RT&A, No 4 (89)
Volume 20, December 2025

AN INVENTORY MODELING FOR DECAYING ITEMS WITH PRICE, STOCK AND RELIABILITY-DEPENDENT DEMAND UNDER MEMORY EFFECTS

Narendra Kumar[1], Liliana Guran[2], Devendra Kumar[3], Sanjeet Kumar[4], Ajay Singh Yadav[5,*], Krishan Kumar Yadav[6,*]

•

[1]Department of Mathematics, IILM University, Greater Noida, Uttar Pradesh, India, 201310
[2]Department of Mathematics, Babes-Bolyai University, Cluj-Napoca, Romania, 400347
[3]Department of Mathematics, University of Technology and Applied Sciences-Shinas, Sultanate of Oman, Oman, 324
[4]Department of Mathematics, Lakshmi Narain College of Technology & Science, Bhopal, Madhya Pradesh, India, 462022
[5]Department of Mathematics, SRM Institute of Science and Technology, Delhi-NCR Campus, Modinagar, Uttar Pradesh, India, 201204
[6]Department of Mathematics, School of Sciences, JECRC University, Jaipur, Rajasthan, India, 303905
drnk.cse@gmail.com, liliana.guran@ubbcluj.ro, Devendra.kumar@shct.edu.om, sanjeetkumarmath@gmail.com, ajaysiny@srmist.edu.in, krishan.yadav@jecrcu.edu.in
*Corresponding author

Abstract

A generalized Economic Order Quantity (EOQ) framework is employed to investigate memory effects within an inventory system. Fractional calculus provides an effective mathematical tool for reflecting such memory characteristics in economic model. In this study, a fractional-order inventory model without shortages is developed, where demand is depends on price, on-hand stock, and product reliability, while deterioration is assumed to occur at a constant rate. The developing fractional differential equation is formulated in the Caputo sense, and a memory-kernel approach is used to incorporate dependence on past system states. By applying the Laplace transform technique together with Mittag-Leffler functions, an analytical solution to the model is obtained. The results indicate that the degree of memory in the system can be modulated through the order of the fractional derivative or integral. Furthermore, a sensitivity analysis is performed for both short-memory and long-memory scenarios to determine the key parameters affecting system behavior under varying conditions with the help of MATLAB software.

Keywords: Inventory modeling, deteriorating items, multi variables dependent demand, Caputo derivative, Laplace transform method, memory effect.

1. Introduction and Literature Survey

In operations research, inventory systems inherently exhibit memory-dependent behavior. Customer demand is often influenced by a retailer's service quality and the reliability of the products offered. As a result, purchasing decisions are shaped not only by current conditions but also

DOI: https://doi.org/10.24412/1932-2321-2025-489-51-57

by customers past experiences with the product. Negative experiences can significantly reduce the likelihood of future purchases, highlighting the crucial role of memory effects in inventory dynamics. Motivated by these observations, this study aims to incorporate memory or past experience into the inventory framework, offering a fresh perspective and contributing a new dimension to the existing inventory literature. In current era, the most promising field of mathematical research is based on the idea of Fractional Calculus (FC) that deals with the generalization of arbitrary order of our traditional classical differentiation, integration, differential equation [12]. The applicability of this branch of mathematics was very rare till a few days ago from its first introduction in 1695 due to the lack of significant physical interpretation of differential equations of fractional order. Later the extensive knowledge of FC amazingly encourages to analyze the dynamical nature of our physical real world problem. It had been propagated effectively to tackle the complexities arrived at in the formations of mathematical modeling in applied science and technology. It is quite comprehensive that FC has significant potential to interpret the physical phenomena in its generalized form though the formulation of basic concepts and the physical interpretations are continuously being modified. The various features and aspects of Fractional Differential Equation (FDE) have been elegantly presented by [12], [13]. The application of fractional calculus in inventory modeling has been widely explored in recent era due to the memory effects parameter. Kumar, Singh, and Kumar [[19]] investigated space-time fractional telegraph equations and developed both analytical and approximate solutions using the Laplace transform method. In another related contribution, Singh, Kumar, and Kumar [[20]] introduced a new fractional formulation of a nonlinear shock wave equation in gas flow dynamics. Their work highlighted that the FC approach provides robustness in modeling. More recently, Kumar and Gupta [[21]] proposed an efficient FC numerical methodology for solving fractional coupled Boussinesq equations, and provides the robustness for nonlinear systems such as inventory modeling.

The study on inventory systems for decaying items has an important and crucial topic in whole operations research due to its applicability in industries such as pharmaceuticals, food industries, technological industries, and many more. Firstly work by Ghare and Schrader [1] introduced a foundational inventory system with exponential decay rate. After this worked many authors on that topic. Urban [2] considered price-reliant demand rate, while Mandal and Phaujdar [3] explored the impact of stock levels on demand behavior by consumer. Additionally the models were extended by Yadav [4], who examined the effect of demand due to multiple market parameters considering stock and price reliant in deteriorating environments. Deterioration combined with shortage strategies was addressed in details by Abad [5], who developed a pricing and lot-sizing stock model assuming partial shortage. Jaggi [6] and later Jaggi et al. [7] contributed to the field by assuming time-reliant and reliability-based demand functions, and also combining both of them providing more realistic to real - world inventory scenarios. Despite these advancements, most traditional models overlook the impact of system memory " the effect of past inventory states on current decisions. This limitation has motivated researchers to explore fractional calculus, which provides a natural way to consider memory effects through non-integer order derivatives. Tarasov [8] and Atangana and Baleanu [9] have developed foundational theories in this domain. More recently, Bera et al. [10] and Sahoo and Mishra [11] applied fractional-order approaches to inventory systems with decaying items, assuming stock and reliability-reliant demand rate. Their findings demonstrate the practical advantages of using fractional calculus for modeling assuming memory-effect systems, particularly in capturing long-term dependency and dynamic adjustment. Mebarek-Oudina, F [14-17] are also used the advanced techniques for enhancement and some other factors. These models highlight the significance of managing decaying-related costs to optimize inventory levels and enhance overall system performance. This paper builds upon these contributions by developing a fractional-order inventory model that integrates price, stock, and reliability-reliant demand rate taking along with deterioration and memory effects.

This work not only bridges the gap between theory and practical demands but also provides a reliable conceptual theory for decision-making in perishable inventory systems with memory-reliant characteristics.

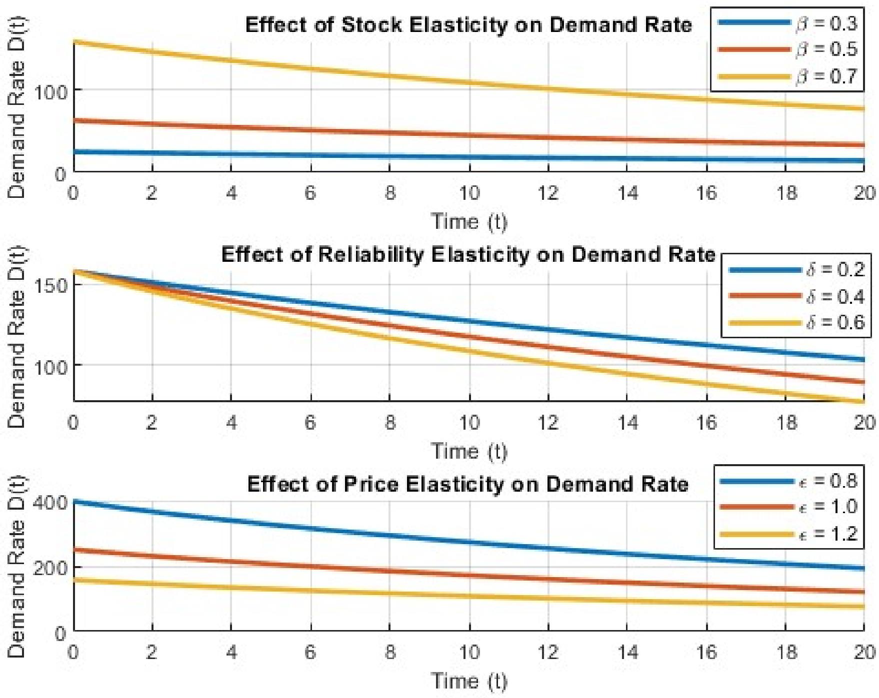

Figure 1: *Effect of Elasticities on Demand Rate.*

2. Notations and assumptions

2.1. Notations

The following notations are used in this model.

1. $I(t)$: Inventory level at time t.
2. Q: Order quantity.
3. T: Cycle time.
4. h_f, h_b: Holding costs for frontroom and backroom.
5. θ: Deterioration rate.
6. α: Fractional order ($0 < \alpha \leq 1$).
7. c: Unit purchasing cost.

2.2. Assumptions

The following assumptions are used in this paper.

1. Products deteriorate over time and are modeled using Caputo fractional derivative.
2. Demand depends on price (p), visible (frontroom) stock (S_f), and reliability (R).
3. Backroom inventory (S_b) is used to replenish frontroom stock.

4. No lead time and replenishment is instantaneous.
5. Shortages are not allowed.
6. Reliability decay function:
$$R(t) = R_0 e^{-\gamma t^{\alpha}} \tag{1}$$
7. Demand Function:
$$D(p, S_f, R) = a S_f^{\beta} R^{\delta} p^{-\epsilon} \tag{2}$$
Where a: base demand, β: stock elasticity, δ: reliability elasticity, ϵ: price elasticity.

3. Fractional Inventory Formulation and Solution

Inventory deterioration is modeled using a Caputo fractional derivative as follows.

$$D_t^{\alpha} I(t) + \theta I(t) = -D(p, S_f(t), R(t)), \quad 0 \le t \le T, \tag{3}$$

with the initial condition $I(0) = I_0$.
Here, D_t^{α} denotes the Caputo fractional derivative of order α defined as follows.

$$D_t^{\alpha} I(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{I'(\tau)}{(t-\tau)^{\alpha}} \, d\tau, \quad 0 < \alpha < 1. \tag{4}$$

The analytical solution of the Fractional differential equation involving the Caputo derivative of order α is equation (6). Since, equation (3) is a linear fractional differential equation and its solution can be derived using the Laplace transform method. Taking the Laplace transform of both sides, then we getting the following equations.

$$\mathcal{L}\{D_t^{\alpha} I(t)\} + \theta \mathcal{L}\{I(t)\} = -\mathcal{L}\{D(p, S_f(t), R(t))\},$$
$$s^{\alpha} \tilde{I}(s) - s^{\alpha-1} I(0) + \theta \tilde{I}(s) = -\tilde{D}(s),$$

where $\tilde{I}(s)$ and $\tilde{D}(s)$ are the Laplace transforms of $I(t)$ and $D(p, S_f(t), R(t))$, respectively.
Rewriting the equation, then we getting

$$\tilde{I}(s) = \frac{s^{\alpha-1} I_0 - \tilde{D}(s)}{s^{\alpha} + \theta}. \tag{5}$$

Taking the inverse Laplace transform, then we getting

$$I(t) = I_0 E_{\alpha}(-\theta t^{\alpha}) - \int_0^t (t-\tau)^{\alpha-1} E_{\alpha,\alpha}(-\theta (t-\tau)^{\alpha}) D(p, S_f(\tau), R(\tau)) \, d\tau, \tag{6}$$

where $E_{\alpha}(\cdot)$ and $E_{\alpha,\alpha}(\cdot)$ are the Mittag-Leffler functions defined by:

$$E_{\alpha}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + 1)}, \tag{7}$$
$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}. \tag{8}$$

Thus, the general solution to the fractional inventory model equation is given by equation (6), which combines the memory effect captured by the fractional derivative and the dynamic demand function.

Next, we calculate all the associated inventory costs as follow:

1. Ordering Cost (OC):
$$OC = A \tag{9}$$

2. Purchasing Cost (PC):

$$PC = cQ \tag{10}$$

3. Holding Cost (HC):

$$HC == h_f \int_0^T S_f(t)dt + h_b \int_0^T S_b(t)dt \tag{11}$$

4. Deterioration Cost (DC):

$$DC = c\theta \int_0^T I(t)dt \tag{12}$$

The Total Cost Function is

$$TC(T) = \frac{1}{T}(OC + PC + HC + DC) \tag{13}$$

4. Optimal Solution Procedure

In this section, we examine the convexity of the total cost function in order to determine the optimal cycle. Following an analytical optimization approach similar to the Negi et al. [18], and the necessary condition for minimizing the total cost is given as follows:

$$\frac{d\,TC(T)}{dT} = 0. \tag{14}$$

To compute the optimal cycle length T by solving to above equation. The conditions as mention below that must be satisfied in order to minimize TC(T):

$$\frac{d^2\,TC(T)}{dT^2} > 0. \tag{15}$$

5. Numerical Example

Assume the input data as follows $\alpha = 0.9$, $\theta = 0.05$, $c = 20$, $h_f = 1.5$, $h_b = 1.0$, $s = 5$, $v = 3$, $a = 100$, $\beta = 0.4$, $\delta = 0.3$, $\epsilon = 1.1$, $R_0 = 1$, $\gamma = 0.05$.
The obtained optimal values are as follows:

$$T^* = 6.5,\ Q^* = 870,\ p^* = 14.8,\ C_p = 17400,\ C_h = 520,$$
$$C_d = 880,\ C_s = 340,\ SV = 1250,\ TC = \$2884.6.$$

6. Graphical Representation

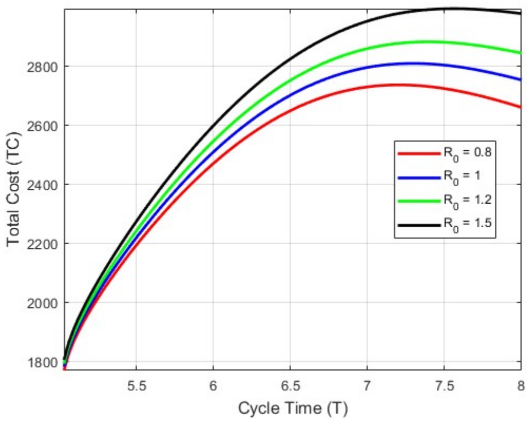

Figure 2: *Total Cost vs Cycle Time for Different Reliability Parameters*

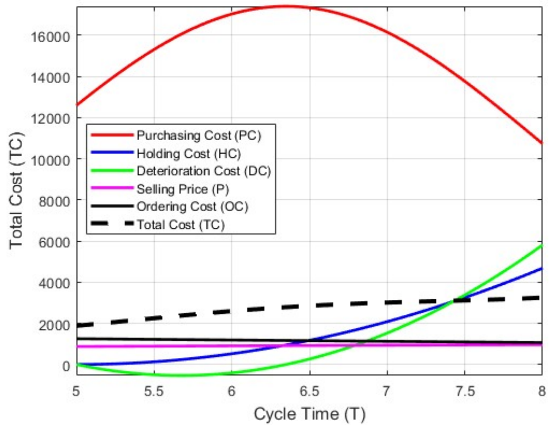

Figure 3: *Total Cost vs Cycle Time for Different Costs and Price*

The graphical depictions in this part provide a comprehensive grasp of the dynamics within the deteriorated inventory model. Figure 2 and Figure 3 explains the link between demand rates and time, while Figure 2 illustrates how memory effect impacts inventory management.

7. Conclusion

In this study, we explored the significant and practical role of the memory effect in inventory systems to minimizing the total costs. The memory effect, which is include the past good and bad experiences in decision making to develop more practical business models. The proposed inventory system considered constant deterioration rate and price, stock and reliability reliant demand, reflecting more realistic inventory systems under memory effect. A numerical example is illustrate for the applicability of the proposed model using analytical optimization method with the help of MATLAB software version R2021b. The results highlights the practical advantages of memory effects, it also acknowledges that practical considerations and real-world variability in the proposed system. Future research directions is that the consideration of environmental consideration by assuming memory effect in inventory models.

References

[1] Ghare, P. M., & Schrader, G. F. (1963). A model for an exponentially decaying inventory. *Journal of Industrial Engineering*, **14**(5), 238–243.

[2] Urban, T. L. (1992). Pricing and ordering policy for a product subject to risk of obsolescence. *International Journal of Production Economics*, **27**, 263–272.

[3] Mandal, B., & Phaujdar, S. (2001). An inventory model for deteriorating items with stock-dependent demand and time-dependent partial backlogging. *Applied Mathematical Modelling*, **25**(12), 1149–1155.

[4] Yadav, S. P. (2013). A perishable inventory model with stock, time, and price-dependent demand under inflation. *International Journal of Management Science and Engineering Management*, **8**(2), 132–138.

[5] Abad, P. L. (2001). Optimal pricing and lot-sizing under conditions of perishability and partial backordering. *Management Science*, **47**(6), 813–826.

[6] Jaggi, C. K. (2008). Retailer™s optimal replenishment decisions with credit-linked demand under permissible delay in payments. *Applied Mathematical Modelling*, **32**(10), 2012–2025.

[7] Jaggi, C. K., Sharma, A., & Mittal, M. (2011). A fuzzy model for deteriorating inventory items with reliability-dependent demand under permissible delay. *Mathematical and Computer Modelling*, **53**(5"6), 1423–1433.

[8] Tarasov, V. E. (2011). *Fractional dynamics: Applications of fractional calculus to dynamics of particles, fields and media*. Springer.

[9] Atangana, A., & Baleanu, D. (2016). New fractional derivatives with nonlocal and non-singular kernel: Theory and application to heat transfer model. *Thermal Science*, **20**(2), 763–769.

[10] Bera, U., Sahoo, S., & Mishra, B. K. (2020). A fractional order deteriorating inventory model with reliability-dependent demand. *Computers & Industrial Engineering*, **147**, 106662.

[11] Sahoo, S., & Mishra, B. K. (2021). A fractional inventory model with stock and reliability dependent demand under trade credit policy. *Journal of Computational and Applied Mathematics*, **390**, 113362.

[12] Miller, K.S., Ross, B.: An Introduction to the Fractional Calculus and Fractional Differential Equations. Wiley, New York (1993).

[13] Diethelm, K.: The Analysis of Fractional Differential Equations. Springer, Verlag (2010).

[14] Mebarek-Oudina, F. "Exploring Passive Heat Transfer Enhancement Techniques: Applications, Benefits, and Challenges." Babylonian Journal of Mechanical Engineering 2024 (2024): 122-127.

[15] Mebarek-Oudina, F., et al. "Thermal performance of MgO-SWCNT/water hybrid nanofluids in a zigzag walled cavity with differently shaped obstacles." Modern Physics Letters B (2025): 2550163.

[16] Kumar, M. Anil, et al. "The impact of Soret Dufour and radiation on the laminar flow of a rotating liquid past a porous plate via chemical reaction." Modern Physics Letters B 39.10 (2025): 2450458.

[17] Mebarek-Oudina, Fateh. "Numerical modeling of the hydrodynamic stability in vertical annulus with heat source of different lengths." Engineering science and technology, an international journal 20.4 (2017): 1324-1333.

[18] Negi, A., Singh, O., Yadav, A.S. et al. An Inventory Model for Perishable Goods with Demand that Varies Probabilistically with Selling Price Using a Pentagonal Fuzzy Framework. Oper. Res. Forum 6, 150 (2025). https://doi.org/10.1007/s43069-025-00555-5

[19] KUMAR, Devendra, Jagdev SINGH, and Sunil KUMAR. "Analytic and approximate solutions of space-time fractional telegraph equations via Laplace transform." Walailak Journal of Science and Technology (WJST) 11.8 (2014): 711-728.

[20] Singh, Jagdev, Devendra Kumar, and Sunil Kumar. "A New Fractional Model of Nonlinear Shock Wave Equation Arising in Flow of Gases." Nonlinear Engineering 3.1 (2014).

[21] Kumar, Saurabh, and Vikas Gupta. "An Efficient Numerical Approach to Solve Fractional Coupled Boussinesq Equations." Journal of Computational and Nonlinear Dynamics 19.12 (2024): 121002.