FERMATEAN QUADRIPARTITIONED NEUTROSOPHIC FUZZY GRAPH

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Abstract

A fuzzy graph is a mathematical technique that represents relationship of an objects with uncertainty or imprecision, using nodes and edges with membership degrees to indicate the strength of connections. It provides an important paradigm for modelling and handling real world optimization tasks. Neutrosophic graphs extend classical graph theory to handle uncertain, imprecise and inconsistent information, enabling the modeling of complex relationships with varying degrees of truth, indeterminacy and falsity. This framework provides a more nuanced understanding of complex systems, allowing for improved accuracy and increased flexibility in decision-making, network analysis and optimization problems. Neutrosophic graphs can be applied to various fields, including multi-criteria decision-making, supply chain management and logistics, where uncertain or imprecise data is common. The extension of Neutrosophic set is the Quadripartitioned Neutrosophic Set. The division of indeterminacy function of the Neutrosophic set into the contradiction and ignorance component is termed as Quadripartitioned Neutrosophic Set. Associated with each vertex are values of truth, contradiction, ignorance and falsity that reflects it characteristics of a graph. Similarly, for each edge, these values signify the strength or reliability of the relationship between the connected vertices. Quadripartitioned Neutrosophic Fuzzy graph gives an accurate representation of uncertainty which leads to a greater clarity of complex systems and relationships. It has an emerging applications in social network analysis, image processing and decision making systems. Fermatean Neutrosophic Graph is a hybrid model of Fermatean and Neutrosophic graph. This enhancement provides an increased capacity to handle uncertain and unclear data. In contrast to traditional neutrosophic values, this framework operates with truth, indeterminacy and falsity membership degrees constrained by the condition that the sum of their third powers is less than or equal to two. In this article, a new graph is defined called Fermatean Quadripartitioned Neutrosophic Fuzzy Graph (FQNFG). We proposed the order, size, complete, complement and strong of Fermatean Quadripartitioned Neutrosophic Fuzzy Graph. Furthermore, the paper establishes operations for FQNFG such as composition, Cartesian product, Cross product and lexicographic product are also studied.

Keywords: Neutrosophic Fuzzy graph, Quadripartitioned Neutrosophic Fuzzy graph, Fermatean Neutrosophic Fuzzy graph, Complete and Complement Fermatean Quadrapartitioned Neutrosophic Fuzzy graph..

1. Introduction

In 1736, Graph theory was developed by Euler. Graph theory is a section of mathematics which studies the characteristics and applications of graphs, consisting of vertices (nodes) and connected by edges (links). It models relationships between objects and is widely used in networks, structures and various fields. Applications span cryptography, data mining and machine learning etc. This theory is a prominent solving technique in a variety of domains, including geometry, algebra, number theory and computer science. The idea of Fuzzy Set theory was proposed by L.A. Zadeh [1]. Azriel Rosenfeld initiated the Fuzzy Graph theory, which paves the way for new research in mathematics, communication systems, economics and a variety of other subjects [2]. Neutrosophic Fuzzy Sets were used to generate new concepts for Neutrosophic Fuzzy Graphs [3, 4]. Senapati et al. introduced a novel idea called the Fermatean Fuzzy Set [5, 6]. Thamizhendi et al. introduced the notion of Fermatean Fuzzy Hyper-Graphs and its characteristics [7]. Later, Antony et al, [8] suggested a new emergent idea of Fermatean Neutrosophic, which combines Neutrosophic sets and Fermatean fuzzy sets. Said Broumi et al. introduced a new paradigm for Fermatean Neutrosophic graphs and their applications based on the Fermatean Neutrosophic fuzzy set [9] and also presented the Interval-valued Fermatean neutrosophic graphs and regularity [18].

The Quadripartitioned Neutrosophic Graph denotes the division of the Neutrosophic set's indeterminacy component into contradiction and ignorance components. Fermatean Quadripartitioned Neutrosophic Fuzzy Graph is the result of combining Fermatean and Quadripartitioned Neutrosophic Fuzzy Graphs. In this article will see the new novel idea Fermatean Quadripartitioned Neutrosophic Fuzzy Graphs. Here, section 2 provides basic definitions. Section 3 presents the definition of FQNFG, order, size, complete, complement and strong of FQNFG. Section 4 introduces the concept of composition, cartesian product, cross product and lexicographic product of FQNFG etc., along with theorems and examples. Section 5 concludes the paper with future scope of research.

2. Preliminaries

Definition 2.1. [2] A Fuzzy Graph G is a pair of functions $G(\sigma,\mu)$ where σ is a fuzzy subset of a non empty set S and μ is a symmetric fuzzy relation on σ. The underlying crisp graph of $G(\sigma,\mu)$ is denoted by $G^* : (\sigma^*,\mu^*)$.

Definition 2.2. [16] Consider a fuzzy graph $G : (\sigma,\mu)$, then order of G is termed as $O(G) = \sum_{u\in V}\sigma(u)$

Definition 2.3. [16] Consider a fuzzy graph $G : (\sigma,\mu)$, then size of G is termed as $S(G) = \sum_{u,v\in V}\mu(u,v)$

Definition 2.4. [17] A Neutrosophic Fuzzy Graph $G = (\sigma,\mu)$ where $\sigma = (\sigma_1,\sigma_2,\sigma_3)$ such that $\sigma_1,\sigma_2,\sigma_3$ from $\sigma \to [0,1]$ represent value of truth, indeterminacy and falsity membership value with $0 \le \sigma_1(u)+\sigma_2(u)+\sigma_3(u) \le 3$, $\mu \subseteq \sigma\times\sigma$ with μ_1,μ_2 and μ_3 from $\sigma\times\sigma$ to $[0,1]$ such that
$\mu_1(uv) \le min(\sigma_1(u),\sigma_1(v))$
$\mu_2(uv) \le min(\sigma_2(u),\sigma_2(v))$
$\mu_3(uv) \le max(\sigma_3(u),\sigma_3(v))$
$0 \le \mu_1(uv)+\mu_2(uv)+\mu_3(uv) \le 3$, for every $(uv) \in \mu$.

Definition 2.5. [9] A Fermatean Neutrosophic Graph (FNG) on a universal set X is a pair $G = (P,Q)$ where P is Fermatean neutrosophic set on X and Q is a Fermatean Neutrosophic relation on X so that:
$T_Q(u,v) \le T_p(u) \wedge T_p(v)$
$I_Q(u,v) \ge I_p(u) \vee I_p(v))$
$F_Q(u,v) \ge F_p(u) \vee F_p(v)$
and $0 \le T_Q^3(u,v)+I_Q^3(u,v)+F_Q^3(u,v) \le 2\ \forall u,v \in X$, where, $T_Q : X\times X \to [0,1]$, $I_Q : X\times X \to [0,1]$ and $F_Q : X\times X \to [0,1]$ indicates degree of membership, degree of indeterminacy-membership and degree of non-membership of Q, Correspondingly. Here, P and Q are the Fermatean Neutrosophic vertex and edge set of G.

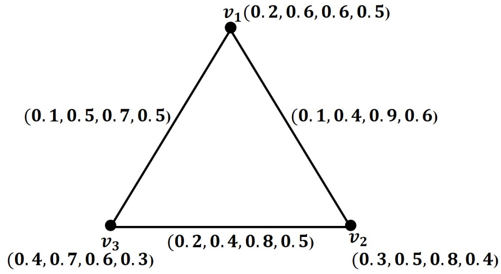

Figure 1: *Fermatean Quadripartitioned Neutrosophic Fuzzy Graph*

3. Fermatean Quadripartitioned Neutrosophic Fuzzy Graph

Definition 3.1. A Fermatean Quadripartitioned Neutrosophic Fuzzy Graph (FQNFG) an Universal Set X is a pair $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$, $\sigma_{FQ} : X \to [0, 1]$ is a Fermatean Quadripartitioned Neutrosophic Set on X and $\mu_{FQ} : X \times X \to [0, 1]$ is a Fermatean Quadripartitioned Neutrosophic mapping on $X \times X$ so that

$$\begin{aligned} T\mu_{FQ}(uv) &\leq \min(T_{\sigma_{FQ}}(u), T_{\sigma_{FQ}}(v)) \\ C_{\mu_{FQ}}(uv) &\leq \min(C_{\sigma_{FQ}}(u), C_{\sigma_{FQ}}(v)) \\ I_{\mu_{FQ}}(uv) &\geq \max(I_{\sigma_{FQ}}(u), I_{\sigma_{FQ}}(v)) \\ F_{\mu_{FQ}}(uv) &\geq \max(F_{\sigma_{FQ}}(u), F_{\sigma_{FQ}}(v)) \end{aligned}$$

with $0 \leq T^3_{\mu_{FQ}}(uv) + C^3_{\mu_{FQ}}(uv) + I^3_{\mu_{FQ}}(uv) + F^3_{\mu_{FQ}}(uv) \leq 3\ \forall u, v \in X$, where $T_{\mu_{FQ}} : X \times X \to [0, 1]$, $C_{\mu_{FQ}} : X \times X \to [0, 1]$, $I_{\mu_{FQ}} : X \times X \to [0, 1]$, $F_{\mu_{FQ}} : X \times X \to [0, 1]$, indicates degree of truth, contradiction, ignorance and false membership of μ_{FQ}. σ_{FQ} and μ_{FQ} is the Fermatean Quadripartitioned Neutrosophic vertex and edge set of G_{FQ}.

Definition 3.2. A Fermatean Quadripartitioned Neutrosophic Fuzzy Graph $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$ is said to be complete if the following conditions are satisfied.

$$\begin{aligned} T_{\mu_{FQ}}(uv) &= (T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v)) \\ C_{\mu_{FQ}}(uv) &= (C_{\sigma_{FQ}}(u) \wedge C_{\sigma_{FQ}}(v)) \\ I_{\mu_{FQ}}(uv) &= (I_{\sigma_{FQ}}(u) \vee I_{\sigma_{FQ}}(v)) \\ F_{\mu_{FQ}}(uv) &= (F_{\sigma_{FQ}}(u) \vee F_{\sigma_{FQ}}(v)) \end{aligned}$$

where $u, v \in \sigma_{FQ}$.

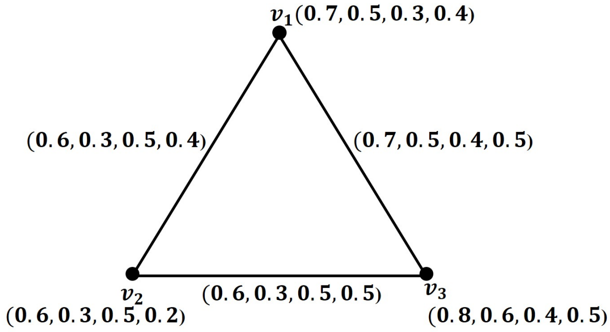

Figure 2: *Complete FQNFG*

Definition 3.3. Let $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$ be a FQNFG, then Size of G_{FQ} is defined as $S(G_{FQ}) = (\sum_{uv \in \mu_{FQ}} T_{\mu_{FQ}}(uv), \sum_{uv \in \mu_{FQ}} C_{\mu_{FQ}}(uv), \sum_{uv \in \mu_{FQ}} I_{\mu_{FQ}}(uv), \sum_{uv \in \mu_{FQ}} F_{\mu_{FQ}}(uv))$.
From the below figure, $S(G) = (1.4, 0.7, 2.6, 2)$

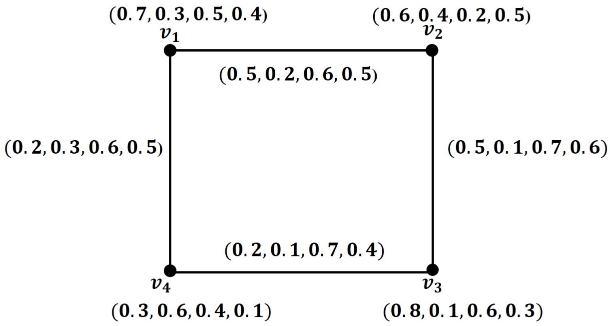

Figure 3: *Fermatean Quadripartitioned Neutrosophic Fuzzy Graph*

Definition 3.4. Let $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$ be a Fermatean Quadripartitioned Neutrosophic Fuzzy Graph. The Complement of FQNFG is $\overline{G_{FQ}}$= $(\overline{\sigma_{FQ}},\overline{\mu_{FQ}})$, where $\overline{\sigma_{FQ}}$=$(\overline{T_{\sigma_{FQ}}}$,$\overline{C_{\sigma_{FQ}}}$, $\overline{I_{\sigma_{FQ}}}$, $\overline{F_{\sigma_{FQ}}})$ and $\overline{\mu_{FQ}}$=$(\overline{T_{\sigma_{FQ}}}$,$\overline{C_{\sigma_{FQ}}}$, $\overline{I_{\sigma_{FQ}}}$, $\overline{F_{\sigma_{FQ}}})$ defined by

$$\begin{aligned}
\overline{T_{\mu_{FQ}}} &= |T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v) - T_{\mu_{FQ}}(uv)| \\
\overline{C_{\mu_{FQ}}} &= |C_{\sigma_{FQ}}(u) \wedge C_{\sigma_{FQ}}(v) - C_{\mu_{FQ}}(uv)| \\
\overline{I_{\mu_{FQ}}} &= |I_{\sigma_{FQ}}(u) \vee I_{\sigma_{FQ}}(v) - I_{\mu_{FQ}}(uv)| \\
\overline{F_{\mu_{FQ}}} &= |F_{\sigma_{FQ}}(u) \vee F_{\sigma_{FQ}}(v) - F_{\mu_{FQ}}(uv)|
\end{aligned}$$

for all $u, v \in \sigma_{FQ}$ and $uv \in \mu_{FQ}$.

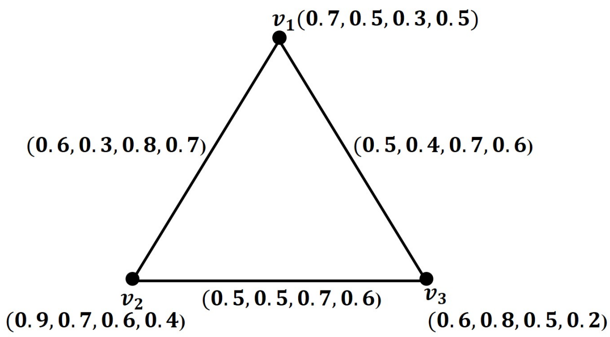

Figure 4: *Fermatean Quadripartitioned Neutrosophic Fuzzy Graph*

Definition 3.5. Let $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$ be a FQNFG, then order of G_{FQ} is termed as $O(G_{FQ}) = (\sum_{u \in \sigma_{FQ}} T_{\sigma_{FQ}}(u), \sum_{u \in \sigma_{FQ}} C_{\sigma_{FQ}}(u), \sum_{u \in \sigma_{FQ}} I_{\sigma_{FQ}}(u), \sum_{u \in \sigma_{FQ}} F_{\sigma_{FQ}}(u))$.
From figure 3, $O(G) = (2.4, 1.4, 1.7, 1.3)$

Definition 3.6. A Fermatean Quadripartitioned Neutrosophic Fuzzy Graph $G_{FQ} = (\sigma_{FQ}, \mu_{FQ})$ is called as strong, when following criteria are met.

$$\begin{aligned}
T_{\mu_{FQ}}(u, v) &= (T_{\sigma_{FQ}}(u) \wedge T_{\sigma_{FQ}}(v)) \\
C_{\mu_{FQ}}(u, v) &= (C_{\sigma_{FQ}}(u) \wedge C_{\sigma_{FQ}}(v)) \\
I_{\mu_{FQ}}(u, v) &= (I_{\sigma_{FQ}}(u) \vee I_{\sigma_{FQ}}(v)) \\
F_{\mu_{FQ}}(u, v) &= (F_{\sigma_{FQ}}(u) \vee F_{\sigma_{FQ}}(v))
\end{aligned}$$

where $u, v \in \sigma_{FQ}$.

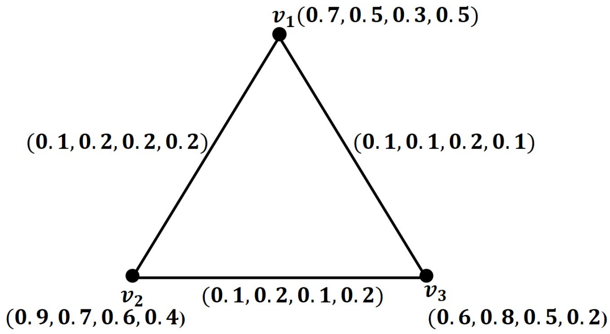

Figure 5: *Complement of FQNFG*

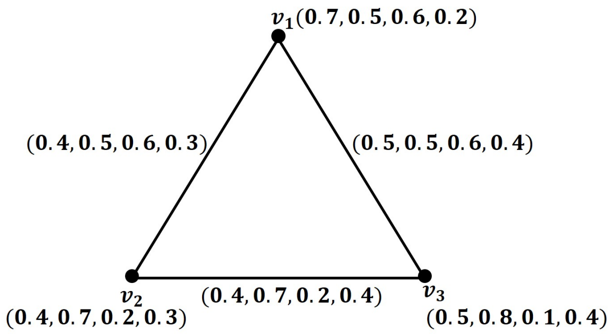

Figure 6: *Strong Fermatean Quadripartitioned Neutrosophic Fuzzy Graph*

4. Products on Fermatean Quadripartitioned Neutrosophic Fuzzy Graph

Definition 4.1. A Composition of two FQNFG G_{FQ1}, G_{FQ2} represented by $G_{FQ1} \circ G_{FQ2}$ and termed as $G_{FQ1} \circ G_{FQ2} = (\sigma_{FQ1} \circ \sigma_{FQ2}, \mu_{FQ1} \circ \mu_{FQ2})$ where

1. $$\begin{aligned} T_{\sigma_{FQ1} \circ \sigma_{FQ2}}(uv) &= T_{\sigma_{FQ1}}(u) \wedge T_{\sigma_{FQ2}}(v) \\ C_{\sigma_{FQ1} \circ \sigma_{FQ2}}(uv) &= C_{\sigma_{FQ1}}(u) \wedge C_{\sigma_{FQ2}}(v) \\ I_{\sigma_{FQ1} \circ \sigma_{FQ2}}(uv) &= I_{\sigma_{FQ1}}(u) \vee I_{\sigma_{FQ2}}(v) \\ F_{\sigma_{FQ1} \circ \sigma_{FQ2}}(uv) &= F_{\sigma_{FQ1}}(u) \vee F_{\sigma_{FQ2}}(v) \end{aligned}$$
   for all $(u, v) \in \sigma_{FQ1} \circ \sigma_{FQ2}$.

2. $$\begin{aligned} T_{\mu_{FQ1} \circ \mu_{FQ2}}(uv_1)(uv_2) &= T_{\sigma_{FQ1}}(u) \wedge T_{\mu_{FQ2}}(v_1v_2) \\ C_{\mu_{FQ1} \circ \mu_{FQ2}}(uv_1)(uv_2) &= C_{\sigma_{FQ1}}(u) \wedge C_{\mu_{FQ2}}(v_1v_2) \\ I_{\mu_{FQ1} \circ \mu_{FQ2}}(uv_1)(uv_2) &= I_{\sigma_{FQ1}}(u) \vee I_{\mu_{FQ2}}(v_1v_2) \\ F_{\mu_{FQ1} \circ \mu_{FQ2}}(uv_1)(uv_2) &= F_{\sigma_{FQ1}}(u) \vee F_{\mu_{FQ2}}(v_1v_2) \end{aligned}$$
   for all $u \in \sigma_{FQ1}, v_1v_2 \in \mu_{FQ2}$

3. $$\begin{aligned} T_{\mu_{FQ1} \circ \mu_{FQ2}}(u_1v)(u_2v) &= T_{\mu_{FQ1}}(u_1u_2) \wedge T_{\sigma_{FQ2}}(v) \\ C_{\mu_{FQ1} \circ \mu_{FQ2}}(u_1v)(u_2v) &= C_{\mu_{FQ1}}(u_1u_2) \wedge C_{\sigma_{FQ2}}(v) \\ I_{\mu_{FQ1} \circ \mu_{FQ2}}(u_1v)(u_2v) &= I_{\mu_{FQ1}}(u_1u_2) \vee I_{\sigma_{FQ2}}(v) \\ F_{\mu_{FQ1} \circ \mu_{FQ2}}(u_1v)(u_2v) &= F_{\mu_{FQ1}}(u_1u_2) \vee F_{\sigma_{FQ2}}(v) \end{aligned}$$
   for all $v \in \sigma_{FQ2}, u_1u_2 \in \mu_{FQ1}$

4. $$\begin{aligned} T_{\mu_{FQ1} \circ \mu_{FQ2}}(u_1v_1)(u_2v_2) &= T_{\mu_{FQ1}}(u_1u_2) \wedge T_{\sigma_{FQ2}}(v_1) \wedge T_{\sigma_{FQ2}}(v_2) \\ C_{\mu_{FQ1} \circ \mu_{FQ2}}(u_1v_1)(u_2v_2) &= C_{\mu_{FQ1}}(u_1u_2) \wedge C_{\sigma_{FQ2}}(v_1) \wedge C_{\sigma_{FQ2}}(v_2) \\ I_{\mu_{FQ1} \circ \mu_{FQ2}}(u_1v_1)(u_2v_2) &= I_{\mu_{FQ1}}(u_1u_2) \vee I_{\sigma_{FQ2}}(v_1) \vee T_{\sigma_{FQ2}}(v_2) \\ F_{\mu_{FQ1} \circ \mu_{FQ2}}(u_1v_1)(u_2v_2) &= F_{\mu_{FQ1}}(u_1u_2) \vee F_{\sigma_{FQ2}}(v_1) \vee F_{\sigma_{FQ2}}(v_2) \end{aligned}$$
   for all $u_1u_2 \in \mu_{FQ1}, v_1v_2 \in \mu_{FQ2}$.

Theorem 4.1. A Composition of two FQNFG is also the Fermatean Quadripartitioned Neutrosophic Fuzzy graph.

Proof. Case I: For $u \in \sigma_{FQ1}, v_1v_2 \in \mu_{FQ2}$

$$\begin{aligned}
T_{\mu_{FQ1}\circ\mu_{FQ2}}((uv_1)(uv_2)) &= T_{\sigma_{FQ1}}(u) \wedge T_{\mu_{FQ2}}(v_1v_2) \\
&\leq T_{\sigma_{FQ1}}(u) \wedge [T_{\sigma_{FQ2}}(v_1) \wedge T_{\sigma_{FQ2}}(v_2)] \\
&= [T_{\sigma_{FQ1}}(u) \wedge T_{\sigma_{FQ2}}(v_1)] \wedge [T_{\sigma_{FQ1}}(u) \wedge T_{\sigma_{FQ2}}(v_2)] \\
&= T_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u, v_1) \wedge T_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u, v_2)
\end{aligned}$$

$$\begin{aligned}
C_{\mu_{FQ1}\circ\mu_{FQ2}}((uv_1)(uv_2)) &= C_{\sigma_{FQ1}}(u) \wedge C_{\mu_{FQ2}}(v_1v_2) \\
&\leq C_{\sigma_{FQ1}}(u) \wedge [C_{\sigma_{FQ2}}(v_1) \wedge C_{\sigma_{FQ2}}(v_2)] \\
&= [C_{\sigma_{FQ1}}(u) \wedge C_{\sigma_{FQ2}}(v_1)] \wedge [C_{\sigma_{FQ1}}(u) \wedge C_{\sigma_{FQ2}}(v_2)] \\
&= C_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u, v_1) \wedge C_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u, v_2)
\end{aligned}$$

$$\begin{aligned}
I_{\mu_{FQ1}\circ\mu_{FQ2}}((uv_1)(uv_2)) &= I_{\sigma_{FQ1}}(u) \vee I_{\mu_{FQ2}}(v_1v_2) \\
&\geq I_{\sigma_{FQ1}}(u) \vee [I_{\sigma_{FQ2}}(v_1) \vee I_{\sigma_{FQ2}}(v_2)] \\
&= [I_{\sigma_{FQ1}}(u) \vee I_{\sigma_{FQ2}}(v_1)] \vee [I_{\sigma_{FQ1}}(u) \vee I_{\sigma_{FQ2}}(v_2)] \\
&= I_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u, v_1) \vee I_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u, v_2)
\end{aligned}$$

$$\begin{aligned}
F_{\mu_{FQ1}\circ\mu_{FQ2}}((uv_1)(uv_2)) &= F_{\sigma_{FQ1}}(u) \vee F_{\mu_{FQ2}}(v_1v_2) \\
&\geq F_{\sigma_{FQ1}}(u) \vee [F_{\sigma_{FQ2}}(v_1) \vee F_{\sigma_{FQ2}}(v_2)] \\
&= [F_{\sigma_{FQ1}}(u) \vee F_{\sigma_{FQ2}}(v_1)] \vee [F_{\sigma_{FQ1}}(u) \vee F_{\sigma_{FQ2}}(v_2)] \\
&= F_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u, v_1) \vee F_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u, v_2)
\end{aligned}$$

for all $(uv_1, uv_2) \in \mu_{FQ1} \circ \mu_{FQ2}$
Case II: For $v \in \sigma_{FQ2}, (u_1u_2) \in \mu_{FQ1}$

$$\begin{aligned}
T_{\mu_{FQ1}\circ\mu_{FQ2}}((u_1v)(u_2v)) &= T_{\mu_{FQ1}}(u_1u_2) \wedge T_{\sigma_{FQ2}}(v) \\
&\leq [T_{\sigma_{FQ1}}(u_1) \wedge T_{\sigma_{FQ1}}(u_2)] \wedge T_{\sigma_{FQ2}}(v) \\
&= [T_{\sigma_{FQ1}}(u_1) \wedge T_{\sigma_{FQ2}}(v)] \wedge [T_{\sigma_{FQ1}}(u_2) \wedge T_{\sigma_{FQ2}}(v)] \\
&= T_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u_1, v) \wedge T_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u_2, v)
\end{aligned}$$

$$\begin{aligned}
C_{\mu_{FQ1}\circ\mu_{FQ2}}((u_1v)(u_2v)) &= C_{\mu_{FQ1}}(u_1u_2) \wedge C_{\sigma_{FQ2}}(v) \\
&\leq [C_{\sigma_{FQ1}}(u_1) \wedge C_{\sigma_{FQ1}}(u_2)] \wedge C_{\sigma_{FQ2}}(v) \\
&= [C_{\sigma_{FQ1}}(u_1) \wedge C_{\sigma_{FQ2}}(v)] \wedge [C_{\sigma_{FQ1}}(u_2) \wedge C_{\sigma_{FQ2}}(v)] \\
&= C_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u_1, v) \wedge C_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u_2, v)
\end{aligned}$$

$$\begin{aligned}
I_{\mu_{FQ1}\circ\mu_{FQ2}}((u_1v)(u_2v)) &= I_{\mu_{FQ1}}(u_1u_2) \vee I_{\sigma_{FQ2}}(v) \\
&\geq [I_{\sigma_{FQ1}}(u_1) \vee I_{\sigma_{FQ1}}(u_2)] \vee I_{\sigma_{FQ2}}(v) \\
&= [I_{\sigma_{FQ1}}(u_1) \vee I_{\sigma_{FQ2}}(v)] \vee [I_{\sigma_{FQ1}}(u_2) \vee I_{\sigma_{FQ2}}(v)] \\
&= I_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u_1, v) \vee I_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u_2, v)
\end{aligned}$$

$$\begin{aligned}
F_{\mu_{FQ1}\circ\mu_{FQ2}}((u_1v)(u_2v)) &= F_{\mu_{FQ1}}(u_1u_2) \vee F_{\sigma_{FQ2}}(v) \\
&\geq [F_{\sigma_{FQ1}}(u_1) \vee F_{\sigma_{FQ1}}(u_2)] \vee F_{\sigma_{FQ2}}(v) \\
&= [F_{\sigma_{FQ1}}(u_1) \vee F_{\sigma_{FQ2}}(v)] \vee [F_{\sigma_{FQ1}}(u_2) \vee F_{\sigma_{FQ2}}(v)] \\
&= F_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u_1, v) \vee F_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u_2, v)
\end{aligned}$$

for all $(u_1v, u_2v) \in G_{FQ1} \circ G_{FQ2}$
Case 3: For all $(u_1v_1) \in \mu_{FQ1}, (u_2v_2) \in \mu_{FQ2}$

$$\begin{aligned}
T_{\mu_{FQ1}\circ\mu_{FQ2}}(u_1v_1)(u_2v_2) &= T_{\mu_{FQ1}}(u_1u_2) \wedge T_{\sigma_{FQ2}}(v_1) \wedge T_{\sigma_{FQ2}}(v_2) \\
&\leq [T_{\sigma_{FQ1}}(u_1) \wedge T_{\sigma_{FQ1}}(u_2)] \wedge [T_{\sigma_{FQ2}}(v_1) \wedge T_{\sigma_{FQ2}}(v_2)] \\
&= [T_{\sigma_{FQ1}}(u_1) \wedge T_{\sigma_{FQ2}}(v_1)] \wedge [T_{\sigma_{FQ1}}(u_2) \wedge T_{\sigma_{FQ2}}(v_2)] \\
&= T_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u_1, v_1) \wedge T_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u_2, v_2)
\end{aligned}$$

$$\begin{aligned}
C_{\mu_{FQ1}\circ\mu_{FQ2}}(u_1v_1)(u_2v_2) &= C_{\mu_{FQ1}}(u_1u_2) \wedge C_{\sigma_{FQ2}}(v_1) \wedge C_{\sigma_{FQ2}}(v_2) \\
&\leq [C_{\sigma_{FQ1}}(u_1) \wedge C_{\sigma_{FQ1}}(u_2)] \wedge [C_{\sigma_{FQ2}}(v_1) \wedge C_{\sigma_{FQ2}}(v_2)] \\
&= [C_{\sigma_{FQ1}}(u_1) \wedge C_{\sigma_{FQ2}}(v_1)] \wedge [C_{\sigma_{FQ1}}(u_2) \wedge C_{\sigma_{FQ2}}(v_2)] \\
&= C_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u_1, v_1) \wedge C_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u_2, v_2)
\end{aligned}$$

$$\begin{aligned}
I_{\mu_{FQ1}\circ\mu_{FQ2}}(u_1v_1)(u_2v_2) &= I_{\mu_{FQ1}}(u_1u_2) \vee I_{\sigma_{FQ2}}(v_1) \vee I_{\sigma_{FQ2}}(v_2) \\
&\geq [I_{\sigma_{FQ1}}(u_1) \vee I_{\sigma_{FQ1}}(u_2)] \vee [I_{\sigma_{FQ2}}(v_1) \vee I_{\sigma_{FQ2}}(v_2)] \\
&= [I_{\sigma_{FQ1}}(u_1) \vee I_{\sigma_{FQ2}}(v_1)] \vee [I_{\sigma_{FQ1}}(u_2) \vee I_{\sigma_{FQ2}}(v_2)] \\
&= I_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u_1, v_1) \vee I_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u_2, v_2)
\end{aligned}$$

$$\begin{aligned}
F_{\mu_{FQ1}\circ\mu_{FQ2}}(u_1v_1)(u_2v_2) &= F_{\mu_{FQ1}}(u_1u_2) \vee F_{\sigma_{FQ2}}(v_1) \vee F_{\sigma_{FQ2}}(v_2) \\
&\geq [F_{\sigma_{FQ1}}(u_1) \vee T_{\sigma_{FQ1}}(u_2)] \wedge [F_{\sigma_{FQ2}}(v_1) \vee T_{\sigma_{FQ2}}(v_2)] \\
&= [F_{\sigma_{FQ1}}(u_1) \vee F_{\sigma_{FQ2}}(v_1)] \vee [F_{\sigma_{FQ1}}(u_2) \vee F_{\sigma_{FQ2}}(v_2)] \\
&= F_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u_1, v_1) \vee F_{\sigma_{FQ1}\circ\sigma_{FQ2}}(u_2, v_2)
\end{aligned}$$

for all $(u_1v_1, u_2v_2) \in \sigma_{FQ1} \circ \sigma_{FQ2}$ ■

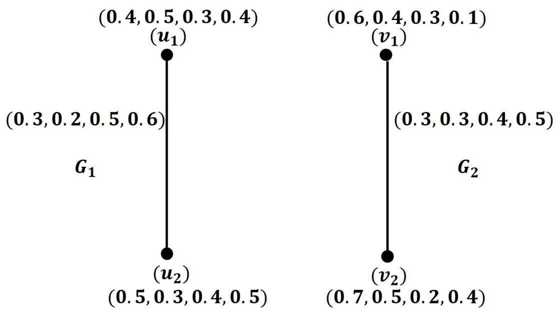

Figure 7: *Fermatean Quadripartitioned Neutrosophic Fuzzy Graph*

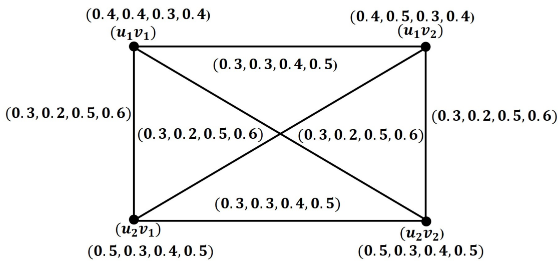

Figure 8: *Composition of FQNFG*

Definition 4.2. The lexicographic product of two FQNFG G_1,G_2 is represented by $G_1 \cdot G_2$ and termed as $G_1 \cdot G_2 = (A_1 \cdot A_2, B_1 \cdot B_2)$

1. $T_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(uv) = T_{\sigma_{FQ1}}(u) \wedge T_{\sigma_{FQ2}}(v)$
$C_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(uv) = C_{\sigma_{FQ1}}(u) \wedge C_{\sigma_{FQ2}}(v)$
$I_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(uv) = I_{\sigma_{FQ1}}(u) \vee I_{\sigma_{FQ2}}(v)$
$F_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(uv) = F_{\sigma_{FQ1}}(u) \vee F_{\sigma_{FQ2}}(v)$
for all $(u,v) \in \sigma_{FQ1}\cdot\sigma_{FQ2}$.

2. $T_{\mu_{FQ1}\cdot\mu_{FQ2}}(uv_1)(uv_2) = T_{\sigma_{FQ1}}(u) \wedge T_{\mu_{FQ2}}(v_1v_2)$
$C_{\mu_{FQ1}\cdot\mu_{FQ2}}(uv_1)(uv_2) = C_{\sigma_{FQ1}}(u) \wedge C_{\mu_{FQ2}}(v_1v_2)$
$I_{\mu_{FQ1}\cdot\mu_{FQ2}}(uv_1)(uv_2) = I_{\sigma_{FQ1}}(u) \vee I_{\mu_{FQ2}}(v_1v_2)$
$F_{\mu_{FQ1}\cdot\mu_{FQ2}}(uv_1)(uv_2) = F_{\sigma_{FQ1}}(u) \vee F_{\mu_{FQ2}}(v_1v_2)$
for all $u \in \sigma_{FQ1}, v_1v_2 \in \mu_{FQ2}$.

3. $T_{\mu_{FQ1}\cdot\mu_{FQ2}}(u_1v_1)(u_2v_2) = T_{\mu_{FQ1}}(u_1u_2) \wedge T_{\mu_{FQ2}}(v_1v_2)$
$C_{\mu_{FQ1}\cdot\mu_{FQ2}}(u_1v_1)(u_2v_2) = C_{\mu_{FQ1}}(u_1u_2) \wedge C_{\mu_{FQ2}}(v_1v_2)$
$I_{\mu_{FQ1}\cdot\mu_{FQ2}}(u_1v_1)(u_2v_2) = I_{\mu_{FQ1}}(u_1u_2) \vee I_{\mu_{FQ2}}(v_1v_2)$
$F_{\mu_{FQ1}\cdot\mu_{FQ2}}(u_1v_1)(u_2v_2) = F_{\mu_{FQ1}}(u_1u_2) \vee F_{\mu_{FQ2}}(v_1v_2)$
for all $u_1u_2 \in \mu_{FQ1}, v_1v_2 \in \mu_{FQ2}$.

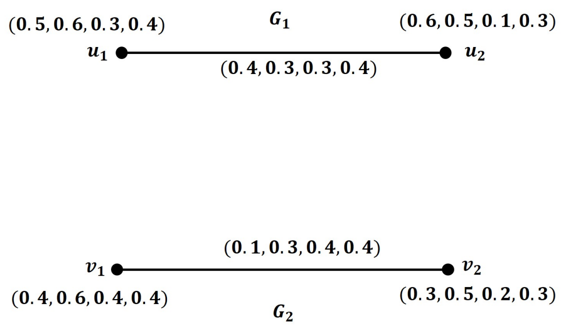

Figure 9: *Fermatean Quadripartitioned Neutrosophic Fuzzy Graph*

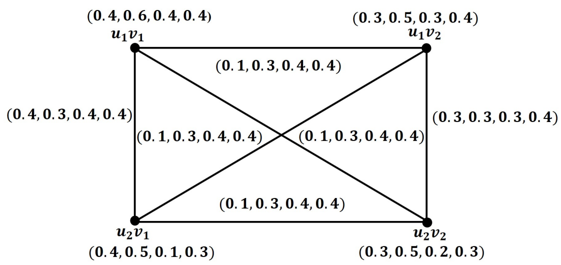

Figure 10: *Lexicographic produt of FQNFG*

Theorem 4.2. A lexicographic product of two FQNFG is also the Fermatean Quadripartitioned Neutrosophic Fuzzy graph.

Proof. To prove the theorem we use the following two cases.
Case I: For $u \in \sigma_{FQ1}, v_1v_2 \in \mu_{FQ2}$

$$\begin{aligned} T_{\mu_{FQ1}\cdot\mu_{FQ2}}((uv_1)(uv_2)) &= T_{\sigma_{FQ1}}(u) \wedge T_{\mu_{FQ2}}(v_1v_2) \\ &\leq T_{\sigma_{FQ1}}(u) \wedge [T_{\sigma_{FQ2}}(v_1) \wedge T_{\sigma_{FQ2}}(v_2)] \\ &= [T_{\sigma_{FQ1}}(u) \wedge T_{\sigma_{FQ2}}(v_1)] \wedge [T_{\sigma_{FQ1}}(u) \wedge T_{\sigma_{FQ2}}(v_2)] \\ &= T_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(u,v_1) \wedge T_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(u,v_2) \end{aligned}$$

$$
\begin{aligned}
C_{\mu_{FQ1}\cdot\mu_{FQ2}}((uv_1)(uv_2)) &= C_{\sigma_{FQ1}}(u) \wedge C_{\mu_{FQ2}}(v_1v_2) \\
&\leq C_{\sigma_{FQ1}}(u) \wedge [C_{\sigma_{FQ2}}(v_1) \wedge C_{\sigma_{FQ2}}(v_2)] \\
&= [C_{\sigma_{FQ1}}(u) \wedge C_{\sigma_{FQ2}}(v_1)] \wedge [C_{\sigma_{FQ1}}(u) \wedge C_{\sigma_{FQ2}}(v_2)] \\
&= C_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(u,v_1) \wedge C_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(u,v_2)
\end{aligned}
$$

$$
\begin{aligned}
I_{\mu_{FQ1}\cdot\mu_{FQ2}}((uv_1)(uv_2)) &= I_{\sigma_{FQ1}}(u) \vee I_{\mu_{FQ2}}(v_1v_2) \\
&\geq I_{\sigma_{FQ1}}(u) \vee [I_{\sigma_{FQ2}}(v_1) \vee I_{\sigma_{FQ2}}(v_2)] \\
&= [I_{\sigma_{FQ1}}(u) \vee I_{\sigma_{FQ2}}(v_1)] \vee [I_{\sigma_{FQ1}}(u) \vee I_{\sigma_{FQ2}}(v_2)] \\
&= I_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(u,v_1) \vee I_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(u,v_2)
\end{aligned}
$$

$$
\begin{aligned}
F_{\mu_{FQ1}\cdot\mu_{FQ2}}((uv_1)(uv_2)) &= F_{\sigma_{FQ1}}(u) \vee F_{\mu_{FQ2}}(v_1v_2) \\
&\geq F_{\sigma_{FQ1}}(u) \vee [F_{\sigma_{FQ2}}(v_1) \vee F_{\sigma_{FQ2}}(v_2)] \\
&= [F_{\sigma_{FQ1}}(u) \vee F_{\sigma_{FQ2}}(v_1)] \vee [F_{\sigma_{FQ1}}(u) \vee F_{\sigma_{FQ2}}(v_2)] \\
&= F_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(u,v_1) \vee F_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(u,v_2)
\end{aligned}
$$

for all $(uv_1, uv_2) \in \mu_{FQ1} \cdot \mu_{FQ2}$
Case II: For all $(u_1v_1) \in \mu_{FQ1}, (u_2v_2) \in \mu_{FQ2}$

$$
\begin{aligned}
T_{\mu_{FQ1}\cdot\mu_{FQ2}}((u_1v_1)(u_2v_2)) &= T_{\mu_{FQ1}}(u_1u_2) \wedge T_{\mu_{FQ2}}(v_1v_2) \\
&\leq [T_{\sigma_{FQ1}}(u_1) \wedge T_{\sigma_{FQ1}}(u_2)] \wedge [T_{\sigma_{FQ2}}(v_1) \wedge T_{\sigma_{FQ2}}(v_2)] \\
&= [T_{\sigma_{FQ1}}(u_1) \wedge T_{\sigma_{FQ2}}(v_1)] \wedge [T_{\sigma_{FQ1}}(u_2) \wedge T_{\sigma_{FQ2}}(v_2)] \\
&= T_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(u_1,v_1) \wedge T_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(u_2,v_2)
\end{aligned}
$$

$$
\begin{aligned}
C_{\mu_{FQ1}\cdot\mu_{FQ2}}((u_1v_1)(u_2v_2)) &= C_{\mu_{FQ1}}(u_1u_2) \wedge C_{\mu_{FQ2}}(v_1v_2) \\
&\leq [C_{\sigma_{FQ1}}(u_1) \wedge C_{\sigma_{FQ1}}(u_2)] \wedge [C_{\sigma_{FQ2}}(v_1) \wedge C_{\sigma_{FQ2}}(v_2)] \\
&= [C_{\sigma_{FQ1}}(u_1) \wedge C_{\sigma_{FQ2}}(v_1)] \wedge [C_{\sigma_{FQ1}}(u_2) \wedge C_{\sigma_{FQ2}}(v_2)] \\
&= C_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(u_1,v_1) \wedge C_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(u_2,v_2)
\end{aligned}
$$

$$
\begin{aligned}
I_{\mu_{FQ1}\cdot\mu_{FQ2}}((u_1v_1)(u_2v_2)) &= I_{\mu_{FQ1}}(u_1u_2) \vee I_{\mu_{FQ2}}(v_1v_2) \\
&\geq [I_{\sigma_{FQ1}}(u_1) \vee I_{\sigma_{FQ1}}(u_2)] \vee [I_{\sigma_{FQ2}}(v_1) \vee I_{\sigma_{FQ2}}(v_2)] \\
&= [I_{\sigma_{FQ1}}(u_1) \vee I_{\sigma_{FQ2}}(v_1)] \vee [I_{\sigma_{FQ1}}(u_2) \vee I_{\sigma_{FQ2}}(v_2)] \\
&= I_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(u_1,v_1) \vee I_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(u_2,v_2)
\end{aligned}
$$

$$
\begin{aligned}
F_{\mu_{FQ1}\cdot\mu_{FQ2}}((u_1v_1)(u_2v_2)) &= F_{\mu_{FQ1}}(u_1u_2) \vee F_{\mu_{FQ2}}(v_1v_2) \\
&\geq [F_{\sigma_{FQ1}}(u_1) \vee F_{\sigma_{FQ1}}(u_2)] \vee [F_{\sigma_{FQ2}}(v_1) \vee F_{\sigma_{FQ2}}(v_2)] \\
&= [F_{\sigma_{FQ1}}(u_1) \vee F_{\sigma_{FQ2}}(v_1)] \vee [F_{\sigma_{FQ1}}(u_2) \vee F_{\sigma_{FQ2}}(v_2)] \\
&= F_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(u_1,v_1) \vee F_{\sigma_{FQ1}\cdot\sigma_{FQ2}}(u_2,v_2)
\end{aligned}
$$

for all $(u_1v_1, u_2v_2) \in \sigma_{FQ1} \cdot \sigma_{FQ2}$
This completes the proof. ∎

Definition 4.3. A Cartesian product of two FQNFG G_{FQ1},G_{FG2} and also represented by G_{FQ1} cartesian product G_{FQ2} is termed as
$G_{FQ1} \times G_{FQ2}$ =$(\sigma_{FQ1} \times \sigma_{FQ2}, \mu_{FQ1} \times \mu_{FQ2})$

1. $T_{\sigma_{FQ1}\times\sigma_{FQ2}}(uv) = T_{\sigma_{FQ1}}(u) \wedge T_{\sigma_{FQ2}}(v)$
   $C_{\sigma_{FQ1}\times\sigma_{FQ2}}(uv) = C_{\sigma_{FQ1}}(u) \wedge C_{\sigma_{FQ2}}(v)$
   $I_{\sigma_{FQ1}\times\sigma_{FQ2}}(uv) = I_{\sigma_{FQ1}}(u) \vee I_{\sigma_{FQ2}}(v)$
   $F_{\sigma_{FQ1}\times\sigma_{FQ2}}(uv) = F_{\sigma_{FQ1}}(u) \vee F_{\sigma_{FQ2}}(v)$
   for all $(u,v) \in \sigma_{FQ1} \times \sigma_{FQ2}$
   The membership value of the links $\mu_{FQ1} \times \mu_{FQ2}$ is determined as

2. $T_{\mu_{FQ1}\times\mu_{FQ2}}(uv_1)(uv_2) = T_{\sigma_{FQ1}}(u) \wedge T_{\mu_{FQ2}}(v_1v_2)$
$C_{\mu_{FQ1}\times\mu_{FQ2}}(uv_1)(uv_2) = C_{\sigma_{FQ1}}(u) \wedge C_{\mu_{FQ2}}(v_1v_2)$
$I_{\mu_{FQ1}\times\mu_{FQ2}}(uv_1)(uv_2) = I_{\sigma_{FQ1}}(u) \vee I_{\mu_{FQ2}}(v_1v_2)$
$F_{\mu_{FQ1}\times\mu_{FQ2}}(uv_1)(uv_2) = F_{\sigma_{FQ1}}(u) \vee F_{\mu_{FQ2}}(v_1v_2)$
for all $u \in \sigma_{FQ1}, (v_1, v_2) \in \mu_{FQ2}$

3. $T_{\mu_{FQ1}\times\mu_{FQ2}}(u_1v)(u_2v) = T_{\mu_{FQ1}}(u_1u_2) \wedge T_{\sigma_{FQ2}}(v)$
$C_{\mu_{FQ1}\times\mu_{FQ2}}(u_1v)(u_2v) = C_{\mu_{FQ1}}(u_1u_2) \wedge C_{\sigma_{FQ2}}(v)$
$I_{\mu_{FQ1}\times\mu_{FQ2}}(u_1v)(u_2v) = I_{\mu_{FQ1}}(u_1u_2) \vee I_{\sigma_{FQ2}}(v)$
$F_{\mu_{FQ1}\times\mu_{FQ2}}(u_1v)(u_2v) = F_{\mu_{FQ1}}(u_1u_2) \vee F_{\sigma_{FQ2}}(v)$
for all $v \in \sigma_{FQ2}, (u_1u_2) \in \mu_{FQ1}$

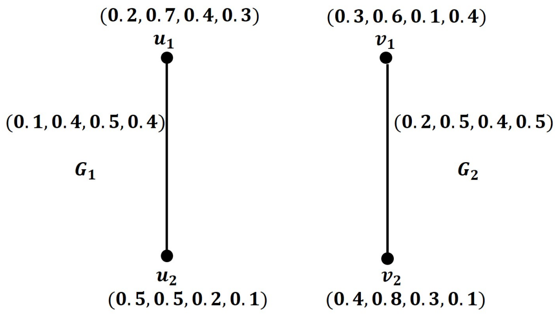

Figure 11: *Fermatean Quadripartitioned Neutrosophic Fuzzy Graph*

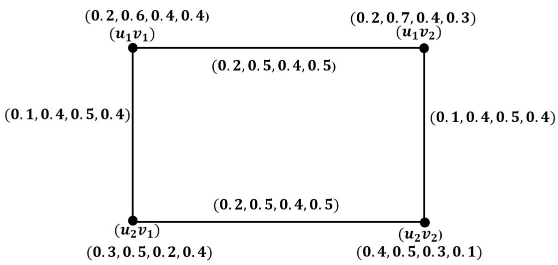

Figure 12: *Cartesian Product of FQNFG*

Theorem 4.3. A Cartesian Product of two FQNFG is a Fermatean Quadripartitioned Neutrosophic Fuzzy Graph.

Proof. To prove the theorem we use the following two cases.
Case I: For $u \in \sigma_{FQ1}, (v_1v_2) \in \mu_{FQ2}$

$$\begin{aligned} T_{\mu_{FQ1}\times\mu_{FQ2}}((uv_1)(uv_2)) &= T_{\sigma_{FQ1}}(u) \wedge T_{\mu_{FQ2}}(v_1v_2) \\ &\leq T_{\sigma_{FQ1}}(u) \wedge [T_{\sigma_{FQ2}}(v_1) \wedge T_{\sigma_{FQ2}}(v_2)] \\ &= [T_{\sigma_{FQ1}}(u) \wedge T_{\sigma_{FQ2}}(v_1)] \wedge [T_{\sigma_{FQ1}}(u) \wedge T_{\sigma_{FQ2}}(v_2)] \\ &= T_{\sigma_{FQ1}\times\sigma_{FQ2}}(u, v_1) \wedge T_{\sigma_{FQ1}\times\sigma_{FQ2}}(u, v_2) \end{aligned}$$

$$\begin{aligned} C_{\mu_{FQ1}\times\mu_{FQ2}}((uv_1)(uv_2)) &= C_{\sigma_{FQ1}}(u) \wedge C_{\mu_{FQ2}}(v_1v_2) \\ &\leq C_{\sigma_{FQ1}}(u) \wedge [C_{\sigma_{FQ2}}(v_1) \wedge C_{\sigma_{FQ2}}(v_2)] \\ &= [C_{\sigma_{FQ1}}(u) \wedge C_{\sigma_{FQ2}}(v_1)] \wedge [C_{\sigma_{FQ1}}(u) \wedge C_{\sigma_{FQ2}}(v_2)] \\ &= C_{\sigma_{FQ1}\times\sigma_{FQ2}}(u, v_1) \wedge C_{\sigma_{FQ1}\times\sigma_{FQ2}}(u, v_2) \end{aligned}$$

$$\begin{aligned} I_{\mu_{FQ1} \times \mu_{FQ2}}((uv_1)(uv_2)) &= I_{\sigma_{FQ1}}(u) \vee I_{\mu_{FQ2}}(v_1v_2) \\ &\geq I_{\sigma_{FQ1}}(u) \vee [I_{\sigma_{FQ2}}(v_1) \vee I_{\sigma_{FQ2}}(v_2)] \\ &= [I_{\sigma_{FQ1}}(u) \vee I_{\sigma_{FQ2}}(v_1)] \vee [I_{\sigma_{FQ1}}(u) \vee I_{\sigma_{FQ2}}(v_2)] \\ &= I_{\sigma_{FQ1} \times \sigma_{FQ2}}(u, v_1) \vee I_{\sigma_{FQ1} \times \sigma_{FQ2}}(u, v_2) \end{aligned}$$

$$\begin{aligned} F_{\mu_{FQ1} \times \mu_{FQ2}}((uv_1)(uv_2)) &= F_{\sigma_{FQ1}}(u) \vee F_{\mu_{FQ2}}(v_1v_2) \\ &\geq F_{\sigma_{FQ1}}(u) \vee [F_{\sigma_{FQ2}}(v_1) \vee F_{\sigma_{FQ2}}(v_2)] \\ &= [F_{\sigma_{FQ1}}(u) \vee F_{\sigma_{FQ2}}(v_1)] \vee [F_{\sigma_{FQ1}}(u) \vee F_{\sigma_{FQ2}}(v_2)] \\ &= F_{\sigma_{FQ1} \times \sigma_{FQ2}}(u, v_1) \vee F_{\sigma_{FQ1} \times \sigma_{FQ2}}(u, v_2) \end{aligned}$$

for all $(uv_1, uv_2) \in G_{FQ1} \times G_{FQ2}$
Case II: For $v \in \sigma_{FQ2}, (u_1u_2) \in \mu_{FQ1}$

$$\begin{aligned} T_{\mu_{FQ1} \times \mu_{FQ2}}((u_1v)(u_2v)) &= T_{\mu_{FQ1}}(u_1u_2) \wedge T_{\sigma_{FQ2}}(v) \\ &\leq [T_{\sigma_{FQ1}}(u_1) \wedge T_{\sigma_{FQ1}}(u_2)] \wedge T_{\sigma_{FQ2}}(v) \\ &= [T_{\sigma_{FQ1}}(u_1) \wedge T_{\sigma_{FQ2}}(v)] \wedge [T_{\sigma_{FQ1}}(u_2) \wedge T_{\sigma_{FQ2}}(v)] \\ &= T_{\sigma_{FQ1} \times \sigma_{FQ2}}(u_1, v) \wedge T_{\sigma_{FQ1} \times \sigma_{FQ2}}(u_2, v) \end{aligned}$$

$$\begin{aligned} C_{\mu_{FQ1} \times \mu_{FQ2}}((u_1v)(u_2v)) &= C_{\mu_{FQ1}}(u_1u_2) \wedge C_{\sigma_{FQ2}}(v) \\ &\leq [C_{\sigma_{FQ1}}(u_1) \wedge C_{\sigma_{FQ1}}(u_2)] \wedge C_{\sigma_{FQ2}}(v) \\ &= [C_{\sigma_{FQ1}}(u_1) \wedge C_{\sigma_{FQ2}}(v)] \wedge [C_{\sigma_{FQ1}}(u_2) \wedge C_{\sigma_{FQ2}}(v)] \\ &= C_{\sigma_{FQ1} \times \sigma_{FQ2}}(u_1, v) \wedge C_{\sigma_{FQ1} \times \sigma_{FQ2}}(u_2, v) \end{aligned}$$

$$\begin{aligned} I_{\mu_{FQ1} \times \mu_{FQ2}}((u_1v)(u_2v)) &= I_{\mu_{FQ1}}(u_1u_2) \vee I_{\sigma_{FQ2}}(v) \\ &\geq [I_{\sigma_{FQ1}}(u_1) \vee I_{\sigma_{FQ1}}(u_2)] \vee I_{\sigma_{FQ2}}(v) \\ &= [I_{\sigma_{FQ1}}(u_1) \vee I_{\sigma_{FQ2}}(v)] \vee [I_{\sigma_{FQ1}}(u_2) \vee I_{\sigma_{FQ2}}(v)] \\ &= I_{\sigma_{FQ1} \times \sigma_{FQ2}}(u_1, v) \vee I_{\sigma_{FQ1} \times \sigma_{FQ2}}(u_2, v) \end{aligned}$$

$$\begin{aligned} F_{\mu_{FQ1} \times \mu_{FQ2}}((u_1v)(u_2v)) &= F_{\mu_{FQ1}}(u_1u_2) \vee F_{\sigma_{FQ2}}(v) \\ &\geq [F_{\sigma_{FQ1}}(u_1) \vee F_{\sigma_{FQ1}}(u_2)] \vee F_{\sigma_{FQ2}}(v) \\ &= [F_{\sigma_{FQ1}}(u_1) \vee F_{\sigma_{FQ2}}(v)] \vee [F_{\sigma_{FQ1}}(u_2) \vee F_{\sigma_{FQ2}}(v)] \\ &= F_{\sigma_{FQ1} \times \sigma_{FQ2}}(u_1, v) \vee F_{\sigma_{FQ1} \times \sigma_{FQ2}}(u_2, v) \end{aligned}$$

for all $(u_1v, u_2v) \in G_{FQ1} \times G_{FQ2}$
Hence the proof. ■

Definition 4.4. A Cross Product of two FQNFG is represented by $G_{FQ1} \star G_{FQ2} = (\sigma_{FQ1} \star \sigma_{FQ2}, \mu_{FQ1} \star \mu_{FQ2})$ and defined as

1. $T_{\sigma_{FQ1} \star \sigma_{FQ2}}(uv) = T_{\sigma_{FQ1}}(u) \wedge T_{\sigma_{FQ2}}(v)$
   $C_{\sigma_{FQ1} \star \sigma_{FQ2}}(uv) = C_{\sigma_{FQ1}}(u) \wedge C_{\sigma_{FQ2}}(v)$
   $I_{\sigma_{FQ1} \star \sigma_{FQ2}}(uv) = I_{\sigma_{FQ1}}(u) \vee I_{\sigma_{FQ2}}(v)$
   $F_{\sigma_{FQ1} \star \sigma_{FQ2}}(uv) = F_{\sigma_{FQ1}}(u) \vee F_{\sigma_{FQ2}}(v)$
   for all $(u, v) \in \sigma_{FQ1} \star \sigma_{FQ2}$

2. $T_{\mu_{FQ1} \star \mu_{FQ2}}(u_1v_1)(u_2v_2) = T_{\mu_{FQ1}}(u_1u_2) \wedge T_{\mu_{FQ2}}(v_1v_2)$
   $C_{\mu_{FQ1} \star \mu_{FQ2}}(u_1v_1)(u_2v_2) = C_{\mu_{FQ1}}(u_1u_2) \wedge C_{\mu_{FQ2}}(v_1v_2)$
   $I_{\mu_{FQ1} \star \mu_{FQ2}}(u_1v_1)(u_2v_2) = I_{\mu_{FQ1}}(u_1u_2) \vee I_{\mu_{FQ2}}(v_1v_2)$
   $F_{\mu_{FQ1} \star \mu_{FQ2}}(u_1v_1)(u_2v_2) = F_{\mu_{FQ1}}(u_1u_2) \vee F_{\mu_{FQ2}}(v_1v_2)$
   for all $(u_1u_2) \in \mu_{FQ1}, (v_1v_2) \in \mu_{FQ2}$

Theorem 4.4. A Cross Product of two FQNFG is a replica of the Fermatean Quadrapartitioned Neutrosophic Fuzzy Graph.

Proof. For all $(u_1v_1, u_2v_2) \in G_{FQ1} \star G_{FQ2}$

$$\begin{aligned}
T_{\mu_{FQ1}\star\mu_{FQ2}}(u_1v_1)(u_2v_2) &= T_{\mu_{FQ1}}(u_1u_2) \wedge T_{\mu_{FQ2}}(v_1v_2) \\
&\leq [T_{\sigma_{FQ1}}(u_1) \wedge T_{\sigma_{FQ1}}(u_2)] \wedge [T_{\sigma_{FQ2}}(v_1) \wedge T_{\sigma_{FQ2}}(v_2)] \\
&= [T_{\sigma_{FQ1}}(u_1) \wedge T_{\sigma_{FQ2}}(v_1)] \wedge [T_{\sigma_{FQ1}}(u_2) \wedge T_{\sigma_{FQ2}}(v_2)] \\
&= T_{\sigma_{FQ1}\star\sigma_{FQ2}}(u_1, v_1) \wedge T_{\sigma_{FQ1}\star\sigma_{FQ2}}(u_2, v_2)
\end{aligned}$$

$$\begin{aligned}
C_{\mu_{FQ1}\star\mu_{FQ2}}(u_1v_1)(u_2v_2) &= C_{\mu_{FQ1}}(u_1u_2) \wedge C_{\mu_{FQ2}}(v_1v_2) \\
&\leq [C_{\sigma_{FQ1}}(u_1) \wedge C_{\sigma_{FQ1}}(u_2)] \wedge [C_{\sigma_{FQ2}}(v_1) \wedge C_{\sigma_{FQ2}}(v_2)] \\
&= [C_{\sigma_{FQ1}}(u_1) \wedge C_{\sigma_{FQ2}}(v_1)] \wedge [C_{\sigma_{FQ1}}(u_2) \wedge C_{\sigma_{FQ2}}(v_2)] \\
&= C_{\sigma_{FQ1}\star\sigma_{FQ2}}(u_1, v_1) \wedge C_{\sigma_{FQ1}\star\sigma_{FQ2}}(u_2, v_2)
\end{aligned}$$

$$\begin{aligned}
I_{\mu_{FQ1}\star\mu_{FQ2}}(u_1v_1)(u_2v_2) &= I_{\mu_{FQ1}}(u_1u_2) \vee I_{\mu_{FQ2}}(v_1v_2) \\
&\geq [I_{\sigma_{FQ1}}(u_1) \vee I_{\sigma_{FQ1}}(u_2)] \vee [I_{\sigma_{FQ2}}(v_1) \vee I_{\sigma_{FQ2}}(v_2)] \\
&= [I_{\sigma_{FQ1}}(u_1) \vee I_{\sigma_{FQ2}}(v_1)] \vee [I_{\sigma_{FQ1}}(u_2) \vee I_{\sigma_{FQ2}}(v_2)] \\
&= I_{\sigma_{FQ1}\star\sigma_{FQ2}}(u_1, v_1) \vee I_{\sigma_{FQ1}\star\sigma_{FQ2}}(u_2, v_2)
\end{aligned}$$

$$\begin{aligned}
F_{\mu_{FQ1}\star\mu_{FQ2}}(u_1v_1)(u_2v_2) &= F_{\mu_{FQ1}}(u_1u_2) \vee F_{\mu_{FQ2}}(v_1v_2) \\
&\geq [F_{\sigma_{FQ1}}(u_1) \vee F_{\sigma_{FQ1}}(u_2)] \vee [F_{\sigma_{FQ2}}(v_1) \vee F_{\sigma_{FQ2}}(v_2)] \\
&= [F_{\sigma_{FQ1}}(u_1) \vee F_{\sigma_{FQ2}}(v_1)] \vee [F_{\sigma_{FQ1}}(u_2) \vee F_{\sigma_{FQ2}}(v_2)] \\
&= F_{\sigma_{FQ1}\star\sigma_{FQ2}}(u_1, v_1) \vee F_{\sigma_{FQ1}\star\sigma_{FQ2}}(u_2, v_2)
\end{aligned}$$

Hence the proof. ∎

5. Conclusion

The Quadripartitioned Neutrosophic Set is a generalization of Neutrosophic Set. A novel concept called the Fermatean Quadripartitioned Neutrosophic Fuzzy Graph (FQNFG) is defined. Its fundamental properties and products were discussed. The proposed concept is also applicable to bipolar Fermatean Quadrapartitioned Neutrosophic graph, Interval vaued Fermatean Quadrapartitioned Neutrosophic graph and so on. The FQNFG can be applied to a variety of applications, including expert systems, image processing, computer networks and social systems.

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