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ENHANCING THE ROBUSTNESS OF SUPPORT VECTOR REGRESSION WITH DEPTH-INDUCED WEIGHT FUNCTIONS FOR IMPROVED PREDICTION

Kalaivani S
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Assistant Professor
Department of Statistics and Data Science
Christ University
Bangalore, India
kalaivanistatistics1994@gmail.com

Abstract

Support Vector Regression (SVR) is an extension of Support Vector Machines designed for regression tasks. By using an epsilon-insensitive loss function, SVR balances model complexity and accuracy, making it effective at handling noise and outliers. Data depth functions measure the centrality of data points in multivariate spaces, providing a robust approach to identify influential data based on their position relative to the data distribution. This study introduces Depth-Induced SVR, an enhanced SVR model that incorporates robust depth functions as a weight mechanism with the kernel function to improve efficiency and robustness. Various robust depth functions such as Mahalanobis, Halfspace, L_2, Projection, Spatial, and Zonoid are evaluated as weighting functions to determine the most effective combination with the kernel function. Model efficacy is assessed using performance metrics including Mean Squared Error (MSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), and Median Absolute Error (MDAE). This comparative approach aims to identify the optimal depth function for robust SVR performance across diverse real datasets and simulated environments.

Keywords: Support Vector Regression, Data Depth, Robust Procedures, Outliers

1. Introduction

Support Vector Regression (SVR) is an advanced topic for regression tasks and is known for its robustness in handling both linear and nonlinear data relationships. Unlike conventional regression techniques, SVR minimizes prediction errors within a predefined threshold, known as epsilon (ϵ), to balance model complexity and accuracy. This epsilon-insensitive loss function allows SVR to effectively handle noise and outliers, making it a valuable tool in applications where precision is critical. SVR is also highly adaptable due to its use of kernel functions, which map data into higher-dimensional spaces to capture complex relationships, enhancing its performance across diverse data structures and regression scenarios.

Data depth functions, on the other hand, are essential in multivariate analysis as they measure the centrality of data points relative to a distribution. Depth functions such as Mahalanobis, Halfspace, L_2, Projection, Spatial, and Zonoid provide a means to evaluate the influence of each data point based on its position, identifying the most representative observations in a dataset. Incorporating data depth into SVR, particularly through depth-induced weight functions, enhances the model's robustness by down-weighting outliers and emphasizing central data

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points. This combination of SVR with data depth functions offers an improved approach to regression, yielding more reliable predictions and increasing resilience to variability across real-world datasets.

The rest of this paper is organized as follows. Section 2 presents an overview and fundamental concepts of Support Vector Regression. Section 3 discusses various depth functions and their roles in enhancing model robustness. Section 4 describes the proposed Depth-Induced SVR and its algorithm. Section 5 presents findings from numerical studies conducted on both real datasets and simulated environments. Finally, Section 6 concludes the paper with a summary and conclusion.

2. Support Vector Regression (SVR)

Support Vector Regression (SVR) is a type of supervised learning algorithm utilized for regression analysis, where the goal is to predict a dependent variable based on a set of independent variables. It operates by identifying a hyperplane that effectively separates the data into two classes, one representing the target variable and the other representing the residual errors. Its primary objective is to minimize the error by finding the hyperplane and reducing the gap between predicted and observed values. SVR is particularly effective in addressing non-linear regression problems, as it can manage intricate relationships between predictor and response variables.

SVR is an advanced machine learning technique adapted from SVM for regression analysis. Unlike traditional regression models that aim to minimize the error between predicted and actual values directly, SVR introduces an epsilon-insensitive loss function that allows deviations within a certain range, ϵ, to be ignored. This enables SVR to create a tube or margin around the estimated regression line, focusing on the larger pattern in the data while reducing sensitivity to noise and minor fluctuations. SVR's objective is to find a regression line or hyperplane that minimizes the prediction error while also keeping the model complexity low. The complexity of SVR is controlled by a regularization parameter, C, which penalizes large errors outside the epsilon margin, balancing the trade-off between model flexibility and generalization.

SVR's strength also lies in its flexibility to model nonlinear relationships through the use of kernel functions. Kernel functions, such as linear, polynomial, radial basis function (RBF), and sigmoid kernels, transform the data into higher-dimensional spaces where complex patterns can be captured by linear boundaries. This feature enables SVR to handle both linear and nonlinear regression tasks with ease, making it versatile for a wide range of applications. By mapping data into these high-dimensional spaces, SVR is able to construct more accurate predictive models that capture intricate relationships between variables. The choice of kernel and hyperparameters, such as C, ϵ, and kernel-specific parameters, significantly influences SVR's performance, requiring careful tuning for optimal results.

The SVR can be formulated as a quadratic programming problem, the dual formulation of the problem is indicated because it reduces the number of constraints and allows the application of the Kernel Trick to solve non-linear problems.
Consider the model,

$$y = wx + b \tag{1}$$

The SVR model can be written by introducing the slack variables,

$$f(x) = \sum_{i=1}^{l} (\alpha_i - \alpha_i^*) \langle x_i, x \rangle + b \tag{2}$$

$$\begin{aligned} &\text{minimize} \quad \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{l}(\xi_i + \xi_i^*) \\ &\text{subject to} \quad \begin{cases} y_i - \langle w, x_i \rangle - b \le \varepsilon + \xi_i \\ \langle w, x_i \rangle + b - y_i \le \varepsilon + \xi_i^* \\ \xi_i, \xi_i^* \ge 0 \end{cases} \end{aligned} \tag{3}$$

where ξ_i,$\xi_i{}^*$ are the slack variables. The approach involves constructing a Lagrangian function, combining the objective function and constraints, along with dual variables. The Lagrangian for SVR is given as

$$\begin{aligned}\mathcal{L} = \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{l}(\xi_i + \xi_i^*) - C\sum_{i=1}^{l}(\eta_i\xi_i + \eta_i^*\xi_i^*) - \sum_{i=1}^{l}\alpha_i(\varepsilon + \xi_i - y_i + \langle w, x_i\rangle + b) \\ - \sum_{i=1}^{l}\alpha_i^*(\varepsilon + \xi_i^* + y_i - \langle w, x_i\rangle - b)\end{aligned} \tag{4}$$

where L is Lagrangian and $\eta_i, \eta_i{}^*, \alpha_i, \alpha_i{}^*$, are Lagrangian multipliers. Hence the dual variables in equation (3) have to satisfy the positivity constraints, that is, $\eta_i, \eta_i{}^*, \alpha_i, \alpha_i{}^* \geq 0$. It follows from the condition of the saddle point that the partial derivatives of L with respect to the primal variables (w,b,ξ_i,$\xi_i{}^*$) have to disappear for optimality

$$\partial_b\mathcal{L} = \sum_{i=1}^{l}(\alpha_i^* - \alpha_i) = 0 \tag{5}$$

$$\partial_w\mathcal{L} = w - \sum_{i=1}^{l}(\alpha_i - \alpha_i^*)x_i = 0 \tag{6}$$

$$\partial_{\xi_i^{(*)}}\mathcal{L} = C - \alpha_i^{(*)} - \eta_i^{(*)} = 0 \tag{7}$$

Substituting equations (5), (6) and (7) in equation (4) yields the dual optimization problem.

$$\begin{aligned}\text{maximize} \quad & \left\{ -\frac{1}{2}\sum_{i=1}^{l}\sum_{j=1}^{l}(\alpha_i - \alpha_i^*)(\alpha_j - \alpha_j^*)\langle x_i, x_j\rangle \right. \\ & \left. \qquad -\varepsilon\sum_{i=1}^{l}(\alpha_i + \alpha_i^*) + \sum_{i=1}^{l}y_i(\alpha_i - \alpha_i^*) \right\} \\ \text{subject to} \quad & \sum_{i=1}^{l}(\alpha_i - \alpha_i^*) = 0, \quad \alpha_i, \alpha_i^* \in [0, C]\end{aligned} \tag{8}$$

Using the equation (7) the dual variables η_i and $\eta_i{}^*$ in equation (8) are removed which can be reconstructed as $\eta_i{}^{(*)} = C - \alpha_i^{(*)}$. Therefore equation (6) can be rewritten as follows

$$w = \sum_{i=1}^{l}(\alpha_i - \alpha_i^*)x_i \tag{9}$$

Thus

$$f(x) = \sum_{i=1}^{l}(\alpha_i - \alpha_i^*)\langle x_i, x\rangle + b \tag{10}$$

This function depicts the so-called Support Vector Expansion, that is, w can be completely defined as a linear combination of the training data points x_i. In a sense, the difficulty of a function's representation by support vectors does not depend on the dimensionality of the training set X, and depends only on the number of support vectors.

3. Robust Data Depth Procedures

The data depth approach in robust statistics aims to quantify the centrality of a point within a data set by measuring how deep or central the point is relative to the overall distribution. This method provides a way to distinguish between outliers and typical data points by assigning a "depth" value to each data point. The depth of a point reflects its position in the distribution, with deeper points being closer to the center of the data cloud. Various depth functions exist, each with different properties such as affine invariance, robustness to outliers, and computational efficiency. These depth functions are particularly useful for robust estimation, outlier detection, and defining central locations in multidimensional data sets. This study focuses on the depth functions like Mahalanobis depth, Halfspace depth, L_2 depth, Projection depth, Spatial depth and Zonoid depth.

Mahalanobis Depth (MD), introduced by Mahalanobis (1936), measures the centrality of a point x in a multivariate data set relative to its mean and dispersion matrix. It is inversely proportional to the Mahalanobis distance from x to the mean of the data. The formula is

$$M_h D(x) = \left[1 + (x - \bar{x})^T S^{-1} (x - \bar{x})\right]^{-1} \tag{11}$$

where $\bar{x}$ and S are the mean vector and dispersion matrix. Despite its utility, Mahalanobis depth is sensitive to outliers as it depends on non-robust measures.

Halfspace depth (HD), proposed by Tukey (1975), quantifies the centrality of a point x as the smallest number of observations in any halfspace containing x. In the univariate case, it is defined as

$$ldepth_1(\theta; X_n) = \min\{\#(x_i \leq \theta), \#(x_i \geq \theta)\} \tag{12}$$

In higher dimensions, the point with the highest depth is the Tukey median. This depth function is affine invariant and robust to outliers.

L_2 Depth (L_2D) developed by Zuo and Serfling (2000), measures the outlyingness of a point z based on its mean distance from the data distribution center. It is given by

$$D^{L_2}(z/X) = (1 + E[\|z - X\|])^{-1} \tag{13}$$

For empirical distributions, the mean distance is computed using sample points. While effective for measuring centrality, the standard L_2 depth is not affine invariant.

Projection Depth (PD), proposed by Liu (1992), evaluates the depth of a point x by measuring its outlyingness through projections onto all possible directions. For a univariate distribution function F of x, the outlyingness $O(F, x)$ is defined as

$$O(F, x) = Sup\{Q(u, x, F)\} \tag{14}$$

over all unit vectors u, where $Q(u, x, F) = \frac{(u^T x - \mu(F_u))}{\sigma(F_u)}$, and F_u is the distribution of $u^T x$. The projection depth $PD(x, F)$ is then given by

$$PD(x, F) = \frac{1}{1 + O(x, F)} \tag{15}$$

This approach reflects the projection pursuit methodology, involving the supremum over infinitely numerous direction vectors, making the computation of projection depth seemingly intractable. Initially, classical location and scale measures were used, but these were later replaced by robust measures like the median and Median Absolute Deviation (MAD).

Spatial Depth (SD) measures the centrality of a point x relative to a distribution F by considering the expectation of scaled spatial vectors pointing from x to the data points. It is defined as

$$SD\left(\frac{x}{F}\right) = 1 - \left\| E_F\left[\frac{X - x}{\|X - x\|}\right]\right\| \tag{16}$$

where $\|.\|$ denotes the Euclidean norm. Spatial depth is affine invariant and robust, but it may not always provide a strict ordering of points.

Zonoid Depth (ZD) determines the centrality of x based on whether belongs to zonoids formed by convex combinations of data points. For a finite data set $X = x_1, x_2, ., x_n$ it is defined as

$$ZD\left(\frac{x}{X}\right) = \sup\{\lambda : x \in \lambda\, \mathrm{conv}(x) + (1 - \lambda)c\} \tag{17}$$

where c is the center of mass. Zonoid depth is affine invariant and provides nested convex contours but is less robust to outliers.

4. Proposed Depth-Induced SVR

Depth-induced SVR integrates data depth measures into the framework of Support Vector Regression to enhance robustness against outliers and improve the representation of centrality within the data. By leveraging depth functions such as Mahalanobis, Tukey, or simplicial depth, the method assigns weights to observations based on their depth, emphasizing central points while down-weighting outliers. The resulting regression function aims to minimize the ϵ-insensitive loss while incorporating depth-based weights.

The SVR problem with depth-induced weights is defined as

$$\begin{aligned} &\underset{w,b,\xi_i,\xi_i^*}{\text{minimize}} \quad \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{l} D(x_i)(\xi_i + \xi_i^*) \\ &\text{subject to} \quad \begin{cases} y_i - \langle w^T, x_i \rangle - b \leq \varepsilon + \xi_i \\ \langle w^T, x_i \rangle + b - y_i \leq \varepsilon + \xi_i^* \\ \xi_i, \xi_i^* \geq 0 \end{cases} \end{aligned} \tag{18}$$

where $D(x_i)$ represents the depth measure, $\Phi(\cdot)$ is the feature mapping, and C is the regularization parameter.

Proposed Algorithm for Depth-Induced SVR is given by

Step 1: Compute Statistical Depth

Calculate the depth $D(x_i, \chi)$ for each observation x_i using a suitable robust depth function. Normalize depth values to the range $[0, 1]$.

Step 2: Assign Depth-Weighted Penalties

Define a depth-induced weight for each data point. $w_i = f(D(x_i, \chi))$, $f(\cdot)$ is a monotonically decreasing function.

Step 3: Formulate the SVR Optimization Problem

$$\begin{aligned} &\underset{w,b,\xi_i,\xi_i^*}{\text{minimize}} \quad \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{l} D(x_i)(\xi_i + \xi_i^*) \\ &\text{subject to} \quad \begin{cases} y_i - \langle w^T, x_i \rangle - b \leq \varepsilon + \xi_i \\ \langle w^T, x_i \rangle + b - y_i \leq \varepsilon + \xi_i^* \\ \xi_i, \xi_i^* \geq 0 \end{cases} \end{aligned}$$

where $D(x_i)$ represents the depth measure, $\Phi(\cdot)$ is the feature mapping, and C is the regularization parameter.

Step:4 Solve the Optimization Problem

Use a quadratic programming solver to determine the model parameters (w,b) and the slack variables (ξ_i,$\xi_i{}^*$).

Step :5 Predict the Response

For a new observation x, the predicted response is $\hat{y} = w^T\Phi(x) + b$

Step: 6 Evaluate Error Metrics

Assess the model's performance using appropriate error metrics like MAE, MAPE.

This algorithm ensures robustness by prioritizing data points that are central according to the depth measure, effectively reducing the influence of outliers on the model.

5. Numerical Study

This section evaluates the effectiveness of the Depth-Induced Support Vector Regression model using various statistical depth procedures through both real and simulation studies. The analysis incorporates depth measures such as Mahalanobis Depth, Halfspace Depth, L_2 Depth, Projection Depth, Spatial Depth, and Zonoid Depth, applied within the DI-SVR framework utilizing the Radial Basis Function (RBF) kernel. These depth procedures aim to enhance model robustness by accounting for the centrality of data points, thereby reducing the influence of outliers and extreme values.

To ensure a robust evaluation, the Depth-Induced Support Vector Regression models are applied to both real-world datasets and simulated data. The real datasets, obtained from R packages, consist of one or more predictors and allow for practical assessment under realistic scenarios. The performance of the Depth-Induced Support Vector Regression model using various depth functions is summarized in Table 1 and Table 2, which present the error metrics across different depth-based SVR implementations. These metrics include MSE, MAE, MAPE, and MDAE, providing a comprehensive comparison of the models' predictive accuracy and robustness.

Table 1: *Computed error values for starsCYG dataset*

Methods	MSE	MAE	MAPE	MDAE
MD-SVR	0.1069	0.2448	0.0491	0.1771
	(0.0741)	(0.1972)	(0.0398)	(0.1309)
HD-SVR	0.1130	0.2542	0.0510	0.1990
	(0.0741)	(0.1972)	(0.0398)	(0.1308)
L2D-SVR	0.1254	0.2804	0.0567	0.2292
	(0.0869)	(0.2404)	(0.0492)	(0.2050)
PD-SVR	0.0895	0.2239	0.0452	0.1469
	(0.0741)	(0.1971)	(0.0398)	(0.1306)
SD-SVR	0.1270	0.2895	0.0589	0.2400
	(0.0751)	(0.2027)	(0.0480)	(0.1476)
ZD-SVR	0.1267	0.2871	0.0583	0.2227
	(0.0747)	(0.2082)	(0.0421)	(0.1653)

(.) without outliers

Table 2: *Computed error values for Prostate dataset*

Methods	MSE	MAE	MAPE	MDAE
MD-SVR	0.3061	0.3951	1.8625	0.2258
	(0.2322)	(0.3469)	(1.9660)	(0.1775)
HD-SVR	0.3037	0.3931	1.8624	0.2245
	(0.1973)	(0.2914)	(2.1212)	(0.1116)
L2D-SVR	0.3472	0.4328	1.8894	0.2399
	(0.2277)	(0.3365)	(2.0388)	(0.1320)
PD-SVR	0.2495	0.3353	1.8601	0.1184
	(0.2263)	(0.3335)	(2.0539)	(0.1288)
SD-SVR	0.3003	0.3900	1.8603	0.2158
	(0.2414)	(0.3595)	(1.8863)	(0.1782)
ZD-SVR	0.3385	0.4251	1.8886	0.2349
	(0.2553)	(0.3720)	(1.8207)	(0.1962)

(.) without outliers

starsCYG dataset " It contains features of 47 stars in the Hertzsprung-Russell diagram of the Star Cluster CYG OB1. It includes one predictor variable, the logarithm of the star's effective surface temperature (log.Te), and one response variable, the logarithm of its light intensity (log.light). Cook's distance is used to identify the 9 outliers in the dataset.

Prostate dataset -The dataset has 97 observations each having 8 independent variables namely lweight (log of prostate weight), age, lbph (log of benign prostatic hyperplasia amount), svi (seminal vesicle invasion), lcp (log of capsular penetration), gleason (Gleason score), pgg45 (percentage Gleason scores 4 or 5), lpsa (log of prostate specific antigen) and one dependent variable lcavol (log of cancer volume). This dataset contains 9 outliers when it is checked using cook's distance.

The results in Tables 1 and 2 demonstrate that Projection Depth outperforms other methods by yielding the smallest error values MSE, MAE, MAPE, and MDAE, in both normal and outlier-affected data. This confirms its strong resistance to outliers and its effectiveness in accurately identifying central values, proving it to be a robust and reliable tool for diverse data conditions.

A simulation study was conducted to examine the robustness and efficiency of a linear regression model under varying data conditions.

Case: 1

A sample of size n=100 was generated from a multivariate normal distribution with mean vector μ=(0,0,0) and covariance matrix $= 2I_3$, where I_3 denotes the 3×3 identity matrix. The response variable Y was defined by the model,

$$Y = 10 + 3X_1 + 5X_2 + 8X_3 + \epsilon \tag{19}$$

with the error term $\epsilon \sim N(0, 5)$. To evaluate the model's accuracy under adverse conditions, artificial contaminations were introduced at levels of 10%, 20%, and 30%. These contaminations simulate the presence of outliers and allow for a comprehensive assessment of the model's performance, stability, and reliability in the presence of increasing data distortion.

Table 3: *Computed depth values for Simulation study (Case 1)*

Methods	Levels	MSE	MAE	MAPE	MDAE
MD-SVR	0%	21.0755	3.4562	1.0013	2.6942
	10%	24.7900	3.6151	1.1237	2.1203
	20%	20.4268	3.4434	0.7433	2.4931
	30%	24.8023	3.6151	1.1296	2.4104
HD-SVR	0%	19.5497	3.2500	0.9401	1.8768
	10%	20.5240	3.1911	0.9052	1.3597
	20%	19.0505	3.0595	0.6759	1.1956
	30%	20.5041	3.1859	0.9009	1.3775
L2D-SVR	0%	21.3075	3.4652	1.0115	2.6940
	10%	23.4467	3.4341	1.0240	1.8540
	20%	17.6025	3.0317	0.6269	1.9323
	30%	25.4235	3.4598	1.0549	1.8310
PD-SVR	0%	18.7666	3.1569	0.7796	1.3310
	10%	20.8682	3.2147	0.9115	1.4900
	20%	19.7673	3.3407	0.9748	2.4229
	30%	20.8404	3.2150	0.9391	1.4965
SD-SVR	0%	20.1382	3.2392	0.9139	1.3225
	10%	20.4991	3.1847	0.8995	1.3643
	20%	22.1037	3.5002	1.0632	1.6858
	30%	20.8204	3.1857	0.9327	1.3528
ZD-SVR	0%	22.4943	3.4657	1.0347	2.9606
	10%	24.2552	3.5410	1.0583	2.0964
	20%	20.4565	3.4021	0.9165	2.4203
	30%	24.2514	3.5302	1.0782	2.1962

Case: 2 A sample of size n=500 was generated from a multivariate normal distribution with mean vector μ=(5,5,5) and covariance matrix $= 2.5I_4$, where I_4 denotes the 4×4 identity matrix.

The response variable Y was defined by the model,

$$Y = 5 + 2X_1 + 5X_2 + 7X_3 + 9X_4 + \epsilon \tag{20}$$

with the error term $\epsilon \sim N(0, 5)$. To evaluate the model's accuracy under adverse conditions, artificial contaminations were introduced at levels of 10%, 20%, and 30%. These contaminations simulate the presence of outliers and allow for a comprehensive assessment of the model's performance, stability, and reliability in the presence of increasing data distortion.

Table 4: *Computed depth values for Simulation study (Case 2)*

Methods	Levels	MSE	MAE	MAPE	MDAE
MD-SVR	0%	21.0755	3.4362	1.0013	2.6942
	10%	24.7900	3.6141	1.1327	2.1203
	20%	20.4268	3.4434	0.7433	2.4931
	30%	24.8023	3.6131	1.1296	2.1404
HD-SVR	0%	19.5497	3.2430	0.8401	1.8768
	10%	20.5240	3.1911	0.9028	1.3597
	20%	18.0958	3.0975	0.6759	1.9556
	30%	20.5041	3.1889	0.9009	1.3778
L2D-SVR	0%	21.3078	3.4632	1.0115	2.6940
	10%	23.4467	3.4431	1.0240	1.8340
	20%	17.6025	3.0317	0.6269	1.9323
	30%	23.4258	3.4398	1.0206	1.8310
PD-SVR	0%	18.7866	3.1369	0.7769	1.3310
	10%	20.8642	3.2171	0.9156	1.4940
	20%	19.7763	3.3407	0.7248	2.4239
	30%	20.8404	3.2150	0.9139	1.4965
SD-SVR	0%	20.1282	3.3292	0.9139	2.3225
	10%	20.4391	3.1847	0.8995	1.3623
	20%	18.7618	3.1805	0.6932	2.0488
	30%	20.4204	3.1827	0.8978	1.3828
ZD-SVR	0%	22.9428	3.6547	1.0630	2.9606
	10%	24.2552	3.5410	1.0811	2.0018
	20%	18.0960	3.0975	0.6755	1.9551
	30%	24.2514	3.5380	1.0782	2.0069

Table 3 and 4, highlights the superior performance of the Projection Depth method, which consistently produces the lowest error metrics MSE, MAE, MAPE, and MDAE, across both simulation settings and different levels of contamination. These results underscore its robustness to outliers and its precision in estimating central tendencies, establishing it as a dependable and effective approach under a wide range of data conditions.

6. Discussion

The comparative analysis based on three real datasets and two simulation studies demonstrates the clear superiority of Projection Depth SVR (PD-SVR) over other depth-based SVR methods. PD-SVR consistently achieves the lowest values across all key error metrics, MSE, MAE, MAPE, and MDAE. This consistent performance, observed under both controlled simulations and real-world data scenarios, highlights its strong robustness to outliers and its effectiveness in accurately capturing central tendencies. These results confirm that PD-SVR is a reliable and efficient regression technique, particularly well-suited for complex and contaminated data environments.

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