SYSTEM RELIABILITY AND PROFITABILITY CONSIDERING CONCEPTS OF WARRANTY AND INSURANCE: AN OVERVIEW

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Abstract

The concepts of reliability, warranty, and insurance are interwoven in certain respects. System warranties and insurance coverage, whether provided by the manufacturer or a third party, reflect the system's potential reliability and offer consumers a sense of security. Conversely, enhancing system reliability during the development process can help reduce warranty and insurance costs. The purpose of this study is to compile the existing literature and provide a structured review of system reliability evaluation, incorporating the concepts of warranty, pricing, and insurance policies. Synthesizes the ideas, models, and methodologies that various authors have included in their study. To achieve this, a detailed review of 111 journal articles published between 1992 and the present was conducted. The primary focus areas of the review include system reliability, warranty policies, insurance mechanisms, and cost analysis. In addition, the study highlights potential directions for future research in this domain.

Keywords: reliability, warranty, insurance, cost benefit analysis.

1. Introduction

The recent development in numerous technical areas has opened the door for many innovations and technologies to enter the mainstream, compelling manufacturers to offer a wide range of customer alternatives. Despite the countless benefits of technological advancement, the complexity of the system is one of its disadvantages. As a result, consumers are concerned about system reliability, which could negatively impact the profits of the developed product. Reliability is the probability that system operates satisfactorily for intended period under given operating conditions. Redundancy is one of the vital tools that can be used to improve system reliability, but cost is always an issue to consider. Therefore, the manufacturer offers warranty, an extended warranty, and insurance on his products to alleviate consumer worries and apprehensions while also ensuring system reliability. Review papers [1, 2, 3, 4, 5] contain extensive literature on warranty. A product warranty is a written assurance from the manufacturer offered to the user to ensure system performance and to provide free repair/replacement if the product malfunctions. It is a legal settlement between the system user and the system provider. The contract addresses product effectiveness and user benefits in the event of failure. The conditional warranty policy states that if any defect occurs that is covered by the warranty terms stated in the contract at the time of purchase, the product is repaired or replaced at no cost to the user; otherwise, the user will pay the cost for the same. Contrarily, insurance is a legal agreement between a person or a group (the policyholder) and an insurance firm (the insurer). The insurer accepts payment of a premium from the policyholder in exchange for an agreement to offer reimbursement for certain risks detailed in the insurance policy. Insurance is designed to protect policyholders from unexpected events or accidents that could result in financial hardship. Policyholders rely on insurance companies to provide financial protection when unexpected events occur. Throughout the design and development of the system, consideration is given to insurance policies and system warranties, both of which rely significantly on the system's reliability and efficacy.Figure 1 illustrates the connections between these ideas. Costs associated with warranties can be reduced by enhancing system reliability during the process of development.

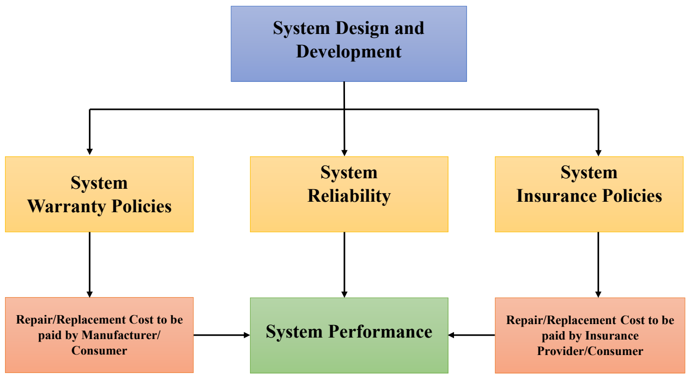

Figure 1: *Relation between system reliability, warranty and insurance*

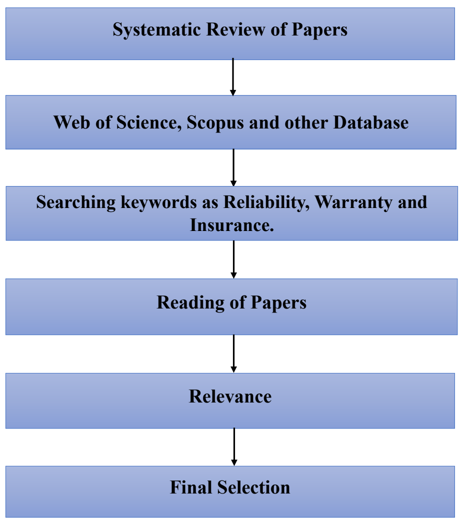

Figure 2: *Methodology for Systematic Review*

This study's goal is to compile the most recent literature and provide a structured assessment of system reliability evaluation, including the ideas of warranty, pricing, and insurance policies.

After a thorough study of the bibliography, 111 papers are chosen. A conceptual framework that represents the methodology used in the current investigation is shown in Figure 2. Initially, year-by-year searches from 1992 to the present in Web of Science, Scopus and other databases have been conducted using terms like insurance, warranty, and reliability. After reading those papers, select the pertinent article from the list. The article is structured in the following way. The discussion begins with an exploration of the literature on warranty policies and their various types. The overview of reliability with warranty concepts and insurance-related literature is done in next section. After that, the literature with a joint focus on reliability, warranty, insurance, and cost-benefit analysis is provided. Finally, the paper concludes along with future scope.

2. Warranty policies and warranty types

In this section, we briefly present the literature on various warranty policies and types explored by different researchers. Warranties serve as a marketing tool for manufacturers, allowing consumers to compare similar brands based not only on product attributes like quality, features, price, and reliability but also on the warranties offered at the time of purchase. For consumers, warranties provide the benefit of repair or replacement in case of a breakdown, potentially saving them significant maintenance costs. This creates a stronger connection between manufacturers and consumers, benefiting both parties.
Figure 3 illustrates several types of warranty policies based on cost, duration, and other variables. Under a free replacement warranty policy, the user incurs no cost for repairing or replacing a defective item, with the manufacturer covering all expenses within the warranty term. In contrast, a pro-rata warranty policy requires the system provider to repair or replace a failed product during the warranty period by paying a charge proportional to the product's age and sale price, with the user covering the remaining costs.
Warranties can also be categorized as one-dimensional or two-dimensional based on the variables involved. A one-dimensional warranty is defined by a single factor, such as duration, age, or usage. A two-dimensional warranty is defined by an area characterized by two variables, often one representing lifetime and the other representing item usage.
Additionally, warranties can be renewing or non-renewing. In a renewing warranty, if an item fails, it is repaired or replaced, and a new warranty is issued under the same conditions as the original. In a non-renewing warranty, the defective item is repaired or replaced for a specified period after the original purchase, but the replacement product does not receive a new warranty. Long-term warranties, which extend beyond the standard warranty period, can be purchased separately for an additional cost.

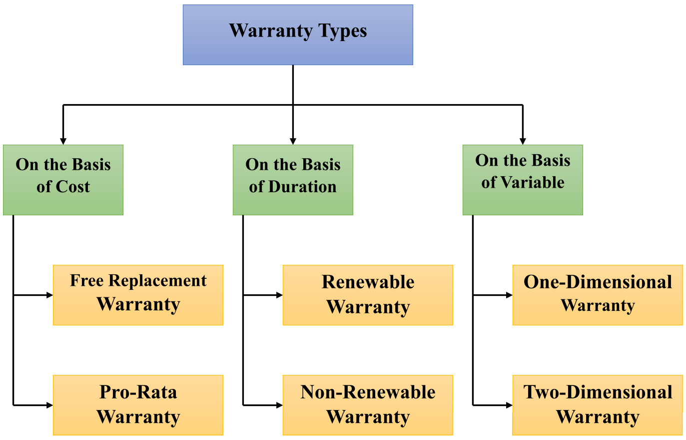

Figure 3: *Different types of Warranty Policies*

Table 1 provides a comprehensive summary of the findings from papers published in previous years by various researchers on different warranty policies.

Table 1: *Detailed Survey of Papers Reviewed pertaining to different Warranty Policies and Types*

Sr. No.	Author(s)	Year	Methods/Warranty Types	Findings
1	Murthy and Hussain [33]	1995	Free replacement warranty policy	Explored active (hot standby) and passive (cold, warm standby) redundancy strategies to lower the manufacturer's overall expenses, encompassing both production and warranty service costs.

Table 1: *Continued*

Sr. No.	Author(s)	Year	Methods/Warranty Types	Findings
2	Eliashberg et al. [14]	1997	Two-dimensional free repair/replacement warranty	Proposed a two-dimensional free repair/replacement warranty for the automotive industry, based on usage within a specific time frame.
3	Balachander [28]	2001	Long-term warranty	Provided a signalling-based explanation for the empirical finding that lower-quality products might come with longer warranties. Depending on whether the new entrant is more or less reliable than the established competitor, they demonstrated a negative or positive correlation between the length of the warranty and the product's reliability.
4	Mi [6]	2002	Bathtub-shaped failure rate function	Optimized the burn-in period, replacement frequency, and warranty terms. Determined the minimum duration for the burn-in period and the maximum limit for the most appropriate replacement.
5	Baik et al. [15]	2004	Two-dimensional warranty	Proposed a two-dimensional model based on a bivariate failure distribution with limited repair capabilities, where degradation occurs due to both age and usage. Addressed scenarios where failures could only be resolved through replacement.
6	Yang and Zaghati [43]	2004	Two-dimensional warranty	Outlined a two-dimensional warranty censoring technique for assessing reliability, illustrated using automotive onboard diagnostics as a case study.
7	Bai and Pham [7]	2005	Free repair policy	Developed a new warranty policy that imposes a limit on the number of repairs allowed for a system within a specified time period. Once this limit is reached, the system will be replaced instead of repaired.
8	Rai and Singh [16]	2005	Two dimensional warranty	Introduced a streamlined method for setting mileage warranty limits to influence the volume and cost of claims for new products.
9	Manna et al.[17]	2006	Fixed as well as flexible warranty plans	Identified the optimal warranty coverage area, balancing the warranty cost budget with customer needs for both fixed and flexible warranty plans across different regions.
10	Jung and Bai [18]	2007	Two-dimensional warranty	Calculated the lifespan distribution of a system covered under a two-dimensional warranty, which considers both age and usage simultaneously. This was done using a bivariate technique, assuming that age and usage are statistically correlated.

Table 1: *Continued*

Sr. No.	Author(s)	Year	Methods/Warranty Types	Findings
11	Matis et al. [8]	2008	Non-renewing combined warranty policy	Investigated the optimal product pricing, pro-rata warranty length, repair option selection, and profitability of a system in scenarios where replacement or minimal/general repair plans are offered during the standard warranty period.
12	Chien [9]	2008	Imperfect free replacement warranty renewal policy	Investigated a cost model that considered the consumer's perspective for product replacement and preventive maintenance.
13	Huang and Yen [19]	2009	Two-dimensional warranty	Optimized manufacturers' revenues by implementing a two-dimensional warranty model that included periodic preventive maintenance and an optimal warranty policy.
14	Lawless et al. [20]	2009	Two-dimensional warranty	Developed models that consider both usage and age within a heterogeneous population of products. Estimated the parameters using field data, incorporating information from American car warranties.
15	Jack et al. [21]	2009	Two-dimensional warranty	Proposed a repair-replace method for a two-dimensional warranty model in which the product is replaced upon the first failure, and subsequent failures are addressed with minimal repairs. Utilized an accelerated failure time model to assess the impact of item degradation on the usage rate.
16	Kadyan and Ramnivas [58]	2013	Free repair/replacement warranty	Analyzed a single-unit system and evaluated various system metrics, including reliability, Mean Time to System Failure (MTSF), and availability.
17	Gallego et al. [29]	2014	Extended warranty	Demonstrated the viability of an extended warranty in both homogeneous and diverse markets.
18	Yeh and Fang [10]	2014	Pro-rata warranty	Proposed a Bayesian decision-making model to assist manufacturers for pricing and production decisions.
19	Dai et al. [23]	2017	Two-dimensional warranty	Investigated the impact of usage rate on product failure using an accelerated failure time model. Recommended a stochastic expectation-maximization approach to estimate reliability model parameters for both censored and field data.
20	Jiao and Zhu [11]	2018	Minimal repair-replacement policy	Optimized the product's selling price, preventive maintenance intervals, and warranty period duration.
21	Arnold et al. [30]	2019	Non-renewing and renewing free repair warranties	Developed a geometric procedure to model the warranty claim process and assess warranty costs. The model was validated using a simulation approach and actual data from the automobile industry.

Table 1: *Continued*

Sr. No.	Author(s)	Year	Methods/Warranty Types	Findings
22	Wang et al. [12]	2019	Free repair/replacement warranty	Reduced the overall estimated service cost by determining the extent to which a component could be upgraded under a free repair or replacement warranty.
23	He et al. [31]	2020	Extended warranty	Proposed an extended warranty model that benefits all parties by optimizing the purchase period, maintenance procedures, and consumer usage rate. The model identifies the product failure process using the generalized Polya process failure mode.
24	Zheng and Su [24]	2020	Two-dimensional warranty	Proposed a flexible two-dimensional basic warranty policy comprising two rectangular regions. The first region accounts for the utilization rate, while the second region applies this usage rate in practice.
25	Taleizadeh and Mokhtarzadeh [25]	2020	Two-dimensional warranty	Applied the value-at-risk method to evaluate how a two-dimensional warranty coverage product, offered through both online and offline channels, would generate warranty claims over time. Optimized the product's pricing, warranty period, and usage parameters for each channel to maximize profit.
26	Wang et al. [26]	2021	Two dimensional warranty	Established the system's failure rate function under imperfect preventive maintenance using a two-dimensional warranty policy, employing an accelerated failure time model.
27	Gupta and Bhattacharya [27]	2022	Pro rata warranty	Utilized a non-parametric technique to evaluate the mean residual life and reliability from incomplete two-dimensional warranty data. This evaluation was based on the density function, assuming that age and usage are independent variables.
28	Mabrouk and Chelbi [32]	2022	Extended warranty	Enhanced the maintenance policy, extended the warranty period, and offered a free system replacement warranty.
29	Shang et al. [13]	2022	Free repair warranty with rebate and charge	Examined a free repair warranty with rebate and charge options to ensure reliability throughout the warranty period.

The study aimed to analyses various warranty policies, including free replacement, two-dimensional, minimal repair, long-term, renewing, non-renewing, feasible extended, and pro-rata policies, while exploring their characteristics and impact on both manufacturers and consumers.

3. System reliability

3.1. With warranty

This section highlights the literature that explores the combined concepts of warranty and reliability. Table 2 provides a detailed summary of research papers published in recent years, examining the relationship between reliability and warranty as studied by various researchers.

Table 2: *Detailed Survey of Papers Reviewed pertaining to relation between reliability and warranty*

Sr. No.	Author(s)	Year	Findings
1	Agarwal et al. [34]	1996	Explored the connection between warranty and reliability for home electronics and appliances, focusing on factors like product age and market trends. They concluded that this relationship weakens when there is significant variability in reliability and minimal market change.
2	Lu [35]	1998	Presented a mathematical model for vehicle customer mileage distribution using the log-normal distribution. They proposed a method for predicting reliability based on field failure records collected during the first four to five months of a vehicle's warranty period.
3	Vintr [36]	1999	Enhanced product reliability using two distinct approaches. One focused on minimizing manufacturing costs for extended warranty periods, while the other aimed at maximizing the warranty period based on known production costs.
4	Wood [37]	2001	Examined various types of failures from multiple perspectives, including component replacement failures from a hardware engineer's viewpoint, part return failures for the manufacturer, warranty claim failures for the finance department, maintenance issues for service organizations, and product degradation from the users' perspective. They discussed numerous reliability metrics and explored both theoretical and practical relationships between them.
5	Wu and Meeker [38]	2002	Utilized warranty databases and statistical methods to identify early major issues, allowing them to take proactive measures to address reliability concerns.
6	Yang and Zaghati [39]	2002	Established a connection between reliability, service time, and mileage. They recorded various warranty and non-warranty claims and developed a model for mileage accumulation using the sequential regression method based on warranty data.
7	Rai and Singh [40]	2003	Addressed the issue of insufficient data that hindered the reporting of failures beyond the warranty period. They utilized a two-parameter estimation method to generate a complete dataset from the incomplete warranty mileage data.
8	Kleyner and Bender [41]	2003	Introduced an effective technique for enhancing reliability by applying a conventional military approach, utilizing failed components identified through manufacturer warranty data.
9	Kim et al. [82]	2004	Established a model to analyse preventative maintenance time when a system was sold with a warranty from the viewpoints of the customer and the manufacturer, where failure was either repaired minimally or replaced at no cost to the user.
10	Yang and Cekecek [42]	2004	Evaluated the performance of industrial systems under a product warranty policy, taking into account user errors in system operation and manufacturing defects.

Table 2: *Continued*

Sr. No.	Author(s)	Year	Findings
11	Kleyner and Sandborn [44]	2005	Optimized the cost of achieving the desired level of reliability by implementing the two-dimensional warranty censoring technique.
12	Pecht [45]	2006	Examined the relationship between reliability and warranty in the automotive industry.
13	Mannar et al. [46]	2006	Proposed a novel fault zone localization methodology, using a case study of warranty data from the electronics industry to explore the relationship between warranty failures and design variables in product manufacturing.
14	Ion et al. [47]	2007	Examined the effect of the warranty call rate and estimated the parameters of the failure distribution using data from the first year of the warranty. Their findings indicated that the Weibull failure distribution best represented the data.
15	Huang and Fang [87]	2008	Developed a Bayesian decision model incorporating warranty and preventive maintenance to mitigate product deterioration.
16	Yun et al. [48]	2008	Investigated two warranty service procedures: the first strategy considered the age of the item when an incomplete repair was performed, while the second strategy did not take the item's age into account when assessing the level of improvement.
17	Vazquez and Rey-Stolle [49]	2008	Developed a reliability model for degraded photovoltaic modules, establishing justifiable power warranties and other system parameters based on field data.
18	Christozov et al. [50]	2009	Proposed a mathematical model to assess both the risk of product malfunction and the risk of making an incorrect purchase decision.
19	Murthy et al. [51]	2009	Focused on the reliability and diversity of product levels.
20	Lee et al. [22]	2010	Employed a fuzzy technique to develop a model for determining the optimal warranty period and analyzing warranty claim data.
21	Saidi-Mehrabad [52]	2010	Explored strategies for enhancing the reliability of used goods under various warranty policies, such as free replacement, pro-rata warranty, and their combinations, using a virtual age approach and screening test method.
22	Summit [54]	2011	Assessed the time required to accurately estimate parameters using a failure model based on the Weibull distribution. This model simulated the occurrence of warranty claims resulting from the failure of a single component over a period of up to five years.
23	Wu [55]	2012	Provided a summary of warranty research and explored potential future directions for the analysis of warranty data.
24	Zhou et al. [56]	2012	Examined the statistical hazard rate model to detect reliability issues using warranty data and information from the upstream supply chain.
25	Gurel and Cakmakci [57]	2013	Introduced a parametric Weibull model based on warranty claims and service data, designed to forecast the potential warranty period and the future lifespan of the product.
26	Hsu et al. [59]	2015	Using a Bayesian framework, developed a hierarchical model for warranty prediction.
27	Mackelprang et al. [60]	2015	Introduced a variable called "unexpected product failure costs," measuring the uncertainty associated with product reliability outcomes.

Table 2: *Continued*

Sr. No.	Author(s)	Year	Findings
28	Limon et al. [61]	2016	Assessed reliability using field data, including warranty claims, field failures, recall information, and maintenance records, along with censored data derived from consumer usage rate profiles.
29	Gupta et al. [62]	2017	Proposed a bivariate reliability function based on usage rates, along with corresponding reliability measures for warranty claims data, employing a conditional approach.
30	Shang et al. [64]	2018	Proposed an Inverse Gaussian degradation model paired with a conditional renewal replacement warranty policy, along with an optimal warranty duration and sale price from the manufacturer's perspective. Using prior user-observed degradation levels, they recommended the most effective post-warranty maintenance policy.
31	He et al. [65]	2018	Collected warranty data from manufacturers and applied a log-linear regression model to examine the impact on reliability during the early stages of production.
32	Aljazea and Wu [66]	2019	Investigated warranty risks within the automotive industry and developed a comprehensive taxonomy of warranty hazards.
33	Wang et al. [67]	2019	Developed an accelerated reliability growth model and a multiphase reliability testing program from the manufacturer's viewpoint, aiming to optimize the trade-off between warranty costs and reliability improvements.
34	Zhu et al. [68]	2019	Developed a reliability model to forecast costs, warranty policies, sales duration, and promotional strategies.
35	Jie et al. [69]	2020	Proposed a reliability model that considered regional variations in warranty terms and pricing. Their approach aimed to maximize the effectiveness of a regional warranty differential pricing strategy by using both unified and segmented warranty systems.
36	Li et al.[70]	2021	Developed a branched power law model to characterize the deterioration behavior of a system over multiple life cycles, considering both lifecycle warranty costs and analysis for several remanufacturing cycles. By using field time-to-failure data, they assessed the system's reliability and examined four potential warranty policies to evaluate warranty costs and environmental impacts, providing insights to support decision-making.
37	Shang et al. [71]	2021	Examined maintenance practices aimed at ensuring long-term product reliability beyond the warranty period. They explored two types of periodic maintenance strategies: one where replacement is performed first, followed by preventive maintenance, and another where preventive maintenance is conducted first, with replacement occurring later.
38	Wang et al. [72]	2021	Analyzed the reliability of individual components with irregular imperfect preventive maintenance and investigated the optimal maintenance strategy for multi-component systems under warranty.
39	Dong et al. [73]	2022	Employed a genetic algorithm to estimate the reliability threshold for opportunistic repair of power transmission devices.
40	Shang et al. [74]	2023	Focused on the application of advanced digital technologies in maintaining product reliability, while also exploring the characteristics of random warranty models and their implications for product managers and manufacturers.

Table 2: *Continued*

Sr. No.	Author(s)	Year	Findings
41	Qiao et al. [75]	2023	Proposed a new generalized post-warranty maintenance strategy designed to reduce maintenance costs, incorporating a repair time limit within an age-based preventive replacement policy. They also examined the impact of the manufacturer's warranty terms on consumers' joint warranty-maintenance decisions.
42	Hussien and El-Sherbeny [111]	2024	Evaluated the performance of industrial systems under a product warranty policy, taking into account user errors in system operation and manufacturing defects.

3.2. With warranty and insurance

Following section provides a concise summary of the literature on system reliability, insurance, and warranty. Insurance is designed to protect policyholders from unexpected events or accidents that could result in financial hardship. Policyholders rely on insurance companies to provide financial protection when unexpected events occur. Table 3 presents a concise literature review encompassing the concepts of reliability, warranty, and insurance.

Table 3: *Concise summary of the literature on system reliability, insurance, and warranty*

Sr. No.	Author(s)	Year	Findings
1	Fumagalli et al. [76]	2004	Developed a reliability insurance model for electrical power distribution systems, aiming to enhance service quality and achieve the desired level of system reliability.
2	Billimoria and Poudineh [77]	2019	Proposed a novel model, the "insurer-of-last-resort," which integrates insurance-based risk management concepts. This model aligns incentives for centralized decision-making, addressing the challenges energy-only markets face in ensuring resource adequacy and reliability.
3	Liu et al. [78]	2020	Used a semi-Markov process and a sequential Monto-Carlo simulation framework to optimize a new insurance stochastic model that takes risk interdependence for the use of cyber insurance in power systems.
4	Lau et al. [79]	2021	Suggested a new coalitional platform-based cyber insurance design that uses cyber security modelling and reliability implication analysis to estimate insurance rates.
5	Sachdeva et al. [80, 109]	2022 & 2024	Investigated a system under an extended conditional warranty with both long-term and short-term insurance.

4. Cost-benefit analysis

Manufacturers take into account development costs, warranty costs, and insurance costs when developing and manufacturing a system. Therefore, system profitability and various cost models are being investigated by many researchers, which are briefly addressed in this section. Table 4 offers a concise literature review that emphasizes cost as a critical factor alongside the concepts of reliability, warranty, and insurance.

Table 4: *Concise summary of the literature on system reliability, insurance, and warranty*

Sr. No.	Author(s)	Year	Findings
1	Hussain and Murthy [81]	2003	Developed a model that optimizes the costs associated with manufacturing units and the provision of warranty services.
2	Kleyer et al. [83]	2004	Determined the desired level of reliability by minimizing the overall life cycle costs in the large-volume automotive electronics sector.
3	Sheu and Chien [84]	2005	Investigated the optimal burn-in period to balance manufacturing and warranty costs. They also considered the impact of renewing or not renewing free replacement and rebate policies.
4	Bai and Pham [85]	2006	Developed cost analysis models for complex systems, including parallel, series, series-parallel, and parallel-series configurations, from the manufacturer's perspective. Their models focused on a full-service warranty renewal policy, which ensures that consumers are not charged for replacing a system with a new one in the event of failure or during preventive maintenance.
5	Wu et al. [86]	2007	Analyzed a cost model to determine the optimal burn-in duration and warranty period for a fully renewable combination of free replacement and pro-rata warranty.
6	Sgarbossa and Pham [88]	2010	Developed a cost model aimed at eliminating failures during both testing and the warranty period.
7	Park and Pham [89]	2010	Developed warranty cost models based on quasi-renewal processes for single and multi component systems with imperfect repair services. They also evaluated the reliability and other system performance measures for these systems.
8	Taneja [90]	2010	Developed a cost-benefit model for a system under warranty, where the repair cost is borne by either the manufacturer or the user, depending on whether the warranty claim is covered or not.
9	Wang et al. [53]	2010	Developed reliability-based design models for perfect maintenance, non-repairable products, and low maintenance to reduce the overall cost of production, maintenance, and warranty.
10	Gonzalez and Bueno [91]	2011	Analyzed a discounted warranty cost policy for a repairable, identical-series system, where components remain operational until they reach their critical failure level.
11	Jalali Naini and Shafiee [92]	2011	Developed a decision model to identify the optimal pricing strategy and upgrading plan for warranted second-hand products. Their model was based on real-world data from used electronic devices.
12	Shafiee et al. [93]	2011	Investigated a cost model for used products through a two-dimensional free repair/replacement strategy. Their study compared the warranties provided for both new and used products.
13	Park and Pham [94]	2012	Developed cost models for both warranty and non-warranty periods, considering minimal repairs with free repair/replacement warranties, as well as pro-rata warranties for k-out-of-n systems. To minimize long-term expected costs, they incorporated a two-dimensional warranty function based on repair and failure times, alongside a periodic preventive maintenance strategy.
14	Huang et al. [95]	2013	Used a reliability-based preventive maintenance approach to determine optimal warranty limits for a two-dimensional warranty policy. They also incorporated non-periodic preventive maintenance strategies to maximize manufacturer profit.

Table 4: *Continued*

Sr. No.	Author(s)	Year	Findings
15	Williams [96]	2013	Applied a change-point software reliability growth model to identify optimal release practices, estimate delivery costs, and assess the reliability of warranted software systems.
16	Tong et al. [97]	2014	Optimized the pricing strategy for an automobile product by offering a two-dimensional extended warranty service, which could be purchased both at the time of sale and upon the expiration of the base warranty.
17	Xie et al. [98]	2014	Analyzed the increasing profit of a new durable product sold at a fixed price over a set sales duration, with non-renewable free post-warranty service. Their study focused on the scenario where customers did not request warranty or post-warranty repairs from the manufacturer.
18	Niwas et al. [99]	2015	Investigated two single-unit guaranteed systems, where repair is free of charge to the user with the constraint that failure is not the result of user negligence and preventive maintenance is performed beyond warranty time. The first unit works as new after being repaired beyond the warranty period, however, the second unit degraded and was replaced with a new one.
19	Darghouth et al. [63]	2017	Evaluated several maintenance strategies, such as no Preventive Maintenance, periodic multiphase Preventive Maintenance, and product Preventive Maintenance with constant efficiency within warranty duration, and established a cost analysis model for improvement of reliability for warranted second-hand goods.
20	Zhao et al. [100]	2018	Developed a novel one-dimensional warranty cost optimization technique by utilizing degradation data, accounting for imperfect repairs, uncertainties in experimental data, and variations in field data. They used this approach to predict warranty cost intervals.
21	Alkahtani et al. [101]	2019	Created a decision support system that integrates stepwise approaches to identify manufacturing issues, map them to design information, and ultimately improve design parameters to achieve maximum reliability at the lowest cost.
22	Wang et al. [102]	2019	Examined periodic and imperfect preventive maintenance strategies for a product under a two-dimensional warranty, aiming to enhance customer satisfaction. They reduced the expected total cost per product by optimizing preventive maintenance actions and comparing their proposed strategy with a minimal repair corrective maintenance approach.
23	Solkhe and Taneja [103, 104, 105]	2018 & 2019	Analyzed a single-unit system under conditional warranty and examined the system's profitability both before and after the warranty period expired. They noted that conditional warranties can lead to disputes between the manufacturer and the customer. In light of this, they extended their work by considering the involvement of a higher authority, such as a court of law, to resolve such disputes. Additionally, they conducted a cost-benefit analysis for both the manufacturer and the user within the context of each of these models.
24	Koschnick and Hartman [106]	2020	Developed performance-based warranties that guarantee product operation up to a specified threshold. They also proposed an algorithm for designing such warranties, taking into account the associated operating costs.

Table 4: *Continued*

Sr. No.	Author(s)	Year	Findings
25	Cha et al. [107]	2022	Proposed a new renewable warranty policy aimed at maximizing the probability of success while minimizing associated costs. Their approach considered different populations of items, categorizing them into weak and strong groups.
26	Sachdeva et al. [108]	2023	Conducted a cost-benefit analysis of a system for both conditional and extended warranty models.
27	Huang et al. [110]	2024	Focused on two major factors: burn-in (run-in), a prelaunch testing process to detect faults, and periodic preventive maintenance, which attempts to eliminate unexpected failures during product use.

Numerous cost models for complex systems under warranty and insurance, with a focus on system reliability, have been developed by various researchers.

5. Conclusion and future scope

Both professionals and academics have recently become more interested in warranty and insurance. This article offered a thorough synthesis and review of warranties types, pricing and insurance by summarizing the existing literature and outlining some intriguing areas for further study.This review will be beneficial to academics conducting studies in this field because it neatly compiles practically all warranty and insurance policies, models, and approaches. In conclusion, several innovative and challenging research directions are proposed:

- **Exploration of Flexible Warranties**:The concept of flexible warranties warrants further study. Customers could have the option to customize warranty contracts that best suit their needs. It would be intriguing to model consumer preferences, product reliability, maintenance requirements, and other relevant factors simultaneously under these policies.
- **Cost Analysis of Multi-Component Systems with 2D Warranties**:There is a need for more in-depth research on the cost analysis of multi-component systems covered by two-dimensional warranties.
- **Focus on Real-World Warranty Challenges**:Researchers should prioritize addressing practical issues in warranties by gathering real-world data from manufacturers and consumers. This approach would help tackle real-life challenges in warranty claims and services more effectively.

References

[1] Murthy DN and Djamaludin I. (2002). New product warranty: A literature review. *International Journal of Production Economics*, 79(3):231-60.
[2] Shafiee M and Chukova S. (2013). Maintenance models in warranty: A literature review. *European Journal of Operational Research*, 229(3):561-72.
[3] Wang X and Xie W. (2018). Two-dimensional warranty: A literature review *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, 232(3):284-307.
[4] Karim R and Suzuki K.(2005). Analysis of warranty claim data: a literature review *International Journal of Quality & Reliability Management*, 22(7):667-86.
[5] Murthy DN and Blischke WR. (1992). Product warranty management III: A review of mathematical models *European Journal of Operational Research*, 63(1):1-34.
[6] Mi J. (2002). On bounds for some optimal policies in reliability *Journal of applied probability*, 39(3):491-502.

[7] Bai J and Pham H. (2005). Repair-limit risk-free warranty policies with imperfect repair *IEEE Transactions on Systems, Manand Cybernetics-Part A: Systems and Humans*, 35(6):765-72.
[8] Matis TI, Jayaraman R and Rangan A.(2008). Optimal price and pro rata decisions for combined warranty policies with different repair options *IIE Transactions*, 40(10):984-91.
[9] Chien YH.(2008). Optimal age-replacement policy under an imperfect renewing free-replacement warranty *IEEE Transactions on Reliability*,57(1):125-33.
[10] Yeh CW and Fang CC.(2014). Optimal pro-rata warranty decision with consideration of the marketing strategy under insufficient historical reliability data *The International Journal of Advanced Manufacturing Technology*, 71:1757-72.
[11] Jiao C and Zhu X.(2018). Optimal design of sales and maintenance under the renewable warranty *RAIRO-Operations Research*, 52(2):529-42.
[12] Wang X, Xie M and Li L.(2019). On optimal upgrade strategy for second-hand multi-component systems sold with warranty *International journal of production research*, 57(3):847-64.
[13] Shang L, Yu X, Du Y et al.(2022). An optimal random hybrid maintenance policy of systems under a warranty with rebate and charge *Mathematics*, 10(18):3229.
[14] Eliashberg J, Singpurwalla ND and Wilson SP. (1997). Calculating the reserve for a time and usage indexed warranty *Management Science*, 43(7):966-75.
[15] Baik J, Murthy DP and Jack N. (2004). Two dimensional failure modeling with minimal repair *Naval Research Logistics (NRL)*, 51(3):345-62.
[16] Rai B and Singh N. (2005). A modeling framework for assessing the impact of new time/mileage warranty limits on the number and cost of automotive warranty claims *Reliability Engineering & System Safety*, 88(2):157-69.
[17] Manna DK, Pal S and Sinha S.(2006). Optimal determination of warranty region for 2D policy: A customers' perspective *Computers & Industrial Engineering*, 50(1-2):161-74.
[18] Jung M and Bai DS.(2007). Analysis of field data under two-dimensional warranty *Reliability Engineering & System Safety*, 92(2):135-43.
[19] Huang YS and Yen C. (2009). A study of two-dimensional warranty policies with preventive maintenance *IIE transactions*, 41(4):299-308.
[20] Lawless JF, Crowder MJ and Lee KA. (2009). Analysis of reliability and warranty claims in products with age and usage scales *Technometrics*, 51(1):14-24.
[21] Jack N, Iskandar BP and Murthy DP. (2009). A repair replace strategy based on usage rate for items sold with a two-dimensional warranty *Reliability Engineering & System Safety*, 94(2):611-7.
[22] Lee SH, Lee SJ and Moon KI.(2010). A fuzzy logic-based approach to two-dimensional automobile warranty system *Journal of circuits, Systems, and Computers*, 19(01):139-54.
[23] Dai A, He Z, Liu Z et al.(2017). Field reliability modeling based on two-dimensional warranty data with censoring times *Quality Engineering*, 29(3):468-83.
[24] Zheng R and Su C. (2020). A flexible two dimensional basic warranty policy with two continuous warranty regions *Quality and Reliability Engineering International*, 36(6):2003-18.
[25] Taleizadeh AA and Mokhtarzadeh M. (2020). Pricing and two-dimensional warranty policy of multi-products with online and offline channels using a value-at-risk approach *Computers & Industrial Engineering*, 148:106674.
[26] Wang R, Cheng Z, Rong L et al. (2021). Availability optimization of two-dimensional warranty products under imperfect preventive maintenance *IEEE Access*, 9:8099-109.
[27] Gupta SK and Bhattacharya D. (2022). Non-parametric estimation of bivariate reliability from incomplete two-dimensional warranty data *Reliability Engineering & System Safety*, 222:108385.
[28] Subramanian B. (2001). Warranty signaling and reputation *Management Science*, 47(9):1302-9.
[29] Gallego G, Wang R, Ward J et al. (2014). Flexible duration extended warranties with dynamic reliability learning *Production and Operations Management*, 23(4):645-59.
[30] Arnold R, Chukova S, Hayakawa Y et al. (2019). Warranty cost analysis with an alternating geometric process *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, 233(4):698-715.

[31] He Z, Wang D, He S et al. (2020). Two-dimensional extended warranty strategy including maintenance level and purchase time: A win-win perspective *Computers & Industrial Engineering*, 141:106294.

[32] Ben Mabrouk A and Chelbi A. (2023). Optimal maintenance policy for equipment leased with base and extended warranty *International Journal of Production Research*, 61(3):898-909.

[33] Murthy DN and Hussain AZ. (1995). Warranty and optimal redundancy design *Engineering Optimization+ A35*, 23(4):301-14.

[34] Agrawal J, Richardson PS and Grimm PE. (1996). The relationship between warranty and product reliability *Journal of Consumer Affairs*, 30(2):421-43.

[35] Lu MW. (1998). Automotive reliability prediction based on early field failure warranty data *Quality and Reliability Engineering International*, 14(2):103-8.

[36] Vintr Z. (1999). Optimization of reliability requirements from manufacturer's point of view *InAnnual Reliability and Maintainability. Symposium* 1999 Proceedings (Cat. No. 99CH36283), 183-189. IEEE.

[37] Wood AP. (2001). Reliability-metric varieties and their relationships. InAnnual Reliability and Maintainability Symposium. Proceedings *International Symposium on Product Quality and Integrity (Cat. No. 01CH37179)* 2001 Jan 22, 110-115. IEEE.

[38] Wu H and Meeker WQ.(2002). Early detection of reliability problems using information from warranty databases *Technometrics*, 44(2):120-33.

[39] Yang G and Zaghati Z.(2002). Two-dimensional reliability modeling from warranty data *InAnnual Reliability and Maintainability Symposium* 2002 Proceedings (Cat. No. 02CH37318), 272-278. IEEE.

[40] Rai B and Singh N. (2003). Hazard rate estimation from incomplete and unclean warranty data *Reliability Engineering & System Safety*, 81(1):79-92.

[41] Kleyner A and Bender M.(2003). Enhanced reliability prediction method based on merging military standards approach with manufacturer's warranty data *InAnnual Reliability and Maintainability Symposium, 2003* ,202-206. IEEE.

[42] Yang K and Cekecek E.(2004). Design vulnerability analysis and design improvement by using warranty data *Quality and reliability engineering international* , 20(2):121-33.

[43] Yang G and Zaghati Z. (2004). Reliability and robustness assessment of diagnostic systems from warranty data *InAnnual Symposium Reliability and Maintainability*,146-150. IEEE.

[44] Kleyner A and Sandborn P. (2005). A warranty forecasting model based on piecewise statistical distributions and stochastic simulation *Reliability Engineering & System Safety*, 88(3):207-14.

[45] Pecht MG. (2006). Establishing a relationship between warranty and reliability *IEEE Transactions on Electronics Packaging Manufacturing*, 29(3):184-90.

[46] Mannar K, Ceglarek D, Niu F et al. (2006). Fault region localization: Product and process improvement based on field performance and manufacturing measurements *IEEE Transactions on Automation Science and Engineering*, 3(4):423-39.

[47] Ion RA, Petkova VT, Peeters BH et al. (2007). Field reliability prediction in consumer electronics using warranty data *Quality and Reliability Engineering International*, 23(4):401-14.

[48] Yun WY, Murthy DN and Jack N. (2008). Warranty servicing with imperfect repair *International Journal of Production Economics*, 111(1):159-69.

[49] Vazquez M and Rey Stolle I. (2008). Photovoltaic module reliability model based on field degradation studies *Progress in photovoltaics: Research and Applications*, 16(5):419-33.

[50] Christozov D, Chukova S and Mateev P. (2009). On two types of warranties: Warranty of malfunctioning and warranty of misinforming *Asia-Pacific Journal of Operational Research*, 26(03):399-420.

[51] Murthy DN, Osteras T and Rausand M.(2009). Component reliability specification *Reliability Engineering & System Safety*, 94(10):1609-17.

[52] Saidi-Mehrabad M, Noorossana R and Shafiee M. (2010). Modeling and analysis of effective ways for improving the reliability of second-hand products sold with warranty *The international journal of advanced manufacturing technology*, 46:253-65.

[53] Wang Z, Huang HZ and Du X.(2010). Optimal design accounting for reliability, maintenance, and warranty *Journal of Mechanical Design*, 132(1): 011007.

[54] Summit RA.(2011). Estimating the reliability model parameters through a simulation of warranty claims: How much data is needed? *ANZIAM Journal*, 53:C451-64.
[55] Wu S.(2012). Warranty data analysis: A review *Quality and Reliability Engineering International*, 28(8):795-805.
[56] Zhou C, Chinnam RB and Korostelev A.(2012). Hazard rate models for early detection of reliability problems using information from warranty databases and upstream supply chain *International Journal of Production Economics*, 139(1):180-95.
[57] Gurel U and Cakmakci M.(2013). Impact of reliability on warranty: A study of application in a large size company of electronics industry *Measurement*, 46(3):1307-310.
[58] Kadyan MS.(2013). Cost benefit analysis of a single-unit system with warranty for repair *Applied mathematics and Computation*, 223:346-53..
[59] Hsu NJ, Tseng ST and Chen MW.(2015). Adaptive warranty prediction for highly reliable products *IEEE Transactions on Reliability*, 64(3):1057-67.
[60] Mackelprang AW, Habermann M and Swink M.(2015). How firm innovativeness and unexpected product reliability failures affect profitability *Journal of Operations Management*, 38:71-86.
[61] Limon S, Yadav OP, Zuo MJ et al.(2016). Reliability estimation considering usage rate profile and warranty claims *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, 230(3):297-308.
[62] Gupta SK, De S and Chatterjee A.(2017). Some reliability issues for incomplete two-dimensional warranty claims data *Reliability Engineering & System Safety*, 157:64-77.
[63] Darghouth MN, Chelbi A and Ait-Kadi D.(2017). Investigating reliability improvement of second-hand production equipment considering warranty and preventive maintenance strategies *International Journal of Production Research*, 55(16):4643-61.
[64] Shang L, Si S, Sun S et al.(2018). Optimal warranty design and post-warranty maintenance for products subject to stochastic degradation *IISE Transactions*, 50(10):913-27.
[65] He S, Zhang Z, Jiang W et al. (2018). Predicting field reliability based on two-dimensional warranty data with learning effects *Journal of Quality Technology* , 50(2):198-206.
[66] Aljazea AM and Wu S.(2019). Managing risk for auto warranties *International Journal of Quality & Reliability Management*, 36(7):1088-105.
[67] Wang Y, Liu Y, Li X et al.(2019). Multi-phase reliability growth test planning for repairable products sold with a two-dimensional warranty *Reliability Engineering & System Safety*, 189:315-26.
[68] Zhu X, Jiao C and Yuan T.(2019). Optimal decisions on product reliability, sales and promotion under nonrenewable warranties *Reliability Engineering & System Safety*, 192:106268.
[69] Jie L, Liu W, Li M et al.(2020). Sub-Region Warranty Differential Pricing Optimization Strategy Based on Regional Granularity of Use Reliability *IEEE Access*, 8:95523-39.
[70] Li M, Nemani VP, Liu J et al. (2021). Reliability-informed life cycle warranty cost and life cycle analysis of newly manufactured and remanufactured units *Journal of Mechanical Design*, 143(11):112001.
[71] Shang L, Wang H, Wu C et al.(2021). The post warranty random maintenance policies for the product with random working cycles *Eksploatacja i Niezawodnosc*, 23(4).
[72] Wang R, Cheng Z, Dong E et al.(2021). Reliability-based opportunistic maintenance modeling for multi-component systems with economic dependence under base warranty *Discrete Dynamics in Nature and Society*,21:1-6.
[73] Dong E, Gao T, Cheng Z et al.(2022). Opportunistic Maintenance Strategy for Complex Equipment with a Genetic Algorithm Considering Failure Dependence: A Two-Dimensional Warranty Perspective *Sensors*, 22(18):6801.
[74] Shang L, Zhang N, Yang L et al.(2023). Study of a Random Warranty Model Maintaining Fairness and a Random Replacement Next Model Sustaining Post-Warranty Reliability *Axioms*, 12(3):258.
[75] Qiao P, Luo M, Ma Y et al. (2023). Optimal warranty option and post-warranty maintenance strategy under a warranty menu: from a consumer perspective *International Journal of Production Research*, 7:1-23.

[76] Fumagalli E, Black JW, Vogelsang I et al.(2004). Quality of service provision in electric power distribution systems through reliability insurance *IEEE Transactions on Power Systems*, 19(3):1306-93.

[77] Billimoria F and Poudineh R.(2019). Market design for resource adequacy: A reliability insurance overlay on energy-only electricity markets *Utilities Policy*, 60:100935.

[78] Liu Z, Wei W, Wang L et al.(2020). An actuarial framework for power system reliability considering cybersecurity threats *IEEE Transactions on Power Systems*, 36(2):851-64.

[79] Lau P, Wang L, Liu Z et al. (2021). A coalitional cyber-insurance design considering power system reliability and cyber vulnerability *IEEE Transactions on Power Systems*, 36(6):5512-24.

[80] Sachdeva K, Taneja G and Manocha A.(2022). Sensitivity and Economic Analysis of an Insured System with Extended Conditional Warranty *Reliability: Theory & Applications*, 17(3 (69)):315-27.

[81] Hussain AZ and Murthy DN.(2003). Warranty and optimal reliability improvement through product development *Mathematical and computer modelling*, 38(11-13):1211-7.

[82] Kim CS, Djamaludin I and Murthy DN.(2004). Warranty and discrete preventive maintenance *Reliability Engineering & System Safety*, 84(3):301-9.

[83] Kleyner A, Sandborn P and Boyle J. (2004). Minimization of life cycle costs through optimization of the validation program-a test sample size and warranty cost approach *InAnnual Symposium Reliability and Maintainability*, 553-558. IEEE.

[84] Sheu SH and Chien YH. (2005). Optimal burn-in time to minimize the cost for general repairable products sold under warranty *European Journal of Operational Research*, 163(2):445-61.

[85] Bai J and Pham H. (2006). Cost analysis on renewable full-service warranties for multi-component systems *European Journal of Operational Research*, 168(2):492-508.

[86] Wu CC, Chou CY and Huang C.(2007). Optimal burn-in time and warranty length under fully renewing combination free replacement and pro-rata warranty *Reliability Engineering & System Safety*, 92(7):914-20.

[87] Huang YS and Fang CC.(2008). A cost sharing warranty policy for products with deterioration *IEEE Transactions on Engineering Management*, 55(4):617-27.

[88] Sgarbossa F and Pham H.(2010). A cost analysis of systems subject to random field environments and reliability *IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews)*, 40(4):429-37.

[89] Park M and Pham H.(2010). Warranty cost analyses using quasi-renewal processes for multicomponent systems *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, 40(6):1329-40.

[90] Taneja G.(2010). Reliability Model for a System with Conditional Warranty and various types of Repair/Replacement *Advances in Information Theory and Operations Research*, 214-20.

[91] Gonzalez NG and Bueno V. (2011). Estimating the Discounted Warranty Cost of a Minimally Repaired Coherent System *Revista Colombiana de Estadistica*, 34(3):513-43.

[92] Jalali Naini SG and Shafiee M. (2011). Joint determination of price and upgrade level for a warranted second-hand product *The International Journal of Advanced Manufacturing Technology*, 54:1187-98.

[93] Shafiee M, Chukova S, Saidi-Mehrabad M et al. (2011). Two-dimensional warranty cost analysis for second-hand products *Communications in statistics-theory and methods*, 40(4):684-701.

[94] Park M and Pham H. (2012). Warranty Cost Analysis for k-out-of-n Systems With 2-D Warranty *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, 42(4):947-57.

[95] Huang YS, Chen E and Ho JW. (2013). Two-dimensional warranty with reliability-based preventive maintenance *IEEE Transactions on Reliability*, 62(4):898-907.

[96] WILLIAMS DP. (2013). Optimal release policies for a software system with warranty cost and change-point phenomenon *Turkish Journal of Electrical Engineering and Computer Sciences*, 21(1):234-45.

[97] Tong P, Liu Z, Men F et al. (2014). Designing and pricing of two-dimensional extended warranty contracts based on usage rate *International Journal of Production Research*, 52(21):6362-80.

[98] Xie W, Liao H and Zhu X.(2014). Estimation of gross profit for a new durable product considering warranty and post-warranty repairs *IIE Transactions*, 46(2):87-105.

[99] NiwAs R, Kadyan MS and Kumar J.(2015). Probabilistic analysis of two reliability models of a single-unit system with preventive maintenance beyond warranty and degradation *Eksploatacja i Niezawodnosc*, 17(4).

[100] Zhao X, He S and Xie M.(2018). Utilizing experimental degradation data for warranty cost optimization under imperfect repair *Reliability Engineering & System Safety*, 177:108-19.

[101] Alkahtani M, Choudhary A, De A et al.(2019). A decision support system based on ontology and data mining to improve design using warranty data *Computers & industrial engineering*, 130:1027-39.

[102] Wang Y, Liu Y and Zhang A.(2019). Preventive maintenance optimization for repairable products considering two-dimensional warranty and customer satisfaction *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, 233(4):553-66.

[103] Solkhe M and Taneja G. (2018). A reliability model on a system with conditional warranty and intervention of higher authority on denial of valid claim *International Journal of Applied Engineering Research*, 13:16762-67.

[104] Solkhe M and Taneja G.(2019). A Reliability Model on a System with Conditional Warranty and Possibility of Denial of Valid Claim *Journal of Advanced Research in Dynamical and Control Systems*, 11(1):1302-1302.

[105] Solkhe M and Taneja G. (2019). A Reliability Model for a System with Consideration of Periods Before and After Expiry of Conditional Warranty *International Journal of Agricultural & Statistical Sciences*, 15(1).

[106] Koschnick C and Hartman JC.(2020). Using performance-based warranties to influence consumer purchase decisions *The Engineering Economist*, 65(1):1-26.

[107] Cha JH and Finkelstein M. (2022). A new warranty policy for heterogeneous items subject to monotone degradation processes *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, 236(1):55-65.

[108] Sachdeva K, Taneja G and Manocha A. (2023). Reliability and Sensitivity Analysis of a System with Conditional and Extended Warranty *Reliability: Theory & Application*, 18(3 (74)):689-707.

[109] Sachdeva, K., Taneja, G.and Manocha, A. (2024). Reliability and Sensitivity Analysis of an Insured System with Conditional Warranty Duration Lengthier than Insurance Duration. *International Journal of Reliability, Quality and Safety Engineering*, 31(01): 2350034.

[110] Huang, Y. S., Fang, C. C.and Kuo, C. Y. (2024). A study on two dimensional warranty with a preventive maintenance policy under burn in or run in. *Applied Stochastic Models in Business and Industry*.

[111] Hussien, Z. M. and El-Sherbeny, M. S. (2024). Stochastic analysis of an industrial system with preparation time repair under warranty policy. *Applied Sciences*, 14: 2076-3417.