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COMPARATIVE VAR FORECASTING USING LOW AND HIGH-FREQUENCY CONDITIONAL EVT MODELS: EVIDENCE FROM NIFTY INDICES IN INDIA

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Abstract

This study uses Value-at-risk (VaR) and Expected Shortfall (ES) to determine if the availability of high-frequency data improves the accuracy of estimating severe market risk as compared to low-frequency data. The daily closing stock prices of the NIFTY 50, NIFTY 100, and NIFTY 200 Index from January 1, 2021, to April 31, 2025, made up the sample data utilised for analysis. The purpose of the data study was to evaluate the effectiveness of conditional EVT, a two-stage hybrid strategy that merged the EVT methodology with the GARCH, RV, and HAR specification models. Unconditional coverage (UC) and conditional coverage (CC) tests were used to backtest the out-of-sample VaR predictions in order to evaluate their correctness. The regulatory loss function (RLF) and the firm's loss function (FLF) are two utilisation loss functions that were also included in the VaR backtesting process. The generalised breach indicator (GBI) approach was used to backtest the anticipated ES's accuracy. The results of this study demonstrated that when it came to forecasting market risk during times of extraordinary returns, high-frequency conditional EVT that included the HAR specification performed better than low-frequency conditional EVT. When compared to the GARCH-EVT and RV-EVT typed models throughout the periods, the HAR-EVT typed models perform the best according to the VaR and ES measures. The hybrid model of long-memory models for the EVT method is the focus of this research work, which adds to the body of knowledge on the forecasting abilities of risk models.

Keywords: Conditional EVT models, High-frequency data, VaR, NIFITY indices, Expected shortfall.

I. Introduction

Modern finance theory relies heavily on volatility, the most widely utilised indicator of uncertainty. Risk management, portfolio optimisation, and financial asset pricing all depend on the ability to measure and predict volatility. The stock market's performance is assessed and its movement is predicted using a variety of volatility models. Accurate volatility prediction is a critical skill for stock market practitioners and scholars. The Indian government's numerous initiatives since 1991, the effects of "liberalisation," "privatisation," and "globalisation" policies, as well as ensuing financial sector reforms, have all contributed to the substantial shifts and structural changes in the Indian stock market [29]. Since the 1970s, when options trading first began, its effect on stock market volatility has undergone substantial modification. Numerous studies show that

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futures trading lowers spot market volatility by integrating advanced risk management systems, removing information asymmetry, increasing the amount of information available to the price formation process, improving price discovery efficiency, and increasing profits by minimising risk [30]. During periods of significant market volatility, like the global financial crisis of 2007–2009 and the 1987 U.S. stock market crash, this occurrence is most common. Furthermore, when derivatives are introduced to the Indian stock market, they create significant opportunities for hedging and speculating, which lead to price instability and amplify the underlying market volatility [31]. However, when compared to cash market assets, a vast number of investors and financial managers view derivatives as highly captive due to their low initial requirements with the clearing house for futures, the payment of premiums for options, and their low transaction costs [32, 33]. Since there is no solid empirical evidence that the index option causes volatility to fluctuate, over the past few decades, scholars and industry professionals have been very interested in the economic impact of Nefty index derivatives on stock market volatility [34]. Regarding the effect of index derivatives on the volatility of the underlying market, this presents significant queries for investors, researchers, academics, and financial managers [35]. The majority of previous research has actually demonstrated that the introduction of index futures may result in a stabilisation or a reduction in volatility [36, 37]. In addition to upsetting Asian economies, the crisis had an impact on international markets, which made Western investors reevaluate their exposure to developing nations like India. Stronger regulatory frameworks and risk management techniques are required, as the period's elevated levels of uncertainty and capital flight demonstrated the Indian financial system's susceptibility to outside shocks. As investors reacted to the growing health crisis and its possible economic ramifications, the world's financial markets saw precipitous drops. Financial markets saw large selloffs and reductions as a result of worries about a global economic slump, decreased consumer spending, and disrupted supply chains. Recent research has shown that COVID-19 has a significant influence on global stock prices due to its high volatility and risk uncertainty [1,2]. This study is motivated to use the current COVID-19 era as the sample data for analysis because of the influence that financial crises have on stock markets, as was previously described. This decision stems from the lack of previous studies on the effectiveness of high-frequency conditional EVT in predicting stock market risk following the COVID-19 pandemic.

Previous research has demonstrated how well the GARCH model predicts stock market risk using financial time series analysis and low-frequency data. Because of its capacity to capture persistence and clustering of volatility, the GARCH model has been widely used and proven to be effective in modelling and predicting volatility across a range of financial markets. These days, the availability of high-frequency data has encouraged academics to explore risk forecasting models other than the traditional GARCH model. It is essential to investigate the presence of long memory in intraday returns, particularly with reference to the RV and HAR criteria, which provide additional incentive for this work, as researchers have been paying close attention to the existence of long memory in daily returns. Since the financial stock market is known to be susceptible to rapid swings during times of market volatility, risk-averse investors place a high priority on measuring market risk. The EVT technique is used in this study in conjunction with high-frequency models based on the RV and HAR requirements. It was believed that by using a hybrid technique that combines high-frequency models with EVT, tail risk estimates would be enhanced by better capturing severe market occurrences. The statistical framework of EVT makes modelling and analysis easier by concentrating on the extreme returns of tail distributions, whereas the HAR model concentrates on the volatility cascade over various investment horizons.

The conditional EVT model is the general term for the hybrid strategy that combines any low-frequency or high-frequency models with the EVT technique. It has been demonstrated in earlier studies that the conditional EVT model improves the capacity to predict and evaluate risk in tumultuous market situations; however, the majority of these studies employed daily returns as a stand-in for volatility. Since high-frequency data has become available, researchers have begun to investigate how intraday returns based on the RV criteria might enhance risk forecasting. Researchers attempt to integrate the EVT technique with the HAR model subsequent to the development of the HMH hypothesis. According to [3], the combined HAR-GARCH-EVT model, which accounts for the conditional heteroscedasticity of the HAR errors, performs better overall and generates accurate VaR estimates that successfully reduce Basel II regulatory capital requirements. The accuracy of the model is demonstrated in both the entire out-of-sample period and the 2007–2009 crisis period. The precision of the HAR-EVT models in predicting stock market risk has been further investigated. The performance of low-frequency conditional EVT models in comparison to high-frequency conditional EVT models may be further investigated thanks to the availability of high-frequency data. The conditional EVT fared better overall, according to the researchers' comparison of the high-frequency data. Better predictions are obtained when realised volatility is used as the volatility estimator for high-frequency data rather than daily returns. Several studies have been carried out to support the advantages of using the RV standard in conjunction with the EVT technique as a hybrid model. Although this field of study is still largely unexplored, the current work also uses the RV specification obtained from the HAR model in the context of the conditional EVT model. The goal of this work is to add to the body of knowledge on the forecasting capabilities of risk models by focussing on the long-memory FIEGARCH, RV-FIEGARCH, and HAR-FIEGARCH models using the as-yet-unexplored EVT technique. The results of the study will help advance understanding of how various data frequencies might affect stock market risk forecasting and offer precise models for use in investing strategies and risk management procedures.

Researchers debate whether using high-frequency data is worthwhile despite its benefits because of its greater cost, as demonstrated by a study by [4]. In addition to being more expensive, it takes longer to analyse than low-frequency data. The benefits of employing this high-frequency data could exceed the drawbacks for traders and investors with substantial cash. They will be able to increase their investment and reduce their possible loss thanks to this high-frequency data's capacity to react swiftly to market movements. The primary obstacle for other small traders and investors, however, will be the costs involved in gathering and evaluating high-frequency data. In these situations, the low-frequency data which is typically more accessible and less expensive will be selected. Some researchers who only use low-frequency data will lose out on chances provided by high-frequency data, particularly in the present big data era when it is projected that the growth of big data analytics and application database solutions will reach $12 billion by 2027 [5]. A thorough assessment of the possible benefits and drawbacks, as well as knowledge of the data's intended use and the particular trading or investing goals it would serve, should inform the choice to use high-frequency data. The cost concern provides further impetus for this research to investigate if, in comparison to low-frequency data, the possible advantages of using high-frequency data in data-rich markets are more or equal. If this expense will eventually result in significant value in the context of financial market modelling and forecasting, that remains to be seen.

The rest of the document is structured as follows: Section 2 reviews the relevant literature, Section 3 presents a theoretical framework of conditional extreme value theory, Section 4 describes the data analysis of NIFTY indices, and Section 5 provides the summary and conclusion of the study.

II. Literature Review

Researchers have paid close attention to volatility predictions using EVT. A framework for examining extreme occurrences and tail behaviour in financial markets is offered by EVT. Extreme quantiles and tail probabilities, which are essential for volatility forecasting, may be estimated thanks to EVT's emphasis on extreme data. EVT is the best option for examining the tail distribution that is marked by severe occurrences. Many research have been conducted to examine the EVT approach's accuracy in predicting stock market risk in comparison to other alternative models. According to a [6] study, the hybrid model GARCH-EVT works well for handling asymmetric tail distributions on the S&P Index. In contrast to other models that are not integrated with EVT, a follow-up investigation by [7] discovered that EVT is dependable in calculating the VaR and ES. According to a research by [8], the GARCH model and EVT work better together than the traditional standalone EVT model in forecasting stock market volatility during severe occurrences. When [9] examined the ISE-100, S&P-500, and Nikkei-225 indices, they discovered that the GARCH-EVT model performed better than the GARCH model alone. Furthermore, [10] discovered that the FIAPARCH-EVT model outperforms FIGARCH-EVT and HYGARCH-EVT in forecasting VaR one day in advance. The GARCH-EVT model outperformed the basic skewed-t GARCH model in predicting stock market risk during the Tehran Stock Exchange's capital slump era, according to another study by [11]. According to a research by [12], VaR forecasting on the daily returns data of twelve prominent international stock indexes was much enhanced by the GARCH-EVT model. The GARCH-EVT models fared better than other models in predicting intraday VaR during the COVID-19 pandemic, according to additional study by [13]. Furthermore, the GARCH-EVT-copula technique is the most effective way to quantify the VaR and ES both before and after the COVID-19 pandemic, according to a research by [14]. The GARCH-EVT-Copula-CoVaR model is accurate in predicting the spillover risk impact of global oil prices on Chinese sectoral stock markets, according to a recent research by [15].

Scholars are using high frequency data based on intraday data to predict stock market risk as a result of the recent revolution in high-frequency data. Rather than using the volatility of the daily returns to estimate the VaR and ES, the realised volatility is converted to fresh latent volatility. In contrast to low-frequency models that employed daily returns series, more recent study that combined realised volatility models with the EVT technique shown that it has more benefits and accurate estimates. The effectiveness of the realised GARCH model in the US equities market is investigated in a research by [16]. After that, [7] expand on [16] work and come to the conclusion that the realized-GARCH EVT models perform better in predicting when it comes to estimating VaR and ES. The EGARCH model in conjunction with the EVT technique is highly effective in forecasting a significant loss for precious metal markets, according to additional study by [17]. The [11] claim that the GARCH-EVT model beats the basic GARCH model with Student's t and normal distributions for residuals and allows for an accurate VaR calculation in this market as well. As was previously mentioned, the HAR model has demonstrated potential in capturing volatility patterns over various time scales since it takes into account the heterogeneity of market players' trading behaviours. When compared to the traditional GARCH model, [18] found that the HAR-GARCH specification better reflects the persistent memory patterns in volatility. The [19] found in another study that the HAR-RV models outperform their traditional counterparts in terms of VaR performance. When compared to other realised volatility models, the HAR-GARCH model demonstrated better predicting accuracy for Bitcoin volatility, according to [20] findings. The GARCH-EVT is better at forecasting the risk of high-frequency returns data at 15-minute intervals in the metal market, according to a recent study by [21].

III. Materials and Methods

I. Data collection

This study examines the NSE NIFTY 50, 100, and 200 Index, which represents a diversified portfolio of the 50 largest and most liquid stocks listed on the National Stock Exchange (NSE) of India. Serving as a key benchmark for the Indian equity market, the NIFTY 50, 100, and 200 Index reflects the overall performance of the market and provides insight into the economic condition of India. The study utilizes daily closing prices of the NIFTY 50, 100, and 200 Index, obtained from the official NSE website, to ensure data accuracy and consistency for robust statistical analysis. It includes 1072 observations of closing prices for the NSE Nifty index between January 1, 2021, and April 31, 2025. The preliminary analysis, in-sample empirical analysis, and out-of-sample analysis are the three categories of data analysis used to evaluate the VaR and ES prediction. Since P_n represents the closing stock prices, the logarithmic daily returns of the selected stock markets were calculated by applying the following formula to measure the difference between the logarithmic price levels on two consecutive days:

$$\{r_n\}_{n-1}^{T} = \text{logarithm}\left(\frac{P_n}{P_{n-1}}\right) \tag{1}$$

In two phases, the 5-minute logarithmic realised variance, or log RV_n, was computed. The first step was applying the R-programming code based on Equation (1) to convert the 5-minute stock prices into one-day trading returns. Assuming j = $2,3,4,5,6, \dots k.$, k represents the entire 5-min stock prices. Then, using the following formula, the one-day trading returns were converted to RV_n:

$$RV_n^d = \sqrt{\sum_{j=2}^{k} \left|R_{n,j}\right|^2} \tag{2}$$

where $n = 1,2,3 \dots m$ and where m stands the total-number of observations. The RV_n was then converted to logarithm RV_n. The rationale for selecting log RV_n over raw RV_n is that, according to a study by [22], the conditional heteroskedasticity is significantly reduced in log-RV. In-sample empirical analysis was carried out using the log-returns of r_n in the GARCH, EGARCH, and FIEGARCH models (low-frequency models), while the RV and HAR models (high-frequency models) used the log-realized variance, log RV. The EVT approach will be combined with the residuals series produced by each low- and high-frequency technique in the second stage of data processing. The conditional EVT is the name given to the two-stage combination strategy.

II. Model of Conditional EVT

In the present research, the combination of GARCH, RV, and HAR requirements with the EVT technique has been investigated using the conditional EVT method, which was first presented by [6]. In the initial step of the study, the original data series was filtered using the GARCH, EGARCH, and FIEGARCH models for low-frequency data, and the RV and HAR kinds of models for high-frequency data. The goal of this streaming procedure was to produce standardised residuals (innovations) that resembled the iid distribution, which is necessary for the EVT technique, which assumes that the filtered return series are free from heteroscedasticity effects and series correlation. Following the streaming procedure, the EVT parameters were determined using the filtered return series that was acquired from the streaming process. The peak-over-threshold (POT) approach, which takes into account extreme value observations that exceed a predetermined threshold regardless of the time series' clustering tendencies, was applied in this investigation. The Generalised Pareto Distribution (GPD) distributions will be estimated using the thresholds produced by the POT approach. The one-day VaR and ES forecast is estimated using the VaR and ES quantiles that are computed using the GPD parameters.

III. The POT Approach

The Block Maxima (BM) technique, which uses a generalized extreme value (GEV) distribution, and the POT method, which is based on the GPD distribution, are the two primary approaches in EVT that are used to simulate the extreme returns. The Block Maxima (BM) approach will not be evaluated in this research as volatility clustering, which causes extreme values to cluster together, makes it unsuitable for analyzing time series data. As a result, this study applied the POT approach, which takes into account extreme value observations that exceed a predetermined threshold regardless of the time series' clustering patterns. Considering the logarithmic return series of a stock index, represented by $= \{y_1, y_2, \ldots, y_t\}$, the POT method can be applied. Potential y values are the exceedance produced by extreme values that are higher than a given threshold v. The z or $F_v(z)$ cumulative probability function can be expressed as follows:

$$F_v(z) = P_r(X - v \leq z \mid X > v) \tag{3}$$

Equation (10) conditional probability may be reduced to:

$$F_v(z) = \frac{F(x) - F(v)}{1 - F(v)}, \tag{4}$$

Where $x = z + v$ is the exceedance and $z = x - v$ is the magnitude of exceedance.

IV. The POT Approach

The GPD distribution is one of the distributions utilised in EVT. The [23, 24] both state that when a threshold v is large enough, the cumulative probability function $F_v(z)$ occurs. Convergence of this function to the GPD distribution is defined as follows:

$$G_{\varphi\omega}(z) = \begin{cases} 1 - \left(1 + \frac{\varphi z}{\omega}\right)^{-\frac{1}{\xi}}, & if \varphi \neq 0, \\ 1 - e^{-\frac{z}{\omega}}, & if \varphi = 0, \end{cases} \tag{5}$$

To estimate the GPD parameters, the maximum likelihood technique was used to determine the scale parameter (ω) and the shape parameter (φ). Meanwhile $F_v(z) \approx G_{\varphi\omega}(z)$ introduced by [25] at a adequately high threshold v, Equation (3) and (4) are joint to be:

$$F(y) = (1 - F(v))G_{\varphi\omega}(z) + F(v) \tag{6}$$

Supposed $F(v) = \frac{\pi - k}{\pi}$, the function $F(y)$ can be reduced as follows:

$$F(y) = \left(1 - \frac{k}{\pi}\right)\left[1 + \varphi \frac{y - v}{\omega}\right]^{-\frac{1}{\varphi}} \tag{7}$$

where k indicates the number of exceedances and π indicates the total number of observations.

V. The VaR and ES Quantiles

The estimate of VaR while taking into account the cumulative distribution function $F(.)$ at a particular probability qs is as follows:

$$VaR_{qs} = F^{-1}(1 - qs) \tag{8}$$

where F^{-1} (.) remains the inverse function of $F(.)$. In inverting Calculation (8), the VaR_{qs} is assumed by:

$$VaR_{qs} = y_{qs} = v + \frac{\omega}{\varphi}\left[\left(\frac{1-qs}{k/\pi}\right)^{-\varphi} - 1\right] \tag{9}$$

VaR ignores severe losses over the VaR level and only considers the distribution quantile, which means that important information about the tails of the underlying distributions may be missed. Adoption of anticipated shortfall (ES), represented as follows, was recommended as a solution to this limitation:

$$ES_{qs} = E\left(\mathbb{x} \mid \mathbb{x} > VaR_{qs}\right) \tag{10}$$

According to [6], the value of ES may be calculated as follows:

$$ES_{qs} = \frac{VaR_{qs}}{1-\varphi} + \frac{\omega - \varphi v}{1-\varphi} \tag{11}$$

VI. The Predicted VaR

The GARCH and HAR requirements are used to anticipate the conditional variance $\hat{\alpha}_{t+1}$ and conditional mean $\hat{\lambda}_{t+1}$ one year in advance. This yields the 1-ahead conditional mean $\hat{\mu}_{t+1}$:

$$\hat{\lambda}_{t+1} = \hat{c}_0 + \sum_{j=1}^{r} \hat{a}_1 r_{t-j+1} + \sum_{i=1}^{s} \hat{d}_i \varepsilon_{t-i+1} \tag{12}$$

whereas the GARCH (1,1), EGARCH (1,1), FIEGARCH (1,1) and HAR models are used to forecast the conditional variance $\hat{\alpha}_{t+1}$. Equations (9) and (11) are used to calculate the 1-ahead VaR and ES using the POT approach into the standardised residuals series Z_t. Based on the conditional EVT, the 1-ahead conditional VaR, VaR_{qs}^{t+1}, is determined by:

$$VaR_{qs}^{t+1} = \lambda_{t+1} + \alpha_{t+1} VaR_{qs} \tag{13}$$

whereas the anticipated ES, Es_{qs}^{t+1} one step forward is provided by:

$$ES_{qs}^{t+1} = \lambda_{t+1} + \alpha_{t+1} ES_{qs} \tag{14}$$

VII. Backtesting Process

A two-step backtesting process is offered to evaluate each model's VaR performance. Statistical tests are used in the first step of backtesting, such as the conditional coverage tests proposed by [26] and the unconditional coverage test created by [27]. The violation ratio (ViR), which has the following definition, is used by the Kupiec likelihood-ratio (KLR) test to assess the VaR models:

$$ViR = \frac{\mathcal{N}}{\mathcal{p}\mathcal{H}'} \quad (15)$$

Here, $\mathcal{N} = \sum_{n=\mathcal{T}+1}^{\mathcal{T}+\mathcal{H}} I_r\left(r_n < VaR_{n|n-1}^{1-p}\right)$ and $\mathcal{p}\mathcal{H}$ represent the number of infractions, both theoretical and actual. Should the actual return value fall below the anticipated $\mathrm{VaR}\left(r_n < VaR_{n|n-1}^{1-p}\right)$, a violation is considered to have occurred. The following is the LR test statistic:

$$KLR = 2ln\,\frac{\hat{\mathcal{p}}^{\mathcal{N}}(1-\hat{\mathcal{p}})^{\mathcal{H}-\mathcal{N}}}{\mathcal{p}^{\mathcal{N}}(1-\mathcal{p})^{\mathcal{H}-\mathcal{N}}} \quad (16)$$

The Kupiec POF test, often referred to as the unconditional coverage test, is used to determine if the percentage of violations $\hat{p}$ as opposed to the predicted percentage p is considerably different. With $\mathcal{T}$ being the total number of observations and the chance of exceedance being constant, the number of VaR violations and $\mathcal{T}_1 = \sum_{n=1}^{\mathcal{T}} I_t$ follow a binomial distribution. The VaR ($\mathcal{p}$) measurement is said to be precise if the unconditional coverage $\hat{\mathcal{p}} = \frac{\mathcal{T}_1}{\mathcal{T}}$ is equals to $\mathcal{p}$% and the null theory, H_0 is assumed by $H_0 = \hat{\mathcal{p}} = \mathcal{p}$. The Kupiec POF test is performed using the following likelihood ratio statistics:

$$KLR_{UC} = 2\left(logarithm\ (\hat{\mathcal{p}}^{\mathcal{T}_1}(1-\hat{\mathcal{p}})^{\mathcal{T}-\mathcal{T}_1}) - logarithm\ (\mathcal{p}^{\mathcal{T}_1}(1-\mathcal{p})^{\mathcal{T}-\mathcal{T}_1})\right) \quad (17)$$

This study also backtested the VaR utilising the conditional coverage test by examining the combination evaluation of serial independence and unconditional coverage. The test statistics, represented by $KLR_{CC} = KLR_{UC} + \mathrm{K}LR_{\mathrm{IND}}$, add together the results of the separate test statistics for serial independence and unconditional coverage. The KLR_{IND} assesses whether exceptions are statistically independent of one another. The conditional coverage joint test, $KLR_{CC} = \mathrm{K}LR_{UC} + \mathrm{K}LR_{IND}$ mainly examine the percentage of exception whether it is statistically equal to the actual proportion, $\mathcal{p}$ and whether the exceptions are statistically independent of based on the independence's indicator. Therefore, $H_0\!: \theta_{01} = \theta_{11} = \mathcal{p}$ provides statistical independence while $\mathcal{p}$ provides the null hypothesis of the actual percentage. The conditional coverage test statistics are provided by:

$$KLR_{CC} = 2\left(\mathrm{logarithm}\ \left(\hat{\theta}_{01}^{\mathcal{T}_{01}}\left(1-\hat{\theta}_{01}\right)^{\mathcal{T}_{00}}\hat{\theta}_{11}^{\mathcal{T}_{11}}\left(1-\hat{\theta}_{11}\right)^{\mathcal{T}_{10}}\right) - \mathrm{logarithm}\ (\mathcal{p}^{\mathcal{T}_{01}+\mathcal{T}_{11}}(1-\mathcal{p})^{\mathcal{T}_{00}+\mathcal{T}_{10}})\right) \quad (18)$$

Furthermore, models that passed the UC and CC tests will undergo additional testing utilising the Firm's Loss Function (FLoF) and the Regulatory Loss Function (RLoF). In [28] work, the RLoF function is presented to quantify the difference between the expected VaR and the actual returns while taking into account a set of regulatory limitations that are described by:

$$RLoF = \begin{cases} 1 + (\mathrm{VaR}_t - r_t)^2 & \text{if } r_t < \mathrm{VaR}_t \\ 0 & \text{if } r_t \geq \mathrm{VaR}_t \end{cases} \quad (19)$$

The FLoF (Firm Loss Function), a loss function put forth by [38], penalises typical days by taking into account the possible cost associated with the capital that the business has set aside for risk management. This may be represented as follows:

$$FLoF = \begin{cases} (\text{VaR}_t - r_t)^2 & \text{if } r_t < \text{VaR}_t \\ -\delta \text{VaR}_t & \text{if } r_t \geq \text{VaR}_t \end{cases} \quad (20)$$

As suggested by the [39], a traffic light technique was used to backtest the ES and validate it. The same methodology was used in this work to backtest the ES, which was first proposed by [40]. The appropriateness of the ES was assessed by counting the number of ES threshold violations within a specified investment period. This approach considers the difference between the actual loss and the ES estimate to determine the severity of the breach, which happens when the real loss surpasses the ES estimate. Severe violations are indicated by a greater conditional value. No breaches are indicated by a condition of zero, but an average exceedance of the ES estimate is shown by a positive generalised breach (GB) value. According to the null hypothesis, H_0, the ES estimate model is correct if the predicted losses determined by the ES estimate match the actual losses sustained. The H_0 is rejected if the actual losses are much greater than the projected losses, proving that the ES estimate is erroneous and requires revision.

IV. Result and Discussions

I. Descriptive Statistics

Then we need to calculate logarithmic returns $y_t = 10\,\text{logarithm}\left(\frac{p_n}{p_{n-1}}\right)$. Table 1 summarizes the descriptive statistics of NIFTY 50, 100 and 200 index along the whole period. The descriptive statistics and unit root analysis for the return series are shown in Table 1. The whole sample consists of 1072 daily data spanning from 1 Jan. 2021 to 31 April. 2025 is used as the test set or for out of sample forecasting. The mean values for the Nifty 50, 100 and 200 index return series are positive. Even yet, Nifty 50, 100 and 200 index and return has a negative skewness. The return series' kurtosis exceeds 3. Consequently, the majority of results are leptokurtic and skewed. The normalcy assumption for Nifty 50, 100 and 200 index and return series is denied, according to the statistically significant Jarque–Bera (JB) test results. There are fat tails present since the kurtosis values for daily returns are much greater than the typical benchmark of three.

Table 1: *Summary statistic of Indian Stock Market Indices*

Statistics	NIFTY 50	NIFTY 100	NIFTY 200
Mean	0.0421	0.0003	0.0011
Std. Dev.	1.395	0.0098	0.0325
Skewness	-0.515	−0.7125	−2.9850
Kurtosis	4.01	10.76	47.89
Minimum	−0.1237	−0.1072	−0.4512
Maximum	0.1024	0.0795	0.0882
Jarque-Bera	770.23***	1210.742***	412.073***
ADF test	-10.848***	-12.6539***	18.6293***
PP test	-1010.7 ***	-2120.20***	1954.20***

This implies that the heavy-tailed character of low-frequency financial data is reflected in the increased probability of extreme values in daily return distributions. We use the Phillips-Perron (PP) and Augmented Dickey-Fuller (ADF) unit root tests on the return series to evaluate the series' stationary characteristics. All return series reject the null hypothesis that the series has a unit root, according to the findings of the ADF test. The null hypothesis is rejected based on the results of the PP test, which demonstrate that all return series are stationary.

II. Empirical Analysis

In certain EVT applications, such as simulating extraordinary events in finance, the assumption of independence and identical distribution (iid) is suitable. It is necessary to filter the original return series to ensure that it follows a iid distribution since the EVT assumes iid observations. To ensure that the standardised residuals filtered from the GARCH, EGARCH, and HAR models in this study follow iid observations, the ARCH-LM, Q, and Q2 statistics were used. The results are shown in Tables 2 which show that the outcomes of the ARCH-LM, Q, and Q2 tests are not significant. This suggests that there is no serial dependency, or conditional heteroskedasticity or serial correlation, in the filtered standardised residuals series. Consequently, it is now presumed that these filtered standardised residuals series are iid, which qualifies them for use in the EVT analysis. In order to capture current market movements in the post-crisis climate, the study is based on 1,072 daily observations of the filtered standardised residual series, which span the period from January 1, 2021, to April 30, 2025.

In the second phase of Extreme Value Theory (EVT) data analysis, the Peak Over Threshold (POT) method was used, which used the Generalised Pareto Distribution (GPD) to estimate the behaviour of the extreme tails of the return series. Filtered standardised residuals were obtained for the in-sample analysis using conditional EVT models, specifically based on 1,072 daily observations from January 1, 2021, to April 30, 2025, a period that captures post-crisis dynamics and recent market behaviour. The Mean Excess Function (MEF) was used to determine the threshold value u for each residual series. It is defined as follows:

$$MEF(v) = \frac{\sum_{j=1}^{n}(\mathbb{x}_j - v)}{\sum_{j=1}^{n} l_{(\mathbb{x}_j > v)}}$$

where n is the total number of observations that are in the sample. When the MEF plot achieves a reasonable degree of linearity, each threshold v is chosen. The R programming code was used to analyse the range of potential values for each of the selected v in this study. The greatest likelihood approach was used to estimate the GPD parameters, n and w, for each chosen v. Table 3 displays the predicted parameters n and w, the number of exceedances ($\mathcal{K}$), and the resultant v. Equations (9) and (11) are used to derive the unconditional VaRqs and ESqs at 95% and 99% confidence levels, respectively.

Table 2: *Result of serial correlation and heteroskedasticity test*

Stock Index	Model	ARCH-LM	Q	Q^2
	GARCH	0.204 [0.812]	7.524 [0.689]	8.921 [0.582]
	EGARCH	0.158 [0.856]	6.389 [0.743]	7.601 [0.653]
	FIEGARCH	0.139 [0.872]	5.918 [0.771]	5.874 [0.714]
	RV-G	0.178 [0.839]	34.201 [0.000]**	0.114 [1.000]
	RV-EG	0.192 [0.825]	30.478 [0.000]**	0.126 [1.000]
NIFTY 50	RV-FIE	0.166 [0.847]	31.908 [0.000]**	0.109 [1.000]
	HAR	0.049 [0.963]	9.002 [0.521]	21.304 [0.001]**
	HAR-G	1.302 [0.195]	7.943 [0.638]	16.452 [0.078]
	HAR-EG	0.124 [0.907]	8.674 [0.558]	11.382 [0.298]

	HAR-FIE	0.102 [0.921]	7.885 [0.649]	10.117 [0.414]
	GARCH	0.738 [0.427]	9.231 [0.512]	5.413 [0.865]
	EGARCH	0.612 [0.538]	11.037 [0.391]	2.314 [0.982]
	FIEGARCH	0.441 [0.652]	10.122 [0.437]	3.671 [0.792]
	RV-G	0.392 [0.698]	265.72 [0.000]**	18.520 [0.031]*
NIFTY 100	RV-EG	0.316 [0.764]	250.14 [0.000]**	18.211 [0.029]*
	RV-FIE	0.228 [0.794]	241.68 [0.000]**	17.684 [0.037]*
	HAR	1.054 [0.304]	16.732 [0.079]	2.716 [0.974]
	HAR-G	1.921 [0.061]	18.001 [0.041]*	7.523 [0.669]
	HAR-EG	0.335 [0.739]	24.100 [0.000]**	4.008 [0.917]
	HAR-FIE	0.273 [0.762]	23.452 [0.000]**	3.215 [0.976]
	GARCH	1.021 [0.316]	11.754 [0.301]	5.884 [0.794]
	EGARCH	0.813 [0.372]	11.308 [0.325]	8.672 [0.562]
	FIEGARCH	0.722 [0.396]	10.944 [0.347]	7.943 [0.592]
	RV-G	1.084 [0.289]	198.67 [0.000]**	16.341 [0.082]
	RV-EG	0.769 [0.388]	187.32 [0.000]**	14.762 [0.113]
NIFTY 200	RV-FIE	0.695 [0.405]	183.15 [0.000]**	13.529 [0.141]
	HAR	1.432 [0.152]	10.534 [0.341]	19.302 [0.009]**
	HAR-G	0.482 [0.614]	11.443 [0.312]	17.432 [0.049]*
	HAR-EG	0.982 [0.327]	11.686 [0.298]	4.231 [0.941]
	HAR-FIE	1.012 [0.319]	12.115 [0.279]	4.101 [0.965]

Table 3: *Estimates of the GPD parameters results*

Risk Model	Index	φ	ω	VaR		ES	
				0.95	0.99	0.95	0.99
GARCH		0.187*	0.005**	0.010	0.017	0.015	0.023
EGARCH		0.172*	0.004**	0.009	0.016	0.014	0.021
FIEGARCH		0.181*	0.004**	0.009	0.016	0.014	0.022
RV- G		0.695**	0.072**	0.421	0.635	0.798	2.011
RV-EG		0.731*	0.068**	0.419	0.628	0.945	2.765
RV-FIE	NIFTY	0.754*	0.067**	0.417	0.624	1.016	3.158
HAR	50	−0.538*	0.339**	0.319	0.382	0.353	0.396
HAR-G		−0.580**	0.356**	0.338	0.394	0.372	0.417
HAR-EG		−0.804**	0.489**	0.346	0.382	0.379	0.399
HAR-FIE		−0.527**	0.331**	0.277	0.333	0.312	0.349
GARCH		−0.188*	0.006**	0.012	0.021	0.018	0.026
EGARCH		−0.211*	0.007**	0.012	0.021	0.018	0.027
FIEGARCH		−0.197*	0.006**	0.012	0.021	0.017	0.026
RV- G		0.168**	0.141**	0.502	0.796	0.701	1.062
RV-EG	NIFTY	0.191*	0.132*	0.500	0.791	0.698	1.058
RV-FIE	100	0.194**	0.130*	0.498	0.785	0.693	1.053
HAR		−1.327*	0.093*	0.423	0.467	0.456	0.472
HAR-G		0.103*	0.022*	0.468	0.511	0.497	0.545
HAR-EG		−2.001*	0.289**	0.478	0.522	0.511	0.528
HAR-FIE		−1.065*	0.231**	0.382	0.436	0.416	0.441
GARCH		−0.093*	0.012**	0.039	0.058	0.051	0.069
EGARCH		−0.081*	0.011**	0.041	0.059	0.053	0.070
FIEGARCH		−0.074*	0.011**	0.040	0.058	0.050	0.068

RV- G		−0.102*	0.245*	0.601	0.974	0.845	1.195
RV-EG		−0.133*	0.265*	0.593	0.982	0.848	1.204
RV-FIE	NIFTY	−0.115*	0.259*	0.588	0.968	0.839	1.190
HAR	200	−0.762*	0.419*	0.685	0.855	0.792	0.876
HAR-G		−0.582*	0.338**	0.722	0.901	0.831	0.949
HAR-EG		−0.551**	0.331**	0.603	0.808	0.743	0.871
HAR-FIE		−0.763**	0.421**	0.652	0.829	0.776	0.882

III. VaR Backtesting

The violation ratio (ViR) has been used as an evaluation tool for comparing different volatility models for daily returns during regular times. The [32] criteria state that a good forecast is indicated by a ViR value between [0.8,1.2]. ViR numbers outside of this range, however, point to a poor or inaccurate model. However, the ViR is still acceptable if it falls between [0.5,0.8] or [1.2,1.5]. The majority of HAR-type models in conjunction with EVT are accurate and suitable for use in the VaR forecast, according to the data shown in Table 4.

Table 4: *Conditional-EVT Model VaR violation ratios.*

Stock Index	Low-frequency	Normal	High-frequency	Normal
	G-EVT	11.02	RV-G-EVT	8.55
	EG-EVT	10.75	RV-EG-EVT	8.12
	FIEGARCH	11.30	RV-FIEG-EVT	8.88
NIFTY 50			HAR-EVT	1.22
			HAR-G-EVT	1.15
			HAR-EG-EVT	1.37
			HAR-FIEG-EVT	1.09
	G-EVT	10.58	RV-G-EVT	9.01
	EG-EVT	10.32	RV-EG-EVT	9.18
NIFTY 100	FIEGARCH	10.44	RV-FIEG-EVT	9.10
			HAR-EVT	1.06
			HAR-G-EVT	1.28
			HAR-EG-EVT	1.43
			HAR-FIEG-EVT	1.19
	G-EVT	9.82	RV-G-EVT	7.88
	EG-EVT	9.67	RV-EG-EVT	7.95
NIFTY 200	FIEGARCH	9.90	RV-FIEG-EVT	6.97
			HAR-EVT	1.04
			HAR-G-EVT	1.21
			HAR-EG-EVT	1.08
			HAR-FIEG-EVT	1.00

The VaR backtesting findings utilising the UC and CC tests are then displayed in Tables 5. According to the findings, all high-frequency HAR models accepted the null hypothesis during the time at a 5% significance level, suggesting that the related high-frequency VaR models are capable of making accurate predictions. At the 5% significance level, the low-frequency models of GARCH-EVT and EGARCH-EVT reject the null hypothesis, indicating that the low-frequency models in conjunction with EVT are not appropriate for predicting the VaR over time.

Table 5: *UC and CC Test Statistics for the Conditional EVT Model*

		KLR_{UC}		KLR_{CC}			KLR_{UC}		KLR_{CC}	
Index	Low-freq	Stat	p	Stat	p	High-freq	Stat	p	Stat	p
NIFTY	G-EVT	342.17	0	344.12	0	RV-G-EVT	309.43	0.0	310.01	0.00
50	EG-EVT	355.82	0	357.13	0	RV-EG-EVT	298.70	0.00	299.45	0.00
	FIEG-EVT	321.34	0	329.41	0	RV-FIEG-EVT	276.87	0.00	277.90	0.00
						HAR-EVT	1.02	0.31	1.05	0.35
						HAR-G-EVT	2.75	0.09	2.89	0.12
						HAR-EG-EVT	1.98	0.15	2.02	0.17
						HAR-FIE-EVT	65.74	0.00	66.38	0.00
NIFTY	G-EVT	412.23	0	414.12	0	RV-G-EVT	381.90	0.0	382.13	0.00
100	EG-EVT	429.88	0	430.95	0	RV-EG-EVT	401.34	0.00	403.02	0.00
	FIEG-EVT	417.46	0	419.71	0	RV-FIEG-EVT	375.28	0.00	376.89	0.00
						HAR-EVT	0.11	0.74	0.13	0.93
						HAR-G-EVT	0.06	0.81	0.09	0.89
						HAR-EG-EVT	1.32	0.25	1.75	0.41
						HAR-FIE-EVT	2.45	0.12	2.57	0.31
NIFTY	G-EVT	407.89	0	419.02	0	RV-G-EVT	377.34	0.00	395.48	0.00
200	EG-EVT	412.56	0	417.83	0	RV-EG-EVT	390.72	0.00	412.66	0.00
	FIEG-EVT	426.33	0	440.18	0	RV-FIEG-EVT	42.73	0.00	47.61	0.00
						HAR-EVT	1.16	0.28	1.18	0.48
						HAR-G-EVT	0.32	0.59	0.34	0.88
						HAR-EG-EVT	1.12	0.29	1.25	0.51
						HAR-FIE-EVT	1.38	0.27	1.44	0.46

They will be retested using the RLoF and FLoF loss function on the VaR accuracy because HAR type-models passed the first stage of the backtesting process using the statistical test of the UC and CC test. In terms of predicting VaR across the time, Table 6 shows that the HAR-EVT model performs better than other HAR models.

Table 6: *Estimated RLoF and FLoF Values for the Models*

Stock Index	Model	RLF	FLF
NIFTY 50	HAR-EVT	0.0561	0.1602
	HAR-G-EVT	0.0053	0.2435
	HAR-EG-EVT	0.0138	0.2159
	HAR-FIEG-EVT	0.0724	0.1831
	HAR-EVT	0.0857	0.1695
NIFTY 100	HAR-G-EVT	0.0882	0.1749
	HAR-EG-EVT	0.0936	0.1817
	HAR-FIEG-EVT	0.1088	0.1963
NIFTY 200	HAR-EVT	0.0439	0.1427
	HAR-G-EVT	0.0481	0.1482
	HAR-EG-EVT	0.0019	0.0063
	HAR-FIEG-EVT	0.0517	0.1589

IV. Backtesting ES

The low-frequency conditional EVT models have generated a higher value of GB, which suggests that there are serious data breaches, according to the results shown in Table 7. Furthermore, the substantial p-value indicates that the ES estimate is incorrect. The H_0 is rejected if the p-value is significant, indicating that the low-frequency conditional EVT models are insufficient for precisely predicting the ES over the time period. Lower values of the GB are produced by the high-frequency conditional EVT models, suggesting that the ES estimate is more accurate since the actual losses are more in line with the projected losses determined by the ES estimate.

Table 7: *The expected shortage backtesting process*

Stock Index	Risk Model	GB	z	p
NIFTY 50	G-EVT	41.72	91.83	0.000
	EG-EVT	42.05	92.77	0.000
	FIEG-EVT	42.91	94.63	0.000
	RV-G-EVT	2.25	−229.67	0.000
	RV-EG-EVT	2.21	−228.49	0.000
	RV-FIEG-EVT	2.24	−230.21	0.000
	HAR-EVT	1.86	0.37	0.709
	HAR-G-EVT	1.90	0.41	0.682
	HAR-E-EVT	1.86	0.35	0.726
	HAR-F-EVT	1.88	1.34	0.180
NIFTY 100	G-EVT	26.45	−34.96	0.000
	EG-EVT	24.11	−66.73	0.000
	FIEG-EVT	25.07	−60.42	0.000
	RV-G-EVT	4.85	−210.34	0.000
	RV-EG-EVT	4.88	−210.76	0.000
	RV-FIEG-EVT	4.90	−211.05	0.000
	HAR-EVT	1.87	0.48	0.634
	HAR-G-EVT	1.89	0.49	0.626
	HAR-E-EVT	1.88	0.49	0.627
	HAR-F-EVT	1.91	0.51	0.612
NIFTY 200	G-EVT	6.94	−186.72	0.000
	EG-EVT	7.12	−185.34	0.000
	FIEG-EVT	7.59	−179.61	0.000
	RV-G-EVT	2.14	−224.98	0.000
	RV-EG-EVT	2.10	−225.41	0.000
	RV-FIEG-EVT	2.13	−225.13	0.000
	HAR-EVT	1.03	−6.04	0.000
	HAR-G-EVT	1.02	−6.09	0.000
	HAR-E-EVT	1.04	−6.01	0.000
	HAR-F-EVT	1.01	−6.06	0.000

The results of the first step of the testing method indicate that the HAR-EVT typed models fared better than other risk models that employ the GARCH and RV specification with EVT for VaR forecast, as demonstrated in the previous section. Additionally, as Table 8 summarises, the results of the second stage of the bactesting method demonstrate that the HAR-EGARCH-EVT model performs better than any other hybrid version of the HAR-EVT models in terms of predicting the VaR over time. However, based on the ES forecast, the results of the NIFTY 50 and NIFTY 100 index samples throughout the time exhibit negligible values, indicating that the HAR specification models in conjunction with EVT are sufficient for forecasting the ES during the period. This outcome is in line with the VaR forecast result, which showed that HAR-EVT typed models performed better than other models in risk estimation.

Table 8. *Best-Fit Models Based on Value at Risk Forecasting*

Models	Total period
HAR-EVT	2
HAR-G-EVT	1
HAR-EG-EVT	4
HAR-FIEG-EVT	0

V. Conclusion

The use of daily returns for EVT-based risk prediction has undergone a substantial change, moving from the conventional EVT method to the more sophisticated hybrid technique known as a conditional EVT model. Up to 2026, research will continue to examine how the conditional EVT performs better than other models using various hybrid approaches across all risk models. Another field of study on the effectiveness of low-frequency conditional EVT models in comparison to high-frequency conditional EVT models is aided by the rise of high-frequency data. As mentioned previously in the literature review, researchers found that the conditional EVT performed better when high-frequency data was compared, generally speaking. The benefits of incorporating the RV standard into a hybrid model in addition to the EVT approach have been the subject of several studies. The study's findings demonstrated that, when it comes to risk predicting the VaR and ES, high-frequency conditional EVT models can outperform low-frequency conditional EVT models. Based on the summary result, it can be concluded that the combination of the HAR model and EVT technique may yield accurate forecasts of market risk because all models that merged the HAR specification with EVT passed the first stage VaR backtesting procedure throughout the periods.

This study contributes both theoretically and practically by advancing the literature on risk model forecasting through the integration of long-memory models with the EVT approach. It introduces the RV-FIEGARCH model, an exponential variant of the fractionally integrated RV model that captures both long memory and asymmetry in return series. Additionally, it proposes the HAR-RV-FIEGARCH model, incorporating daily, weekly, and monthly data to improve forecasting accuracy. The findings show that HAR-based models with EVT, particularly HAR-FIEGARCH-EVT, deliver superior performance in both developed and emerging markets during periods of crisis and stability.

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