lambda-1}(1-x^{\lambda})^{\beta-1}[1-(1-x^{\lambda})^{\beta}]^{\theta-1}\left[1+\frac{\bar{e}}{e}[1-(1-x^{\lambda})^{\beta}]^{\theta}\right]^{-1}}{1-\log\left[e+\bar{e}(1-(1-x^{\lambda})^{\beta})^{\theta}\right]}$$

4.3. Hazard Rate

The hazard rate is a fundamental concept in reliability theory and survival analysis. It describes the instantaneous rate at which an event (such as failure or death) occurs, given that the subject has survived up to a certain time. The expression for the hazard rate of the LEK distribution is obtained as

$$h(x;\alpha,\beta,\lambda) = \frac{(\frac{e-1}{e})\beta\lambda\theta x^{\lambda-1}(1-x^{\lambda})^{\beta-1}[1-(1-x^{\lambda})^{\beta}]^{\theta-1}\left[1+\frac{\bar{e}}{e}[1-(1-x^{\lambda})^{\beta}]^{\theta}\right]^{-1}}{\log\left[e+\bar{e}(1-(1-x^{\lambda})^{\beta})^{\theta}\right]}$$

Figure 2 displays different shapes of hazard rate for different parameter combinations viz., constant, bath-tub, increasing and J-shaped. The pattern of the hazard rate plots in Figure 2 reveals utility of the proposed LEK distribution in describing varying lifetime events.

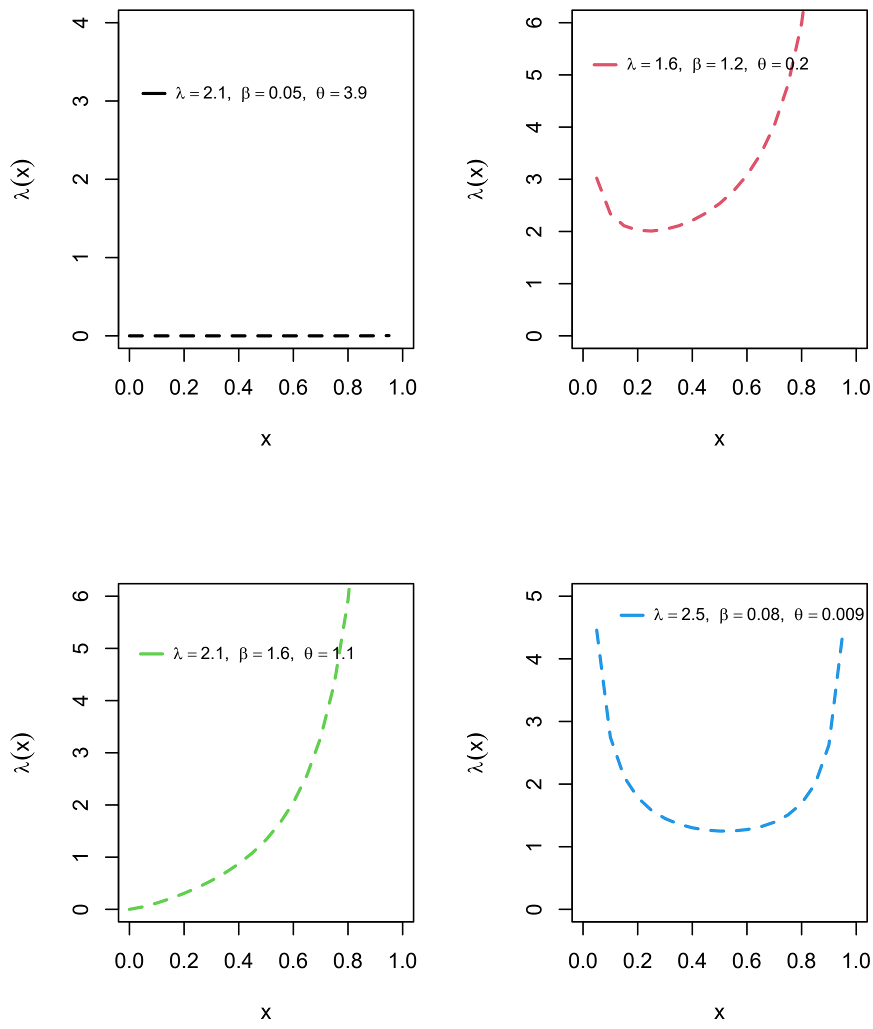

Figure 2: *Hazard rate Plots of LEK distribution*

4.4. Mean Residual life

The mean residual life of a component following LEK distribution can be obtained as

$$\mu(x) = \frac{1}{R(x)}\left[E(x) - I_1(x)\right] - x$$

Where

$$E(x) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} \delta_{p,q}\beta B\left[\left(\frac{1}{\lambda}+1\right), \beta(1+q)\right]$$

and

$$I_1(x) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} \delta_{p,q}\beta B\left[t^{\lambda}; \left(\frac{1}{\lambda}+1\right), \beta(1+q)\right]$$

$$\mu(x) = \frac{\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} \delta_{p,q}\beta B\left[\left(\frac{1}{\lambda}+1\right), \beta(1+q)\right] - \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} \delta_{p,q}\beta B\left[t^{\lambda}; \left(\frac{1}{\lambda}+1\right), \beta(1+q)\right]}{\log\left[e + \bar{e}(1-(1-x^{\lambda})^{\beta})^{\theta}\right]}$$

4.5. Mean Waiting Time

The expression for mean waiting time is given as

$$\bar{\mu}(x) = x - \frac{I_1(x)}{F(x)}$$

$$\bar{\mu}(x) = x - \frac{\sum_{p=0}^{\infty}\sum_{q=0}^{\infty} \delta_{p,q}\beta B\left[t^{\lambda}; \left(\frac{1}{\lambda}+1\right), \beta(1+q)\right]}{1-\log\left[e+\bar{e}(1-(1-x^{\lambda})^{\beta})^{\theta}\right]}$$

5. Generating Functions

The moment generating function of X $\sim$ LEK (λ, β, θ) distribution can be obtained using the relation

$$M_x(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!}\mu_r'$$

$$M_x(t) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty} \frac{t^r}{r!}\frac{t^r}{r!}\delta_{p,q}\beta B\left[(\frac{r}{\lambda}+1), \beta(j+1)\right]$$

The characteristic function of LEK distribution is given by

$$\phi_x(t) = \sum_{r=0}^{\infty} \frac{(it)^r}{r!}\mu_r'$$

$$\phi_x(t) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty} \frac{(it)^r}{r!}\frac{t^r}{r!}\delta_{p,q}\beta B\left[(\frac{r}{\lambda}+1), \beta(j+1)\right]$$

6. Estimation of Parameters

Let $x_1, x_2, x_3, ...x_n$ be a random sample of size n from LEK distribution. The likelihood and log-likelihood functions are respectively given by equations below

$$L(x;\lambda,\beta,\theta) = \prod_{i=1}^{n} \frac{(e-1)\theta\lambda\beta x_i^{\lambda-1}(1-x_i^{\lambda})^{\beta-1}[1-(1-x_i^{\lambda})^{\beta}]^{\theta-1}}{e+\bar{e}\left[1-(1-x_i^{\lambda})^{\beta}\right]^{\theta}}$$

$$l = n\log(e-1) + n\log\theta + n\log\lambda + n\log\beta + (\lambda-1)\log x + (\beta-1)\log(1-x^{\lambda})$$
$$+ (\theta-1)\log\left[1-(1-x^{\lambda})^{\beta}\right] - \sum_{i=1}^{n}\left(e+\bar{e}\left[[1-(1-x^{\lambda})^{\beta}\right]^{\theta}\right)$$

$$\frac{\partial l}{\partial\theta} = \frac{n}{\theta} + \log\left[1-(1-x^{\theta})^{\beta}\right] + \sum_{i=1}^{n}\log\left(e+\bar{e}\left[1-(1-x^{\lambda})^{\beta}\right]^{\theta}\right)$$

$$\frac{\partial l}{\partial\lambda} = \frac{n}{\lambda} + \log x - \frac{(\beta-1)x^{\lambda}\log x}{1-x^{\lambda}} + \frac{\beta x^{\lambda}\log x(1-x^{\lambda})^{\beta-1}(\theta-1)}{1-(1-x^{\lambda})^{\beta}}$$
$$-\frac{\sum_{i=1}^{n}\bar{e}\theta\beta\left[(1-(1-x^{\lambda})^{\beta}\right]^{\theta-1}(1-x^{\lambda})^{\beta-1}x^{\lambda}\log x}{e+\bar{e}\left[1-(1-x^{\lambda})^{\beta}\right]}$$

$$\frac{\partial l}{\partial\beta} = \frac{n}{\beta} + \sum_{i=1}^{n}\log(1-x^{\lambda}) - \frac{(\theta-1)(1-x^{\lambda})^{\beta}\log(1-x^{\lambda})}{1-(1-x^{\lambda})^{\beta}} + \sum_{i=1}^{n}\frac{\bar{e}\theta\log(1-x^{\lambda})(1-x^{\lambda})^{\beta}\left[1-(1-x^{\lambda})^{\beta}\right]^{\theta-1}}{e+\bar{e}\left[1-(1-x^{\lambda})^{\beta}\right]}$$

Since, above equations are non-linear, we will use R software to solve these equations and estimate the parameters.

7. Simulation Study

In this section two comprehensive simulation studies to assess the accuracy of the maximum likelihood procedure is discussed. Based on quantile function described in equation 7 and using R package random samples of size 20, 50, 125, 500 are generated for two parameter combinations $\lambda = 0.01, \beta = 1.5, \theta = 0.1$ and $\lambda = 0.03, \beta = 0.1, \theta = 1.6$. For each sample size and both parameter combinations ,the MLEs, Biases and Mean Square Errors (MSEs) are computed and the numerical results are given in table 1.

Table 1: *Simulation results for* $\lambda = 0.01, \beta = 1.5, \theta = 0.1$

Sample size	Parameters			MLE			Bias			MSE		
n	λ	β	θ	$\hat{\lambda}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{\lambda}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{\lambda}$	$\hat{\beta}$	$\hat{\theta}$
20	0.01	1.5	0.1	0.8079	1.3259	0.2182	0.7980	0.1741	0.1188	0.7857	0.1826	0.0756
50				0.7600	1.4586	0.1524	0.7500	0.0414	0.0524	0.7857	0.1826	0.0756
125				0.4632	1.4751	0.1317	0.4532	0.0249	0.0317	0.4514	0.0014	0.0022
500				0.0498	1.4978	0.1028	0.0398	0.0022	0.0028	0.0397	0.0001	0.0002
20	0.03	0.1	1.6	0.0647	0.1810	1.6667	0.0544	0.0812	0.8020	0.0186	0.0090	4.2664
50				0.0272	0.1683	1.4353	0.0206	0.0683	0.2730	0.0013	0.0063	0.2450
125				0.0195	0.1496	1.5241	0.0129	0.0496	0.0789	0.0004	0.0044	0.0446
500				0.0264	0.1112	1.5983	0.0036	0.0112	0.0017	0.0001	0.0010	0.0004

From table 1 it is observed that the MLEs closely approach to true parameter values for both the parameter combinations as the sample size increases from 20 to 500. Further the bias and Mean Square Error (MSE) of each estimate decrease as the sample size (n) increases, indicating enhanced precision and improved accuracy of the maximum likelihood estimation method.

8. Applications

This section presents real data analysis of two real life data sets, demonstrating the practical applicability and effectiveness of the proposed LEK Distribution. For performance comparison purposes with the proposed distribution, we used basic Kumaraswamy distribution [6], Transmuted Kumaraswamy distribution [13] and Kumaraswamy Inverse Exponential distribution [14]. For comparison, we use Goodness of fit measures namely Akaike Information Criterion (AIC), Akaike Information Criterion Corrected (AICC), Hannan - Quinn information Criterion (HQIC) , Kolmogorov-Smirnov (KS) and P-value statistics calculated using R programming. Decision rule is the distribution having the lowest numerical value of aforementioned measures is considered as the best fit.

Data set I: The first data set has been taken from [15] , and this data set relates to 50 observations on burr and are given as

0.04, 0.02, 0.06, 0.12, 0.14, 0.08, 0.22, 0.12, 0.08, 0.26, 0.24, 0.04, 0.14, 0.16, 0.08, 0.26, 0.32, 0.28, 0.14, 0.16, 0.24, 0.22, 0.12, 0.18, 0.24, 0.32, 0.16, 0.14,0.08, 0.16, 0.24, 0.16, 0.32, 0.18, 0.24, 0.22, 0.16, 0.12, 0.24, 0.06, 0.02, 0.18, 0.22, 0.14, 0.06, 0.04, 0.14, 0.26, 0.18, 0.16.

Data Set II: This data set shows water capacity of Shasta reservoir in California, USA for August and December from 1975 to 2016. This data has been previously used by [16] .

0.667157, 0.287785, 0.126977, 0.768563, 0.703119, 0.729986, 0.767135, 0.811159, 0.829569, 0.726164, 0.423813, 0.715158, 0.640395, 0.363359, 0.463726, 0.371904, 0.291172, 0.414087, 0.650691, 0.538082, 0.744881, 0.722613, 0.561238, 0.813964, 0.709025, 0.668612, 0.524947, 0.605979, 0.715850, 0.529518, 0.824860, 0.742025, 0.468782, 0.345075, 0.425334, 0.767070, 0.679829, 0.613911, 0.461618, 0.294834, 0.392917, 0.688100.

Table 2: *Performance measures for Data Set I*

Model	$\hat{\lambda}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{\alpha}$	AIC	AICC	HQIC	K-S	P-value
LEKD	2.8472	88.9604	0.5123	-	-108.5281	-108.0063	-220.8718	0.0910	0.8023
KUMD	2.0774	33.1375	-	-	-108.1374	-107.882	-218.8185	0.11025	0.5777
TKD	-0.2911	-	30.1874	1.9335	-106.5026	-105.9809	-216.821	0.1052	0.6375
KIED	0.4590	2.5266	-	0.3905	-77.9185	-77.3967	-159.6527	0.2407	0.0061

Table 3: *Performance measures for Data Set II*

Model	$\hat{\lambda}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{\alpha}$	AIC	AICC	HQIC	K-S	P-value
LEKD	16.1956	48.6481	0.1320	-	-31.2110	-30.5795	-66.5113	0.0989	0.7692
KUMD	3.4355	3.7681	-	-	-27.2620	-26.9543	-57.2501	0.1362	0.3826
TKD	-0.2970	-	3.8196	3.1568	-25.6987	-25.0672	-55.4867	0.1283	0.4565
KIED	2.0064	17.7055	-	0.8960	-5.0926	-4.4610	-14.2745	0.1918	0.0794

From numerical results in table 2 and table3 it is observed that the Log Exponentiated Kumaraswamy distribution has the least value of all the Goodness of fit measures under consideration against its comparative models. Hence, LEK distribution could be considered as the best model that fits the given data sets. The histogram and density plots of data sets for the LEK distribution against its comparative distributions are also displayed in figure 3 and figure 4. From figures it is also evident that LEK distribution provides a better fit than its comparative distributions.

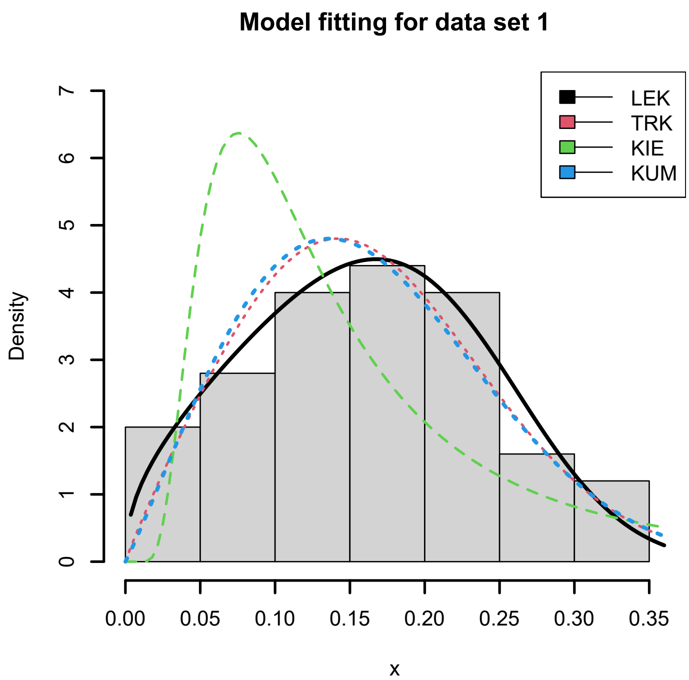

Figure 3: *Histogram and Density plots for dataset I*

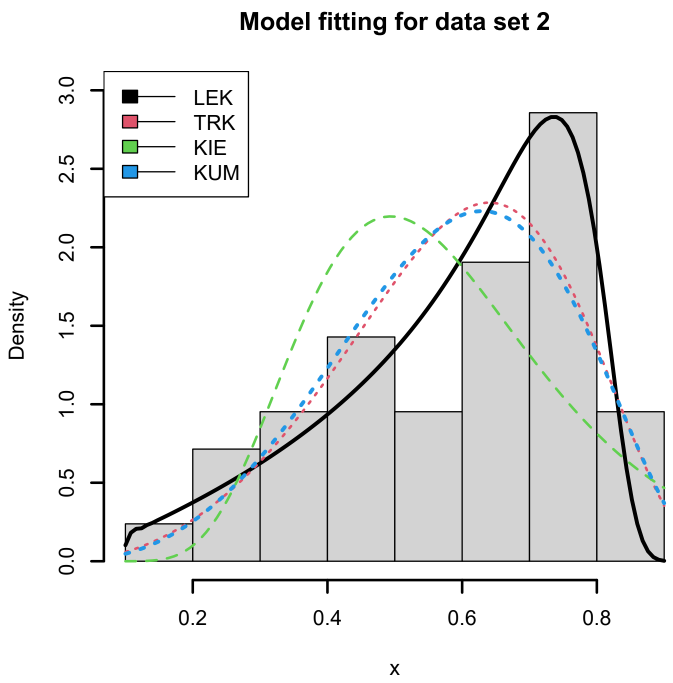

Figure 4: *Histogram and Density plots for dataset II*

9. Conclusion

This paper introduces a novel three-parameter generalization of Kumaraswamy distribution referred to as Log-Exponentiated Kumaraswamy distribution (LEK), which serves as a better alternative to model data from various domains by means of real-life applications. Statistical and reliability characteristics of the proposed model are discussed in detail and the parameters of its estimates are obtained using Maximum likelihood estimation method. A simulation study is performed, using different parameter combinations and all the estimators perform better and with in crease in sample size (n), bias and measn square error of each estimate decreases indicating improved accuracy. To illustrate the applicability and effectiveness of the proposed model , two real life data sets have been analyzed and compared with some probability distributions. The numerical and graphical results suggest that the proposed LEK distribution performs better in modeling the data sets considered than comparative probability distributions. we hope that the proposed LEK distribution can be regarded as the one of the best for modeling data sets in various applied fields like engineering, life sciences, finance, hydrology etc.

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