CONSTRUCTION OF A NEW ATTRIBUTE CONTROL CHART BASED ON RAYLEIGH DISTRIBUTION UNDER HYBRID CENSORING - A BAYESIAN APPROACH

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Abstract

Statistical Process Control (SPC) is a quality control approach that makes use of statistical tools to understand, monitor, and improve a process. A control chart is a tool for monitoring process performance that consists of a visual indication to detect anomalous deviations because of assignable causes. This chart compares the values of a quality attribute to the control limits. In the quality control process, the control chart is often generated by ignoring parameter uncertainty. The identification of changes in the parameter(s) within the probability distribution of one or more process-related variables is an essential part of monitoring. Estimating the parameters is essential since this might have an impact on the control chart's long-term performance in a controlled or out-of-control condition. This article provides a novel attribute control chart in the form of a Bayesian approach based on the Rayleigh lifetime distribution and the Hybrid censoring technique. A Bayesian approach will be used to calculate the control chart parameters and average run length. The control chart parameters are determined for various combinations of values, and the performance of the developed control chart is evaluated using the Average Run Length (ARL). Numerical examples are used to explain the proposed control chart, and simulated data is used to show how it might be used.

Keywords: Rayleigh Distribution, Predictive Distribution, Attribute Control Chart, Hybrid Censoring, Average Run Length

1. Introduction

Nowadays manufacturing industry is facing a lot due to process variation which leads to producing defects and facing huge costs. During a continuous manufacturing process, we want to know whether the process is in control or not and to know if there is any presence of variation. Variation may be due to chance or assignable causes. Quality control involves monitoring, measuring, and correcting the performance of a product against predefined standards or specifications. One of the most common and effective tools for quality control is the control chart, which is a graphical representation of the data that has been collected from the manufacturing industries. A control chart always has a central line (CL) for the average, an upper line for the upper control limit (UCL), and a lower line for the lower control limit (LCL).

In order to monitor the production process in various circumstances, many new control charting techniques have been developed. There are various types of control charts which are broadly similar and have been developed to suit particular characteristics of the quality attribute being analyzed. Two broad categories of charts exist, which are based on if the data being monitored is "variable" or "attribute" in nature. Variable control charts are a type of graph that evaluates data with precise measurement. The attribute control charts are used to distinguish between conforming and non-conforming items.

Lifetime is regarded as a quality characteristic for some products. Life tests are used to monitor the manufacturing process for these products. The product may be classed as conforming or non-conforming based on the results of the life test. This form of product testing takes a considerable amount of time because the duration of the test is lengthy. In this circumstance, the censoring technique is essential and cannot be ignored. Some of the censoring techniques used in life testing include Type-I censoring, Type-II censoring, and hybrid censoring. According to the hybrid censoring, the life test terminates at the earliest of the specified test time t or the time at which the (UCL + 1)th failure is discovered. When the number of failures observed during the life test is between the UCL and LCL at time t, the production process is in control; otherwise, the production process is out of control. If the product fails the life test before the test termination time t, it may be regarded as a nonconforming product.

The attribute control chart, such as the np control chart, is based on the fraction non-conforming, which is determined by assuming that the quality characteristic follows the normal distribution. In reality, the distribution of the quality characteristics may not be normal. In this situation, using the present control chart may mislead industrial engineers and cause the non-conforming items to increase. Many works on constructing attribute control chart using various lifetime distributions may be found in literature, which include [1] – Weibull, [2] – Pareto, [3] – Birnbaum Saunder, [4] – Weibull, [5] – Inverse Rayleigh, [6] - Burr X & XII, Inverse Gaussian & Exponential, [7] – Lognormal, [8] – Exponentiated Half Logistic, [9] – Dagum, [10] - Pareto, [11] – Weibull Pareto, [12] – Gamma, [13] – Log–Logistic, [14] - Weibull, [15] – Generalized Log Logistic, [16] – Generalized Exponential, [17] – Inverse Weibull, [18] – Length-Biased Weighted Lomax, [19] - Rayleigh, and [20] – Half Normal & Half Exponential Power.

The attribute control charts discussed in the earlier works are based on the implicit assumption that the mean number of defectives is constant throughout the time when the process is in control. Control charts based entirely on sample variance, such as the np - chart, may generate several false alarms if the real number of defectives is not consistent over time while an in-control scenario occurs. False alarms will arise because the observed number of nonconforming items includes variability because of both process and sampling variation, while the control limits are based only on sampling variance. Any process variations become more evident in larger samples. The actual number of nonconforming items is expected to differ from sample to sample according to an underlying distribution.

The Bayesian approach is commonly employed for dealing with uncertainty about the parameter of interest. For attributes, [21] initially proposed a Bayesian process control chart. Numerous authors, including [22] - [31] have contributed to the literature and worked on a variable control chart for various lifetime distributions under Bayesian approach.

The Rayleigh distribution is frequently employed as a lifetime distribution in research relating to electro vacuum devices and communication engineering. Because the hazard function of the Rayleigh distribution is a linearly increasing function, it appears to be an appropriate lifetime distribution for items with a faster rate of deterioration. This is a specific type of Weibull distribution, which is one of the statistical distributions that may be employed efficiently in reliability engineering and life testing. Reference [32] proposed it as the Rayleigh Power (and Amplitude) distribution. Later, [33] and [34] updated its name to Rayleigh distribution and investigated its properties, estimation procedures, and applications. Numerous authors studied this skewed distribution because of its significance in engineering-related aspects. The Rayleigh distribution may also be used for modelling the lifetime of products in the marketplace that are the result of a manufacturing process. According to the literature study, there has been no research on control charts which include a life test for a non-normal distribution, such as a Rayleigh distribution with Hybrid censoring under the Bayesian approach.

In this paper, the design of an attribute control chart based on a predictive distribution of the lifetime is considered under the assumption that the lifetime follows a Rayleigh distribution and the process parameter is modelled by an Inverse-Rayleigh distribution under a hybrid censoring. The control chart coefficient was determined, and the performance of the proposed control chart was discussed with the average run length (*ARL*). When the process mean changes, the proposed control chart is designed. With the use of simulated data, the proposed control chart's applicability is described. Simulated data is used to illustrate the proposed control chart. Section 2 presents designing the Bayesian control chart and provides a procedure for obtaining the control chart for given strength. The real-life application of the proposed control chart through simulation study is described in Section 3. The final section provides a summary of the overall findings.

2. Designing a Bayesian Attribute control chart

Let "T" be the lifetime of the products produced in a production process. Assume that "T" follows a Rayleigh distribution with parameter θ given for $t > 0$ by its density function $f_{T|\{\theta\}}$:

$$f_{T|\{\theta\}}(t) = \frac{te^{-\frac{t^2}{2\theta^2}}}{\theta^2}; t > 0, \theta > 0 \tag{1}$$

The distribution function of $F_{T|\{\theta\}}$ is given by:

$$F_{T|\{\theta\}}(t) = 1 - e^{-\frac{t^2}{2\theta^2}}; t > 0, \theta > 0 \tag{2}$$

According to [35], Consider the process parameter θ with $\theta > 0$ as random variable Θ with prior distribution given by the inverse Rayleigh distribution with density function for $\theta > 0$:

$$h_{\Theta|\{\lambda\}}(\theta) = \frac{1}{\theta^3\lambda^2} e^{-\frac{1}{2\lambda^2\theta^2}}; \theta > 0, \lambda > 0 \tag{3}$$

The updated lifetime of the product is described by the predictive distribution of "T" whose density function can be obtained from the prior distribution (3) and the sampling distribution (1) as:

$$g_{T|\{\lambda\}}(t) = \frac{2\lambda^2 t}{[1 + \lambda^2 t^2]^2}; t > 0 \tag{4}$$

Hereafter, the above predictive distribution referred as "Rayleigh-Inverse-Rayleigh" distribution and the distribution function of $T|\{\lambda\}$ is obtained as:

$$G_{T|\{\lambda\}}(t) = 1 - \frac{1}{[1 + \lambda^2 t^2]}; t > 0 \tag{5}$$

The product's mean lifetime under Rayleigh-Inverse-Rayleigh distribution is given by

$$\mu = \frac{\pi}{2\lambda} \tag{6}$$

The target mean life of the product is denoted by μ_0 when the process is in-control and the experiment time t_0 can be written in terms of in-control process mean such as $t_0 = a\mu_0$ ("a" is an experiment termination ratio). Let p be the failure probability of an item before the experiment time t_0 and can be written as follows:

$$p = 1 - \frac{1}{\left[1 + \left[\frac{\mu}{2\mu}\right]^2 [a\mu_0]^2\right]} \tag{7}$$

The process is declared to be in-control if there is no difference between the process mean and the target mean, i.e., $\mu = \mu_0$. Now, let p_0 be the failure probability of an item when the process is in-control and is obtained by

$$p_0 = 1 - \frac{1}{\left[1 + \left[\frac{a\mu}{2}\right]^2\right]} \tag{8}$$

Reference [36] pointed out that "Realistically, however, when a special cause influences a process, it may cause a shift in more than one parameter (location, scale, skewness, etc.) of the process distribution". It means that, due to the occurrence of an assignable cause, the process may be affected and subsequently the process mean or shape parameter of the distribution will be shifted from the specified target level. In addition, this parameter shift doesn't greatly affect the distribution so it reduces from one to another. But in such a situation, the process will be declared as out-of-control and stopped until the cause of process variations is removed. Also, it is assumed that the process mean is shifted to $\mu_1 = f\mu_0$, where f is a shift constant. Then the failure probability of an item when the process is out of control, denoted by p_1, is given as

$$p_1 = 1 - \frac{1}{\left[1 + \left[\frac{a\mu}{2f}\right]^2\right]} \tag{9}$$

Usually, for the given values of $a\&f$, the failure probabilities are calculated by using equations (8) and (9).

We develop a new np control chart for the lifetime of the product distributed according to the Rayleigh-Inverse-Rayleigh distribution based on the number of products in each subgroup. The following is the designed np control chart's operational procedure:

- Select a sample of n items randomly from the subgroup of the production process and put the sample items for the life test of specified termination time t_0.
- Observe the number of failed items and denotes it as "d".
- Terminate the life test either after reached at time t_0 or $d > UCL$ before reaching time t_0, whichever is earlier.
- If $d < LCL$ or $d > UCL$, then declare the production process as out-of-control. Otherwise, declare the production process as in control.

The proposed control chart is considered to be an np control chart since the number of failed items in a subgroup of fixed sample size is plotted against the subgroup. The number of failed items in the life test follows a binomial (n, p_0) distribution when the production process is in control. The probability of an item failing before test termination time t_0, is denoted as p_0. As a result, the following equations are used to determine the control limits for the proposed control chart:

$$UCL = np_0 + k\sqrt{np_0(1-p_0)} \tag{10}$$

$$LCL = max[0, np_0 - k\sqrt{np_0(1-p_0)}] \tag{11}$$

Where "k" is the control limits coefficient and the failure probability p_0 of the product before the time t_0 is calculated using equation (8).

The probability that the process is declared to be in control is denoted by P_{in}^0 and given as

$$p_{in}^0 = P(LCL \leq D \leq UCL|p_0)$$

$$p_{in}^0 = \sum_{d=[LCL]+1}^{[UCL]} \binom{n}{d} p_0^d (1-p_0)^{n-d} \tag{12}$$

Generally, the average run length is used to evaluate the performance of the control chart. The in-control ARL of the proposed control chart denoted by ARL_0, is determined by

$$ARL_0 = \frac{1}{1 - [\sum_{d=[LCL]+1}^{[UCL]} \binom{n}{d} p_0^d (1-p_0)^{n-d}]} \tag{13}$$

Assume the process mean has been changed from μ_0 to μ_1. The probability in equation (9) now becomes

$$p_1 = 1 - \frac{1}{\left[1 + \left[\frac{a\mu}{2f}\right]^2\right]}$$

The probability that the process is declared to be in control after shifting to μ_1 is now determined by

$$p_{in}^1 = P(LCL \leq D \leq UCL | p_1)$$

$$p_{in}^1 = \sum_{d=[LCL]+1}^{[UCL]} \binom{n}{d} p_1^d (1-p_1)^{n-d} \tag{14}$$

The ARL for the shifted process (out-of-control) is denoted by ARL_1 and is obtained by:

$$ARL_1 = \frac{1}{1 - [\sum_{d=[LCL]+1}^{[UCL]} \binom{n}{d} p_1^d (1-p_1)^{n-d}]} \tag{15}$$

To analyse the performance of the proposed control chart in detecting the process shift, we determine the optimal parameters LCL, UCL, a and k for the specified values of in-control ARL. For this purpose, we considered four cases of in-control ARL such as 200, 300, 400 and 500 with fixed sample size n = 25. Also, we considered four cases of sample size such as 20, 25, 30 and 35 with fixed in-control ARL = 370. The optimal parameters are selected so that the in-control ARL is as near as to the specified ARL values and the out-of-control $ARLs$ are reported for different values of shift constant. Table 1 and Table 2 show such optimal parameters along with corresponding out-of-control $ARLs$.

From Table 1 and Table 2, we can observe that the out-of-control ARL (i.e., ARL_1) decreases if the shift constant "f" decreases. It represents the ability to quickly detection of process shifts by the proposed control chart. The following algorithm can be used to determine the optimal parameters for designing the proposed control chart for different combinations of specified in-control ARL and sample size.

1. Specify the values of ARL say r_0 and sample size n.
2. Set a = 0.001 and k = 0.001.
3. Determine the failure probability of the product (i.e., p_0) when the process is in control by using equation (8) and also determine the control chart parameters (LCL and UCL) by substituting the values of n, a and k in equation (10) and (11).
4. Calculate for in-control ARL by using equation (13) and compare it with r_0. Continue this process, for different combinations of (a and k) until getting an in-control ARL is very close to the specified ARL, r_0.
5. Whenever such ARL exist, then the corresponding parameters (LCL, UCL, a, k) are the required optimal parameters. Then calculate the out-of-control ARL (i.e., ARL_1) using these optimal parameters in equation (15).

Table 1: *ARLs when the process mean shifted for the fixed sample size $n = 25$*

$n = 25$				
Parameter	$r_0 = 200$	$r_0 = 300$	$r_0 = 400$	$r_0 = 500$
LCL	6	6	5	5
UCL	20	21	20	20
a	0.6633	0.6758	0.6365	0.6583
k	2.7900	2.9070	2.8010	2.9691
$Shift(f)$	ARL	ARL	ARL	ARL
1.00	200.0989	300.1445	400.2567	500.0598
0.90	190.2520	224.5374	353.2836	236.5317
0.80	50.5499	142.0953	81.9755	55.2572
0.70	13.6761	32.2751	19.7207	14.5899
0.60	4.6098	8.9289	5.9207	4.8163
0.50	2.0606	3.2263	2.3788	2.1125
0.40	1.2676	1.6087	1.3453	1.2804
0.30	1.0399	1.1226	1.0534	1.0422
0.20	1.0017	1.0099	1.0024	1.0017
0.10	1.0000	1.0000	1.0000	1.0000

Table 2: *ARLs when the process mean shifted for the fixed value of $r_0 = 370$*

$r_0 = 370$				
Parameter	$n = 20$	$n = 25$	$n = 30$	$n = 35$
LCL	4	5	8	14
UCL	17	20	24	31
a	0.6909	0.6316	0.7012	0.8638
k	2.8343	2.8397	2.9146	2.8954
$Shift(f)$	ARL	ARL	ARL	ARL
1.00	370.0466	370.3559	370.0958	370.0734
0.90	295.5639	282.9189	171.0958	240.0764
0.80	86.1301	90.0271	38.1100	95.0159
0.70	24.8906	21.2006	10.0689	21.9703
0.60	8.3369	6.2245	3.4980	6.5196
0.50	3.4030	2.4493	1.6885	2.6029
0.40	1.7687	1.3622	1.1482	1.4403
0.30	1.1988	1.0563	1.0155	1.0852
0.20	1.0273	1.0025	1.0003	1.0064
0.10	1.0006	1.0000	1.0000	1.0000

3. Application of Proposed Control Chart

3.1. Real-Life Application

In this section, a design example is given to demonstrate a real-life application of the proposed control chart under the Bayesian approach. Suppose that a manufacturer is interested in enhancing the quality of its product. It is known that the failure time of the product follows the Rayleigh distribution with the parameter following Inverse-Rayleigh distribution and the target mean life of the product is 1000 hours. Let a sample of size $n = 25$ be taken from each subgroup and subjected to a censored life test. The target in-control ARL, r_0 is 400. From Table 1, the constant

a = 0.636, k = 2.8010, LCL = 5 and UCL = 20. Therefore, the control chart to be set up by the manufacturer is designed as follows:

Step 1: Select a sample of 25 products from each subgroup and conduct a life test for 636 hours. Record the number of failed products during the life test and denote it as "d".
Step 2: Terminate the life test either after reaching 636 hours or the number of failed products greater than twenty before reaching 636 hours whichever is earlier.
Step 3: If $d > 20$ or $d < 5$, declare the process out of control. Declare the process to be in control if $5 \leq d \leq 20$.

3.2. Simulation Study

In this section, the application of the proposed control chart is demonstrated using simulated data. The data are generated from a Rayleigh distribution when the process is in-control with the parameters following Inverse-Rayleigh distribution and the targeted average lifetime of 1000 hours. We consider a random sample of size n = 20 for each sample batch. The first fifteen samples are generated from the in-control process, and the next fifteen samples are from a shifted process with f = 0.50 to achieve 30 sample batches. Then, from Table 2, when n = 20 and ARL_0 = 370, we have constants k = 2.8343, a = 0.691, UCL = 17 and LCL = 4. The number of products having the lifetime below 691 hours is noted ("D") and are: 10, 15, 12, 12, 12, 11, 12, 10, 10, 10, 14, 13, 12, 8, 10, 16, 17, 16, 18, 17, 17, 17, 20, 19, 20, 18, 17, 19, 19, 15. The values of the nonconforming items are plotted with two control limits (LCL = 4 and UCL = 17) in Figure 1.

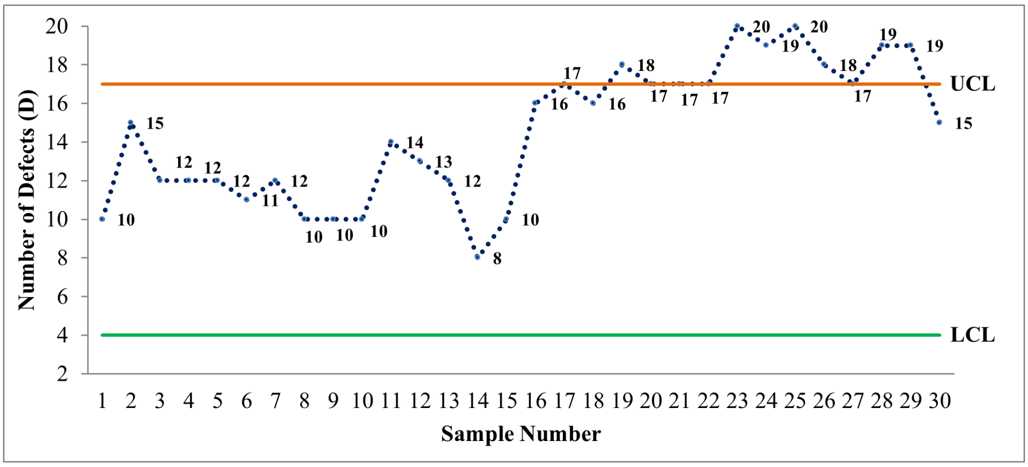

Figure 1: *Control Chart for Simulation Data*

From Figure 1, we observed that the proposed control chart detects a shift at the 17th sample (2nd observation after the shift), with a tabulated ARL of 3.40. As a result, the produced control chart recognises the shift in the manufacturing process efficiently. From this, it is concluded that the proposed control chart will detect the process shift quicker.

4. Discussion

In this paper, we have developed a methodology for a new Bayesian attribute control chart for the lifetime of products. The life tests are performed by hybrid censoring assuming a Rayleigh distribution for the lifetime and an Inverse-Rayleigh distribution as a prior. The proposed control chart is to ensure the mean lifetime of the product as the quality criterion. The newly established control chart is highly adaptable and may be used to monitor the lifetime of quality products. The tables are provided for industrial usage and are explicated with the use of simulated data. It is generated using R software from a Rayleigh-Inverse-Rayleigh distribution. The performance of the proposed control chart is expressed in terms of *ARLs* for various shift constants (f). It should be noted that if the hybrid censoring scheme is used to carry out the life test, executing the sampling inspection will reduce the time and cost of conducting the life test. The developed attribute control chart has the potential to be extended for use with various other statistical distributions as part of a continuing research study.

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