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EXPLORING SOME NEW CONTRIBUTION TO RAM AWADH DISTRIBUTION WITH COMPARATIVE PROPERTIES AND ITS APPLICATIONS

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Abstract

In this study, a novel extension of the Ram Awadh distribution referred to as length biased Ram Awadh distribution is proposed and systematically developed. This distribution is constructed by applying length-biased transformation to the baseline Ram Awadh distribution, making it more suitable for modeling data where longer durations or larger values are more likely to be observed. A comprehensive exploration of the distribution's statistical properties is presented, including the derivation of moments, harmonic mean, reliability function, failure rate function, reverse hazard rate function, order statistics, entropy as well as the bonferroni and Lorenz curves. Parameter estimation is conducted through the maximum likelihood estimation method, ensuring robustness and efficiency. The practical utility and goodness of fit of the proposed model are demonstrated through its application to three real-life datasets, highlighting its flexibility and improved performance in empirical context.

Keywords: Ram Awadh distribution, length biased distribution, order statistics, reliability analysis

I. Introduction

In probability and statistics the weighted distributions retains a significant popularity because the classical distributions may not provide a proper fit to different datasets occurred from diversified fields. There are various situations where classical distributions may not provide the best fit in such cases, we generalize the standard probability models by adding a shape or additional parameter to it and this shape parameter is added through one of reputed weighted technique. The weighted distributions provide a collective approach for model stipulation and data interpretation problems. The weighted distributions are useful in distribution theory because it provides a new

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method to extend the existing classical distribution for modeling lifetime data due to introduction of additional parameter in the model which creates flexibility. The weighted distributions are applicable only if the observations recorded without any experiment, repetition and random process. Fisher [8] introduced the weighted distribution to study the ascertainment bias which later on Rao [16] formulated in more general way with respect to modeling statistical data where usual practice of using standard distributions for purpose was found to be inappropriate. The weighted distribution reduced to length biased distribution particularly when weight function considers only length of units of interest. The concept of length biased distribution was introduced by Cox [5] in renewal theory and hence the length biased sampling situation occurs were a proper sampling frame is absent. In such situation items are sampled at a rate proportional to their lengths so that larger values could be sampled with higher probability. The length biased distribution has applications in biomedical areas such as survival analysis, family history, disease and intermediate events. The length biased distribution is a particular case of weighted distributions and may arise if there is not proper selection among the sample observations.

A magnificent contribution done by various authors to introduce several important length biased probability distributions along with their applications. Alzoubi [1] derived the length-biased Loai distribution with statistical properties and application. Abd-Elfattah et al. [3] presented the length biased Burr-XII distribution and obtain its properties and application. Chaito and Khamkong [6] proposed the length-biased Weibull-Rayleigh distribution with application to hydrological data. Das and Roy [7] discussed on some length biased weighted Weibull distribution. Ganaie and Rajagopalan [9] obtained the length biased power quasi Lindley distribution with properties and applications of lifetime data. Ganaie et al. [10] described the length biased Pratibha distribution and illustrate its applications. Ganaie and Rajagopalan [11] studied the length biased weighted new quasi Lindley distribution with statistical properties and applications. Kersey and Oluyede [13] constructed the theoretical properties of the length biased inverse Weibull distribution. Mustafa and Khan [14] studied the length biased powered inverse Rayleigh distribution with applications. Oluwafemi and Olalekan [15] presented length and area biased exponentiated weibull distribution based on forest inventories. Reyad et al. [17] derived the length biased weighted frechet distribution and studied its properties and estimation. Sharma et al. [18] discussed on the length and area biased Maxwell distributions.

Ram Awadh distribution is a recently proposed new one parametric distribution introduced by Shukla [19] and its few statistical properties have been discussed. Additionally its parameters are estimated by using method of moments and method of maximum likelihood estimation.

II. Length Biased Ram Awadh (LBRA) Distribution

The probability density function of Ram Awadh distribution is given by

$$f(x; \lambda) = \frac{\lambda^6}{\left(\lambda^6 + 120\right)}\left(\lambda + x^5\right) e^{-\lambda x} \; ; x > 0, \lambda > 0 \tag{1}$$

and cumulative distribution function of Ram Awadh distribution is given by

$$F(x; \lambda) = 1 - \left(1 + \frac{\lambda x\left(\lambda^4 x^4 + 5\lambda^3 x^3 + 20\lambda^2 x^2 + 60\lambda x + 120\right)}{\left(\lambda^6 + 120\right)}\right) e^{-\lambda x} \; ; x > 0, \lambda > 0 \tag{2}$$

Consider random variable X constitutes non-negative condition with probability density function $f(x)$. Suppose its weight function be $w(x)$ that is non-negative, then probability density of weighted

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random variable X_w is given by

$$f_w(x) = \frac{w(x)f(x)}{E(w(x))}, \quad x > 0$$

Where $w(x)$ is weight function of $\mathrm{non-negative}$ condition, hence $E(w(x)) = \int w(x)f(x)dx < \infty.$

In this paper, we have to get the length biased version of Ram Awadh distribution. Consequently the weight function as $w(x) = x$, the resulting distribution is termed as length biased distribution and therefore its probability density function is given by

$$f_l(x; \lambda) = \frac{x\,f(x; \lambda)}{E(x)} \tag{3}$$

Here $E(x) = \int\limits_0^\infty x\,f(x; \lambda)\,dx$

$$E(x) = \frac{\left(\lambda^6 + 720\right)}{\lambda\left(\lambda^6 + 120\right)} \tag{4}$$

After substituting the equation (1) and (4) in equation (3), we have obtained the probability density function of length biased Ram Awadh distribution as

$$f_l(x; \lambda) = \frac{\lambda^7 x}{\left(\lambda^6 + 720\right)}\left(\lambda + x^5\right)e^{-\lambda x} \tag{5}$$

and cumulative distribution function of length biased Ram Awadh distribution will be obtained as

$$F_l(x; \lambda) = \int\limits_0^x f_l(x; \lambda)dx$$

$$= \int\limits_0^x \frac{\lambda^7 x}{\left(\lambda^6 + 720\right)}\left(\lambda + x^5\right)e^{-\lambda x}dx \tag{6}$$

After the simplification of equation (6), we have obtained the cumulative distribution function of length biased Ram Awadh distribution

$$F_l(x; \lambda) = \frac{1}{\left(\lambda^6 + 720\right)}\left(\lambda^6\gamma(2, \lambda x) + \gamma(7, \lambda x)\right) \tag{7}$$
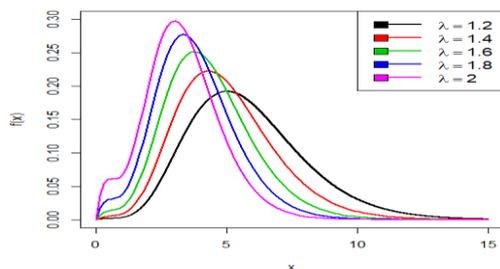

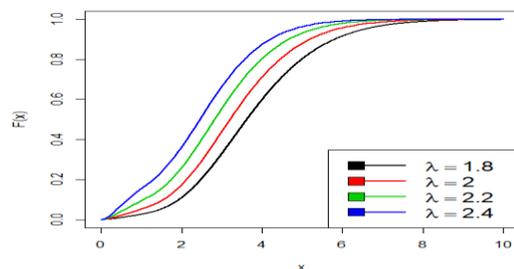**Figure 1:** *Pdf plot of LBRA distribution*

Figure 2: *Cdf plot of LBRA distribution*

III. Reliability analysis

In this portion, we have derived and presented some particular properties of length biased Ram Awadh distribution. The reliability function of length biased Ram Awadh distribution is given by

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$$R(x) = 1 - F_l(x; \lambda)$$

$$R(x) = 1 - \frac{1}{\left(\lambda^6 + 720\right)}\left(\lambda^6 \gamma(2, \lambda x) + \gamma(7, \lambda x)\right)$$

The hazard function of length biased Ram Awadh distribution is given by

$$h(x) = \frac{f_l(x; \lambda)}{1 - F_l(x; \lambda)}$$

$$h(x) = \frac{\lambda^7 x\left(\lambda + x^5\right)e^{-\lambda x}}{(\lambda^6 + 720) - (\lambda^6 \gamma(2, \lambda x) + \gamma(7, \lambda x))}$$

The reverse hazard rate function of length biased Ram Awadh distribution is given by

$$h_r(x) = \frac{f_l(x; \lambda)}{F_l(x; \lambda)}$$

$$h_r(x) = \frac{\lambda x\left(\lambda + x^5\right)e^{-\lambda x}}{\left(\gamma(2, \lambda x) + \gamma(7, \lambda x)\right)}$$

The Mills Ratio of length biased Ram Awadh distribution is given by

$$M.R = \frac{1}{h_r(x)} = \frac{\left(\gamma(2, \lambda x) + \gamma(7, \lambda x)\right)}{\lambda x\left(\lambda + x^5\right)e^{-\lambda x}}$$
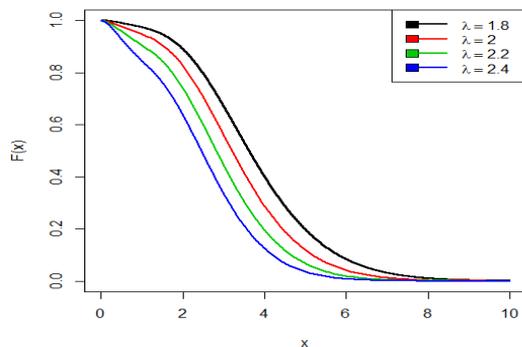

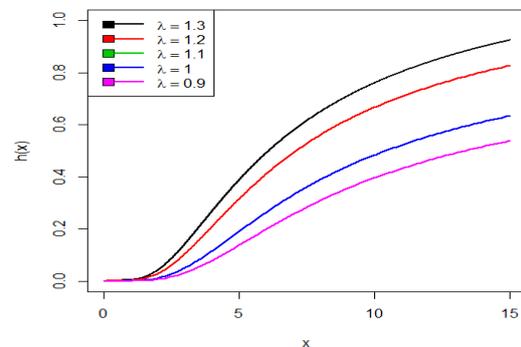**Figure 3:***Reliability plot of LBRA distribution*

Figure 4:*Hazard plot of LBRA distribution*

IV. Order Statistics

Order statistics is a fundamental component in statistics which has wide range of applications in reliability and life testing. Consider order statistics $X_{(1)}$, $X_{(2)}$, …, $X_{(n)}$ of a random sample $X_{(1)}$, $X_{(2)}$, …, $X_{(n)}$ from a continuous distribution has its probability density function $f_x(x)$ and its cumulative distribution function $F_x(x)$, then probability density function of rth order statistics $X_{(r)}$ is given by

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_X(x)\left(F_X(x)\right)^{r-1}\left(1 - F_X(x)\right)^{n-r} \tag{8}$$

After applying the equation (5) and (7) in equation (8), we will get the probability density function of rth order statistics $X_{(r)}$ of length biased Ram Awadh distribution as

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$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} \left(\frac{\lambda^7 x}{\left(\lambda^6 + 720\right)} \left(\lambda + x^5\right) e^{-\lambda x} \right) \left(\frac{1}{\left(\lambda^6 + 720\right)} \left(\lambda^6 \gamma(2, \lambda x) + \gamma(7, \lambda x)\right) \right)^{r-1}$$

$$\times \left(1 - \frac{1}{\left(\lambda^6 + 720\right)} \left(\lambda^6 \gamma(2, \lambda x) + \gamma(7, \lambda x)\right) \right)^{n-r}$$

Therefore probability density function of higher order statistics $X_{(n)}$ of length biased Ram Awadh distribution is given by

$$f_{x(n)}(x) = \left(\frac{nx \lambda^7}{\left(\lambda^6 + 720\right)} \left(\lambda + x^5\right) e^{-\lambda x} \right) \left(\frac{1}{\left(\lambda^6 + 720\right)} \left(\lambda^6 \gamma(2, \lambda x) + \gamma(7, \lambda x)\right) \right)^{n-1}$$

and therefore probability density function of first order statistics $X_{(1)}$ of length biased Ram Awadh distribution is given by

$$f_{x(1)}(x) = \left(\frac{nx \lambda^7}{\left(\lambda^6 + 720\right)} \left(\lambda + x^5\right) e^{-\lambda x} \right) \left(1 - \frac{1}{\left(\lambda^6 + 720\right)} \left(\lambda^6 \gamma(2, \lambda x) + \gamma(7, \lambda x)\right) \right)^{n-1}$$

V. Statistical Properties

In this portion, we have derived some specified features of length biased Ram Awadh distribution.

I. Moments

Suppose the random variable X represents the length biased Ram Awadh distribution, then the rth order moment of proposed distribution will be obtained as

$$\mu_r' = E(X^r) = \int_0^\infty x^r f_l(x; \lambda) \, dx$$

$$= \int_0^\infty \frac{\lambda^7 x^{r+1}}{\left(\lambda^6 + 720\right)} \left(\lambda + x^5\right) e^{-\lambda x} dx \tag{9}$$

After the simplification of equation (9), we get

$$\mu_r' = E(X^r) = \frac{\left(\lambda^6 \Gamma(r+2) + \Gamma(r+7)\right)}{\lambda^r \left(\lambda^6 + 720\right)} \tag{10}$$

Therefore substituting $r = 1, 2, 3$ and 4 in equation (10), we will get the first four moments of length biased Ram Awadh distribution

$$\mu_1' = \frac{\left(2\lambda^6 + 5040\right)}{\lambda \left(\lambda^6 + 720\right)}$$

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$$\mu_2' = \frac{\left(6\lambda^6 + 40320\right)}{\lambda^2\left(\lambda^6 + 720\right)}$$

$$\mu_3' = \frac{\left(24\lambda^6 + 362880\right)}{\lambda^3\left(\lambda^6 + 720\right)}$$

$$\mu_4' = \frac{\left(120\lambda^6 + 3628800\right)}{\lambda^4\left(\lambda^6 + 720\right)}$$

$$\text{Variance} = \frac{\left(2\lambda^{12} + 24480\lambda^6 + 3628800\right)}{\lambda^2\left(\lambda^6 + 720\right)^2}$$

$$\text{S.D} = \frac{\sqrt{\left(2\lambda^{12} + 24480\lambda^6 + 3628800\right)}}{\lambda\left(\lambda^6 + 720\right)}$$

II. Harmonic mean

The harmonic mean of length biased Ram Awadh distribution should be determined as

$$\text{H.M} = E\left(\frac{1}{x}\right) = \int_0^\infty \frac{1}{x} f_l(x; \lambda)dx$$

$$= \int_0^\infty \frac{\lambda^7}{\left(\lambda^6 + 720\right)}\left(\lambda + x^5\right)e^{-\lambda x}dx \qquad (11)$$

After simplification of equation (11), we get

$$\text{H.M} = \frac{\lambda\left(\lambda^6 + 120\right)}{\left(\lambda^6 + 720\right)}$$

III. Moment generating function and characteristic function

The moment generating function of length biased Ram Awadh distribution will be obtained as

$$M_X(t) = E\left(e^{tx}\right) = \int_0^\infty e^{tx} f_l(x; \lambda)dx$$

$$= \int_0^\infty \frac{\lambda^7 x}{\left(\lambda^6 + 720\right)}\left(\lambda + x^5\right)e^{-(\lambda - t)x}dx \qquad (12)$$

After the simplification of equation (12), we get

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$$M_X(t) = \frac{\lambda^7}{(\lambda - t)^7 \left(\lambda^6 + 720\right)} \left(\lambda(\lambda - t)^5 + 720\right)$$

Obviously, the characteristic function of length biased Ram Awadh distribution is given by

$$M_X(it) = \frac{\lambda^7}{(\lambda - it)^7 \left(\lambda^6 + 720\right)} \left(\lambda(\lambda - it)^5 + 720\right)$$

VI. Bonferroni and Lorenz Curves

The bonferroni and Lorenz curves were applied to illustrate the graphical description of inequality in income or poverty. The bonferroni and Lorenz curves should be defined as

$$B(p) = \frac{1}{p\mu_1'} \int_0^q x\, f_l(x; \lambda)\, dx$$

$$L(p) = pB(p) = \frac{1}{\mu_1'} \int_0^q x\, f_l(x; \lambda)\, dx$$

$$\text{Where } \mu_1' = \frac{\left(2\lambda^6 + 5040\right)}{\lambda\left(\lambda^6 + 720\right)}$$

$$B(p) = \frac{\lambda\left(\lambda^6 + 720\right)}{p\left(2\lambda^6 + 5040\right)} \int_0^q \frac{\lambda^7 x^2}{(\lambda^6 + 720)} \left(\lambda + x^5\right) e^{-\lambda x}\, dx \tag{13}$$

After the simplification of equation (13), we get

$$B(p) = \frac{1}{p\left(2\lambda^6 + 5040\right)} \left(\lambda^6 \gamma(3,\ \lambda q) + \gamma(8,\ \lambda q)\right)$$

$$L(p) = \frac{1}{\left(2\lambda^6 + 5040\right)} \left(\lambda^6 \gamma(3,\ \lambda q) + \gamma(8,\ \lambda q)\right)$$

VII. Parameter Estimation and Fisher's Information Matrix

In this portion, we have applied the technique of parameter estimation to estimate the parameter of length biased Ram Awadh distribution. Consider the random sample $X_1,\ X_2,\ …,\ X_n$ of size n from the length biased Ram Awadh distribution, then the likelihood function should be defined as

$$L(x) = \prod_{i=1}^n f_l(x; \lambda)$$

$$= \prod_{i=1}^n \left(\frac{\lambda^7 x_i}{\left(\lambda^6 + 720\right)} \left(\lambda + x_i^5\right) e^{-\lambda x_i}\right)$$

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$$= \frac{\lambda^{7n}}{\left(\lambda^6 + 720\right)^n} \prod_{i=1}^{n}\left(x_i \left(\lambda + x_i^5 \right) e^{-\lambda x_i} \right)$$

The log likelihood function should be defined as

$$logL = 7nlog\,\lambda - nlog\left(\lambda^6 + 720\right) + \sum_{i=1}^{n} log\,x_i + \sum_{i=1}^{n} log\left(\lambda + x_i^5\right) - \lambda \sum_{i=1}^{n} x_i \qquad (14)$$

Therefore differentaiting the log likelihood equation (14) with respect to λ. The following normal equation must be satisfied

$$\frac{\partial logL}{\partial \lambda} = -\frac{7n}{\lambda} - n\left(\frac{6\lambda^5}{\left(\lambda^6 + 720\right)} \right) + \sum_{i=1}^{n}\left(\frac{1}{\left(\lambda + x_i^5\right)} \right) - \sum_{i=1}^{n} x_i = 0$$

The above likelihood equation is too complicated to solve it algebraically. Therefore we use R and wolfram mathematics for estimating the parameter of proposed distribution.

To apply the asymptotic normality results for attaining confidence interval. We state that $(\hat{\beta} = \hat{\lambda})$ which represents the maximum likelihood estimate of $(\beta = \lambda)$. The result can be analyzed as

$$\sqrt{n}(\hat{\beta} - \beta) \to N\left(0, I^{-1}(\beta)\right)$$

Here $I^{-1}(\beta)$ is Fisher's Information Matrix

$$I(\beta) = -\frac{1}{n}\left(E\left(\frac{\partial^2 logL}{\partial \lambda^2} \right) \right)$$

$$E\left(\frac{\partial^2 logL}{\partial \lambda^2} \right) = -\frac{7n}{\lambda^2} - n\left(\frac{\left(\lambda^6 + 720\right)\left(30\lambda^4\right) - \left(6\lambda^5\right)^2}{\left(\lambda^6 + 720\right)^2} \right) - \sum_{i=1}^{n}\left(\frac{1}{\left(\lambda + x_i^5\right)^2} \right)$$

Therefore β is not known hence $I^{-1}(\beta)$ should be estimated by $I^{-1}(\hat{\beta})$ and it is applied to attain asymptotic confidence interval for λ.

VIII. Entropy

The term entropy is a scientific concept which is commonly associated with uncertainty or disorder of a system and is also applied in various fields.

I. Renyi Entropy

The Renyi entropy is named after Alfred Renyi is a quantity which generalizes several forms of entropy. The Renyi entropy should be derived as

$$R_T(\alpha) = \frac{1}{1-\alpha} log\left(\int f_1^{\alpha}(x; \lambda)\,dx \right)$$

$$= \frac{1}{1-\alpha} log\left(\left(\frac{\lambda^7}{\left(\lambda^6 + 720\right)} \right)^{\alpha} \int_{0}^{\infty} x^{\alpha} e^{-\alpha\lambda x}\left(\lambda + x^5 \right)^{\alpha} dx \right) \qquad (15)$$

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After the simplification of equation (15), we get

$$R_T(\alpha) = \frac{1}{1-\alpha} \log\left(\left(\left(\frac{\lambda^{6\alpha-5k-1}}{\left(\lambda^6+720\right)^\alpha}\right) \sum_{k=0}^{\infty} \binom{\alpha}{k} \lambda^{\alpha-k} \frac{\Gamma(\alpha+5k+1)}{\alpha^{\alpha+5k+1}}\right)\right)$$

Clearly, the Tsallis entropy of length biased Ram Awadh distribution will be obtained as

$$T_s(\xi) = \frac{1}{\xi-1}\left(1 - \int_0^\infty f_l^{\xi}(x;\lambda)\,dx\right)$$

$$= \frac{1}{\xi-1}\left(1 - \left(\frac{\lambda^7}{\left(\lambda^6+720\right)}\right)^\xi \int_0^\infty x^\xi e^{-\lambda\xi x}\left(\lambda+x^5\right)^\xi dx\right) \quad (16)$$

After the simplification of equation (16), we get

$$T_s(\xi) = \frac{1}{\xi-1}\left(1 - \left(\frac{\lambda^{6\xi-5j-1}}{\left(\lambda^6+720\right)^\xi}\right) \sum_{j=0}^{\infty} \binom{\xi}{j} \lambda^{\xi-j} \frac{\Gamma(\xi+5j+1)}{\xi^{\xi+5j+1}}\right)$$

IX. Likelihood Ratio Test

Consider random sample X_1, X_2, …, X_n of size n from length biased Ram Awadh distribution, then the proposed test is applied to analyze its flexibility

$$H_0 : f(x) = f(x;\lambda) \quad \text{against} \quad H_1 : f(x) = f_l(x;\lambda)$$

In order to identify whether the random sample of size n comes from length biased Ram Awadh distribution or Ram Awadh distribution, the proposed rule is applied

$$\Delta = \frac{L_1}{L_0} = \prod_{i=1}^{n}\left(\frac{f_l(x;\lambda)}{f(x;\lambda)}\right)$$

$$= \prod_{i=1}^{n}\left(\frac{\lambda x_i\left(\lambda^6+120\right)}{\left(\lambda^6+720\right)}\right)$$

$$= \left(\frac{\lambda\left(\lambda^6+120\right)}{\left(\lambda^6+720\right)}\right)^n \prod_{i=1}^{n} x_i$$

The null hypothesis should be refused to accept, if

$$\Delta = \left(\frac{\lambda\left(\lambda^6+120\right)}{\left(\lambda^6+720\right)}\right)^n \prod_{i=1}^{n} x_i > k$$

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$$\Delta^* = \prod_{i=1}^{n} x_i > k \left(\frac{\left(\lambda^6 + 720 \right)}{\lambda \left(\lambda^6 + 120 \right)} \right)^n$$

$$\Delta^* = \prod_{i=1}^{n} x_i > k^*, \quad \text{where} \quad k^* = k \left(\frac{\left(\lambda^6 + 720 \right)}{\lambda \left(\lambda^6 + 120 \right)} \right)^n$$

X. Application

In this portion, we have analyzed and examined the goodness of fit of length biased Ram Awadh distribution by applying three real data sets and then performance has been established in order to reveal that the length biased Ram Awadh distribution shows a better fit in comparison over Ram Awadh, Lindley and Pranav distributions.

Data set 1: The following real lifetime data set given below constitutes the relief times (in minutes) of 20 patients receiving an analgesic reported by Gross and Clarke [12] and the observations are
1.1, 1.4, 1.3, 1.7, 1.9, 1.8, 1.6, 2.2, 1.7, 2.7, 4.1, 1.8, 1.5, 1.2, 1.4, 3.0, 1.7, 2.3, 1.6, 2.0

Data set 2: The second real data set reported by Bader and Priest [4] represents the tensile strength measured in GPa of 69 carbon fibers tested under the tension at gauge lengths of 20mm and the values are given as
1.312, 2.726, 2.382, 1.958, 2.954, 2.535, 2.140, 3.585, 2.684, 2.301, 1.865, 2.848, 2.511, 2.063, 3.233, 2.642, 2.274, 1.803, 2.818, 2.478, 2.027, 3.096, 2.629, 2.270, 1.552, 2.800, 2.434, 2.006, 3.084, 2.570, 2.240, 1.314, 2.770, 2.382, 1.966, 3.012, 2.554, 2.179, 3.858, 2.697, 2.359, 1.944, 2.880, 2.514, 2.098, 3.433, 2.648, 2.301, 1.861, 2.821, 2.490, 2.055, 3.128, 2.633, 2.272, 1.700, 2.809, 2.435, 2.021, 3.090, 2.586, 2.253, 1.479, 2.773, 2.426, 1.997, 3.067, 2.566, 2.224

Data set 3: The third real lifetime data set consists of 40 patients suffering from blood cancer (leukemia) reported from one of ministry of health hospitals in Saudi Arabia see Abouammah et al. [2]. The observations of ordered lifetimes (in years) is given as
0.315, 0.496, 0.616, 1.145, 1.208, 1.263, 1.414, 2.025, 2.036, 2.162, 2.211, 2.37, 2.532, 2.693, 2.805, 2.91, 2.912, 3.192, 3.263, 3.348, 3.348, 3.427, 3.499, 3.534, 3.767, 3.751, 3.858, 3.986, 4.049, 4.244, 4.323, 4.381, 4.392, 4.397, 4.647, 4.753, 4.929, 4.973, 5.074, 5.381

To compute the model comparison criterions along with the unknown parameters are estimated thoroughly by applying the technique of R Software. To regulate the performance of length biased Ram Awadh distribution in comparison over Ram Awadh, Lindley and Pranav distributions, the considered criterions such as Akaike Information Criterion *(AIC)*, Bayesian Information Criterion *(BIC)*, Akaike Information Criterion Corrected *(AICC)*, Consistent Akaike Information Criterion *(CAIC)*, Shannon's entropy *H(X)* and *-2logL* have been applied. The distribution performs obviously better if it has the smaller criterion values of *AIC, BIC, AICC, CAIC, -2logL* and *H(X)* as compared over other specified distributions. The following formulas were applied for determining the criterion values

$$AIC = 2k - 2\log L, \qquad BIC = k \log n - 2 \log L, \qquad AICC = AIC + \frac{2k(k+1)}{(n-k-1)}$$

$$CAIC = -2 \log L + \frac{2kn}{n-k-1} \qquad\qquad H(X) = -\frac{2 \log L}{n}$$

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Here *k* is the number of parameters in the statistical model, *n* is the sample size and *-2logL* is the maximized value of log- likelihood function.

Table 1: *shows MLE and S. E of Data set 1, Data set 2 and Data set 3 of performed distributions*

Data set 1			Data set 2		
Distribution	MLE	S.E	Distribution	MLE	S.E
LBRA	$\hat{\lambda} = 2.69338$	$\hat{\lambda} = 0.166145$	LBRA	$\hat{\lambda} = 2.3812762$	$\hat{\lambda} = 0.0841710$
Ram Awadh	$\hat{\lambda} = 2.045874$	$\hat{\lambda} = 0.130715$	Ram Awadh	$\hat{\lambda} = 1.8504152$	$\hat{\lambda} = 0.0663762$
Lindley	$\hat{\lambda} = 0.816118$	$\hat{\lambda} = 0.136092$	Lindley	$\hat{\lambda} = 0.6545057$	$\hat{\lambda} = 0.0580316$
Pranav	$\hat{\lambda} = 1.401400$	$\hat{\lambda} = 0.124708$	Pranav	$\hat{\lambda} = 1.2251389$	$\hat{\lambda} = 0.0597324$

Data set 3		
Distribution	MLE	S.E
LBRA	$\hat{\lambda} = 2.076465$	$\hat{\lambda} = 0.105233$
Ram Awadh	$\hat{\lambda} = 1.6618903$	$\hat{\lambda} = 0.084099$
Lindley	$\hat{\lambda} = 0.5269213$	$\hat{\lambda} = 0.06074$
Pranav	$\hat{\lambda} = 1.08000$	$\hat{\lambda} = 0.071233$

Table 2: *Shows analysis, comparison and performance of fitted distributions*

Data sets	Distribution	-2logL	AIC	BIC	AICC	CAIC	H(X)
1	LBRA	56.17852	58.17852	59.17425	58.40074	58.4007	2.8089
	Ram Awadh	68.52522	70.52522	71.52095	70.74744	70.7474	3.4262
	Lindley	60.4991	62.4991	63.49483	62.6991	62.7213	3.0249
	Pranav	62.38652	64.38652	65.38225	64.60874	64.6087	3.1193
2	LBRA	174.4082	176.4082	178.6423	176.4679	176.4679	2.5276
	Ram Awadh	207.1334	209.1334	211.3675	209.1931	209.1931	3.0019
	Lindley	238.3805	240.3805	242.6146	240.4402	240.4402	3.4547
	Pranav	217.1224	219.1224	221.3565	219.1821	219.1821	3.1467
3	LBRA	137.724	139.724	141.4129	139.8292	139.8292	3.4431
	Ram Awadh	141.6505	143.6505	145.3394	143.7557	143.7557	3.5412
	Lindley	160.5012	162.5012	164.19	162.6064	162.6064	4.0125
	Pranav	146.9727	148.9727	150.6616	149.0779	149.0779	3.6743

It is clearly realized from results given above in table 2 that length biased Ram Awadh distribution has the smaller criterion values of *AIC, BIC, AICC, CAIC, H(X)* and *-2logL* as compared over Ram Awadh, Lindley and Pranav distributions which illustrates that the length biased Ram Awadh distribution provides a better fit as compared over Ram Awadh, Lindley and Pranav distributions.

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XI. Conclusion

In this manuscript, we have established a novel extension of Ram Awadh distribution referred to as length biased Ram Awadh distribution which is introduced by incorporating the length biased technique to its classical distribution. The formulated new distribution has been accomplished and examined with several statistical properties those are moments, mean and variance, harmonic mean, shape of pdf and cdf, coefficient of variation, Renyi entropy measure, order statistics, bonferroni and Lorenz curves have been obtained and presented. Additionally reliability function, hazard function, reverse hazard function, moment generating function and characteristic function have been derived and described. In addition parameter of length biased Ram Awadh distribution is estimated by applying the technique of maximum likelihood estimation. Further, the capability and flexibility of presented length biased Ram Awadh distribution has been investigated and inspected by applying the three real data sets and hence therefore it is commenced from the result that length biased Ram Awadh distribution leads to a better fit as compared over Ram Awadh, Lindley and Pranav distributions.

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