

# CONSTRUCTION OF COMPLETE STOCHASTIC LIFE CYCLES OF HIGHLY CRITICAL INFRASTRUCTURES

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## Abstract

*The article describes a risk-based approach to constructing the full life cycle (LC) of HCI with accounting for climate change. In this case, the objective function (OF) of HCI risk management as a function of time is reduced to the generalized cost of operating a strategically important object in the time interval "from (birth) cradle / to grave".*

*The life cycle, built on the basis of these data, makes it possible to evaluate and predict: (1) the inherent design reliability of the HCI, (2) the probabilities of all types of failures of the HCI; (3) the monetary consequences of these failures; (4) risks associated with random times of diagnostics, repairs and restoration of HCI and with the moments of its failures; (5) the overall risk of HCI at each moment of its existence with accounting for climate change.*

**Keywords:** life cycle, risk, critical infrastructure, offshore pipelines

## I. Introduction

The modern concept of creating critical and strategic infrastructures is that their design should: 1) cover the entire life cycle (LC); and 2) be based on modern concepts of managing complex systems according to safety and risk criteria in conditions of dynamic multifactorial uncertainty [1].

In traditional infrastructure design, life cycle analysis (LCA) is used primarily to obtain design parameters and define maintenance strategies for the facility. In the design of critical infrastructures, LCA is naturally needed to provide a qualified assessment of the environmental and/or economic impact of the product or service produced by these infrastructures throughout their entire service life.

The list of components of the life cycle of large infrastructure projects usually looks like this: 1) extraction of raw materials necessary for the creation of the infrastructure; 2) processing of materials, manufacturing and construction (construction) of the facility; 3) use and operation of the system; and 4) disposal or restoration of the infrastructure after its useful life.

Modern life cycle analysis of infrastructures imperatively requires an expansion of the number of central/key design components. Knowledge of the system structure and its physical and mechanical behavior, determined by the strength and reliability, resilience and safety of its elements (fundamental characteristics of the system) is fundamentally insufficient. No less important components of LCA are: 1) financial factors (the cost of future investments in diagnostics, monitoring, maintenance, repair and renovation of infrastructure), discount rates); 2)

responsibility between generations; 3) environmental aspects and sustainability of development.

The emerging global consensus regarding the value of the life cycle concept in the design and operation of infrastructures is based on the fact that: 1) leading enterprises of all types have recognized de facto that LCA is the key to continuous improvement of production efficiency; 2) continuous and long-term environmental protection has become a key criterion in consumer markets and in government procurement guidelines; 3) government regulations around the world are moving towards LCA accountability [2].

LCA has thus become a valuable decision support tool for all categories of decision makers (technocrats, legislators, lawyers and politicians) in assessing the impact of a product or process on the service life of an infrastructure. It also plays an important role in defining environmental policies and strategies that contribute to the sustainable development of an enterprise or region.

Currently, LCA is widely used to assess the impact of large projects on the environment (global climate change/carbon footprint, depletion of the ozone layer and natural resources, acidification, eutrophication (excessive saturation of water bodies with harmful biogenic elements), human health, ecotoxicity, etc.).

Additional requirements for the design of complex infrastructures are: 1) creation of digital quasi-twins of the designed infrastructures; 2) modeling of a quasi-complete group of scenarios for their operation and accidents under normal and extreme conditions using an interdisciplinary approach.

## II. Formulation of life cycle cost

In general, the objective function, taking into account the benefits and costs of the life cycle associated with an object (system), is determined by the expression [3]:

$$U(z) = B(z) - C(z), \quad (1)$$

where  $B(z)$  is the benefit from operating the asset;  $C(z)$  is the life cycle cost;  $z$  is a vector of parameters that describes the decision variables, which include aspects such as design criteria (e.g. geometry, material properties, external requirements, etc.), estimated service life, diagnostic and maintenance strategies, etc., depending on the problem under consideration.

Typical life cycle costs include the initial costs associated with planning, designing and constructing the facility; diagnostic and maintenance costs; operating costs to keep the facility functioning; and finally, the probable costs associated with failure, as well as the dismantling costs that arise at the end of the structure's useful life.

Since benefits and costs are uncertain, decisions are made based on expected utility in accordance with decision theory [3]. The objective function in terms of expected utility is expressed as

$$E[U(z)] = E[B(z)] - E[C(z)]. \quad (2)$$

The life cycle cost of an object, taking into account the above, should take into account all costs incurred during its service life. Thus, the life cycle cost can be written as follows:

$$E[C(z)] = -E[C_0(z)] - E[C_I(z)] - E[C_R(z)] - E[C_F(z)] - E[C_{OP}(z)] - E[C_D(z)], \quad (3)$$

where  $C(z)$  is the total life cycle cost;  $C_0(z)$  is the costs associated with planning, design and construction of the facility;  $C_I(z)$  is the total cost of all inspections (diagnostics) throughout the life cycle;  $C_R(z)$  is the total cost of all repairs/restorations during maintenance throughout the life cycle;  $C_F(z)$  is the total costs associated with the asset failure;  $C_{OP}(z)$  is the total operating costs (insurance coverage, payment of interest on the loan, tax on the carbon footprint of the system operation, etc.);  $C_D(z)$  is the cost of dismantling, describes the cost of decommissioning (if any) at the end of the life cycle  $T_s$ .

The total costs of failure can be divided into direct and indirect [4]. The direct cost of failure takes into account the costs directly associated with the failure of the system:

- costs incurred during repairs (restoration) as a result of failure;
- economic costs associated with the loss of system functionality (the cost of product lost due to leakage, the cost of non-produced products (production), fines associated with non-delivery to the consumer, etc.);
- cost of environmental damage;
- the cost of consequences for life safety (impacts on humans) – the cost of restoring health and compensation for the possible loss of human lives.

The indirect cost of failure includes costs associated with damage to the company's reputation, loss of public trust, etc.

The costs of diagnostics and maintenance, as well as the costs in case of system failure, significantly increase the life cycle cost. Therefore, reasonable planning is required to reduce the overall cost while maintaining the safety and operability of the system. Therefore, the optimal diagnostics and maintenance plan is selected in such a way as to minimize the overall life cycle costs in the decision-making process.

The integrity of the system during its service life can be ensured in different ways. On the one hand, it is possible to create, due to the increased material intensity of high-quality material, a very reliable, robust, but also expensive system that will require minimal maintenance. On the other hand, it is possible to create a much cheaper system with minimal reserves of strength and robustness, but compensate for this disadvantage with adequate maintenance (diagnostics, monitoring, repair and restoration), and a subsystem for its protection, which will require significant costs for the care of the system throughout the entire life cycle.

Let us consider the construction of a complete stochastic life cycle using the example of strategically important critical infrastructures – offshore underwater pipelines (OUP).

To quantitatively assess the risk of operating an OUP as a function of time, it is necessary to:

- Design the duration of its life cycle (LC);
- Determine its initial (normative) reliability;
- Have robust quantitative models of its degradation;
- Select formalized strategies, technologies and methods of:
  - instrumental diagnostics [3], [4];
  - monitoring; and
  - maintenance (technical service, repair and restoration of mechanical engineering equipment);
- Determine the method of assessing and discounting all costs and damages distributed randomly over the life cycle interval of the object.

The presence of these strategies allows constructing a quasi-complete group of scenarios for the operation of the OUP using the Monte Carlo method, simulating the real conditions of its loading, degradation and maintenance in working condition, and, as a result, to obtain consistent estimates of the probability of failure (for each type of impact on the pipeline) before and after each intervention. It follows from the above that the results of modeling will directly depend on the accuracy of the used models of degradation of the material and design of the OUP, the loads on them, and, which is imperatively important, the accuracy characteristics of the diagnostic technologies used.

For a new OUP being put into operation, it is necessary to simulate the initiation of a certain representative set of defects characteristic of similar pipelines that are already in operation. Since the frequency of in-line diagnostics of domestic pipelines is regulated, it is necessary to predict the sizes of all initiated defects by the time of the first inspection, taking into account the latent period when the defects are still invisible. The creation of such a set of defects on a digital twin of the OUP is possible only by the Monte Carlo method. In this case, it is necessary to take into account the

loads and impacts that a real pipeline actually experiences in the considered section of the oceanic or sea shelf of Russia.

Given the data described above, the general algorithm for constructing the risk function of a designed or commissioned OUP over a period of time equal to the length of its life cycle consists of several stages described below.

To simulate the operation of a new OUP, models of degradation of protective layers, material and design of the pipe itself are used, adequately describing the growth of all types of defects. Here in after non-trivial questions arise: *How many defects, of what type, number and size should be simulated for the entire pipeline and each of its sections? And where should they be located topographically?*

Since for calculation purposes the division of the OUP into sections is performed according to the criterion of homogeneity of the base, loads/impacts, and pipe material, for practical purposes it is necessary to model the degradation of each pipeline section separately. It is also necessary to model all types of defects that are observed in other operating OUPs similar to the one under consideration. The number of modeled defects in each section should be comparable with the actually observed number of defects in similar sections of the OUP, operated under similar conditions. This will allow obtaining the same power and consistency of statistics related to the parameters of the modeled defects.

It is advisable to model each representative ensemble of similar defects as a Poisson field of points, each of which is a three-dimensional defect (depth  $d$ , length  $l$ , width  $b$ ). The size of the set of defects should allow obtaining consistent statistics of the distribution of their sizes. The modeled topography of the field of actual defects can be adjusted, if necessary, taking into account local corrosion features (for example, when the lower part of the pipe shell is more susceptible to corrosion than the upper one).

### III. Modeling the results of in-pipe diagnostics

Non-destructive methods used for OUP diagnostics are subject to various errors, since they are based on an indirect form of measurement. This can lead to seven different types of measurement errors, on the basis of which the quality (accuracy) metrics of each instrument are determined [5, 6]:

- a) Detection errors: The diagnostic tool does not detect any defects or detects non-existent defects.
- b) Sizing errors: The diagnostic tool incorrectly identifies the sizes of defects.
- c) Identification errors: the diagnostic tool incorrectly identifies detected defects according to the classification adopted in the in-line inspection (ILI) industry;
- d) Location errors: The diagnostic tool incorrectly determines the coordinates (distance and angular position) of detected defects.

The detected defects are only a part of the total set of actual defects. In essence, the diagnostic tool acts as a filter that divides the set of real defects into two groups: detected and undetected. In-line inspection data can be used to characterize the pipeline condition by the probability distributions of the depth and length of defects. However, it is believed that short and shallow defects are most likely to be not detected during diagnostics. As a rule, two factors influence the determination of the accuracy of inspection: the probability of detection/false non-detection of defects and the errors their measurements.

#### I. Probability of detection (POD)

This parameter models the imperfect detection capability of the pigging tools. POD is typically an increasing function of defect depth and is defined as an exponential function:

$$\text{POD}(d) = P_D(d) = 1 - e^{-qd}, \quad (4)$$

where  $P_D(d)$  is the probability of detecting a defect of depth  $d$ ,  $q$  is the constant describing the resolution of the inspection instrument. The expression for POD can be interpreted as an exponential probability distribution of the detected depths, where  $(1/q)$  is the average depth of the detected defects.

Using Bayes' theorem, the probability density function (PDF)  $f_D(d)$  of the depths of *detected* defects can be determined from the PDF  $f_F(d)$  depths of *actual* defects:

$$f_D(d) = \frac{P_D(d) f_F(d)}{\int_0^{\infty} P_D(d) f_F(d) dd}. \quad (5)$$

Probability  $p_d$  detection of a randomly selected defect using a control tool is determined by the expression

$$p_d = \int_0^{\infty} P_D(d) f_F(d) dd. \quad (6)$$

Similarly, the PDF  $f_{ND}(d)$  of the depths of *undetected* defects can be obtained as

$$f_{ND}(d) = \frac{[1 - P_D(d)] f_F(d)}{(1 - p_d)}. \quad (7)$$

Using equations (5) and (7), the PDF  $f_{ND}(d)$  of the depths of *undetected* defects can be expressed through the distribution of *detected* depths:

$$f_{ND}(d) = C \frac{[1 - P_D(d)]}{P_D(d)} f_D(d), \quad (8)$$

where  $C = \frac{1}{\int_0^{\infty} \frac{f_D(d)}{P_D(d)} dd} - 1$  is a normalization constant.

## II. Accuracy of defect size measurement

Uncertainty in defect size measurement is modeled by adding a random measurement error to the actual size. Unlike detection error, it is assumed that the error in determining the size of a defect is independent of its actual size. If the instrument measurements do not contain systematic errors (the instrument is well calibrated), then the size errors  $\varepsilon$  are assumed to have a normal distribution with a mean  $\mu_\varepsilon$  equal to zero and a standard deviation  $\sigma_\varepsilon$  depending on the measuring instrument quality. Thus, after a defect is detected, its size is superimposed by a measurement error due to signal noise.

$$d_m = d_d + \varepsilon_d \quad \text{and} \quad l_m = l_d + \varepsilon_l, \quad (9)$$

where  $d_m$  and  $l_m$  are the measured depth and length respectively;  $\varepsilon_d$  and  $\varepsilon_l$  denote random errors associated with depth  $d_d$  and length  $l_d$  measurement of the detected defect, which can be estimated from the inspection supplier's specifications. For example, suppliers of inspection tools often specify that the error  $\varepsilon$  will be within certain limits (e.g.,  $\pm 10\%$  measurements) with a given probability (e.g., with probability 90%). The measurement error can also be obtained from the results of verification excavations of similar operating pipelines.

The PDF of measured defect depths is obtained by convoluting the probability distribution of detected defects with the probability distribution of measurement errors:

$$f_M(d) = \int_0^{\infty} f_D(x) f_{\varepsilon}(d-x) dx. \quad (10)$$

The distribution of measurement errors is described by a normal distribution with zero mean and a given standard deviation  $\sigma_{\varepsilon}$ :

$$f_{\varepsilon}(x) = \frac{1}{\sigma_{\varepsilon} \sqrt{2\pi}} e^{-\frac{x^2}{2\sigma_{\varepsilon}^2}}. \quad (11)$$

Based on equations (9), the mathematical expectation  $\mu_m$  and standard deviation  $\sigma_m$  of the measured defects are:

$$\mu_m = \mu_d, \quad \sigma_m = \sqrt{\sigma_d^2 + \sigma_{\varepsilon}^2}, \quad (12)$$

where  $\mu_d$  and  $\sigma_d$  are the mathematical expectation and standard deviation of the detected defects sizes.

### III. Modeling defects development and the emergence of new defects

The process of occurrence of new defects in a specific section of the OUP is described by a non-uniform Poisson process (NPP) [7, 8], according to which a number of defects  $N(t)$  during the time interval  $[0, t]$  ( $t = 0$  denotes the time of putting the pipeline into operation) with a PDF defined as [9]:

$$f_N(N(t), m(t)) = \frac{m(t)^{N(t)} e^{-m(t)}}{N(t)!}, \quad t \geq 0, \quad (13)$$

where  $m(t)$  is the average number of defects in the time interval  $[0, t]$ , which, according to [7],

obeys a power function of time  $m(t) = \lambda \delta \int_0^t x^{\delta-1} dx = \lambda t^{\delta}$  with parameters  $\lambda$  and  $\delta$  ( $\lambda, \delta > 0$ ), which

can be quantitatively determined using diagnostic data for similar OUP sections, operated under similar conditions. It should be noted that  $N(t)$  degenerates into a homogeneous Poisson process (HPP) at  $\delta = 1$ .

Suppose that  $n$  inspections have been carried out for the pipeline section under consideration over a certain period of time. At the time of the  $i$ -th inspection ( $i = 1, 2, \dots, n$ ),  $t_i$ , the total number of defects in the pipeline section,  $N_i$ , can be divided into those defects that occurred before the  $(i-1)$ -th inspection, and those defects that occurred between the  $(i-1)$ -th and  $i$ -th inspections,  $N_i^g$ . The random variable  $N_i^g$  also follows a Poisson distribution with PDF:

$$f_N(N_i^g | \lambda, \delta) = \frac{m_i^{N_i^g} e^{-m_i}}{N_i^g!}, \quad (14)$$

where  $m_i = \lambda \delta \int_{t_{i-1}}^{t_i} x^{\delta-1} dx = \lambda (t_i^{\delta} - t_{i-1}^{\delta})$ ,  $t_0 = 0$ .

For UT instruments the case is different. Due to high sensitivity of UT tools, often  $N_i^g = N_i^{gd} - N_i^{gu}$  (a lot of false positives occur).

As noted above, the detected number of defects is generally less than the actual number of defects. Let  $N_i^{gd}$  and  $N_i^{gu}$  denote the detected and undetected number of defects, respectively, i.e.  $N_i^g = N_i^{gd} + N_i^{gu}$ . Based on the Poisson distribution property, the random variables (RV)  $N_i^{gd}$  and  $N_i^{gu}$  follow the Poisson distribution with the corresponding PDFs:

$$f_P(N_i^{gd} | \lambda, \delta) = \frac{(\overline{\text{POD}}_i m_i)^{N_i^{gd}}}{N_i^{gd}!} \exp(-\overline{\text{POD}}_i m_i), \quad (15)$$

$$f_P(N_i^{gu} | \lambda, \delta) = \frac{[(1 - \overline{\text{POD}}_i) m_i]^{N_i^{gu}}}{N_i^{gu}!} \exp[-(1 - \overline{\text{POD}}_i) m_i], \quad (16)$$

where  $\overline{\text{POD}}_i$  is the average POD relative to  $N_i^g$  defects.  $\overline{\text{POD}}_i$  can be calculated as  $\overline{\text{POD}}_i = \int \text{POD}(x) f_{N_i^g}(x) dx$ , where  $f_{N_i^g}(x)$  is the PDF of defect depths  $N_i^g$  at time  $t_i$ .

To model the growth of corrosion defect depths, the non-uniform gamma process (NHGP) [10] is used, according to which the defect depth at time  $t$ ,  $D(t)$  follows a gamma distribution with PDF:

$$f_G(d(t) | \alpha(t), \beta) = \frac{\beta^{\alpha(t)} d(t)^{\alpha(t)-1} \exp[-d(t)\beta]}{\Gamma(\alpha(t))} I_{(0,\infty)}(d(t)), \quad (17)$$

where  $\alpha(t)$  is the time-dependent shape parameter and is assumed to be a power function of time, i.e.  $\alpha(t) = \varphi_1(t - t_s)^{\varphi_2}$  ( $t > t_s$ ), where  $t_s$  is the time of defect occurrence;  $\beta$  ( $\beta > 0$ ) is the time-independent rate parameter (or inverse scale parameter) [11];  $\Gamma(\cdot)$  denotes the gamma function, and  $I_{(0,\infty)}(x(t))$  is the indicator function, which is equal to one if  $d(t) > 0$  and zero otherwise. The mean, variance, and coefficient of variation (COV)  $d(t)$  are  $\alpha(t)/\beta$ ,  $\alpha(t)/\beta^2$  and  $1/(\alpha(t))^{0.5}$ , respectively. The quantity  $\varphi_1/\beta$  represents the average depth at the first unit time increment from  $t_s$ ;  $\varphi_2$  reflects the type of average defect growth path and at  $\varphi_2 > 1, \varphi_2 < 1, \varphi_2 = 1$  represents the accelerating, decelerating, and linear average depth growth, respectively. In addition,  $\varphi_2 = 1$  corresponds to a homogeneous gamma process.

The parameters  $\varphi_1$  and  $\varphi_2$  are assumed to be common to all defects, while  $t_s$  and  $\beta$  are specific to each defect to account for spatial variability in growth. Let  $t_{sr}$  and  $\beta_r$  denote the time of occurrence and the corrosion rate parameter for  $r$ -th defect ( $r = 1, 2, \dots$ ), respectively. It should be emphasized here that the index  $r$  is used to list all defects, including detected and undetected defects, to distinguish it from the index  $j$ , which is used to list detected defects. In what follows, the parameter is assumed  $\beta_r$  to be an exponential function of the random effect parameter  $\xi_r$ , that is  $\beta_r = e^{\xi_r}$ . The advantage of expressing  $\beta_r$  as an exponential function is that it guarantees  $\beta_r$ 's positivity.

From the above assumptions it follows that the increment in the depth of the defect  $r$  during the time interval between the  $(i-1)$ th and  $i$ -th inspections, denoted as  $\Delta d_{ir}$ , is a gamma distribution with the shape parameter  $\Delta \alpha_{ir}$  and corrosion rate  $\beta_r$ , where  $\Delta \alpha_{ir}$  is given by the expression

$$\Delta \alpha_{ir} = \begin{cases} \varphi_1 (t_i - t_{sr})^{\varphi_2}, & i = 1 \\ \varphi_1 (t_i - t_{sr})^{\varphi_2} - \varphi_1 (t_{i-1} - t_{sr})^{\varphi_2}, & i = 2, 3, \dots, n \end{cases} \quad (18)$$

The depth of the defect  $r$  at the  $i$ -th inspection,  $d_{ir}$ , is the sum of the depth at the  $(i-1)$ -th inspection and  $\Delta d_{ir}$ ; that is,  $d_{ir} = d_{i-1,r} + \Delta d_{ir}$ . It should be noted that  $d_{ir}$  when  $t = t_{sr}$  is assumed to be zero.

Below is a procedure is described for modeling the depth of corrosion defects and the corresponding diagnostic results, assuming that all newly detected defects occur between the  $(i-1)$ th and  $i$ -th inspections.

1. Calculate the number of newly occurring defects  $N_i^g$  during the  $i$ -th inspection

( $i = 1, 2, \dots, n$ ) using the Poisson distribution given by equation (14).

2. Determine the initial time  $t_{sr}$  ( $r = 1, 2, \dots, N_i^g$ ) for  $N_i^g$  newly occurring defects.
3. Select the parameters of the random effect  $\xi_r$  ( $r = 1, 2, \dots, N_i^g$ ) for  $N_i^g$  defects from a normal distribution with mean equal to zero and variance  $\sigma_\xi^2$ .
4. Generate defect depth:
  - 4.1) For the  $i$ -th inspection, accept  $r = 1$ .
  - 4.2) Generate the depth increment,  $\Delta d_{ir}^g$ , according to the gamma distribution given by equation (17) with shape and scale parameters equal to  $\varphi_1(t_i - t_{sr})^{\varphi_2}$  and  $e^{\xi_r}$ , respectively. It should be noted that  $d_{ir}^g = \Delta d_{ir}^g$  for newly occurring defects in the  $i$ -th inspection interval.
  - 4.3) If  $d_{ir}^g > d_{th}$  ( $d_{th}$  is the resolution of the in-line tool), accept the defect as a detected defect with probability  $\text{POD}(d_{ir}^g)$ . If the defect is accepted as a detected defect, go to step 4.4); otherwise, set  $r = r + 1$  and go to step 4.2).
  - 4.4) Generate depth increment associated with  $k$ -th inspection ( $k = i + 1, i + 2, \dots, n$ ) for the detected defect,  $\Delta d_{kr}$ , according to the gamma distribution given by equation (17) with shape and scale parameters equal to  $\varphi_1(t_k - t_{sr})^{\varphi_2} - \varphi_1(t_{k-1} - t_{sr})^{\varphi_2}$  and  $e^{\xi_r}$ , respectively. Calculate  $d_{kr} = \sum_{l=i}^k \Delta d_{lr}$  as the actual depth of the detected defect  $r$  during the  $k$ -th inspection.
  - 4.5) Generate the measured depth for the detected defect from the  $i$ -th to the  $n$ -th inspection by adding the random measurement error to the depth of the detected defect. The random measurement error is generated according to the normal distribution with zero mean and standard deviation  $\sigma_\varepsilon$ .
  - 4.6) Accept  $r = r + 1$  and repeat steps 4.2)–4.5) until  $r = N_i^g + 1$ .
  - 4.7) Repeat steps 4.1)–4.6) for all inspections  $i = 1, 2, \dots, n$ .

#### IV. Evaluation of the probability of failure

According to [12], the destruction pressure in the presence of a corrosion defect at time  $t$  is determined by the formula:

$$p_b(t) = \sigma_f \frac{2wt}{D - wt} \left( \frac{1 - \frac{d(t)}{wt}}{1 - \frac{d(t)}{M(t)wt}} \right), \quad (19)$$

where  $\sigma_f$  is the ultimate strength of the pipe material;  $wt$  is the pipe wall thickness;  $D$  is the outer diameter of the pipe;  $d(t)$  and  $l(t)$  are the depth and length of the defect at time  $t$ ; and the Folias parameter is determined by the formula:

$$M(t) = \sqrt{1 + 0.31 \frac{l^2(t)}{D \cdot wt}}. \quad (20)$$

The ultimate strength and geometric properties of the pipe are considered as random variables with their known PDFs. Thus, the strength of the corroded section of the pipe is a nonlinear time dependent function of random variables.

Probability of failure,  $P_i(t)$  on the time interval from 0 to  $t$  can be estimated as the probability that the failure pressure will be less than or equal to the working pressure, or the probability that the corrosion defect depth will be greater than the pipe wall thickness, i.e.

$$P_f(t) = P(p_b(t) - p_o \leq 0) + P(wt - d(t) \leq 0) \quad (21)$$

Working pressure  $p_o$  is modeled as a time-independent random variable because it is controlled by complex pressure control devices.

Thus, the limit state functions (LSF) of the OUP according to the “rupture” and “leak” criteria have the following form:

$$\begin{aligned} g_b(\sigma_f, wt, D, d(t), l(t), p_o) &= p_b(\sigma_f, wt, D, d(t), l(t)) - p_o, \\ g_l(wt, d(t)) &= wt - d(t). \end{aligned} \quad (22)$$

Since the LSF are functions of random arguments, they themselves will be random variables. To find the PDF of the LSF, use the expansion in orthogonal polynomials based on the Gram-Charlier-Edgeworth series [6, 13]. Thus, with known PDFs  $f_b(x)$ ,  $f_l(x)$  functions of limit states, the probability of OUP failure:

$$P_f = F_b(0) + F_l(0) = \int_{-\infty}^0 f_b(x) dx + \int_{-\infty}^0 f_l(x) dx, \quad (23)$$

where  $F_b(x)$ ,  $F_l(x)$  are the distribution functions of the FPS according to the “rupture” and “leak” criteria, respectively.

## V. Repair criterion

The repair criterion for the rupture type is often determined based on the safety factor  $k_b$  for the failure pressure, determined by equation (19), and the maximum allowable working pressure  $p_m$  (MAOP) of the pipeline, i.e. the burst pressure of the pipe section with a defect is less than or equal to the safety factor for the failure, multiplied by MAOP:

$$p_b(t) - k_b p_m \leq 0.$$

Typically,  $k_b$  in the range of or 1.25 до 1.5, and a typical value of 1.25 is intended to provide the same level of integrity as would be found in actual pipeline hydrotesting.

The criterion for repair of the leak type is determined on the basis of the coefficient  $k_l$  at which the depth of the defect is greater than,  $k_l wt$  i.e.:

$$k_l wt - d(t) \leq 0.$$

For the coefficient,  $k_l$  the range is usually from 0.60 to 0.80, which corresponds to 60%-80% of the pipe wall thickness.

## VI. General algorithm for constructing a stochastic life cycle of OUP

**Stage 1.** After receiving a representative ensemble of all types of defects by the time of the first diagnostics, it must be inspected using *virtual* in-line instruments (VII) with metrics of their quality (accuracy), which real equipment used by a particular domestic pipeline organization has. At the same time, the cost of this diagnostic at the time of its implementation is estimated.

The result of this inspection may be: 1) absence of defects equal to or greater than the resolution of the VII; 2) presence of a group of a certain number of defects and anomalies. This group is analyzed to determine whether it contains critical and/or dangerous defects, and whether it is necessary to repair or restore the pipe, immediately or after some time.

**Stage 2.** After this inspection, the probability of missing a dangerous defect that could lead to an accident (leak, rupture) and the cost of restoring the OUP section integrity in the event of such a failure, i.e., the residual risk of the diagnostics performed, are assessed. This requires knowledge of the probability of false non-detection of a dangerous defect, characteristic of the VII used. To

determine the estimated location of a dangerous defect missed during flaw detection, it is recommended to use the approach proposed in [13].

**Step 3.** The result of this step will be one of two decisions: wait until the time of the second inspection (with the risk calculated in step 3) or carry out repairs to the pipe to remove the unacceptable defects, immediately [if the defect(s) are critical] or after some time necessary to carry out the preparatory work, during which the risk under point 3 will continue to increase.

**Stage 4.** If the analysis of the results of the first diagnostics did not lead to a decision to carry out any repair work, the development of all initial defects during the time between the first and second inspections is modeled, as well as the emergence of a group of new defects during the same period of time.

**Stage 5.** This new, enlarged array of defects is again subjected to virtual diagnostics, according to the algorithms of stage 2.

**Stage 6.** The results of the second diagnostics are analyzed to determine what decision should be made regarding the OUP operation, taking into account state regulations and rules in force within the pipeline company.

**Stage 7.** After this, it is possible, using the *quality metrics* of domestic VII, to again estimate the residual risk, taking into account the fact that there is a probability of missing a dangerous defect during the OUP diagnostics. This risk is calculated again using the algorithms of stage 3.

**Stage 8.** If a decision is made to carry out repairs based on the fact that dangerous defects have been identified, the volumes of the predicted repairs, as well as their duration and cost, are assessed. For this purpose, the standards and rules in force in the organizations operating the OUP in question are used.

**Stage 9.** The next step is to assess the most probable condition of the repaired OUP at the time of completion of the repair.

**Stage 10.** When modeling the implementation of the life cycle, a case is possible when, with the growth of defects at one of the phases of the OUP operation, one or more defects reach the maximum size, at which either a leak or a rupture of the pipe inevitably occurs. The time of reaching the maximum size by the first defect (initiating failure) is the beginning of an accident on the pipeline. The material damage from this accident is calculated on the basis of past domestic experience, taking into account discounting. The time and cost of restoring the integrity of the pipeline are estimated taking into account the financial and technological capabilities of the owner/operator of the OUP in question.

The tree of events and control decisions of the OUP life cycle described above consists of random time intervals during which the following occurs respectively: *operation* → *diagnostics* → *repair* → *operation* → *accident* → *restoration* → *operation* → ... (where the phases *diagnostics*, *repair* and *accident* include an element of decision making). The tree is built until the entire OUP life cycle is covered and is presented in Fig. 1.

To account for the influence of the Dragon King (DK) type load, the moment(s) of its impact on the system are played out using the Monte Carlo method. With such modeling, it is also possible to take into account the Black Swan (BS) type load/impact. Despite the fact that an BS type event is fundamentally unpredictable, in the context of this task (only knowing that the BS event can manifest itself) it can be modeled in the same way as the impact of the DK. In this case, the parameters of the hypothetical BS are estimated by one of the brainstorming methods by a group of people with certain specific cognitive abilities, and the time of its implementation is played out as a uniformly distributed RV over the length of the life cycle.

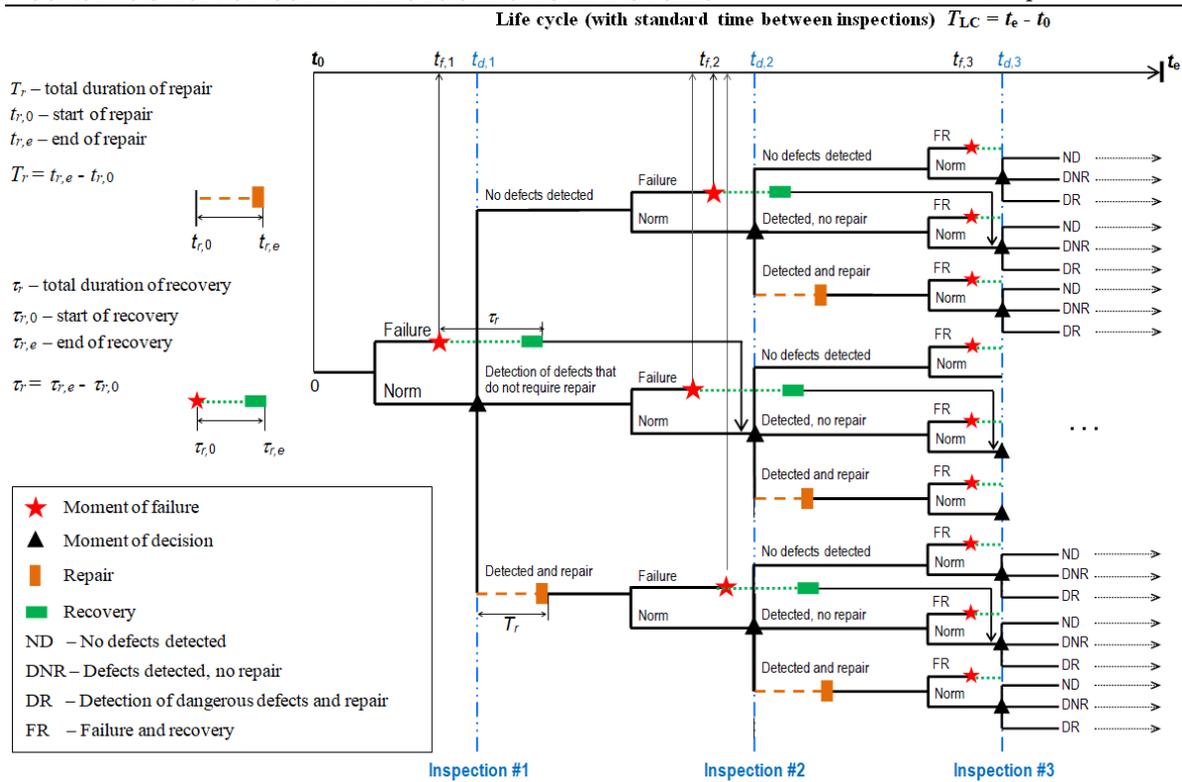


Fig. 1. Stochastic tree of events and decisions “event–decision–event–decision–...” built on the OUP life cycle

An accident of a pipeline near an offshore inhabited platform with fatal outcome(s) and injuries (time of occurrence of the event, its probability and non-economic scale of damage) during its life cycle is also modeled by the MC method. In this case, the target function of risk management of the operation of the offshore inhabited platform includes the cost of health restoration and compensation for possible injuries and loss of years of human life, determined by the method described in [14].

As a result of modeling the implementation of the OUP life cycle, in the general case, random numbers and times of diagnostic work, occurrence of emergency situations and adoption of management decisions on restoring the integrity and operability of the OUP arise. At these moments in time, the risks of operating the OUP change abruptly, which can be assessed quantitatively.

It is advisable to use the constructed full life cycle as a platform for assessing the sensitivity of the sought parameters (inherent reliability, resilience, frequency and quality of inspections, time, volume and cost of repairs, risk, etc.) to any input parameter or their combinations.

Knowing the time moments of planned interventions during the normal OUP operation, as well as the moments of occurrence of emergency situations and accidents, allows constructing a time schedule of discounted operating costs and losses due to OUP failures, as well as the growth and decline of its failure probability. Averaging these schedules over an ensemble of LC implementations gives an estimate of the average values of risk components, and the risk itself as a function of time.

## VI. Conclusion

The article, using the example of the OUP as a strategic infrastructure, presents a methodology for computer modeling of the full stochastic life cycle, based on the technology of diagnostics,

monitoring, maintenance, repair and recovery after an OUP failure, including a full group of accuracy metrics for the diagnostic tools used and allowing one to obtain: consistent estimates of the probability of an undesirable event at each phase of the OUP life cycle; residual risk after each intervention in the functioning of the asset; the magnitude of the risk of making a management decision related to OUP design and operation.

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