

MULTIOBJECTIVE NURSE SCHEDULING PROBLEM IN A FUZZY ENVIRONMENT

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Abstract

Nurse scheduling is vital to hospital quality in almost every aspect, including patient care, bedside assistance, medication administration, support during major surgeries, data collection, reporting, and more. Nurse staffing and recruitment have become critical in the healthcare sector, particularly for staff scheduling, which is essential across industries and services. However, maintaining sufficient nurses is crucial, as understaffing can lead to life-threatening situations with serious financial, social, and legal consequences. This research article aims to minimize scheduling costs by optimizing nurse allocation between departments and shift-wise cost distribution. We assume that the costs associated with various hospital departments are parabolic fuzzy. This study adopts a case study approach to explore the issue of nurse scheduling in a hospital environment, where cost minimization and resource allocation are essential. Fuzzy programming is used to achieve a compromise solution that balances cost efficiency with the required staffing level in hospitals.

Keywords: Multiobjective, Nurse Scheduling, Fuzzy Programming, Defuzzification

1. INTRODUCTION

In the last several years, the managing flow of and demands made for staff, especially nursing personnel, has assumed considerable strategic value. There are many ways that staffing efficiency can be enhanced, as outlined in the following proposals. A general point made in this paper is that many of the scheduling problems that can occur in hospitals can be solved, or nearly solved, by operations research techniques.

Nurses are vital healthcare workers in clinics, private practices, and hospitals. Their key duties include patient and doctor communication, nursing care, medication administration, and supervising aides. These duties put into focus the vital role of nurses in the delivery of health care services, Kingma [1]. It is challenging to name a sphere of a hospital's work where nurses are not indispensable: treatment, assistance at the bedside, administration of medication, necessary surgeries, collection of data, reporting on it, and so on. They are assigned to assess and track the state of patients, together with identifying and managing risks or health problems as soon as possible. Furthermore, post-discharge fed patient support is provided by nurses to help the patient move from the hospital to home care, Allen [2]. The NSP, which is related to this application,

determines the number of nurses to be scheduled in different shifts to meet the hospital's needs most efficiently. One of the major problems is the projected lack of 400,000 nurses in the United States by 2020, complicating the provision of healthcare. Janiszewski Goodin [3]. The primary reasons for the nursing shortage include changes in the hospital work environment, low salaries for nurses, declining enrollment in nursing schools, and decreased job satisfaction among nursing professionals. The primary objective of NSP is to reduce medical expenses while ensuring the optimal allocation of nursing personnel to each shift. Multiple examples of a shortage of nurses and expenses through reducing nursing coverage have occurred, Murray [4]. The NSP concerns allocation of shifts and working hours to nurses while considering some constraints and/or goals. Some constraints are getting the correct number of workers for each shift, working under the state employment acts, and responding to the wishes and availability of the nurses. Targets may include a better calibration of patient treatment, reduced nurse exhaustion, maintaining work-life equilibrium, and restricted staffing expenditures, Hamid et al. [5]. The NSP of healthcare organizations needs to be optimized carefully because inadequate schedule planning has severe outcomes. Lack of staffing could lead to compromised care quality, stress levels, nursing staff turnover, and decreased productivity among the staff. In contrast, there are increasing chances of incurring unnecessary expenses and underutilising the human resource, Patil [6]. In dealing with NSP, practitioners and researchers have tried and tested different solutions from a basic manual system to the most sophisticated computerized scheduling systems, Sarkar et al. [7]. Previously, conventional approaches to assigning students to groups were based on a subjective assessment by the teacher or simple spreadsheet applications, which are relatively ineffective and can result in mistakes and take a lot of time. However, while people had to schedule the meetings through a secretary in the past, with the help of modern technologies, automated scheduling has almost become a trend. These systems also adopt optimization methods like fuzzy programming and the goal programming to create noticeable and efficient plans in scheduling, Mertler [8]. The proposed work focuses on developing a method that handles the NSP, with the primary objective of reducing schedule costs while improving nurse sharing across different hospital departments. The NSP aims to assign a sufficient number of nurses to meet the demands of the hospital. Over the past 50 years, NSP has emerged as a significant challenge in artificial intelligence and operations research. Potential benefits of automated high-quality plan creation include improvements in administrative workload, staff and patient satisfaction, medical resource efficiency, and staff and patient safety. Here we have the following research objectives.

1. We develop a fuzzy programming model to analyse the NSP in hospital demand.
2. Identify the objectives for the Emergency, ICU, and Orthopedic departments and identify the minimal flow.
3. Minimize the costs of different departments and optimize the staff in the different departments.

The initial section offers an introductory overview, followed by the second, which presents a comprehensive literature review. The third section delineates the methodologies employed, while the fourth section concentrates on the formulation of the model. The fifth section defines the problem, while the sixth section highlights the novel contributions of the study. In conclusion, the findings are presented in the final section.

2. LITERATURE REVIEW

Many studies carried out to investigate scheduling practices in healthcare have recently focused on hospital nurses. The methods that have been employed most frequently in solving NSPs are of three types: algorithms, which comprise machine learning and mathematical optimization. This paper enumerates the following approaches to managing NSPs using mathematical programming models: Goal programming, Mixed integer programming, linear programming and network approaches. Warner [9] proposed an MP-based algorithm and suggested a multi-choice

programming model aimed at maximizing nurses' preferences regarding their desired working hours, shift-switching options, and the number of days off they wish to have. Arthur & Ravindran [10] was the first to apply this method with four key objectives: minimizing staff size, reducing the number of staff with unmet requests or preferences, minimizing staff dissatisfaction, and minimizing the gap between planned and required staffing levels. Musa & Saxena [11] presented a simplified GP algorithm with one phase that incorporated the hospital's timing rules as well as the nature of the nurse's decision-making regarding working on weekends.

Ozkarahan & Bailey [12] suggested a more flexible concept that can accommodate both the hospital's and nurses' needs for decision support. The system provided several models to support flexible work schedules while solving the time of the day (TOD) and day of the week (DOW). Chen & Yeung [13] integrated GP and ES, where job types used in the expert system were assigned to nurses. The GP approach helped manage the nurses' busy schedules to meet hiring requirements. Jaumard et al. [14] introduced a general linear programming model of the complex NSP for different nursing skills and skill sets using the approach of creating new columns for the optimized transformation and employing branch and bound. Khan et al. [15] studied estimation of the parameter of inverted Topp-Leone distribution based on records. Millar & Kiragu [16] utilized a network model to assess nurses' organizational requirements for cyclic and non-cyclic shifts. Each point in the network represented potential work and/or off-stretch timing, and the resultant model was transformed into a "shortest path" model with additional complications. Gascon et al. [17] analyzed the issue of rostering nurses working in a particular hospital's flying squad. The problem was developed into a multi-objective programming model, wherein the objectives listed included goals and deviations from these goals. Bilgin et al. [18] developed a single, comprehensive hyper-heuristic approach for managing the NSP. Yilmaz [19] proposed a mathematical programming model for scheduling nurses. The goal was to minimize idle waiting time during a one-week planning period, using Lingo software to construct the model.

Akbari et al.[20] designed a simulated annealing method for the NSP while accounting for possible worker fatigue. Legrain et al.[21] investigated the nurse scheduling process for regular and float teams, solving staffing problems. Two models were introduced to reduce costs and enhance nurse availability through frequent candidate selection, and another utilised a local search technique. Jafari et al.[22] aimed to increase the number of nurses obtaining their preferred shift choices while reducing the excess nursing workforce needed daily. Four separate fuzzy solution approaches were evolved. Ali et al. [23] created a lexicographic goal programming framework emphasising GDP, sustainability, and employment in critical economic sectors to assess and maximize India's progress toward the SDGs. Duenas et al. [24] introduced a multiobjective NSP incorporating nurses' preferences, represented by fuzzy sets, to form an overall preference cost function. Fitriani et al. [25] addressed the NSP using fuzzy goal programming combined with the MINIMAX approach. Adhami & Ahmad [26] proposed a Pythagorean-hesitant fuzzy algorithm for solving the multiobjective transportation problem under uncertainty. Aydas et al.[27] proposed models to analyse fixed and dynamic nurse staffing policies. The dynamic model derived non-stationary patient demand from patient history data. Khalil & Modibbo [28] examined the NSP, considering factors like rotation, length of stay, weekly cycles, and daily shifts. A binary goal programming method was used to solve the problem. Melethil et al.[29] used neutrosophic programming and the time series technique ARIMA to achieve the sustainable development goals of Canada's Agenda 2030. Ali et al. [30] created a healthcare multiobjective issue model with fermatean fuzzy programming and neutrosophic goals.

El Adoly et al.[31] developed a mathematical model for the NSP using a multi-commodity network flow model. Mutingi & Mbohwa [32] introduced a fuzzy genetic algorithm (FGA) approach to address healthcare staff scheduling in fuzzy environments. Tsao et al.[33] focused on the NSP to solve the problem of maximizing nurse satisfaction with working conditions, subject to legal constraints and shift preferences. Melethil et al.[34] proposed a multiobjective optimization problem for the SDGs Agenda 2030 model of India, using lexicographic goal programming. A comparative analysis with fuzzy programming was also conducted. Otero-Caicedo et al.[35] suggested a multiobjective linear model with daily preventive backup nurse plans for staff

scheduling, employing an optimal genetic algorithm for timely solutions.

3. METHODOLOGY

3.1. Multiobjective Programming Problem

The primary goal of Multiobjective Programming Problems (MOPPs) is to determine compromise solutions which satisfy contradictory objectives according to, Ehrgott & Wiecek [36]. Decision-makers in multiple real-world applications must evaluate several simultaneous criteria, including cost, efficiency, and environmental impact. Various objectives cannot agree, making finding one solution that meets all requirements impossible.

The main objective lies in identifying multiple solutions that represent optimal trade-offs since each solution optimization leads to degradation in at least one of the competing objectives. Decision-makers choose a solution by conducting a trade-off analysis to determine the most fitting option based on their established preferences and priorities. The efficient solution of MOPPs can be accomplished using three principal methods: weighted sum methods, goal programming, and fuzzy optimisation. A MOPP solves a mathematical problem using this representation:

$$\begin{aligned} & \max/\min && [f_1(x), f_2(x), \dots, f_k(x)] \\ & \text{sub. to} && g_j(x) = 0, \forall j \\ & && x \geq 0, \end{aligned} \tag{1}$$

where, $f_k(x)$ is k^{th} objective function.

4. FUZZY SET (FS)

The universal set named W contains elements which operators label as w according to, Singh [37]. A fuzzy set X in W generates its membership characteristics through the membership function $\mu_X(w) : W \rightarrow [0, 1]$. This function associates every $w \in W$ with a real-valued $\mu_X(w)$ specified within $[0, 1]$. The value $\mu_X(w)$ represents the extent to which w belongs to set X through its calculation of the membership grade for w within X .

The complete inclusion status matches a membership value of 1, while complete exclusion corresponds to a value of 0. The membership function assigns intermediate values for any element depending on its level between full membership and not being present at all. The ability to define membership grades from complete inclusion at 1 to full exclusion at 0 makes fuzzy sets efficient for dealing with uncertain and imprecise real-world scenarios. Converting a fuzzy number into a precise or crisp value is known as *defuzzification* Adhami et al.[38]. For a parabolic fuzzy number $\tilde{X}(p, q, r)$, the de-fuzzified value is determined using the formula:

$$d(\tilde{X}) = \frac{p + 2q + r}{4}. \tag{2}$$

4.1. Fuzzy Programming

Fuzzy Multiobjective Decision Making originates from, Zimmermann [39] where they used fuzzy sets to develop their solution for resolving multiple optimization objectives. The approach holds such importance because it allows the simultaneous resolution of numerous conflicting goals that appear in all practical problems. A compromise solution exists that serves as a balanced choice after analysing how different objectives affect each other. The fuzzy approach establishes an organized framework to work with this situation through decision systems involving preference determination.

The transformation process in this method transforms every objective function into membership functions, showing the objective satisfaction level. Each objective receives its corresponding aspiration level set by the decision-maker, who then develops the fuzzy membership functions.

The main objective combines maximizing the minimum satisfaction among all objectives with achieving balanced trade-offs. The membership function for minimization-type objective functions with k -th order includes the following mathematical expression:

$$\mu(f_k(\underline{x})) = \begin{cases} 1 & \text{if } f_k(\underline{x}) \leq L_k \\ \frac{U_k - f_k(\underline{x})}{U_k - L_k} & \text{if } L_k < f_k(\underline{x}) < U_k \\ 0 & \text{if } f_k(\underline{x}) \geq U_k \end{cases} \quad (3)$$

The membership function for the k -th objective function in a maximization-type problem can be represented as:

$$\mu(f_k(\underline{x})) = \begin{cases} 1 & \text{if } f_k(\underline{x}) \geq U_k \\ \frac{f_k(\underline{x}) - L_k}{U_k - L_k} & \text{if } L_k < f_k(\underline{x}) < U_k \\ 0 & \text{if } f_k(\underline{x}) \leq L_k \end{cases} \quad (4)$$

The membership function for the k -th objective function of a minimization-type objective can be expressed as follows:

$$U_k = \text{Maximize}[f_k(\underline{x})] \text{ and } L_k = \text{Minimize}[f_k(\underline{x})]$$

$\forall k=1,2,\dots,K$

The general mathematical formulation of the fuzzy optimization technique can be described as follows:

We want to maximize λ subject to the constraints:

$$\begin{aligned} & \text{maximize } \lambda \\ & \text{subject to } \lambda \leq \mu(f_k(\underline{x})) \quad \forall x \\ & \quad \quad \quad g_j(x) = 0 \quad \forall j \\ & \quad \quad \quad x \geq 0 \end{aligned} \quad (5)$$

5. FORMULATION OF MODEL

The Nurse Scheduling Problem (NSP) includes n departments which are designated as D_i and m shifts which use the notation S_j for $i \in 1 \dots n$ and $j \in 1 \dots m$. The cost of placing i -th department nurses at j -th shift operations is denoted as C_{ij} ($i = 1, 2, \dots, n; j = 1, 2, \dots, m$). Also the i -th department needs L_{ij} and U_{ij} ($i = 1, 2, \dots, n; j = 1, 2, \dots, m$) staff members respectively for each shift. A decision variable named X_{ij} shows the number of nurses who will work between department D_i at shift S_j . The goal is to reduce total scheduling costs with nurses distributed enough to fulfil the specified lower and upper nursing requirements across all departments during all shifts.

The general mathematical model for the NSP is given as,

$$\begin{aligned} & \text{Minimize } Z_k = \sum_{j=1}^m c_{ij}x_{ij} \quad \forall j, \\ & \text{Minimize } Z_{k+1} = \sum_{i=1}^n \sum_{j=1}^m x_{ij}, \\ & \text{Subject to } \sum_{i=1}^n \sum_{j=1}^m x_{ij} = f, \\ & \quad \quad \quad \sum_{i=1}^n x_{ij} - S_j = 0 \quad (\text{requirement for the } j\text{-th shift}) \quad \forall j = 1, 2, \dots, m, \end{aligned}$$

$$D_i \sum_{j=1}^m x_{ij} = 0 \quad (\text{availability at the } i\text{-th department}) \quad \forall i = 1, 2, \dots, n,$$

$$x_{ij} \geq 0 \quad \text{and integer} \quad \forall i, j.$$

6. MULTIOBJECTIVE NSP WITH FUZZY PARAMETER

Whether fuzzy or probabilistic, uncertainty plays a significant role in developing decision-making models. The ambiguity of parameters stems from the imprecise estimates provided by decision-makers. Sometimes, it is essential to account for this uncertainty when solving the problem. This study assumes that the cost parameter in the objective function is fuzzy. Adhami et al. [38] To convert this fuzziness into a precise value, ranking functions based on fuzzy set theory are utilised. The following describes the defuzzified NSP being addressed: The general mathematical model for the NSP is as follows:

$$\text{Minimize} \quad Z_k = \sum_{j=1}^m c_{ij} x_{ij} \quad \forall j,$$

$$\text{Minimize} \quad Z_{k+1} = \sum_{i=1}^n \sum_{j=1}^m x_{ij},$$

$$\text{Subject to} \quad \sum_{i=1}^n \sum_{j=1}^m x_{ij} = f,$$

$$\sum_{i=1}^n x_{ij} - S_j = 0 \quad (\text{requirement for the } j\text{-th shift}) \quad \forall j = 1, 2, \dots, m,$$

$$D_i \sum_{j=1}^m x_{ij} = 0 \quad (\text{availability at the } i\text{-th department}) \quad \forall i = 1, 2, \dots, n,$$

$$x_{ij} \geq 0 \quad \text{and integer} \quad \forall i, j.$$

The Equation (2) may be employed to defuzzify the cost parameter, which is presumed to be a parabolic fuzzy number within the objective function of the problem above.

$$\text{Minimize} \quad Z_k = \sum_{j=1}^m \bar{d}(c_{ij}) x_{ij} \quad \forall j,$$

$$\text{Minimize} \quad Z_{k+1} = \sum_{i=1}^n \sum_{j=1}^m x_{ij},$$

$$\text{Subject to} \quad \sum_{i=1}^n \sum_{j=1}^m x_{ij} = f,$$

$$\sum_{i=1}^n x_{ij} - S_j = 0 \quad (\text{requirement for the } j\text{-th shift}) \quad \forall j = 1, 2, \dots, m,$$

$$D_i \sum_{j=1}^m x_{ij} = 0 \quad (\text{availability at the } i\text{-th department}) \quad \forall i = 1, 2, \dots, n,$$

$$x_{ij} \geq 0 \quad \text{and integer} \quad \forall i, j.$$

6.1. The steps for solving the Multiobjective Programming Problem

Fuzzy Programming is a robust and convenient method that can be used to solve several multiple-objective decision-making problems. As a result, we may use this method to determine the formulated model's optimum order amount. The sequential procedure is described as follows:

Step 1: First, defuzzify the fuzzy costs of different departments using the defuzzification formula.

- Step 2: Use the provided constraints to solve the multiple objective optimization problems by focusing solely on a single objective at a time while ignoring the others. The outcome that arises is the ideal solution. Utilizing the ideal solutions, the payoff matrix is constructed. Finally, the payoff matrix builds each objective function's aspiration level.
- Step 3: The k th objective function's aspiration level is chosen as the goal value $g_k (k = 1, 2, \dots, K)$. Find $x = (x_1, x_2, \dots, x_n)$ for optimizing the fuzzy goals $f_k(x) = g_k \forall k = 1, 2, \dots, K$ subject to the constraint $g_j(x) = 0, \forall j$ where, $(g_k) = \text{Max/Min}(f_k(x))$. The symbol ' \leq ' (type of fuzzy-minimum) specifies that the actual value should be less than or equal to the optimum value within a given tolerance range level. When referring to this the symbol ' \geq ' (type of fuzzy-maximum) specifies that the actual value should be approximately greater than or equal to the optimal value up to a specific tolerance limit.
- Step 4: The fuzzy linear membership function should be constructed as Eqn (3) for the minimization type and Eqn (4) for the maximization type objective function.
- Step 5: Using the linear membership function, the multiobjective model of Equation (1) will be converted to the form of Equation (5). Then, the Lingo 16.0 software is used to obtain the compromise solution of the defined multiobjective model.

7. NUMERICAL PROBLEM

The research problem in this paper seeks to understand how the issue of nurse scheduling occurs in a hospital setting, and hereby, this study employs a case study research strategy. The particular case study is based on a New England hospital. The study examines various views regarding nurse schedules in a hospital environment, Khan & Lewis [40]. This particular hospital has an authorised strength of 250 beds, but it's recently been struggling to recruit staff for many existing nursing vacancies because nursing costs are on the rise. Because hospital management's policy decisions directly impact the number of staff in different departments, there has to be effective nurse scheduling. Ways through which management polices affect department staffing include:

1. The general nurse staff in the hospital must meet a minimum requirement based on the hospital's capacity and the number of available beds.
2. The staff levels for each department must remain within the specified minimum and maximum limits for that department.
3. Nurses are allowed to rotate between departments and are considered a homogeneous group, including Registered Nurses (RN), Licensed Practical Nurses (LPN), and nurse aides. This assumption of uniformity is relaxed in a later section of the study.

The approximate eight-hour costs for nurses are available on this website web. The objective is to determine the minimum number of staff needed for hospital management and minimize the costs of nurses in the different departments.

¹ There are three departments, namely the emergency, the neonatal intensive care unit, and the orthopedics department. A hospital emergency department is a specially designated unit structured and operated to provide high-quality emergency care to individuals requiring immediate or acute medical attention, including potential hospital admission. A neonatal intensive care unit (NICU), also known as an intensive care nursery, specializes in treating sick or premature newborns. The orthopedic department primarily performs surgeries on patients with neurological conditions. To ensure emergency preparedness, the minimum staffing level for a given shift is set slightly above the combined minimum staffing requirements of all departments.

¹Nursing Pay Guide

Table 1: Minimal Staff Required Per Day

| Department Info | | |
|-----------------|--------|-----------------------|
| Department | Name | Minimal Staff per Day |
| 1 | Dept 1 | 13 |
| 2 | Dept 2 | 32 |
| 3 | Dept 3 | 22 |

The different departments' costs are fuzzy and given in Table (2).

Table 2: Fuzzy Costs of the Different Departments by Shifts

| Shift Info | | Fuzzy Costs (D1, D2, D3) |
|------------|-----------|---|
| Shift | Time | Values |
| 1 | Morning | (110,120,130); (140,150,160); (90,100,110) |
| 2 | Afternoon | (90,100,110); (120,130,140); (120,130,140) |
| 3 | Night | (100,110,120); (105,115,125); (105,115,125) |

In the Table (2), the fuzzy costs of the different departments are presented as the fuzzy cost values for three different departments over three shifts. Each cell in the table represents a fuzzy number as a triplet, such as (110,120,130), which captures the range of possible costs within that department for a given shift. Using fuzzy numbers makes it possible to deal with uncertainty and variability in cost estimates between departments. Defuzzification (2) is applied using an appropriate method to simplify these fuzzy costs into crisp, single values for further analysis. The results of this defuzzification process are shown in Table (2), where each fuzzy cost is converted into a precise cost figure for more straightforward interpretation and decision making.

Table 3: De-fuzzified Values of Different Departments

| Shift Info | | De-fuzzified Costs (D1, D2, D3) |
|------------|-----------|---------------------------------|
| Shift | Time | De-fuzzified Costs (D1, D2, D3) |
| 1 | Morning | 120; 150; 100 |
| 2 | Afternoon | 100; 130; 130 |
| 3 | Night | 110; 115; 115 |

The Table (3) above shows the defuzzified cost values for each department over different shifts. These values are derived from the fuzzy costs shown in the previous table by applying a defuzzification method. The defuzzified values provide a single, accurate cost figure for each department and shift, making it easier to analyse and compare costs across shifts and departments. For example, in shift 1, the defuzzified costs for departments 1, 2, and 3 are 120, 150, and 100, respectively. This table enables more transparent decision-making by removing uncertainty and providing concrete cost estimates.

Now the staff requirements by shifts and department are given in Table (4).

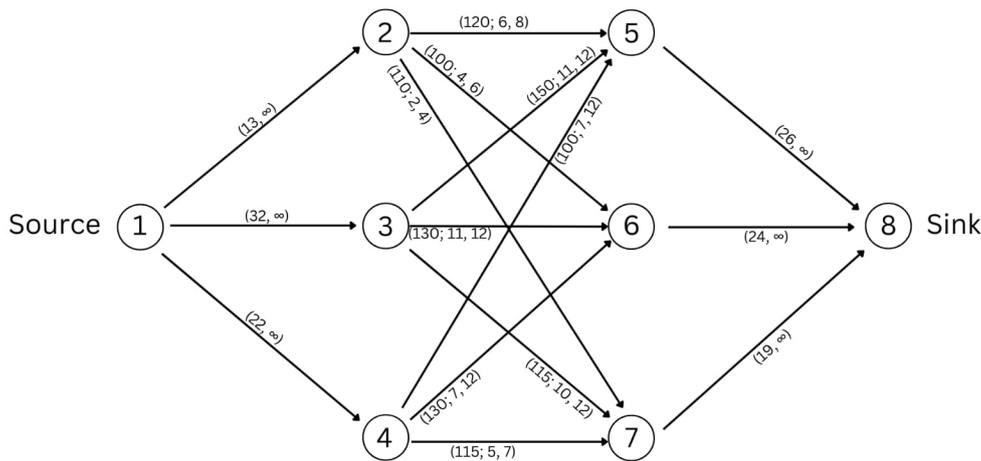


Figure 2

Figure 1: Network graph

Table 4: Staff Requirements by Shift and Department

| Shift and Nurses | | Departments | | |
|------------------|----------------|-------------|---------------|--------------|
| Shift | Minimal Nurses | Dept 1 | Dept 2 | Dept 3 |
| 1 | 26 | 120; (6, 8) | 150; (11, 12) | 100; (7, 12) |
| 2 | 24 | 100; (4, 6) | 130; (11, 12) | 130; (7, 12) |
| 3 | 19 | 110; (2, 4) | 115; (10, 12) | 115; (5, 7) |

We will draw a network using these data based on the problem's requirement and need presented in (4). The Network Staffing Problem will be as follows.

In this network, node 1 represents the source, and node 8 represents the sink. Nodes 2, 3, and 4 correspond to the departments, while nodes 5, 6, and 7 correspond to the shifts. The numbers on the arcs indicate staffing and cost requirements. For instance, the arc (1,3) signifies a need for 13 nurses with no upper limit. Similarly, the arc (2,5) indicates that between 6 and 8 nurses can be assigned to department 1, with a daily cost of \$120 during the first shift. The arc (5,8) specifies a staffing requirement ranging from 1 to 26 nurses, with no upper limit.

The mathematical model for this network is as follows:

$$\begin{aligned}
 \text{Min } Z_1 &= 120x_{25} + 150x_{26} + 100x_{27} && \text{(Emergency Cost)} \\
 \text{Min } Z_2 &= 100x_{35} + 130x_{23} + 130x_{37} && \text{(ICU Cost)} \\
 \text{Min } Z_3 &= 110x_{45} + 115x_{46} + 115x_{47} && \text{(Orthopedics Cost)} \\
 \text{Min } Z_4 &= 13x_{12} + 32x_{13} + 22x_{14} && \text{(Minimal Flow of Nurses)}
 \end{aligned}$$

Subject to

$$\begin{aligned}
 x_{12} + x_{13} + x_{14} &= f \\
 -x_{12} + x_{25} + x_{26} + x_{27} &= 0 \\
 -x_{13} + x_{35} + x_{36} + x_{37} &= 0 \\
 -x_{14} + x_{45} + x_{46} + x_{47} &= 0 \\
 -x_{25} - x_{35} - x_{45} + x_{58} &= 0
 \end{aligned}$$

$$\begin{aligned}
 -x_{26} - x_{36} - x_{46} + x_{68} &= 0 \\
 -x_{58} - x_{68} - x_{78} &= -f
 \end{aligned}$$

$$\begin{aligned}
 x_{12} \geq 13, x_{13} \geq 32, x_{14} \geq 22, x_{58} \geq 26, x_{68} \geq 24, x_{78} \geq 19 \\
 6 \leq x_{25} \leq 8, 4 \leq x_{26} \leq 6, 2 \leq x_{27} \leq 4, \\
 11 \leq x_{35} \leq 12, 11 \leq x_{36} \leq 12, 10 \leq x_{37} \leq 12, \\
 7 \leq x_{45} \leq 12, 7 \leq x_{46} \leq 12, 5 \leq x_{47} \leq 7.
 \end{aligned}$$

8. RESULTS:

We solve each objective by ignoring other objectives to obtain the payoff matrix, x , and using these solutions will be as follows:

Table 5: *Payoff Matrix*

| Objectives | | |
|------------|-----------|----------------------|
| Objective | Condition | Z1; Z2; Z3; Z4 |
| Z1 | Min Z1 | 3270; 3035; 1820; 73 |
| Z2 | Min Z2 | 3960; 2905; 1820; 77 |
| Z3 | Min Z3 | 3960; 3660; 1370; 81 |
| Z4 | Min Z4 | 3340; 2955; 1485; 69 |

Using the payoff matrix, we obtain the lower and upper values of each objective as;

$$3270 \leq Z_1 \leq 3960; \quad 2905 \leq Z_2 \leq 3660; \quad 1370 \leq Z_3 \leq 1820; \quad 69 \leq Z_4 \leq 81$$

We use these goal values for the goal programming to obtain the compromise solution. Using the LINGO 16.O software, we get the result as follows:

$$Z_1 = 3270; \quad Z_2 = 2905; \quad Z_3 = 1370; \quad Z_4 = 69$$

Now, we use the lower and upper values of the objective function to define the membership function as follows:

$$\mu(Z_1(x)) = \begin{cases} 1, & \text{if } Z_1(x) \leq 3270 \\ \frac{3960 - Z_1(x)}{3960 - 3270}, & \text{if } Z_1(x) \in (3270, 3960) \\ 0, & \text{if } Z_1(x) \geq 3960 \end{cases}$$

$$\mu(Z_2(x)) = \begin{cases} 1, & \text{if } Z_2(x) \leq 2905 \\ \frac{3660 - Z_2(x)}{3660 - 2905}, & \text{if } Z_2(x) \in (2905, 3660) \\ 0, & \text{if } Z_2(x) \geq 3660 \end{cases}$$

$$\mu(Z_3(x)) = \begin{cases} 1, & \text{if } Z_3(x) \leq 1370 \\ \frac{1820 - Z_3(x)}{1820 - 1370}, & \text{if } Z_3(x) \in (1370, 1820) \\ 0, & \text{if } Z_3(x) \geq 1820 \end{cases}$$

$$\mu(Z_4(x)) = \begin{cases} 1, & \text{if } Z_4(x) \leq 69 \\ \frac{81 - Z_4(x)}{81 - 69}, & \text{if } Z_4(x) \in (69, 81) \\ 0, & \text{if } Z_4(x) \geq 81 \end{cases}$$

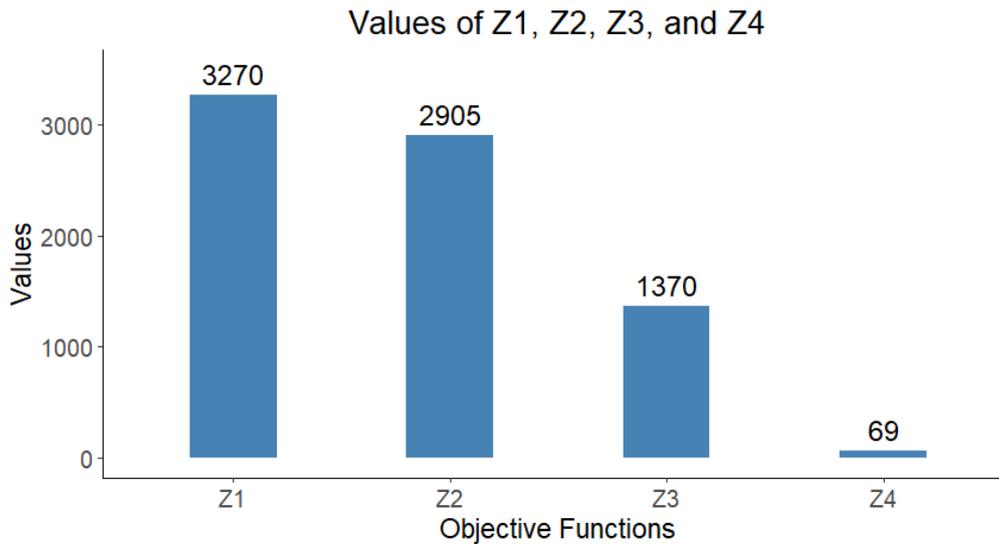


Figure 2: Values of objective function

Using these membership functions, we use fuzzy programming to obtain the compromise solution of the multiobjective programming problem as follows,

Table 6: Objective Values of Z1, Z2, Z3, and Z4

| Objective Functions | |
|---------------------|-------|
| Function | Value |
| Z1 | 3270 |
| Z2 | 2905 |
| Z3 | 1370 |
| Z4 | 69 |

Table (6) shows the costs of different departments in the hospital. For example, Z1 represents the emergency department, which has a minimum price of \$3270; Z2 represents the Intensive Care Unit (ICU), which has a minimum cost of \$2905; Z3 is the Orthopedic Department, which has a price of \$1370; and Z4 represents the minimal flow of nurses, which is 69.

Table (7) presents the optimal nurse scheduling plan, which requires 26 nurses for the first shift. Six nurses are assigned to Department 1, 11 to Department 2, and 9 to Department 3. For the second shift, 24 nurses are assigned, 6 to Department 1, 11 to Department 2, and 7 to Department 3. The third shift involves the allocation of 19 nurses: 2 to Department 1, 10 to Department 2, and 7 to Department 3. Across all three shifts, Department 1 receives 14 nurses, Department 2 has 32 nurses, and Department 3 is assigned 23 nurses, leading to a total of 69 nurses scheduled across all departments. These nurses perform essential duties, including patient care, communication with doctors, and medication administration. Their presence ensures smooth clinical operations and quality care in all departments. In addition to patient responsibilities, they also supervise nursing aides and assist in coordinating department-specific workflows.

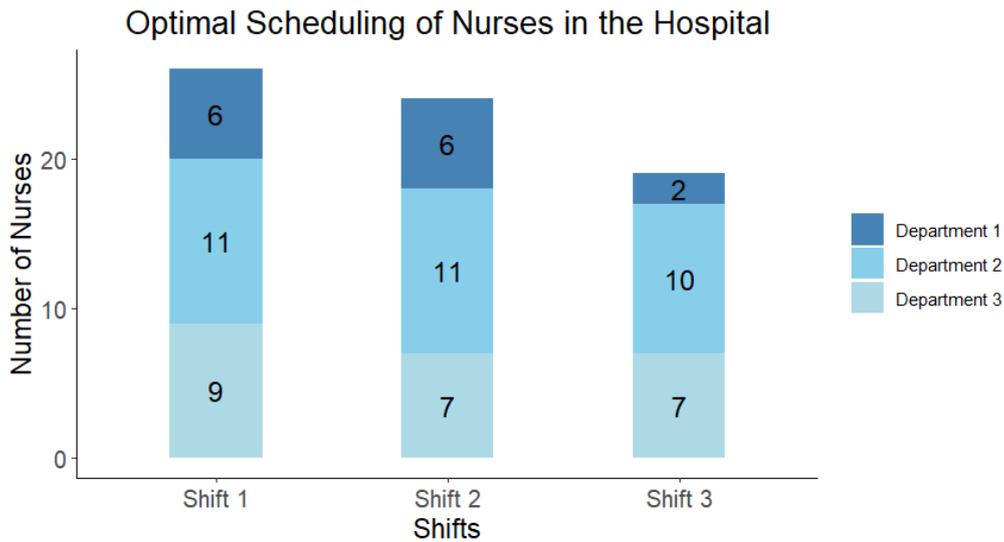


Figure 3: Optimal scheduling of nurses

Table 7: Optimal Scheduling of Nurses in the Hospital

| Shift-wise Distribution | | Departments | | |
|-------------------------|-----------|--------------|--------------|--------------|
| Shift | Time | Department 1 | Department 2 | Department 3 |
| 1 | Morning | 6 | 11 | 9 |
| 2 | Afternoon | 6 | 11 | 7 |
| 3 | Night | 2 | 10 | 7 |
| Total | | 14 | 32 | 23 |

9. CONCLUSION

This study makes a unique contribution by addressing the essential issue of nurse staffing and scheduling in hospitals, where cost minimization and resource allocation are crucial. Using fuzzy programming, we present a compromise approach that efficiently balances the requirement to reduce scheduling expenses while guaranteeing proper personnel distribution across departments. This strategy is essential in the healthcare industry, where insufficient staffing may have serious consequences both economically and in terms of the safety of patients.

Future studies should look at more adaptable models that consider parameter demand, different patient needs, and the application of real-time data to improve the scheduling process. Furthermore, implementing machine learning or optimization algorithms might enhance the decision-making framework, giving a more complete tackle to nurse staffing challenges in a constantly shifting healthcare system.

10. FUTURE WORK

This work provides the foundation for examining nurse staffing and hospital scheduling difficulties via an approach built upon fuzzy and compromise programming. Future research could expand upon this by exploring the following directions:

- **Adaptable Models:** Develop more adaptable scheduling models considering changing patient demands, departmental requirements, and dynamic demand swings.
- **Real-Time Data Integration:** Improve the precision and timeliness of scheduling systems by integrating real-time data from staff availability, patient intakes, and hospital activities.
- **Machine Learning Applications:** Automate the process of forecasting the number of employee requirements and utilising the right resources using ML techniques. These algorithms can also get optimized as trends change and make better decisions about efficiency.
- **Advanced Optimization Techniques:** Use sophisticated optimization techniques, including hybrid methods or meta heuristics, to solve large-scale staffing issues more precisely.

By pursuing these directions, future studies can provide even more robust solutions to the challenges of nurse staffing in the ever-evolving healthcare environment.

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