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EFFICACY OF SINGLE SERVER MARKOVIAN QUEUEING SYSTEM WITH DISSATISFIED CUSTOMERS, BALKING AND NUMEROUS VACATION

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Abstract

In this research work, we derive a single server Markovian encouraged arrival queuing system with dissatisfied customers, balking, and numerous vacations, where we examine that encouraged arrival customers are due to the fewer customers, balking, and dissatisfied customers. Once a system-size enhancement is idle, the system server obtains vacation. Suppose the system server detritus is idle, after the sabbatical vacation. We inferred the probability-generating functions (PGF) and got the closed-form interpretation when the system server is numerous. In addition, we obtain other important performance measures and Little's law is also satisfied for this model.

Keywords: Encouraged arrival, dissatisfied customers, feedback, Numerous vacation, Little's Law.

1. Introduction

The breadth and interest in studying queuing systems involving server vacation have increased in the queuing theory literature. To maximize downtime, server vacations are employed. The manufacturing process, business services, communications networks, online technology, and other scenarios have all seen success with the application of vacation queuing models. Many scholars are eager to explore queuing models that incorporate various vacation regulations, including both single and multiple vacation norms.
On the whole, it is difficult to achieve success in closed-form bulk service queue modelling with servers inactive [1]. The study conducted in [2] examined an M/M/1 line via a working vacation model. Examines the optimal management approach regarding a heterogeneous M/M/1-queue that experiences server outages [3]. M/M/1 queues with several active vacations, including the N-Policy is given in [4] . Working with the matrix approach and the vacation line study in [5] was conducted. An investigation was conducted for the M/M/1 queue, including an individual active vacation is considered in [6]. An M/M/1 queuing systems with server vacation and client impatience were thoroughly analysed by [7], who found that consumers only felt irritated when the servers were used on vacation. The M/M/o queue model with vacations and dissatisfied

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consumers and Numerous flexible vacations, while K is a pre-assigned stable positive number instead of a random variable is taken into account in [8]. This kind of vacation is referred to as a variation of Banik's various vacations in [9]. Papers by [10] and [11] include literature about this kind of vacation. There has been research on an M/M/1/N queue mechanism featuring encouraged arrival [12]. Markovian encouraging arrival queues and then dissatisfied customers were all investigated [13], [14] and [15].

2. Model interpretation

We contemplate a single server Markovian encouraged arrival queuing system with dissatisfied customers, feedback, and a numerous vacation. As per the encouraged arrival procedure, customers engage a server once at a moment $(1+\iota)$, where ι denotes the encouraged arrival rate. μ Follows a service rate identical distribution. η is a feedback rate. Customers follow FIFO discipline. A server does nothing except wait for the next customer to arrive. The vacation rates, denoted by χ are supposed to have an exponential distribution of vacation times. Customers are irritated throughout the vacation. It means that a "Dissatisfied timer" (t) is set off every time a customer accesses the system. It has an identical distribution with ψ as the parameter. When a customer's timer goes off and their service is still unfinished, they leave the line and never reappear.

The closed-form equations for the average system sizes in various server states equations with additional significant performance metrics are also obtained. Assume that the Length of the system T, and then $I(T) = i$ describes at time T, the system executes the $(\mathrm{i}+1)^{\mathrm{th}}$ vacation for $\mathrm{i} = 0, 1, \ldots, \mathrm{H}-1$. Then, the continuous-time Markov process (CTMP) with state $\varphi = \{(m, i) : m \geq 0, i = 0, 1, \ldots, H\}$.

3. The Set of Differential-Difference Equations of the Model

We write the basic equations and derive the probability differential-difference equations Using the model interpretation for the single server Markovian encouraged arrival queuing system with dissatisfied customers, feedback, and a numerous vacation as follows:

$$(\lambda * (1+\iota) + \chi) * P_{00} = \psi * P_{10} + \mu * P_{1H} \tag{1}$$

$$(\lambda * (1+\iota) + \chi + m\psi) * P_{m0} = \lambda * (1+\iota) * P_{m-10} + (m+1) * \psi * P_{m+10}, m \geq 1 \tag{2}$$

$$(\lambda * (1+\iota) + \chi) * P_{0i} = \psi * P_{1i} + \chi * P_{0i-1}, i = 1, 2, \ldots, H-1 \tag{3}$$

$$\begin{aligned} \lambda * (1+\iota) + \chi + m * \psi) * P_{mi} = \lambda * (1+\iota) * P_{m-1i} + (m+1) * \psi * P_{m+1i} \\ i = 1, 2, \ldots, H-1, m \geq 1 \end{aligned} \tag{4}$$

$$\lambda * (1+\iota) * P_{0H} = \chi * P_{0H-1} \tag{5}$$

$$(\lambda * (1+\iota) + \mu) * P_{mH} = \lambda * (1+\iota) * P_{m-1H} + \mu * P_{m+1H} + \chi \sum_{i=0}^{H-1} P_{mi}, m \geq 1 \tag{6}$$

By using the normalizing condition:

$$\begin{aligned} \sum_{m=0}^{\infty} \sum_{i=0}^{H} P_{mH} &= \frac{\lambda * (1+l) * P_{m-1H} + \mu * P_{m+1H} + \chi \sum_{i=0}^{H-1} P_{mi}}{(\lambda * (1+l) + \mu)} \\ &= 1 \end{aligned} \tag{7}$$

Define the (PGFs) as,

$$R_i(Z) = \sum_{m=0}^{\infty} P_{mi} Z^m, 0 \leq Z \leq 1, i = 0, 1, \ldots, H$$

Then, multiply the all equations for m in equations. (1) to (4) via Z^m, we obtain,

$$\psi * (1 - Z) * R_0'(Z) - [\lambda * (1 + \iota) * (1 - Z) + \chi] * R_0(Z) = -\mu * P_{1H} \tag{8}$$

And

$$\psi * (1 - Z) * R_i'(Z) - [\lambda * (1 + \iota) * (1 - Z) + \chi] * R_i(Z) = -\chi * P_{0i-1} \tag{9}$$

Likewise, by using the equations. (5) and (6) we get

$$(1 - Z) * (\lambda * (1 + \iota) * Z - \mu) * R_H(Z) = \chi * Z \sum_{i=0}^{H-1} R_i(Z) + (Z - 1) * \mu * P_{0H} \tag{10}$$

Next, we derive the equations (8) and (9) by closed form.
Equation. (8) will be given as :

$$R_0'(Z) - \left[\frac{\lambda * (1 + \iota)}{\psi} + \frac{\chi}{\psi * (1 - z)} \right] * R_0(Z) = -\frac{\mu * P_{1H}}{\psi * (1 - z)} \tag{11}$$

To solve the differential Equation. (11), by

$$e^{-\frac{\lambda * (1+\iota)}{\psi} * Z} * (1 - Z)^{\frac{\chi}{\psi}}$$

Then, we obtain

$$\frac{d}{dZ} * \left[e^{-\frac{\lambda*(1+\iota)}{\psi} * Z} (1 - Z)^{\frac{\chi}{\psi}} * R_0(Z) \right] = -\frac{\mu}{\psi} * P_{1H} e^{-\frac{\lambda*(1+\iota)}{\psi} * Z} * (1 - Z)^{\frac{\chi}{\psi} - 1}$$

$\int$ ing 0 to Z, we claim that,

$$R_0(Z) = \frac{e^{\frac{\lambda*(1+l)}{\psi} * Z} * \left\{ R_0(0) - \frac{\mu}{\psi} * P_{1H} \int_0^Z (1 - y)^{\frac{\chi}{\psi} - 1} * e^{-\frac{\lambda*(1+l)}{\psi} * y} dy \right\}}{(1 - Z)^{\frac{\chi}{\psi}}} \tag{12}$$

Since $R_0(1) = \sum_{m=0}^{\infty} P_{m0} < \infty$ and $Z = 1$, we have that $z = 1$ must be ought to should be the numerator's value on the right side of Equation (12).

So, we get

$$R_0(0) = \frac{\mu}{\psi} * D * P_{1H} \tag{13}$$

Where,

$$D = \int_0^1 e^{-\frac{\lambda*(1+\iota)}{\psi} * y} * (1 - y)^{\frac{\chi}{\psi} - 1} dy \tag{14}$$

Not $R_0(0) = P_{00}$, Equation. (13) implicit

$$P_{1H} = \frac{\psi}{\mu * D} * P_{00} \tag{15}$$

By substitute the Equation. (15) to (12), we get,

$$R_0(Z) = \frac{e^{\frac{\lambda*(1+l)}{\psi}} * Z}{(1 - Z)^{\frac{\chi}{\psi}}} * \left[1 - \frac{1}{D} \int_0^Z (1 - Y)^{\frac{\chi}{\psi} - 1} e^{-\frac{\lambda*(1+l)}{\psi} * y} dy \right] * P_{00} \tag{16}$$

Eq. (9) will be given as

$$R_i'(Z) - \left[\frac{\lambda * (1+\iota)}{\psi} + \frac{\chi}{\psi * (1-z)}\right] * R_i(Z) = -\frac{\chi * P_{0i-1}}{\psi * (1-Z)} \tag{17}$$

In likewise Equation. (11), implicit

$$R_i(Z) = \frac{e^{\frac{\lambda*(1+l)}{\psi}*Z} * \left\{R_i(0) - \frac{\chi}{\psi} * P_{0i-1} * \int_0^Z (1-y)^{\frac{\chi}{\psi}-1} * e^{-\frac{\lambda*(1+l)}{\psi}*y} dy\right\}}{(1-Z)^{\frac{\chi}{\psi}}} \quad i = 1, 2, \ldots, H-1$$

Since $R_i(1) = \sum_{m=0}^{\infty} P_{mi} < \infty$ and $Z = 1z = 1$ must be ought to should be the numerator's value on the right side of Equation (18).

So, we get

$$P_{0i} = R_i(0) = \frac{\chi}{\psi} * D * P_{0i-1}, i = 1, 2, \ldots, H-1 \tag{19}$$

Where D is determine by Equation. (14). By Use Equation. (19) Again, we get

$$P_{0i} = B^i * P_{00}, i = 1, 2, \ldots, H-1 \tag{20}$$

Where $B = \frac{\chi}{\psi} * D$. By Substitute Equation. (20) into (18), we get

$$R_i(Z) = \frac{e^{\frac{\lambda*(1+\iota)}{\psi}*Z} * B^i}{(1-Z)^{\frac{\chi}{\psi}}} * \left\{1 - \frac{1}{D} * \int_0^Z (1-y)^{\frac{\chi}{\psi}-1} * e^{-\frac{\lambda*(1+\iota)}{\psi}*y} dy\right\} * P_{00} \quad i = 1, 2, \ldots, H-1 \tag{21}$$

By Using Equations. (5) and (20), we get

$$P_{0H} = \frac{\chi}{\lambda * (1+l)} B^{H-1} * P_{00} \tag{22}$$

Next, we procure the P_{00}, and the average service system- size is in numerous states.

For $i = 0, 1, \ldots, H$, let us assume that L_i is the system size in the state i. Hereafter, $E(L_i)$ is the average system size in the state i, which is determined is given by

$$E(L_i) = R_i'(1) = \sum_{m=1}^{\infty} m * P_{mi}, i = 0, 1, \ldots, H$$

i.e., for $i = 0, 1, \ldots, H-1, E(L_i)$ means the average system size, the system-server proceeds the $(i+1)^{\text{th}}$ vacation, and $E(L_H)$ means engaged system is idle or free. Determine $E(L_i)$ for $i = 0, 1, \ldots, H-1$.

Applying the L'Hopital rule to Equation. (11), we obtain

$$R_0'(1) = \lim_{Z \to 1} \frac{[\lambda * (1+\iota) * (1-Z) + \chi] * H_0(Z) - \mu * P_{1H}}{\psi * (1-Z)} = \frac{-\lambda * (1+\iota) * H_0(1) + \chi * H_0'(1)}{-\psi}$$

Thus, we obtain

$$(\chi + \psi) * H_0'(1) = \lambda * (1+\iota) * H_0(1) \tag{23}$$

Likewise, from the Equation. (17), we obtain

$$(\chi + \psi) * H_I'(1) = \lambda * (1+\iota) * H_i(1), i = 1, 2, \ldots, H-1 \tag{24}$$

Equations. (23) & (24) inferred

$$E\left(L_i\right)=\frac{\lambda*(1+\iota)}{\chi+\psi}*H_I(1), I=0,1,\ldots,H-1 \tag{25}$$

For $i=0,1,\ldots,H$, let us assume that $P_{.i}=H_i(1)=\sum_{m=0}^{\infty}P_{im}$. Hereafter, for $i=0,1,\ldots,H-1, P_{.j}$ means the server proceeds the $(i+1)^{\text{th}}$ vacation, and $P_{.H}$ means the server is engaged or free.

Applying the L'Hopital rule, from Equations. (16) & (21), we obtain

$$P_{.i}=R_i(1)=B^{i-1}*P_{00}, i=0,1,\ldots,H-1 \tag{26}$$

We may write Equation. (25) as follows by using Equation. (26).

$$E\left(L_i\right)=\frac{\lambda*(1+\iota)}{\chi+\psi}*B^{i-1}*P_{00} \tag{27}$$

Besides, the average system is on vacation, indicated $E\left(L_U\right)$, as follows:

$$E\left(L_U\right)=\sum_{i=0}^{H-1}E\left(L_i\right)=\frac{\lambda*(1+\iota)}{\chi+\Psi}*\frac{1-B^H}{B(1-B)}*P_{00} \tag{28}$$

Here after, we inferred $P_{.H}\&P_{00}$. Beginning with Equations. (15), (20) & (26), we get $\mu*P_{1H}=\chi*P_{.0}\ \&P_{0i-1}=P_{.i}, i=1,2,\ldots,H-1$. Thus, we obtain

$$\mu*P_{1H}+\chi*\sum_{i=0}^{H-2}P_{0i}=\chi*\sum_{i=0}^{H-1}P_{.i} \tag{29}$$

We may write Equation. (29) as follows by using Equation. (10)

$$R_H(Z)=\frac{\chi*Z}{\lambda*(1+\iota)*Z-\mu}*\frac{\sum_{i=0}^{H-1}\left[R_i(Z)-P_{.i}\right]}{1-Z}-\frac{\mu*P_{0H}}{\lambda*(1+\iota)*Z-\mu} \tag{30}$$

When we use the L'Hopital rule, we obtain

$$R_H(1)=\frac{\chi*\sum_{i=0}^{H-1}R_i'(1)+\mu*P_{0H}}{\mu-\lambda*(1+\iota)} \tag{31}$$

Remarking $R_H(1)=P_{.H}$ and $R_i'(1)=E\left(L_i\right), i=0,1,\ldots,H-1$, beginning with Equation. (31), we get

$$P_{.H}=\frac{\chi*\sum_{i=0}^{H-1}E\left(L_i\right)+\mu*P_{0H}}{\mu-\lambda*(1+\iota)} \tag{32}$$

Equation (32), when we substitute Equations. (22) and (28) yields

$$P_{.H}=\frac{\chi}{\mu-\lambda*(1+l)}*\left[\frac{\lambda*(1+\iota)}{\chi+\psi}*\frac{1-B^H}{B(1-B)}+\frac{\mu}{\lambda*(1+\iota)}*B^{H-1}\right]*P_{00} \tag{33}$$

It is simple to observe that the normalizing condition (7) can instead be expressed via a description of $P_{.i}$.

$$\sum_{i=0}^{H}P_{.i}=1 \tag{34}$$

When we replace Equations (26) and (33) in (34), we obtain

$$P_{00}=\left\{\frac{\mu*\chi+(\mu-\lambda*(1+l))*\psi}{(\mu-\lambda*(1+l))*(\chi+\psi)}*\frac{1-B^H}{B*(1-B)}+\frac{\mu*\chi}{\lambda*(1+l)*(\mu-\lambda*(1+l))}*B^{H-1}\right\}^{-1} \tag{35}$$

Equation (35) is substituted into equation (28), yielding

$$E\left(L_U\right)=\frac{\left(\lambda*(1+l)^2(\mu-\lambda*(1+l))\right.}{\mu*\chi*[\lambda*(1+l)+(\chi+\psi)K(H)]+\lambda*(1+l)*\psi*(\mu-\lambda*(1+l))} \tag{36}$$

Where

$$K(H)=\frac{B^H(1-B)}{1-B^H}$$

We now determine $E\left(L_H\right)$. Applying the L'Hopital rule to Eq. (30), we obtain

$$E\left(L_H\right)=\frac{\chi}{2*(\mu-\lambda*(1+\iota))}*\sum_{i=0}^{H-1}R_i''(1)+\frac{\mu*\chi}{(\mu-\lambda*(1+\iota))^2}\sum_{i=0}^{H-1}R_i'(1) \tag{38}$$

Where $R_i''(1)$ is secure by second derivative $R_i(\mathrm{Z})$ where Z should be 1 . Second derivative Equations. (8) to (9), we get

$$\begin{aligned}-2*\psi*R_i''(\mathrm{Z})+\psi*(1-\mathrm{Z})*\tfrac{d^3}{dZ^3}R_i(\mathrm{Z})=[\lambda*(1+\iota)*(1-\mathrm{Z})+\chi]*R_i''(\mathrm{Z})\\-2*\lambda*(1+\iota)*R_i'(\mathrm{Z}),i=0,1,\ldots,K-1\end{aligned} \tag{39}$$

Putting Z = 1 in Equation. (39), we obtain

$$R_i''(1)=\frac{2*\lambda*(1+\iota)}{\chi+2*\psi}*R_i'(1),i=0,1,\ldots,H-1 \tag{40}$$

When we replace Equation (40) with Equation (38), we get

$$E\left(L_H\right)=\frac{\chi}{\mu-\lambda*(1+l)}*\left(\frac{\mu}{\mu-\lambda*(1+l)}+\frac{\lambda*(1+l)}{\chi+2*\chi}\right)*E\left(L_U\right) \tag{41}$$

In this case, P_{0H} is determined via Equations. (22) and (35) while $E\left(L_U\right)$ is determined by use of Equation. (36).

$$P_{0H}=\frac{\chi*(\mu-\lambda*(1+l))*(\chi+\psi)*K(H)}{\mu*\chi*[\lambda*(1+l)+(\chi+\psi)*K(H)]+\lambda*(1+l)*\psi*(\mu-\lambda*(1+l)} \tag{42}$$

in which Eq. (37) yields K(H).

Let us assume that L be the expected no. of consumers of the queuing system.
The average size of the system $E(L)=E\left(L_U\right)+E\left(L_H\right)$ beginning with Equations. (36) and (41)

4. Special Privilige

Two specific instances of the alternative vacation policy covered in this work are the single v and the numerous vacations.
Privilege 1. Numerous vacation models. If $H=\infty$, hereafter $K(\infty)=0$. Beginning with Equations. (36) and (41), we get

$$E\left(L_U\right)=\frac{\lambda*(1+\iota)*(\mu-\lambda*(1+\iota))}{\mu*\chi+\psi*(\mu-\lambda*(1+\iota))}$$

and

$$E\left(L_H\right)=\frac{\chi}{\mu-\lambda*(1+\iota)}*\left(\frac{\mu}{\mu-\lambda*(1+\iota)}+\frac{\lambda*(1+\iota)}{\chi+2*\psi}\right)*\frac{\lambda*(1+\iota)*\mu}{\mu*\chi+\psi*(\mu-\lambda*(1+\iota))}$$

Privilege 2. Unique vacation method. If H should be one, hereafter $K(1)=B$. Beginning with Equations. (36) and (41), we get

$$E\left(L_U\right)=\frac{\left(\lambda*(1+\iota)^2*(\mu-\lambda*(1+\iota)\right)}{\mu*\chi*[\lambda*(1+\iota)+(\chi+\psi)*B]+\lambda*(1+\iota)*\psi*(\mu-\lambda*(1+\iota))}$$

and

$$E\left(L_H\right)=\frac{\chi}{\mu-\lambda*(1+\iota)}\left(\frac{\mu}{\mu-\lambda*(1+\iota)}+\frac{\lambda*(1+\iota)}{\chi+2*\psi}\right)E\left(L_U\right)+\frac{\lambda*(1+\iota)*\mu}{(\mu-\lambda*(1+\iota))^2}*\frac{\chi*(\mu-\lambda*(1+\iota))(\chi+\psi)*\text{ B}}{\mu*\chi*[\lambda*(1+\iota)+(\chi+\psi)*B]+\lambda*(1+\iota)*\psi*(\mu-\lambda*(1+\iota))}$$

5. Performance Evaluation

We extract some other significant performance metrics in this section.

1. The probability of a server being away on vacation is provided by

$$P_U=\sum_{i=0}^{H-1}P_{.i} \tag{43}$$

When we change the Equation. (26) to Equation. (43), we get

$$P_U=\frac{1-B^H}{B*(1-B)}*P_{00}$$

Applying Equation (28), we obtain

$$P_U=\frac{\chi+\psi}{\lambda*(1+\iota)}*E\left(L_U\right) \tag{44}$$

where $E\left(L_U\right)$ is provided by Equation. (36).

2. The probability that the server is engaged is provided by

$$P_a=\sum_{m=1}^{\infty}P_{mH}=1-P_{0H}-P_U \tag{45}$$

By replacing Equations. (42) and (44) through Eq. (45) and applying Eq. (36), we get

$$P_a=\frac{\lambda*(1+l)*\chi*[\lambda*(1+l)+(\chi+\psi)*K(H)]}{\mu*\chi*[\lambda*(1+l)+(\chi+\psi)*K(H)]+\lambda*(1+l)*\psi*(\mu-\lambda*(1+\iota))} \tag{46}$$

When we replace a continuous variabley, for the numeric H in the R.H.S of Eq. (46), we obtain the f(y), which is represented by $S(y)$. Applying a derivative of $S(y)$ we obtain

$$S'(y)=\frac{(\lambda*(1+\iota))^2\chi*\psi*(\mu-\lambda*(1+\iota))(\chi+\psi)*K'(y)}{\{\mu*\chi*[\lambda*(1+\iota)+(\chi+\psi)*K(y)]+\lambda*(1+\iota)*\psi*(\mu-\lambda*(1+\iota))\}^2}<0$$

The imbalance arises beginning with $\mathrm{H}'(\mathrm{y})<0$. Therefore, S(y) for declined. As that a result, when H increases, P_a lowers.

3. The expected no. of customers served per time is $\mu*P_a$, inferred that the segment of customers served is provided by

$$P_Q=\frac{\mu*P_a}{\lambda*(1+\iota)}$$

Where P_a is provided by Equation. (46).

4. The mean rate of desertion due to dissatisfaction can be described as, the average impatience customers obtain by

$$G_b = \sum_{i=0}^{H-1} \sum_{m=1}^{\infty} m * \psi * P_{mi} = \psi * E\left(L_U\right) \tag{48}$$

Where $E\left(L_U\right)$ is provided by Equation. (36).

6. Numerical Illustration

In this part, we examine the impact of H and ψ on several performance indicators numerically where $\Psi = 0.5, 1.0, 1.5$.

λ	ι	β	μ	χ	H	ψ
4	0.25	0.1	5	2	$1, 2, 3, 4, \ldots, \infty$	$0.5, 1.0, 1.5$

Table 1: *Performance metrics with $\lambda^*(1+\iota), \beta, H$, and ψ fluctuations for ψ values of 0.5, 1.0, and 1.5, ψ values should be minimum values*

H	ψ	$E\left(L_U\right)$	$E\left(L_H\right)$	$E(L)$	P_u	P_a	G_b
1	0.5	0.0419	0.1096	0.1515	0.2095	0.0958	0.0215
	1.0	0.0343	0.1054	0.1397	0.2058	0.0931	0.0343
	1.5	0.0289	0.1027	0.1316	0.2023	0.0913	0.0434
2	0.5	0.0391	0.1080	0.1417	0.1955	0.1843	0.0196
	1.0	0.0323	0.1043	0.1366	0.1938	0.1647	0.0323
	1.5	0.0275	0.1020	0.1295	0.1925	0.1576	0.0413
3	0.5	0.0365	0.1065	0.1430	0.1825	0.1560	0.0183
	1.0	0.0304	0.1033	0.1337	0.1824	0.1500	0.0304
	1.5	0.0261	0.1012	0.1273	0.1827	0.1486	0.0391
4	0.5	0.0341	0.1051	0.1392	0.1705	0.1470	0.0171
	1.0	0.0287	0.1023	0.1310	0.1722	0.1420	0.0287
	1.5	0.0248	0.1005	0.1253	0.1736	0.1323	0.0372
5	0.5	0.0319	0.1039	0.1358	0.1595	0.1298	0.0159
	1.0	0.0271	0.1015	0.1286	0.1626	0.1220	0.0271
	1.5	0.0235	0.0997	0.1231	0.1645	0.1150	0.0353

Table 2: *Little's Law verification*

$\lambda * (\mathbf{1} + \boldsymbol{\iota})$	L	W	L = $\boldsymbol{\lambda} * (\mathbf{1} + \boldsymbol{\iota}) \boldsymbol{W}$
1.25	0.1515	0.1212	0.1515
1.25	0.1397	0.11176	0.1397
1.25	0.1316	0.10528	0.1316
1.25	0.1417	0.11336	0.1417
1.25	0.1366	0.10928	0.1366
1.25	0.1295	0.1036	0.1295
1.25	0.1430	0.1144	0.1430
1.25	0.1337	0.10696	0.1337
1.25	0.1273	0.10184	0.1273
1.25	0.1392	0.11136	0.1392
1.25	0.1310	0.1048	0.1310
1.25	0.1253	0.10024	0.1253
1.25	0.1358	0.10864	0.1358
1.25	0.1286	0.10288	0.1286
1.25	0.1231	0.09848	0.1231

Table 1. shows that E(L_H) and the average system size E(L)decrease with ψ for every value are finite. P_a, the probability that the server is operating, is very much dropping, while P_u, the chance that the server is on vacation, is growing by ψ. Comparison of the Poisson model this method is very much effective.

In this part, we examine the impact of H and ψ on several performance indicators numerically where $\Psi = 2.5, 3.0, 3.5$.

λ	ι	β	μ	χ	H	ψ
4	0.25	0.1	5	2	$1, 2, 3, 4, \ldots, \infty$	$2.5, 3.0, 3.5$

Table 3: *Performance metrics with $\lambda * (1 + \iota), \beta, H$, and ψ fluctuations for ψ values of* 0.5, 1.0, *and* 1.5, ψ *values should be maximum values*

H	ψ	$E(L_U)$	$E(L_H)$	$E(L.$	P_u	P_a	G_b
1	2.5	0.0221	0.0996	0.1217	0.1989	0.0889	0.0553
	3.0	0.0197	0.0985	0.1182	0.1970	0.0882	0.0591
	3.5	0.0178	0.0975	0.1153	0.1958	0.0875	0.0623
2	2.5	0.2116	0.0991	0.1202	0.1904	0.0798	0.0529
	3.0	0.1890	0.0972	0.1144	0.1890	0.0750	0.0567
	3.5	0.0172	0.974	0.1146	0.1892	0.0711	0.0602
3	2.5	0.0203	0.0986	0.1189	0.1827	0.0687	0.0508
	3.0	0.0183	0.0978	0.1161	0.1830	0.0650	0.0549
	3.5	0.0166	0.0971	0.1137	0.1826	0.0632	0.0581
5	2.5	0.0187	0.0978	0.1165	0.1683	0.0584	0.0468
	3.0	0.0169	0.0970	0.1139	0.1690	0.0540	0.0507
	3.5	0.0155	0.0965	0.1120	0.1705	0.0512	0.0543
7	2.5	0.0173	0.0970	0.1143	0.1557	0.0487	0.0483
	3.0	0.0157	0.0964	0.1121	0.1570	0.0444	0.0471
	3.5	0.0145	0.0960	0.1105	0.1595	0.0408	0.0508

Table 4: *Little's law verification*

$\lambda * (\mathbf{1} + \iota)$	L	W	L $= \boldsymbol{\lambda} * (\mathbf{1} + \boldsymbol{\iota})\boldsymbol{W}$
1.25	0.1217	0.09736	0.1217
1.25	0.1182	0.09456	0.1182
1.25	0.1153	0.09224	0.1153
1.25	0.1202	0.09616	0.1202
1.25	0.1144	0.09152	0.1144
1.25	0.1146	0.09168	0.1146
1.25	0.1189	0.09512	0.1189
1.25	0.1161	0.09288	0.1161
1.25	0.1137	0.09096	0.1137
1.25	0.1165	0.0932	0.1165
1.25	0.1139	0.09112	0.1139
1.25	0.1120	0.0896	0.1120
1.25	0.1358	0.10864	0.1358
1.25	0.1286	0.10288	0.1286
1.25	0.1231	0.09848	0.1231

Table 3. shows that $E(L_H)$ and the average system size $E(L)$ all the values dropping with ψ for every value are finite $H.P_u$ and P_a values are growing neither dropping with ψ when $H = 2$

and $H = 3$. That represents that P_v and P_b are not unmodulated functions of ψ when $H \neq \infty$. Comparison of the Poisson model this method is very much effective.

7. Remarks

1. The single-vacation model describes the present model's reduction if $H = 1$. The numerous vacation concept is the reduced version of the existing model if $H = \infty$.
2. It is simple to verify that $\psi - \chi D > 0$, Thus, we obtained $0 < B < 1$.
3. Beginning with Equation. (27) and $0 < B < 1$, it is simple to check that the average system size $E(L_i)$ is a dropping of i for $i = 0, 1, \ldots, H-1$.

Likewise, beginning with Equation. (32), the not equality $P_{.H} > 0$ is exactly equal to $\lambda * (1 + \iota) < \mu$. So, $\lambda * (1 + \iota) < \mu$ is a standard condition for our system. Therefore, let us assume hereafter that $\lambda * (1 + \iota) < \mu$.

8. Conclusion

In this research work, we have developed a single-server Markovian encouraged arrival queuing system with balking, dissatisfied customers, and numerous vacations, where the customer dissatisfaction via the server is on vacation. We have applied a 25% discount on our model, the system size increased, and the waiting time has been reduced. This result is very efficient compared with the Poisson arrival model, and we have obtained the other important performance measures, and Little's law is also satisfied for this model. The effects of the parameters $\lambda^*(1 + \iota)$,β, ψ, and H on some performance measures have been investigated numerically.

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