RELIABILITY BASED AN ATTRIBUTE CONTROL CHART USING EXPONENTIATED EXPONENTIAL-POISSON DISTRIBUTION UNDER HYBRID CENSORING

B. Gokila[1] and A. Sheik Abdullah[1*]

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[1] PG & Research Department of Statistics, Salem Sowdeswari College, Salem-10, India.
[1]gokilastat@gmail.com, [1*]sheik.stat@gmail.com
Correspondence Email: sheik.stat@gmail.com

Abstract

The Exponentiated Exponential-Poisson (EEP) distribution, an advanced extension of traditional lifetime models, offers greater adaptability in representing product lifetimes by more precisely capturing variations in failure rates. To handle incomplete lifetime data, a Hybrid censoring technique is employed, combining elements of both Type-I and Type-II censoring to establish a more robust methodology for lifetime analysis. Naturally, np control charts are used to monitor the count of defective units within a sample. The primary aim is to detect shifts in product lifetime distributions, which are crucial for quality control and reliability assessment. However, this study introduces a modification to the control chart, shifting its focus from simply counting failures to tracking the median lifetime, thereby enhancing its capability to identify changes in product reliability over time. The efficiency of the control chart is measured using the Average Run Length (ARL), which estimates the expected number of observations before an out-of-control signal is triggered. By carefully adjusting the chart's parameters, the study ensures that the in-control ARL aligns closely with a predefined target, thereby improving its reliability. This research is dedicated to developing an np control chart designed to monitor the median lifetime of products that follow the EEP distribution within a Hybrid censoring Scheme. Numerical examples and simulated data are provided to illustrate its real-world applicability. By optimizing control chart parameters based on ARL, the study enhances monitoring precision for product reliability, making it a valuable tool in industries where accurate lifetime assessment is critical.

Keywords: Exponentiated Exponential-Poisson distribution, Hybrid Censoring Scheme, Control Chart, Average Run length.

I. Introduction

In Recent days statistical society, quality has become a given, particularly in the manufacturing sector. To accomplish the objective in such a setting, each product must meet the necessary quality criteria. The process of using statistical methods to systematically monitor and maintain the manufactured product's quality is called statistical quality control (SQC). In manufacturing settings, SQC plays a crucial role in the creation and upkeep of high-quality products. To effectively manage the production process under various conditions, numerous innovative control chart techniques have been introduced. There are two primary types of control charts: attribute control charts and variable control charts. Attribute control charts are utilized to differentiate between conforming and non-conforming items, while variable control charts are employed when analyzing industrial data obtained from the measurement process. A control chart serves as a graphical tool for analysing the fluctuations of a process over time. Each control chart features at least two control limits, designated as the lower control limit (LCL) and the upper control limit (UCL). It is asserted that the process is considered to be in control when the control statistic remains within these limits.

Lifetime is considered a critical quality attribute for certain products. Life testing is employed to manage the manufacturing process of these items. Based on the outcomes of the life test, a product may be categorized as either conforming or non-conforming. This type of testing is time-intensive due to the prolonged duration of the tests. In such cases, the application of censoring techniques is vital and should not be overlooked. Various censoring methods utilized in life testing include Type-I, Type-II, and Hybrid censoring. Under Hybrid censoring, the life test concludes at the earliest of the predetermined test time t or the moment when the (UCL + 1)th failure is recorded. If the number of failures noted during the life test falls between the upper control limit (UCL) and lower control limit (LCL) at time t, the production process is deemed to be in control otherwise, it is considered out of control. The proportion of defective items is monitored using attribute control charts, such as the np control chart, which operates under the assumption that the quality characteristic follows a normal distribution. However, the actual distribution of quality characteristics may deviate from normality. If industrial engineers rely on the current control chart under these circumstances, they may be misled, resulting in an increase in the number of non-conforming products.

II. Review of Literature

While many researchers have contributed to attribute control charts for lifetime distributions, a gap remains in studying control charts for non-normal distributions like the Exponentiated Exponential-Poisson with Hybrid censoring, despite the growing interest in exponentiated models for reliability and life testing. Gunasekaran [7] proposed a control chart for attribute np that utilizes the exponentiated exponential distribution to oversee product lifetimes in the context of accelerated life tests with hybrid censoring, focusing on the optimization of ARL parameters and confirming its efficacy through simulations. Coşkun Kuş [10] paper introduced a two-parameter lifetime distribution with a decreasing failure rate, exploring its statistical properties, estimation via the EM algorithm, simulation-based validation, and real data applications for reliability analysis. Aslam and Jun [3] proposed an optimized attribute control chart based on a Weibull distribution and truncated life test to enhance process monitoring, detect variations efficiently, and improve industrial quality control. Baklizi and Ghannam [5] proposed an attribute control chart for the inverse Weibull distribution under truncated life tests, using failure counts to monitor quality, optimizing parameters via simulations, and validating its performance through ARL analysis and a practical example. Jayadurga et al., [8] proposed an attribute np control chart using repetitive group sampling under truncated life tests to monitor process mean life, optimizing parameters for target ARL, providing ARL tables, and demonstrating its superior shift detection through simulations. Kavitha and Gunasekaran [9] proposed a control chart based on the Exponentiated Exponential Distribution, deriving percentile-based control limits to enhance shift detection and providing tables for practical industrial application.

Al-Marshadi et al., [2] A proposed an advanced attribute control chart using the neutrosophic Weibull distribution to enhance precision in detecting indeterminacy effects, improve monitoring efficiency, and outperform existing methods in industrial quality control. Balamurali and Jeyadurga [6] proposed an attribute np control chart using multiple deferred state sampling and time-truncated life tests to monitor product mean life, optimizing parameters for accurate ARL estimation and validating its effectiveness through simulations. Shaheen et al., [16] developed a control chart for monitoring process variation in lognormal distributions using repetitive sampling, deriving control limits based on distribution parameters and evaluating performance through average run length (ARL) metrics, demonstrating superior detection of small to moderate shifts compared to traditional Shewhart S-charts. Rao and Edwin Paul [18] developed an attribute control chart for time-truncated life tests based on a log-logistic distribution, deriving control limits using failure counts within a specified truncation time and evaluating its performance through Monte Carlo simulations and ARL metrics, demonstrating its superiority in detecting process shifts compared to existing control charts. Nanthakumar and Kavitha [14] proposed an attribute control chart for monitoring product lifetimes under Type-I censoring using the inverse Rayleigh distribution, evaluating its performance through ARL metrics and numerical examples.

Rao [17] developed an attribute control chart for time-truncated life tests based on the exponentiated half logistic distribution, evaluating its performance using average run length (ARL) metrics and Monte Carlo simulations, with results demonstrating its superiority over existing control charts through extensive ARL tables and simulated industrial data. Tibor [20] revisited the three-parameter exponentiated exponential Poisson (EEP) distribution, deriving closed-form expressions for its characteristic function and moment generating function while validating the applicability of existing series and integral expressions for positive integer-order moments under the condition $\nu > 1 - \alpha$, where $\alpha > 0$. Tahir and Cordeiro [13] reviewed univariate distribution compounding methods, introduced new generalized classes, and explored the evolution, trends, and future directions of distribution generalization. Shafqat et al., [15] proposed a Shewhart-type attribute control chart for truncated life tests across various lifetime distributions, evaluating its effectiveness using ARL analysis. Adeoti and ogundipe [1] developed an advanced control chart using a modified chain sampling plan (MchSP) based on a generalized exponential distribution (GED) to enhance industrial quality control by effectively detecting process shifts through real and simulated data analysis. Aldosari and Jun [11] proposed an attribute control chart combining multiple dependent state sampling with repetitive sampling, enhancing sensitivity to small process shifts and outperforming traditional charts in detection efficiency. Aslam and Hyuck [4] propose a variable control chart based on a time-truncated life test for the Weibull distribution, optimizing control limits and run length properties to enhance industrial process monitoring and detect shifts effectively. Rao et al., [19] introduced an attribute control chart for the Dagum distribution under truncated life tests, evaluating its effectiveness using ARL analysis and real-life applications.

Ristic and Nadarajah [12] introduced a three-parameter lifetime distribution, analyzed its statistical properties, proposed univariate and bivariate generalizations, and demonstrated its superiority over existing models using real data. The Exponentiated Exponential-Poisson distribution was initially proposed by Ristic and Nadarajah and it has been demonstrated that this distribution is particularly well-suited for certain types of lifetime data compared to other widely utilized lifetime distributions. The analysis of lifetime data has proven to be notably effective when employing the Exponentiated Exponential-Poisson distribution. This study introduces an attribute control chart based on the Exponentiated Exponential-Poisson distribution within a Hybrid censoring Scheme. The coefficient for the control chart was established, and the performance of the proposed chart was evaluated in terms of the Average Run Length. The design of the control chart is adapted in response to variations in the median of the process scale parameter. The applicability of the proposed control chart is illustrated through the use of simulated data.
The structure of this paper is organized as follows: Section 2 presents a review of the relevant

literature. Section 3 introduces the Exponentiated Exponential-Poisson distribution and details the design of the proposed control chart. Section 4 describes the simulation study, and Section 5 concludes the paper by summarizing the key findings and insights.

III. Design of the control chart

If a random variable *t* has the Exponentiated Exponential-Poisson distribution with parameters *(β, λ, α)* denoting the lifetime of a product in some production process then the density function is given by

$$f(t) = \frac{\alpha\beta\lambda e^{-\beta t}(1-e^{-\beta t})^{\alpha-1} e^{-\lambda(1-e^{-\beta t})^{\alpha}}}{1-e^{-\lambda}} \, , t, \lambda, \alpha, \beta > 0 \tag{1}$$

and the cumulative density function of the model can be obtained as

$$F(t) = \frac{1-e^{-\lambda(1-e^{-\beta t})^{\alpha}}}{1-e^{-\lambda}} \, , t, \lambda, \alpha, \beta > 0 \tag{2}$$

The lifetime of the units follows Exponentiated Exponential – Poisson Distribution with a median life defined as

$$m_0 = \left(-\frac{1}{\beta}\right) log\{1 - [-\frac{1}{\lambda} log[\, 1 - 2^{-1}(1 - e^{-\lambda})]]^{\frac{1}{\alpha}}\} \tag{3}$$

The truncation time t_0 and failure arise from time saving considerations in life testing. In the simulation study for each generated subgroup, we count the number of failures whose lifetimes are ≤ t_0 where the truncation time is determined by

$$t_0 = am_0 \tag{4}$$

where t_0 is the truncated time, *a* is the truncation coefficient and m_0 is the desired median life of the product. The probability of failure before t_0 is given by

$$p = \frac{1-e^{-\lambda(1-e^{-\beta t_0})^{\alpha}}}{1-e^{-\lambda}}, t, \lambda, \alpha, \beta > 0 \tag{5}$$

using equation (3), the probability of failure of a product when the process is in control is

$$p_0 = \frac{1-e^{-\lambda}\left\{1-\left[1-\left[-\frac{1}{\lambda}\log\left(\frac{1+e^{-\lambda}}{2}\right)\right]^{\frac{1}{\alpha}}\right]^{a}\right\}^{\alpha}}{1-e^{-\lambda}} \tag{6}$$

when the median shifts to m_1, the probability of failure of a product becomes
Let $m_1 = fm_0$, where *f* is the shift coefficient, then

$$p_1 = \frac{1-e^{-\lambda}\left(1-\left[1-\left[-\frac{1}{\lambda}\log\left(\frac{1+e^{-\lambda}}{2}\right)\right]^{\frac{1}{\alpha}}\right]^{\frac{a}{f}}\right)^{\alpha}}{1-e^{-\lambda}} \tag{7}$$

The np attribute control chart having Lower Control Limit (LCL) and Upper Control Limit (UCL) for the Exponentiated Exponential-Poisson distribution is explained in the following steps:

Step 1: Select a set of n products randomly from the production process.

Step 2: Conduct the life test on the selected products considering t_0 as the test termination time. Observe the number of failed items D (say).

Step 3: Terminate the life test either after reached at time t_0 or D > UCL before reaching time t_0, whichever is earlier.

Step 4: Declare the process as out of control if D > UCL or D < LCL. Declare the process as in control if LCL ≤ D ≤ UCL.

We are interested in observing the number of failures D in each sample. If the units have failure times that are less than the truncation time t_0, then the units are considered nonconforming or defective. The binomial probability mass function is given by

$$p(D=x)=\binom{n}{x}p^x(1-p)^{n-x}, x=0,1,2,3\ldots \tag{8}$$

The Lower and Upper control limits are given by

$$LCL = max(0, np_0 - k\sqrt{np_0(1-p_o)}) \tag{9}$$

$$UCL = np_0 + k\sqrt{np_0(1-p_o)} \tag{10}$$

where k is the coefficient of the control limits and p_0 is the probability of "nonconforming" when the process is in control. The probability of stating that the process is in control when it is truly in control is obtained using the binomial probability given in equation (8)

$$p_{in}^0 = p(LCL \le D \le UCL|p_0) = \sum_{d=LCL+1}^{UCL}\binom{n}{d}p_0^d(1-p_0)^{n-d} \tag{11}$$

The probability of stating that the process is in control when the median life of the product has shifted to m_1 is given by

$$p_{in}^1 = p(LCL \le D \le UCL|\, p_1) = \sum_{d=LCL+1}^{UCL}\binom{n}{d}p_1^d(1-p_1)^{n-d} \tag{12}$$

It follows from the values p_{in}^0 and p_{in}^1 given in Equation (11) and (12) that the in-control ARL is given by

$$ARL_0 = \frac{1}{1-p_{in}^o} \tag{13}$$

and the out of control ARL are given by

$$ARL_1 = \frac{1}{1-p_{in}^1} \tag{14}$$

To construct the tables for the proposed control chart, we applied the following algorithm:

Step 1: Specify the values of ARL, say r_0 and sample size n.

Step 2: Determine the control chart parameter and truncated time constant a value for which the ARL from equation (13) approach r_0.

Step 3: By using the values of the control chart parameters obtained in step 2, determine the ARL_1 in accordance with shift constant f by using equation (14).

For various values of r_0, n and λ we determine the control chart parameters and ARL_1, which are shown in Tables 1,2,3,4 shows that the ARLs tend to get smaller when the shift constant f gets less.

Table 1: *The Values of ARLs when λ = 1, α=0.5*

				λ = 1, α=0.5					
n	**20**			**25**			**30**		
r_0	**360**	**480**	**580**	**360**	**480**	**580**	**360**	**480**	**580**
a	0.208	0.291	0.685	0.473	0.832	0.283	0.9	0.869	0.732
K	3.5	3.166	3.2162	3.0009	3.1497	3.1005	2.9576	3.1855	3.2284
LCL	0	0	2	2	4	0	6	6	5
UCL	12	12	16	16	19	14	22	23	22
Shift(f)	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
1.00	360.34	479.60	580.25	359.78	480.30	580.30	360.15	480.06	581.00
0.90	448.80	384.18	826.51	283.55	493.20	367.94	300.58	679.93	723.63
0.80	509.32	263.35	960.38	179.88	343.04	216.23	175.29	601.92	559.59
0.70	464.34	159.24	727.67	98.81	178.72	119.20	84.50	311.36	279.96
0.60	310.83	87.33	370.57	49.51	80.53	61.70	37.32	124.60	114.42
0.50	158.82	43.89	151.74	23.07	33.23	29.81	15.61	44.67	42.31
0.40	66.85	20.09	53.83	10.05	12.74	13.33	6.31	14.85	14.50
0.30	23.63	8.27	16.53	4.15	4.64	5.51	2.60	4.75	4.76
0.20	6.82	3.07	4.41	1.75	1.77	2.19	1.28	1.68	1.70
0.10	1.69	1.21	1.28	1.03	1.02	1.08	1.00	1.01	1.01

Table 2: *The Values of ARLs when λ = 2, α=1*

				λ = 2, α=1					
n	**20**			**25**			**30**		
r_0	**360**	**480**	**580**	**360**	**480**	**580**	**360**	**480**	**580**
a	0.526	0.619	0.798	0.644	0.895	0.396	0.42	0.61	0.593
k	2.9514	3.1179	3.2162	3.0004	3.1509	3.2995	3.0871	3.1158	3.0864
LCL	0	1	2	2	4	0	1	3	2
UCL	12	14	16	16	19	14	15	19	18
Shift(f)	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
1.00	359.75	480.01	580.22	359.66	479.77	580.34	359.56	480.21	580.23
0.90	186.66	536.85	945.95	212.63	414.16	346.55	210.31	458.28	252.71
0.80	83.55	300.45	682.06	86.80	169.00	148.63	86.25	186.49	89.51
0.70	35.30	114.36	248.54	32.37	56.58	56.62	32.06	58.48	30.65
0.60	14.57	39.28	76.52	12.04	18.73	20.74	11.81	18.03	10.71
0.50	6.07	13.23	22.84	4.71	6.52	7.61	4.56	5.91	4.05
0.40	2.69	4.63	7.00	2.10	2.58	2.99	2.02	2.28	1.82
0.30	1.42	1.89	2.44	1.21	1.33	1.44	1.18	1.22	1.13
0.20	1.03	1.10	1.20	1.00	1.02	1.03	1.00	1.00	1.00
0.10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 3: ***The Values of ARLs when λ = 5, α= 2***

				λ = 5, α=2				
n	**20**			**25**			**30**	
r_0	**360**	**480**	**580**	**360**	**480**	**580**	**360**	**480**	**580**
a	0.974	0.973	0.988	0.779	0.939	0.663	0.59	0.575	0.899
k	3.0366	3.222	3.2954	3.0023	3.1485	3.1005	2.9521	3.1113	3.2278
LCL	3	3	3	2	4	0	1	0	5
UCL	16	17	17	16	19	14	15	14	22
Shift(f)	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
1.00	360.14	480.04	582.61	360.09	480.88	580.28	359.59	479.80	580.23
0.90	220.62	905.08	812.07	115.46	232.66	129.66	192.99	108.81	366.91
0.80	51.74	223.18	181.59	25.15	44.29	28.60	39.01	23.65	55.65
0.70	12.90	42.49	35.93	6.46	9.81	7.33	8.58	6.07	10.18
0.60	4.02	9.71	8.60	2.26	2.97	2.47	2.57	2.12	2.76
0.50	1.74	3.01	2.80	1.22	1.40	1.26	1.25	1.17	1.29
0.40	1.13	1.42	1.38	1.01	1.04	1.01	1.01	1.00	1.02
0.30	1.00	1.05	1.04	1.00	1.00	1.00	1.00	1.00	1.00
0.20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 4: *The Values of ARLs when λ = 10, α=5*

				λ = 10, α=5				
n	**20**			**25**			**30**	
r_0	**360**	**480**	**580**	**360**	**480**	**580**	**360**	**480**	**580**
a	0.987	0.816	0.937	0.868	0.845	0.77	0.767	0.757	0.751
k	3.0339	3.1713	3.2162	3.0867	3.0745	3.2978	2.9534	3.1127	3.2138
LCL	3	0	2	2	1	0	1	0	0
UCL	16	12	16	16	15	13	15	14	14
Shift(f)	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
1.00	361.18	481.39	580.20	360.32	479.93	579.60	360.17	481.12	579.96
0.90	61.77	49.90	234.75	43.57	33.06	48.19	45.68	27.44	33.28
0.80	6.46	5.63	15.35	4.21	3.65	4.79	4.02	3.12	3.47
0.70	1.67	1.51	2.47	1.27	1.22	1.34	1.21	1.14	1.17
0.60	1.05	1.02	1.13	1.00	1.00	1.00	1.00	1.00	1.00
0.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.40	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

I. Illustration 1

Suppose that the lifetimes of the products follow an Exponentiated Exponential-Poisson distribution with parameter $\lambda= 2$, $\alpha=1$. Consider the following values for the products: $m_0 = 1000$ hours, $ARL_0 = 360$ and $n = 20$. Table 2 provided the following control chart parameters: $k = 2.9514$, $a = 0.526$, $LCL = 0$ and $UCL = 12$. As a consequence, the control chart was established in the following manner.

Step 1: Select a sample of 20 products from each subgroup and submit them to the life test.
Step 2: During the testing, count the number of failed items (D) and fix the time period.
Step 3: Terminate the test if either first failures occur or time elapse whichever comes first.
Step 4: Declare that the process is in control if $0 \leq D \leq 12$.
Step 5: Declare that it is out of control if $D > 12$ or $D < 0$.

II. Illustration 2

Assume that the product lifetimes follow an Exponentiated Exponential-Poisson distribution with parameters $\alpha=2$, $\lambda=5$, Consider the following product values: m_0 =1000 hours, ARL=580, and n = 30. The following control chart parameters were presented in Table 3: $k = 3.2278, a = 0.899$, $LCL = 5$ and $UCL = 22$. As a result, the control chart was established in the following manner: From each subgroup, choose a sample of 30 products, and put them through a life test. Count the number of failed items (D) which is minimum failure during testing or maximum time reached whichever comes first. Now monitor failures and time. Stop the process if the value of D is between 5 and 22 or the time elapse before reaching maximum time then the process is in control; otherwise, it is out of control.

IV. Simulation Study

This section discusses the application of the produced control chart with simulated data. The data was generated and shown in Table 5, using an Exponentiated Exponential-Poisson distribution with an average (median) lifetime (m_0) of 1000 hours, and the value of the shape parameter $\lambda = 2$, $\alpha = 1$. Assuming that the sample size (n) is 20 and the specified ARL (r_0) is 360. At m_0= 1000 hours and $F = 1$, the process is considered to be in control. In-control parameters are employed to construct the first 15 observations of subgroup size 20. Let's now assume that since the median of the Exponentiated Exponential-Poisson distribution has shifted, the process has shifted as well. The value of the shift constant f is 0.5. When f is set to 0.5, the shifted median is used to produce the next 15 observations (shown in Table 5). Take into account that the experiment was conducted at $t = 526$ hours. Table 5, shows the number of failures, indicated by the letter D, for each subgroup. Figure 1, shows the calculated LCL = 0 and UCL = 12 for simulated data.

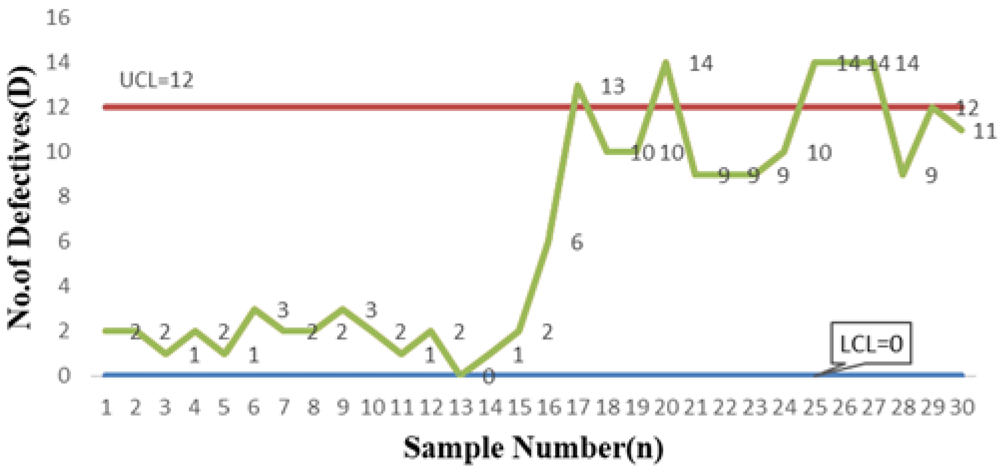

Figure 1: *Control Chart for Simulated Data*

Table 5: *Simulated Data when $m_0 = 1000$ hours, $\lambda = 2$, $\alpha = 1.$, $ARL_0 = 360$, $n = 20$ and $f=1$*

	S. No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Sample	**1**	1177	1096	922	844	798	950	654	475	786	1353	864	696	1056	993	1171
	2	1323	1131	1318	1114	685	819	1502	1809	976	726	599	1156	617	1313	438
	3	1132	964	428	1453	1755	1298	567	1393	490	520	1205	839	966	1750	637
	4	1129	1204	1608	1055	1266	681	811	1129	395	1499	600	501	1097	1000	1308
	5	1145	1600	1314	977	765	1540	926	921	939	1168	396	816	744	227	344
	6	1195	676	854	590	801	729	328	974	1242	1174	1432	639	856	1409	1230
	7	1435	1285	2125	972	789	1160	800	1180	653	1469	1613	837	922	768	1497
	8	1193	359	2244	419	889	351	433	1281	1087	680	1515	1273	995	834	1520
	9	1331	687	1220	948	1157	751	1524	563	1324	637	1499	625	601	1722	1472
	10	1303	876	749	508	993	772	1081	926	1394	939	679	959	1148	1470	1017
	11	828	1302	2026	683	708	958	748	449	1097	1186	1307	2554	1017	1047	1401
	12	456	715	1100	945	1334	1069	662	2205	998	1860	1471	1106	3261	2247	949
	13	543	808	1420	1202	1458	1433	606	843	1021	442	1278	1178	1761	853	588
	14	1017	575	1119	1359	670	506	648	1689	835	468	1915	810	546	1158	1646
	15	858	842	690	862	618	1382	1224	953	846	1383	2240	769	610	989	593
	16	705	1330	1018	825	1030	766	1265	909	1954	1544	1156	485	1028	680	1202
	17	407	959	1131	899	938	498	1116	888	455	1001	1050	1113	1333	1079	1194
	18	1309	321	625	596	1001	1165	1159	813	2257	1083	1103	1462	837	1961	1171
	19	1155	679	1113	754	351	1450	700	1073	1471	1245	915	1846	779	841	1648
	20	2008	1503	714	855	1966	935	931	1551	1265	1194	960	1260	1359	2515	871
	D	2	2	1	2	1	3	2	2	3	2	1	2	0	1	2

Table 5: *Simulated Data when m_0 = 1000 hours, λ = 2, α =1., ARL_0 = 360, n = 20 and f=0.5*

	S. No	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Sample	**1**	325	503	967	321	564	606	401	353	601	387	511	511	458	443	382
	2	688	360	267	577	622	623	1064	689	285	849	299	299	818	731	812
	3	646	433	505	232	430	581	396	495	840	759	388	388	627	395	498
	4	535	228	598	428	832	732	346	379	527	414	546	546	731	429	298
	5	625	653	623	561	347	885	712	195	455	493	619	619	762	646	531
	6	263	1050	704	390	514	321	767	679	527	563	486	486	316	585	892
	7	576	552	296	651	766	951	835	625	500	448	805	805	340	639	500
	8	436	391	659	626	510	489	426	982	825	452	258	258	283	1153	639
	9	572	298	364	456	332	728	892	526	230	434	396	396	633	526	737
	10	569	399	346	581	419	440	331	396	569	302	390	390	195	412	423
	11	316	537	564	316	1115	311	819	190	911	621	215	215	519	236	831
	12	604	424	751	348	318	445	572	598	482	290	506	506	294	415	482
	13	595	334	456	1061	343	532	1082	421	449	467	419	419	371	380	227
	14	210	459	339	541	409	245	531	248	397	389	465	465	605	370	307
	15	820	618	603	792	550	485	647	789	539	492	281	281	762	818	802
	16	682	309	685	450	127	572	515	530	290	627	720	720	1393	385	737
	17	390	352	484	428	517	571	607	345	523	386	786	786	912	319	617
	18	410	400	516	594	500	402	359	782	583	542	540	540	641	652	420
	19	623	586	753	374	438	746	518	584	531	395	284	284	829	400	432
	20	589	613	337	824	472	498	493	588	483	277	298	298	421	569	352
	D	6	13	10	10	14	9	9	9	10	14	14	14	9	12	11

V. Conclusion

In this article, a novel attribute control chart is introduced, utilizing the Exponentiated Exponential-Poisson distribution within a Hybrid censoring Scheme, aimed at ensuring that the median lifetime of the product serves as the quality standard. The newly developed control chart exhibits a high degree of flexibility and can be utilized for monitoring the longevity of quality products. Accompanying tables are designed for industrial applications and are illustrated using simulated data. This data is generated through R software, based on an Exponentiated Exponential-Poisson distribution. The effectiveness of the proposed control chart is quantified in terms of Average Run Lengths (ARLs) for different shift constants (f). It is important to highlight that employing a Hybrid censoring scheme for the life test can lead to reductions in both time and costs associated with the sampling inspection process. Furthermore, the created attribute control chart holds promise for extension to various other statistical distributions as part of ongoing research efforts.

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