EXPERT METHOD OF RANK PENALTIES

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Abstract

Voting methods include a variety of systems and technologies designed to facilitate the electoral process. These methods can range from the use of traditional paper ballots to modern electronic systems, each with unique features and functions. Heuristic peer review methods use simplified decision-making processes to evaluate complex scenarios, especially in areas such as usability and forecasting. These methods improve efficiency and accuracy by combining expert information with heuristic principles to improve results in a variety of applications. While heuristic methods offer significant advantages for expert evaluations, they can also lead to errors if not applied carefully, emphasizing the need for balanced approaches to complex evaluations. We propose an algorithm for finding the most consistent decision of experts in determining the leader in an alternative ranking problem.

Keywords: voting; group choice; coordinated decision; penalty functions; Schulze method; Skating method; expert evaluations; rank statistics

1. Introduction

In practice, there are often situations when there are several decision makers (LPRs), each of which has its own preferences on the same set of compared options, and on the basis of these individual preferences it is necessary to develop a group (collective) preference [1]. For example, the jury needs to allocate places among the contestants, the citizens of a country need to elect a president, etc. Such decision-making problems are called group choice problems, the LPRs participating in them are called electors, and the compared variants are called candidates.

Finding a consensual solution in group decision making (GDM) involves various strategies and models to promote consensus among members. The literature highlights several approaches to increase agreement and optimize decision making.

Consensus models

Liu et al. propose a consensus model using non-reciprocal fuzzy preference relations (NRFPR) to eliminate uncertainty in experts' opinions, using a particle swarm optimization algorithm to reach optimal solutions [2].

Chiclana et al. emphasize the importance of preference consistency by proposing a model for moving from consistency to consensus and thereby improving the quality of group decisions [3].

Overcoming dominance

Maximova et al. introduce the consensus-minus-one principle, which eliminates dominant voices to accelerate consensus, indicating a potential 97% reduction in decision-making time [4].

Dynamics of communication

Zhang and Kasari's research shows that effective communication, especially in lottery selection experiments, can lead to consensual group decisions that often result in extroverted group members [5].

Impact of the decision-making mechanism

Luis et al. find that decision-making mechanisms that promote apparent consensus lead to higher actual levels of agreement, suggesting that process choice significantly affects group dynamics [6].

While these models and strategies provide a framework for consensus building, challenges remain, particularly in managing "leaders" among experts and ensuring that all voices are heard, which can complicate the decision-making process.

2. Summary of methods of harmonized voting

The solution is often considered to be the so-called Schulze method (also Schwartz's method of sequential elimination), a voting system developed bo by Markus Schulze in 1997. Schulze himself calls it the "beatpath method". It allows to determine the winner (with objective counting) using ballots in which voters indicate their preferences for candidates, ranking them. This method can also be used to produce a preference-sorted list of candidates. This method satisfies Condorcet's criterion: if one candidate is the winner when compared to each of the other candidates, then it will be the winner by Schulze's method as well. In addition, Schulze's method allows to formally determine a winner even if there is no winner according to the Condorcet criterion. The winner by Schulze's method always belongs to the Schwarz set.

The Schulze voting method is a complex single-winner voting system that uses the voters' ordinal preferences to determine the winner using a weighted majority graph. This method is characterized by computational efficiency and compliance with various desirable voting criteria.

Key features of the Schulze method

Computational complexity: Schulze's method can be computed in polynomial time, but its simple implementation can be difficult when dealing with large datasets. Recent advances have led to optimized algorithms using parallel computing platforms such as Pregel, which improves scalability for large elections [7].

Axiomatic properties: the method satisfies several important criteria, including monotonicity, inverse symmetry and Condorcet's criterion, making it a robust choice for fair elections [8]. Privacy in computing: innovations in privacy-preserving algorithms allow the Schulze method to be used on encrypted data, guaranteeing the privacy of voter preferences while determining the winner [9].

Although Schulze's method is highly appreciated for its theoretical properties and practical applications, its application in very large electoral districts remains difficult, requiring continuous research on more efficient algorithms [10].

The method of minimizing the number of inversions [11], previously proposed by the authors, is aimed at finding such a compression of all private rankings that would reduce the total inconsistency of expert opinions (based on the equality of all expert participants), measured in inversions of transitions from the initial state to the lexicographic ordering of ranked objects.

Schulze's method [12] optimizes by maximizing the number of wins, while the method of minimizing the number of inversions actually optimizes by the number of defeats, but both methods use the full data set to determine local minima - that is, both methods use the "quantity" of expert opinions, but not their "quality". "Price of victory" in both approaches is measured by the same quantity - the total number of votes received by the object in the caucus. The latter does not correspond to the task of expertise, in which weaker objects have lower "weight" indicators.

The Skating method [13] corrects the situation somewhat. This method is used for scoring in ballroom dancing competitions. It is used to determine the final places of the participants and is based on a method that prioritizes the "majority" according to the judges' scores to solve the problems arising from the subjective determination of the leader of the competition. That is, it uses data from previously determined "leaders" to determine second and subsequent places. The original version of the majority principle was formulated by Arthur Dawson and introduced by the British Official Ballroom Dance Council (now the British Dance Council) in 1937. After some refinements in 1947 and 1948, the system was not changed until 1956. It was first used during the Blackpool Dance Festival and was gradually adopted in ballroom dance competitions around the world, as well as in other dance competitions. In the final round of the competition (when the judges rank each competitor's place), 10 rules are used to calculate points. There must be an odd number of judges for the majority system to work, and each judge can only assign each place to each competitor once. Particularly in dance sport, the judges' ratings give subjective final places for each competitor in the finals, which are then combined to determine the final combined majority opinion of the judges.

This approach is valid for strongly stable solutions, but it does not always work for weakly coordinated, contradictory preferences, which are characteristic of most practical tasks.

3. Rank penalty method

In the proposed method, called "expert method of rank penalty" by the authors, the Schwartz method is taken from the Schulze method to reduce the dimensionality of the problem for the second and further places, and from the Skating method – the principle of sequential determination of the "leader" for subsequent decision-making. However, unlike Skating, the new method has a number of nuances.

In contrast to the Skating method, a qualitative characterization of a place is introduced, taking into account the "penalty points"[1], under-achieved by a candidate when considering his/her place in the ranked list (hence the name of the described method as the method of rank penalties).

The algorithm is simple and consists, in fact, in the sequential application of three rules: search for the maximum of the sum of seats, search for the minimum of "penalties" to reach the maximum determined at the first stage, sequential shift of the calculation and recalculation of voting statistics taking into account this shift. These three operations are repeated cyclically until the last candidate in the ordered list is determined.

Let's demonstrate the work of the method on a classical example[2] of Schulze's method, in which we consider an election in which 45 voters vote for 5 candidates: A, B, C, D, E.

In the Schulze method, each ballot contains a complete list of candidates, and each expert ranks them in order of preference. The most common voting format uses numbers in ascending order, with the expert placing a "1" next to the most desirable candidate, a "2" next to the second most preferred candidate, and so on. Experts may give the same numbers to several candidates, or not fill in this field at all for some candidates (in this case, it is considered that the voter has put

[1] The "penalty" means the weighted sum of votes received by the candidate for the considered place in the ranked list multiplied by the corresponding weighting factor of the place (in the considered example it is the ordinal number of the place from 1 to 5)

[2] Electronic document. Mode of access: https://ru.wikipedia.org/wiki Method_Schulze

such candidates equally below all candidates for which he/she has indicated a number)[3] .

In the example under consideration, the votes were distributed as follows (Table 1), forming eight "fractions" (so we will call the transitive groups formed as a result of the coincidence of the opinion of a group of experts on the ranks of the compared candidates).

Table 1: Initial data (Schulze example)

Fraction number	Order of candidates	Number of votes in the faction
1	ACBED	5
2	ADECB	5
3	BEDAC	8
4	CABED	3
5	CAEBD	7
6	CBADE	2
7	DCEBA	7
8	EBADC	8

The classical solution by Schulze's method leads to the following ordering of candi dates: EACBD, i.e. E ≻ A ≻ C ≻ B ≻ D. The classical solution by the Skating method leads to a different ordering: CEABD, i.e. C ≻ **E** ≻ A ≻ B ≻ D.

As will be shown below, the difference in the obtained solutions is explainable when some qualification rule is taken into consideration, which establishes a criterion for deciding the candidate's place, depending on the proportion of experts who voted in favor of the considered candidate ordering.

In the further description, we assume a 50% decision-making level for "1, 1-2, 1-2-3" and further places. Since the example considers the votes of 45 experts, the decision threshold (cutoff threshold) is greater than or equal to 22.5 votes.

In the first stage the leader is determined by the maximum accumulated sum of places (as in the Skating method), which must be greater than or equal to the cut-off criterion, and in the second stage by the minimum accumulated penalties. So, we construct a table (Table 2)[4] , which reflects the sums of votes gained by candidates A, B, C, D and E taking into account the preferences of experts in all factions (see Table 1). We further consider the accumulated sum of seats for each candidate. There is no clear leader for the first seats - none of the contenders gained the required number of seats to overcome the cut-off threshold.

For 1-2 places, we have a clear leader by the accumulated sum of expert votes - candidate C, who has gained, respectively, 12+12=24 votes. Let's find out whether the found solution is optimal. For this purpose, let's calculate the weighted sums of accumulated places taking into account the "penalties" for not reaching the found max sum of evaluations (max(1-2) = 24) necessary to determine the leader for all candidates for 1-2 places as follows[5] (Table 3):

for A: 1*10 + 2*10 + 3*(max(1-2) -20) = 10 + 20 + 12 = 42,
for B: 1*8 + 2*10 + 3*(max(1-2) -18) = 8 + 20 + 18 = 46,
for C: 1*12 + 2*12 + 3*(max(1-2) -24) = 12 + 24 + 0 = 36,
for D: 1*7 + 2*5 + 3*8+4*(max(1-2) -20) = 7 + 10 + 24+16 = 57 and finally,
for E: 1*8 + 2*8 + 3*(max(1-2) -16) = 8 + 16 + 24 = 48.

[3] Such voting is fundamentally impossible in the Skating system.
[4] For example, candidate A in the 1st place appeared in faction No. 1 (5 voters made this choice) and faction No. 2 (also 5 voters voted for this order). Consequently, cell 1A of the matrix will contain the sum of votes, i.e. 10.
[5] Note that to calculate the rank penalties for candidate D, we had to "plug in" the votes he received for places 1-4 to "get" the necessary 24 votes.

Table 2

	expert voices					cumulative votes				
	1	2	3	4	5	1	1-2	1-3	1-4	1-5
A	10	10	10	8	7	10	20	30	38	45
B	8	10	8	14	5	8	18	26	40	45
C	12	12	0	5	16	12	24	24	29	45
D	7	5	8	10	15	7	12	20	30	45
E	8	8	19	8	2	8	16	35	43	45

Table 3

	expert voices					cumulative votes					accumulated fines				
	1	2	3	4	5	1	1-2	1-3	1-4	1-5	1	1-2	1-3	1-4	1-5
A	10	10	10	8	7	10	20	30	38	45	---	42			
B	8	10	8	14	5	8	18	26	40	45	---	46			
C	12	12	0	5	16	12	24	24	29	45	---	36			
D	7	5	8	10	15	7	12	20	30	45	---	57			
E	8	8	19	8	2	8	16	35	43	45	---	48			

The minimum of "penalties" is again at C (min(1-2) = 36), hence the found solution C is optimal and candidate C takes the 1st place. In the next step, we exclude candidate C from consideration in all factions[6] and update the voting statistics (see Table 4).

Table 4

Fraction number	Order of candidates	Number of votes in the faction
1	ABED	5
2	ADEB	5
3	BEDA	8
4	ABED	3
5	AEBD	7
6	BADE	2
7	DEBA	7
8	EBAD	8

Because of the transitive nature of candidate relationships, we can make such a substitution. Determine a new leader among the remaining candidates. Note that unlike the Skating optimum boy the solution does not always correspond to the leader by the accumulated sum of places. For the first seats, there is again no clear leader - none of the contenders gained the required number of seats to pass the cut-off threshold (22.5). For 1-2 seats, two candidates B(26) and E(30) passed the cut-off threshold. Candidate E(max(1-2) = 30) had the maximum. Next, we calculate the penalties for under-voting for all candidates, respectively:

for A: 1*20 + 2*2 + 3*(max(1-2) -22) = 20 + 4 + 24 = 48,
for B: 1*10 + 2*16 + 3*(max(1-2) -26) = 10 + 32+ 12 = 54,
for D: 1*7 + 2*5 + 3*10 + 4*(max(1-2) -22) = 7 + 10 + 30 + 32 = 79,
for E: 1*8 + 2*22+3*(max(1-2) -30) = 8 + 44 + 0 = 52.

[6] I n fact, we put C in the first place, but we "forget" about it and consider the remaining candidates in the struggle for leadership (1st place, which is actually 2nd in the overall rating).

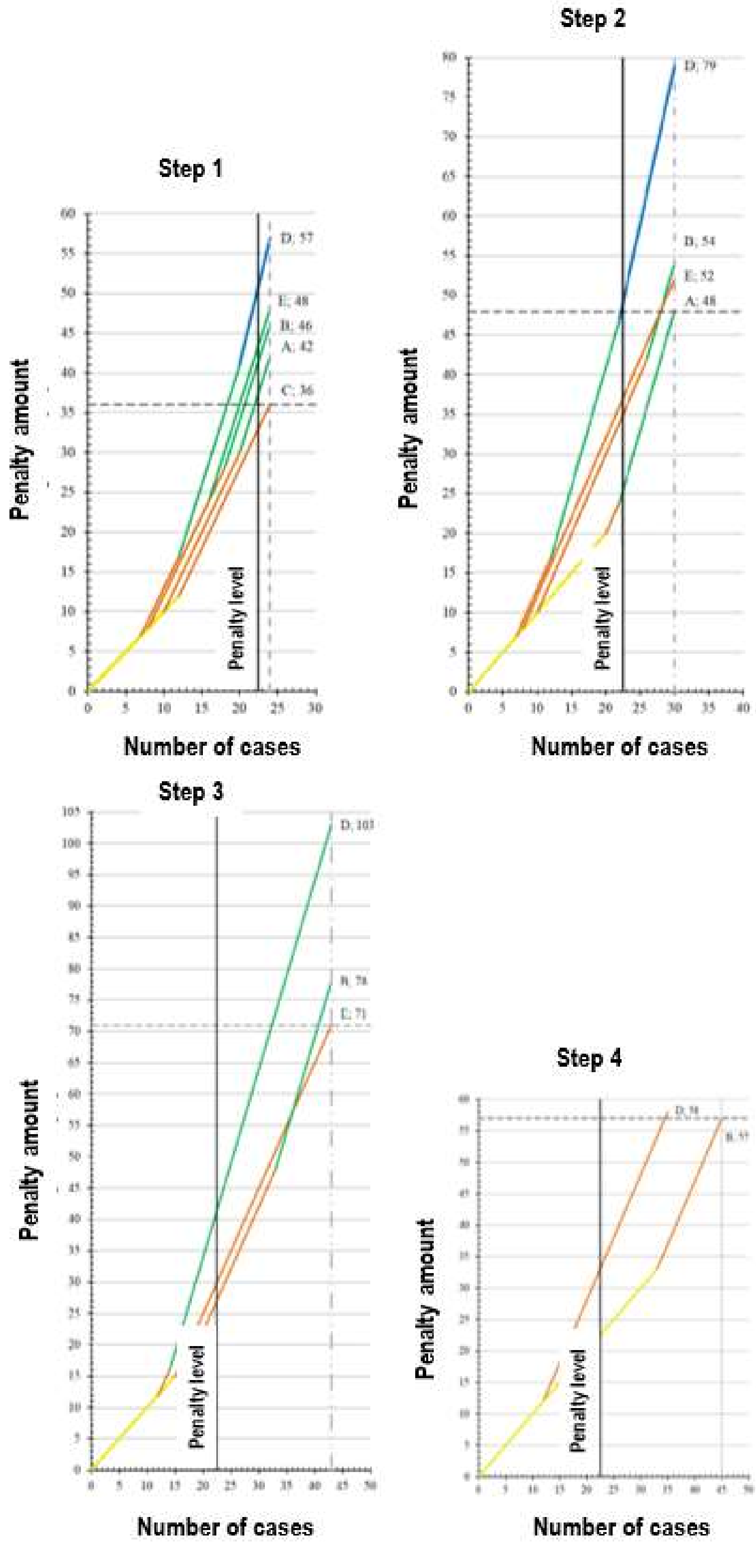

Figure 1. Graphical representation of solution search

Table 5

	expert voices				cumulative votes				accumulated fines			
	1	2	3	4	1	1-2	1-3	1-4	1	1-2	1-3	1-4
A	20	2	8	15	20	22	**30**	45	---	**48**	---	---
B	10	16	14	5	10	**26**	40	45	---	54	---	---
D	7	5	10	23	7	12	22	**45**	---	79	---	---
E	8	22	13	2	8	**30**	43	45	---	52	---	---

The minimum number of accumulated penalties was for candidate A, who becomes the leader at this stage and is excluded from further consideration.

The statistics are updated again and the "struggle for leadership" among candidates B, D, E is considered. This algorithm (with exclusion of the found leader, recalculation of statistics, calculation of accumulated sum of places and accumulated sum of "penalties") is repeated until the last candidate is determined. The scheme in Fig. 1 clearly illustrates the described algorithm.

On the abscissa is the total number of votes. On the ordinate is the sum of penalties. First the abscissa leader is determined, then the penalties for all candidates are determined. Candidate D does not have enough third places - fourth places are used (blue color in the figure - 4th place). Leaders C and E - but E in the second case is inferior to the optimum of A. For candidate A a "step" of 20 in 1 penalty width - height is formed, 2 "steps" - height of 2 penalties in 2 widths, and 3 "steps" of three penalties height and 8 widths. The total number of fines is 20*1+2*2+8*3=48, and the width is (20+2+8)=30 - like leader E. The leader has 2 steps - 8*1 fines + 22*2 fines = 52 fines. But the sum is also: 8+22=30.

The algorithm involves forward and backward permutation between real objects and virtual objects. Outside the scope of this paper, we will leave an interesting cyclic variant - when leaders are equal (equal abscissa values) and their penalties are equal (equal ordinate values) and we choose automatically the "last among equals" object.

Fig. 1 shows the ordered set of seat cases for each candidate date. Penalties for each object is the ordinate of the intersection of the function "its integral of penalties" at a point with abscissa equal to the leader in terms of accumulated sum of votes. The optimum is the minimum value over all objects.

Table 6

Cut-off level	0,5	0,5333	0,6323	0,7777...0,9999
Decision	**CAEBD**	**ACEBD**	**AEBDC**	**EABDC**

The final solution obtained by the rank penalty method is CAEBD, i.e., C ≻ **A** ≻ **E** ≻ B ≻ D. The algorithm is implemented in MS Excel by means of Visual Basic for Applications. As we have already mentioned above - the final solution is sensitive to the adopted cutoff level (Table 6).

4. Conclusions

Existing voting methods work, as a rule, with the number of rank places (as Schulze's method), but for an objective assessment it is necessary to evaluate the "quality" of victories - the importance of the first place is higher than the second and subsequent ones. The resulting qualitative solution is usually weakly stable and depends significantly on the adopted cutoff level.

Unlike the Schulze method, which always searches for a local extremum in the data, the proposed method is preferable for the correct determination of places in the ranked list, because it takes into account the cases when the local extremum does not determine the rank of a candidate and it is necessary to search for a global minimum. The Schulze method for the leader's place determines the candidate by the weighted average of all scores and all fractions without regard to whether he is at the beginning or at the end of the list. In this sense, the minimum inversion search method is more optimal than Schulze's method, but it also does not always determine the places correctly.

A possible direction of improvement of the described method, increasing its stability, may be the introduction of nonlinear convex penalty functions like exponent function or S-shaped logistic curve (since the significance of the last places is usually low).

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