STOCHASTIC MODELING AND ANALYSIS OF A PARALLEL SYSTEM WITH DIFFERING UNIT QUALITIES AND CATASTROPHIC FAILURES

Priya Baloda[1], Amit Kumar[2,*], Vikas Garg[3]

•

[1,2,3] Department of Mathematics, Chandigarh University, Gharuan, Mohali, India, 140413
[1]priyabaloda17@gmail.com, [2,*]amitk251@gmail.com, [3]gargvikas0314@gmail.com

Abstract

In this paper, we conduct a stochastic analysis of a system comprising of two dissimilar units operating in parallel. One unit, characterized by two distinct types of failures, is considered high merit, while the other unit, with only one type of malfunction, is deemed low merit. We introduce the concept of catastrophic failure for the system. It is assumed that a single serviceman is responsible for both repair and replacement tasks. A regenerative point-based approach is employed to estimate various reliability characteristics. Additionally, we analyze the proposed model graphically to illustrate the effects of failure, repair, and replacement rates on the system's mean time to system failure (MTSF), availability, and profit.

Keywords: Parallel units, Semi Markov process, Reliability, Catastrophic Failure.

1. Introduction

Reliability is a key factor in various industries and systems, defining the ability of a system or component to consistently carry out its stated functions without experiencing failure over time. It is particularly important in sectors such as aviation, healthcare, telecommunications, and critical infrastructure, where safety, data integrity, and performance are crucial. Reliable systems reduce threats to operational continuity and human safety across multiple domains by ensuring persistent and flawless operations. Beyond functional efficiency, reliability is also vital for user satisfaction and trust. Additionally, to meet safety and quality criteria, specific industry rules and regulations are often followed when implementing reliability in system design. The reliability of data processing and storage units ensures that data remains secure and accurate, which is critical for everyday operations as well as long-term decision-making. Therefore, reliability is essential for designing systems that customers can truly depend on.

In today's industrial and technological world, achieving greater reliability is crucial for the long-term performance and operational efficiency of systems. Additionally, the setup and design of industrial systems are becoming increasingly complex due to society's constantly changing demands. Experts are continually working to create more profitable and productive models that ensure smooth functioning, user satisfaction, and security across multiple areas. Controlling the intensity and frequency of system breakdowns is also essential, particularly in critical applications such as aerospace and power generation, where operational reliability is paramount. Among the various measures suggested for improving system reliability are redundancy techniques, availability of replacement parts, high-quality components, optimal component arrangement, and appropriate repair options.

Due to minimum costing and functional easiness, various single unit systems are proposed for system models. Several researchers worked on stochastic modeling of systems with single unit. Various reliability models of single-unit systems with different assumptions are being examined by many researchers like [1], [2]. A single unit system with flexible demand is proposed by [3]. Examination of a single unit system following Weibull Failure subjected to server failure is done by [4].

However, single unit system shut down as soon as unit undergoes failure. To overcome this major limitation of single unit models, redundancy is one of the best approaches. The reliance on multiple units offers a promising approach to enhance system robustness and availability. Numerous endeavors have done bunch of work on different redundancies systems. Cold redundant systems have been proposed by many studies. [5] has evaluated the availability and profit of a cold standby system having series parallel configuration. [6] has done a comparative analysis of a single unit system and cold standby system, offering the optimal system configurations as per demand. The impact of maintenance tasks on systems efficiency and performance for a standby system is studied by [7]. A multi component standby system is proposed by [8] and optimal cost and profit optimization strategies had been proposed. A reliability analysis and comparison of four distinct hybrid systems requiring additional supporting devices for operation have been conducted by [9]. Additionally, a significant amount of research (c.f. [10], [11], [12], etc.) describes warm and hot standby systems. A triple-unit system comprising both warm and cold standby units was proposed by [13]. In most of these systems, units have been studied under different types of failures events, with the exception of catastrophic failure. Considering the potential for catastrophic failure helps enhance safety, manage risks, and maintain operational integrity, ultimately leading to more reliable systems. [14] introduced the concept of catastrophic failure for a parallel system consisting of two units with two modes both for operation as well as repair along with timely preventive maintenance of the system. Additionally, some more research work has also been published regarding the concept of catastrophic failure in reliability models (c.f. [15], [16], [17], etc.). These works underline the impact of paying attention to such kinds of sudden failures as these prove to be very crucial for safety and risk management, A complex system further divided between two sub systems; comprising of different number and types of units is deeply investigated by [18].

In the literature on standby systems, studies often focus on either identical units or units with identical failure rates. However, if both units are subjected to similar environmental stresses or operational demands, their failure rates may become interdependent, making the system more susceptible to simultaneous breakdowns. When these failures occur, they can trigger a cascade of adverse effects, overwhelming the system's capacity to manage or mitigate risks. This scenario highlights the critical importance of accurately modeling failure dependencies and incorporating risk assessment strategies to prevent such catastrophic outcomes and ensure system resilience.

This paper seeks to shed light on the intricate dynamics of reliability in parallel configurations comprising dissimilar units with differing failure modes, focusing specifically on catastrophic failure scenarios. Two failure modes for unit 1 and a single failure mode for unit 2 have been considered. To minimize system costs, unit 1 is chosen for its merit quality, while unit 2 is assumed to have sub- merit quality. If the system experiences catastrophic failure, it is replaced and becomes operational only after both units are replaced. A single serviceman is available to perform repairs and replacements of the units. The failure rates of both units and catastrophic failure are assumed to follow a negative exponential distribution, while all other distributions are chosen arbitrarily. Using the Regenerative Point Technique, various reliability measures and their graphical evaluations have been gathered.

2. Assumptions and Notations

In this system, a two-dissimilar-unit parallel system has been analyzed. Unit D_1 is assumed to have two types of failures: partial failure and total failure. Unit D_2 has only one failure mode. It is also assumed that when unit D_1 operates in partial failure mode, its repair is performed simultaneously. However, both units may experience total failure if a catastrophic failure occurs. In such a case, the entire system is replaced by a single repairman assigned to perform all repair and replacement tasks. The failure rates of both units, as well as the catastrophic failure, are modeled using exponential distributions, while all repair and replacement rates are considered to have arbitrary distributions.

All the notations being utilized in this system are listed as:

S_i : Transition state of system model.
$p_{m,n}$: Transition probability from state S_m to S_n.
$p_{m,n,o,p}$: Probability of going from state S_m to S_n via states S_o and S_p.
μ_i : Mean Soujourn Time corresponding to state S_i.
β : Constant catastrophic failure rate.
λ_1 : Constant partial failure rate of D_1.
λ_2 : Total failure rate of D_1.
λ_3 : Failure Rate of D_2
$g_1(t)$: pdf of rate of repair of partially failed D_1.
$g_2(t)$: pdf of rate of repair of total failed D_1.
$g_3(t)$: pdf of repair rate of total failed D_2.
$r(t)$: pdf of replacement rate when system blasts out due to catastrophic failure.
$G_1(t)$: cdf of rate of repair of partially failed D_1.
$G_2(t)$: cdf of rate of repair of total failed D_1.
$G_3(t)$: cdf of repair rate of total failed D_2.
$R(t)$: cdf of replacement rate when system blasts out due to catastrophic failure.
D_{1o}/D_{2o} : D_1 is operative/ D_2 is operative.
$\bar{D}_{1uro}$: D_1 is operative in partial failure mode while repairing is being done simultaneously.
$\bar{D}_{1wro}$: Partially failed D_1 is waiting for repair while being operative.
D_{1ur}/D_{2ur} : Totally failed D_1/ D_2 is under repair.
D_{1wr} : Totally failed Unit D_1 is waiting for repair.
D_{2wr} : D_2 is waiting for repair.
D_{1urp}/D_{2urp} : D_1/D_2 is under replacement.
Ⓢ/© : Symbol used for Stieltjes Convolution / Laplace Convolution.
$*/**$: Symbol used for Laplace Transform (LT)/ Laplace Stieltjes Transform (LST).

Here,

S_0, S_1, S_2, S_3, S_9 are regenerative states while all other are non-regenerative states.

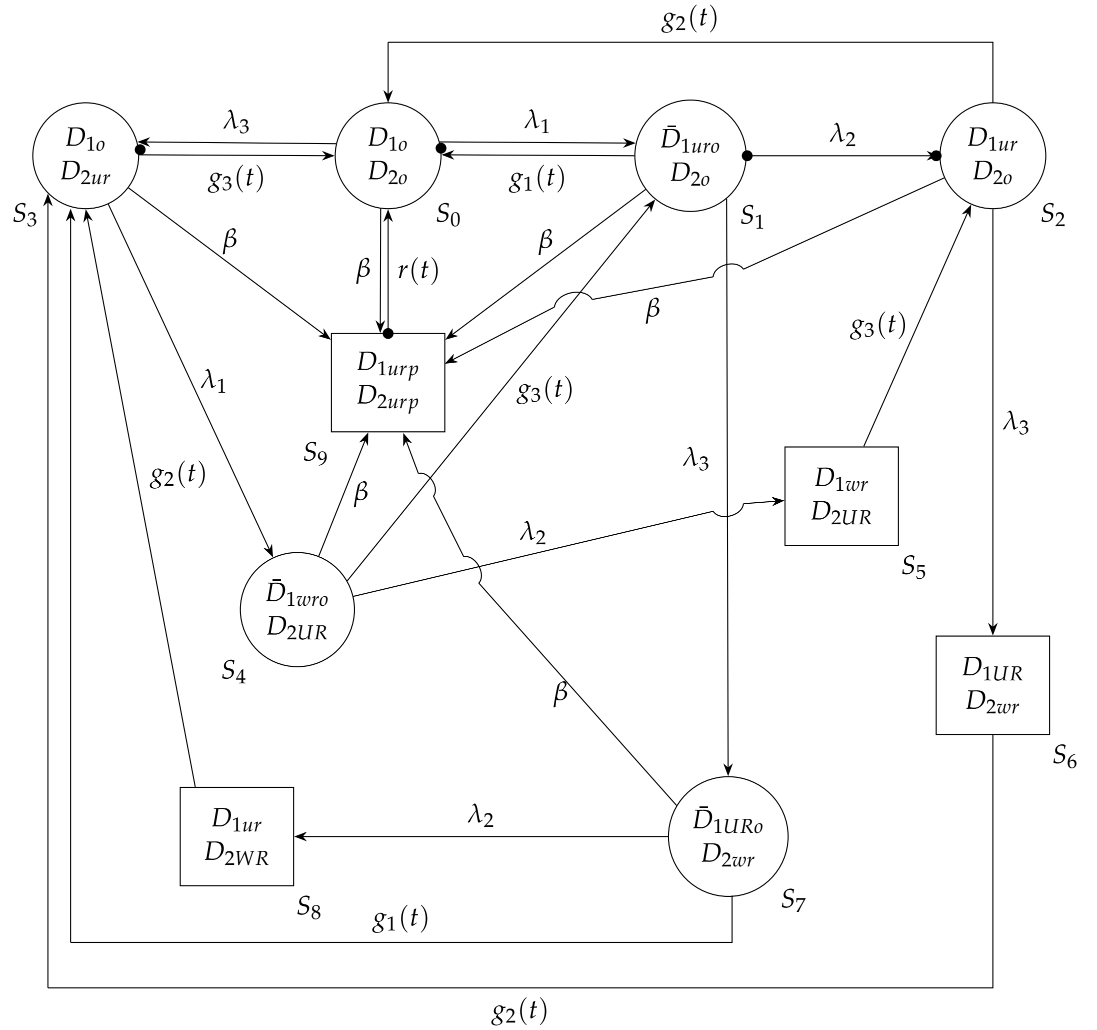


Figure 1: *State Transition Diagram*

• : Regenerative States ○ : Operative State □ : Failed State

3. Reliability Measures

Reliability of any system is measured by various characteristics like Mean Time to System Failure (MTSF), availability of system etc.. The characteristics related to system model are evaluated as:

3.1. Probabilities and Mean Soujourn Time

Assuming that the system progresses from state S_m to S_n within any time interval [0,t), the transition probability $p_{m,n}$ is given by:

$$p_{m,n} = \lim_{t\to\infty} Q_{m,n}(t)dt = Q_{m,n}(\infty) = \int_0^\infty q_{m,n}(t)dt$$

where $q_{m,n}$ represents the probability density function of one step transition from state m to state n and $Q_{m,n}$ represents the cumulative density function of one step transition from state m to state n.

For states $S_i's$, where $i = 0$ to 9, the various transition probabilities are:

$$p_{0,1} = \frac{\lambda_1}{\lambda_1+\lambda_3+\beta},\ p_{0,3} = \frac{\lambda_3}{\lambda_1+\lambda_3+\beta},\ p_{0,9} = \frac{\beta}{\lambda_1+\lambda_3+\beta},\ p_{1,0} = g_1^*(\lambda_2+\lambda_3+\beta),$$
$$p_{1,2} = \frac{\lambda_2}{\lambda_2+\lambda_3+\beta}(1-g_1^*(\lambda_2+\lambda_3+\beta)), p_{1,7} = \frac{\lambda_3}{\lambda_2+\lambda_3+\beta}(1-g_1^*(\lambda_2+\lambda_3+\beta)),$$
$$p_{1,9} = \frac{\beta}{\lambda_2+\lambda_3+\beta}(1-g_1^*(\lambda_2+\lambda_3+\beta)),\ p_{2,0} = g_2^*(\lambda_3+\beta),\ p_{2,6} = \frac{\lambda_3}{\lambda_3+\beta}(1-g_2^*(\lambda_3+\beta)),$$
$$p_{2,9} = \frac{\beta}{\lambda_3+\beta}(1-g_2^*(\lambda_3+\beta)),\ p_{3,0} = g_3^*(\lambda_1+\beta),\ p_{3,4} = \frac{\lambda_1}{\lambda_1+\beta}(1-g_3^*(\lambda_1+\beta)),$$
$$p_{3,9} = \frac{\beta}{\lambda_1+\beta}(1-g_3^*(\lambda_1+\beta)), p_{4,1} = g_3^*(\lambda_2+\beta),\ p_{4,5} = \frac{\lambda_2}{\lambda_2+\beta}(1-g_3^*(\lambda_2+\beta)),$$
$$p_{4,9} = \frac{\beta}{\lambda_2+\beta}(1-g_3^*(\lambda_2+\beta)),\ p_{5,2} = g_3^*(0),\ p_{6,3} = g_2^*(0),$$
$$p_{7,3} = g_1^*(\lambda_2+\beta),\ p_{7,8} = \frac{\lambda_2}{\lambda_2+\beta}(1-g_1^*(\lambda_2+\beta)),\ p_{7,9} = \frac{\beta}{\lambda_2+\beta}(1-g_1^*(\lambda_2+\beta)),$$
$$p_{8,3} = g_2^*(0),\ p_{9,0} = r^*(0), \tag{1}$$

Here, $p_{5,2} = p_{6,3} = p_{8,3} = p_{9,0} = 1$.

Mean Soujourn Time (μ_i) which informs about the average time system remains in any particular state i before proceeding into other state j is given by:

$$\mu_i = \int_0^\infty P(T_i > t)dt = \sum_j m_{ij}$$

where m_{ij} gives the contribution to mean soujourn time in S_i when S_i directly transits to S_j.

$$\mu_0 = \frac{1}{\lambda_1+\lambda_3+\beta},\ \mu_1 = \frac{1-g_1^*(\lambda_2+\lambda_3+\beta)}{\lambda_2+\lambda_3+\beta},\ \mu_2 = \frac{1-g_2^*(\lambda_3+\beta)}{\lambda_3+\beta},$$
$$\mu_3 = \frac{1-g_3^*(\lambda_1+\beta)}{\lambda_1+\beta}, \mu_4 = \frac{1-g_3^*(\lambda_2+\beta)}{\lambda_2+\beta}, \mu_5 = -(g_3^*)'(0), \tag{2}$$
$$\mu_6 = -(g_2^*)'(0),\ \mu_7 = \frac{1-g_1^*(\lambda_2+\beta)}{\lambda_2+\beta},\ \mu_8 = -(g_2^*)'(0),\ \mu_9 = -(r^*)'(0)$$

Also,

μ_i' defining the average time of transition between regenerative states starting from any regenerative state i are given by:

$$\mu_0' = \mu_0,\ \mu_1' = m_{1,0} + m_{1,2} + m_{1,9} + m_{1,9,7} + m_{1,3,7} + m_{1,3,7,8}$$
$$\mu_2' = m_{2,0} + m_{2,9} + m_{2,3,6}.\ \mu_9' = \mu_9$$
$$\mu_3' = m_{3,0} + m_{3,9} + m_{3,9,4} + m_{3,1,4} + m_{3,2,4,5}$$

Additionally, μ_i'' which tells about the time of moving from regenerative state i to either any other regenerative state or to any failed state is given by:

$$\mu_1'' = m_{1,0} + m_{1,2} + m_{1,9} + m_{1,9,7} + m_{1,3,7} + m_{1,8,7}$$
$$\mu_3'' = m_{3,0} + m_{3,9} + m_{3,9,4} + m_{3,1,4} + m_{3,5,4}$$

3.2. Mean Time to System Failure (MTSF)

Consider the transition from any state S_m to any failed state which is further assumed as absorbing state. Let $\varphi_m(t)$ denotes the cdf of above considered transition, so the expression establishing the MTSF of system is given by:

$$\varphi_m(t) = \sum_n Q_{m,n}(t) \circledS \varphi_n(t) + \sum_p Q_{m,p}(t) \tag{3}$$

Here, m is any operative state which is transiting to any operative regenerative state n and any failed state p. Reliability($R(t)$) of the system is governed by performing inverse Laplace Transform of following formula:

$$R^*(s) = \frac{1 - \varphi_0^{**}(s)}{s} \tag{4}$$

where $\varphi_0^{**}(s)$ is obtained by taking LST of (3). Also, MTSF is given by:

$$MTSF = \lim_{s \to 0} R^*(s) = \lim_{s \to 0} \frac{1 - \varphi_0^{**}(s)}{s} \tag{5}$$

Solving the above equations, the MTSF of system is obtained as:

$$MTSF = \frac{N^1}{D^1}$$

where

$$\begin{aligned} N^1 &= \mu_0(1 - p_{1,3,7}p_{3,1,4}) + (p_{0,1} + p_{0,3}p_{3,1,4})(\mu_1'' + p_{1,2}\mu_2) + \mu_3''(p_{0,3} + p_{0,1}p_{1,3,7}) \\ D^1 &= 1 - p_{0,1}(p_{1,0} + p_{1,2}p_{2,0} + p_{1,3,7}p_{3,0}) - p_{0,3}(p_{3,0} + p_{3,1,4}(p_{1,0} + p_{1,2}p_{2,0})) - p_{1,3,7}p_{3,1,4} \end{aligned} \tag{6}$$

3.3. Availability Analysis

At $t = 0$, let the system be in regenerative state S_m, and $\hat{A}_m(t)$ shows the probability of system availability at any instant t. If $\tilde{U}_m(t)$ defines the probability of system being in upstate S_m up to time t without going to another regenerative state, then

$$\begin{aligned} &\tilde{U}_0(t) = e^{-(\lambda_1+\lambda_3+\beta)t}, \tilde{U}_1(t) = e^{-(\lambda_2+\lambda_3+\beta)t}\overline{G_1}(t), \\ &\tilde{U}_2(t) = e^{-(\lambda_3+\beta)t}\overline{G_2}(t), \tilde{U}_3(t) = e^{(-\lambda_1+\beta)t}\overline{G_3}(t) \end{aligned} \tag{7}$$

Now, $\hat{A}_m(t)$ is computed by following relations:

$$\hat{A}_m(t) = \tilde{U}_m(t) + \sum_n q^i_{m,n}(t) \copyright \hat{A}_n(t) \tag{8}$$

where transition from regenerative state m to regenerative state n completes in i number of transitions. Taking LT of (7) to find the value of $\hat{A}_0^*(s)$ and using this results for determining the steady state availability $\hat{A}_0(\infty)$ of the system, we get:

$$\hat{A}_0(\infty) = \lim_{s \to 0} s\hat{A}_0^*(s) = \frac{N^2}{D^2} \tag{9}$$

where

$$\begin{aligned} N^2 &= \mu_0 A + \mu_1 B + \mu_2 C + \mu_3 D \\ D^2 &= \mu_0' A + \mu_1' B + \mu_2' C + \mu_3' D + \mu_9' E \end{aligned} \tag{10}$$

3.4. Server's Busy Period

Let $B_m^R(t)$ and $B_m^{RP}(t)$ denotes the probability that at any instant t, server is busy in repair and replacement respectively, with a restriction that system entered regenerative state S_m at $t = 0$. Let transition takes place between regenerative states m to n, within i transitions, then the relations for $B_m^R(t)$ and $B_m^{RP}(t)$ are :

$$\begin{aligned} B_m^R(t) &= W_m(t) + \sum_{m,n} q_n^i(t) © B_n^R(t) \\ B_m^{RP}(t) &= W_m(t) + \sum_{m,n} q_n^i(t) © B_n^{RP}(t) \end{aligned} \tag{11}$$

Here, $W_m(t)$ is the probability that repairman is busy at state S_m without further going to either any different regenerative state or to same state via one or more non regenerative states.

As LT of (11) will provide us values of $B_0^{R^*}(s)$ and $B_0^{RP^*}(s)$, so the probability of busy time in repair and replacement of server is given by:

$$B_0^R(\infty) = \lim_{s\to 0} sB_0^{R^*}(s) = \frac{N^3}{D^2} \tag{12}$$

$$B_0^{RP}(\infty) = \lim_{s\to 0} sB_0^{RP^*}(s) = \frac{N^4}{D^2} \tag{13}$$

In eqs. (12) and (13), N^3 and N^4 are given by:

$$\begin{aligned} N^3 &= W_1^*(0)B + W_2^*(0)C + W_3^*(0)D \\ N^4 &= W_9^*(0)E \end{aligned} \tag{14}$$

D^2 is defined in (10).

3.5. Expected Number of Visits

Let $R_m(t)$ and $RP_m(t)$ calculates the server's expected number of visits for performing repair and replacement in the state S_m in time interval $(0, t]$. The following recursive relations are used for estimating $R_m(t)$ and $RP_m(t)$:

$$\begin{aligned} R_m(t) &= \sum_{m,n} Q_n^i(t) Ⓢ(\acute{C} + R_n(t)) \\ RP_m(t) &= \sum_{m,n} Q_n^i(t) Ⓢ(\acute{C} + RP_n(t)) \end{aligned} \tag{15}$$

Here, the regenerative state m gets leaded to consecutive regenerative state n through i steps. If the repairman does task afresh , then $\acute{C} = 1$, otherwise $\acute{C} = 0$. LST of (15) will gives us value of $R_0^{**}(t)$ and $RP_0^{**}(t)$. With the help of these, the expected number of times for which server visits for performing repair and replacement is given by:

$$\begin{aligned} R_0(\infty) &= \lim_{s\to 0} sR_0^{**}(s) = \frac{N^5}{D^2} \\ RP_0(\infty) &= \lim_{s\to 0} sRP_0^{**}(s) = \frac{N^6}{D^2} \end{aligned} \tag{16}$$

where, N^5 and N^6 are given by:

$$\begin{aligned} N^5 &= B + C + D \\ N^6 &= E \end{aligned} \tag{17}$$

D^2 is defined in (10).

Here,

$$
\begin{aligned}
A &= 1 - p_{3,1,4}(p_{1,3,7} + p_{1,3,7,8}) - p_{2,3,6}(p_{3,1,4}p_{1,2} + p_{3,2,4,5}) \\
B &= p_{0,1}(1 - p_{3,2,4,5}p_{2,3,6}) + p_{0,3}p_{3,1,4} \\
C &= p_{0,1}(p_{1,2} + p_{3,2,4,5}(p_{1,3,7} + p_{1,3,7,8})) + p_{0,3}(p_{3,2,4,5} + p_{1,2}p_{3,1,4}) \\
D &= p_{0,3} + p_{0,1}(p_{1,3,7} + p_{1,3,7,8} + p_{1,2}p_{2,3,6}) \\
E &= p_{0,1}((p_{1,9} + p_{1,9,7})(1 - p_{2,3,6}p_{3,2,4,5}) + (p_{1,3,7} + p_{1,3,7,8})(p_{3,9} + p_{3,9,4} + p_{2,9}p_{3,2,4,5}) + \\
&\quad p_{1,2}(p_{2,9} + p_{2,3,6}(p_{3,9} + p_{3,9,4}))) + p_{0,3}(p_{3,9} + p_{3,9,4} + p_{3,1,4}(p_{1,9} + p_{1,9,7} + p_{1,2}p_{2,9}) + \\
&\quad p_{3,2,4,5}p_{2,9}) + p_{0,9}(1 - p_{3,1,4}(p_{1,3,7} + p_{1,3,7,8} + p_{1,2}p_{2,3,6}) - p_{3,2,4,5}p_{2,3,6})
\end{aligned}
\tag{18}
$$

3.6. Profit Evaluation

In the steady state, the profit of the system can be exerted from following relation:

$$P = C_1\hat{A}_0 - C_2B_0^R - C_3B_0^{RP} - C_4R_0 - C_5RP_0 \tag{19}$$

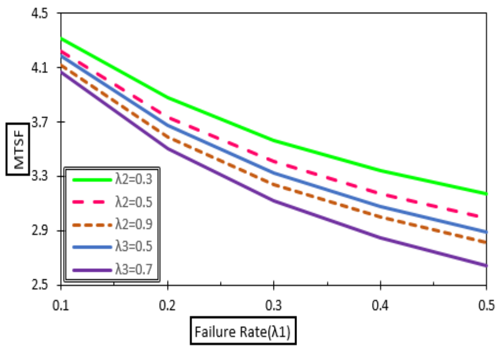
where,
P = Profit of system.
C_1 = Revenue generated by system per unit up time.
C_2 = Server's cost per unit time for performing repair.
C_3 = Server's cost per unit time for performing replacement.
C_4 = Cost per visit for repair charged by server.
C_5 = Cost per visit for replacement charged by server.

4. Graphical Surveying

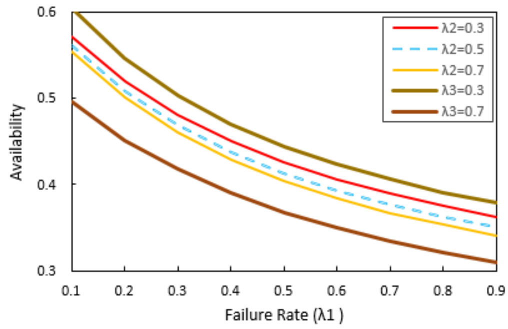
To examine the effect of various parameters involved in system model on reliability variables, graphical analysis has been done by considering a particular case of repair and replacement rates. Assume $g_1(t) = \alpha_1 e^{-\alpha_1 t}, g_2(t) = \alpha_2 e^{-\alpha_2 t}, g_3(t) = \alpha_3 e^{-\alpha_3 t}, r(t) = \gamma e^{-\gamma t}$. For finding the profitability, one particular set of costs has been assumed:
$C_1 = 6000, C_2 = 1000, C_3 = 1500, C_4 = 200, C_5 = 400$

4.1. Impact of Failure Rates

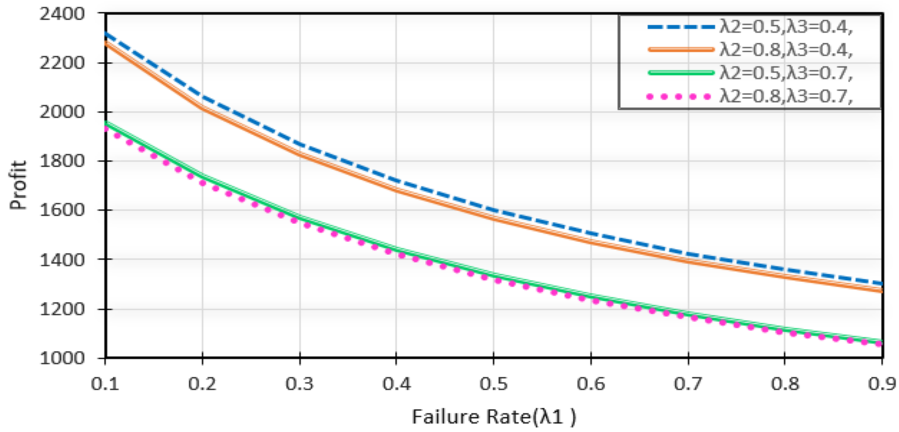
Different failure rate affect the system to different extent. From figure 2, it is obvious that MTSF, availability and profit of the system decreases with increase in value of λ_1 keeping all the other parameters constant. However, the influence is larger in case of MTSF as compared to that of availability and profit. Graph has been created to study the effect of λ_2 and λ_3 on MTSF and availability, keeping the other parameters constant. However, λ_2 does not affect the reliability measures to nice extent. It is evident from 2a that as we increase the value of λ_2, there is minor decrease in the values of availability as well as MTSF. While Change in the value of λ_3 has a dynamic effect on reliability characteristics. Graphical results which are obtained by variation in λ_3 while other parameters as assigned constant values as $\lambda_2 = 0.4, \beta = 0.5, \gamma = 0.6, \alpha_1 = 0.6, \alpha_2 = 0.4, \alpha_3 = 0.5$. This impact can be visualized from 2a. Figure 2b and 2c manifests that there is slight change in the availability and profit of system respectively with increase in λ_2 while increase in λ_3 decreases the profit to an appreciable extent.



(a) *Variation in MTSF with λ_2 and λ_3*



(b) *Variation in Availability with λ_2 and λ_3*



(c) *Effect of λ_2 and λ_3 on Profit*

Figure 2: *Impact of Failure Rates*

4.2. Impact of Catastrophic Failure Rate

Figure 3 supports the fact that catastrophic failure rate(β) has negative influence on profit of system. Also as the replacement rate(γ) goes upwards, profit also follows the same trend if all other parameters are assumed to be constant.

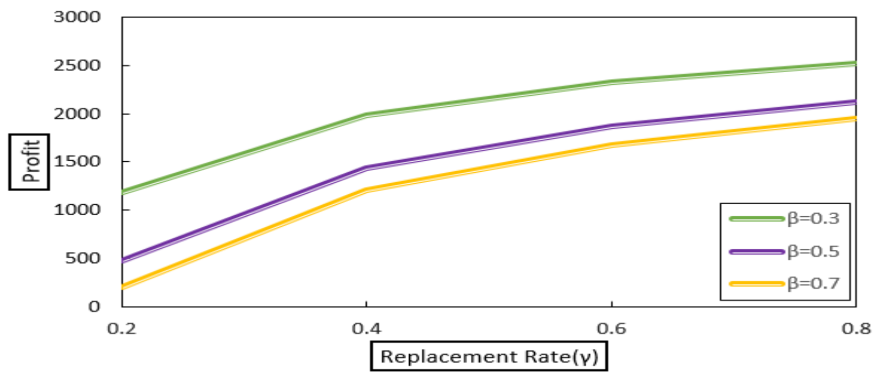
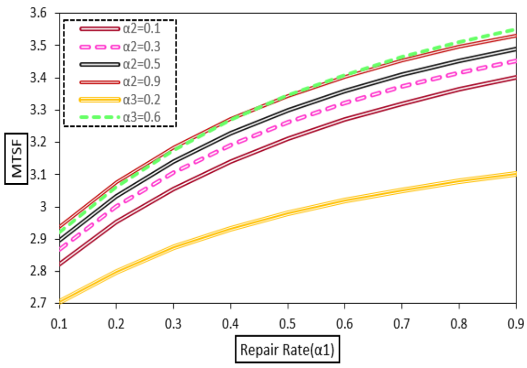


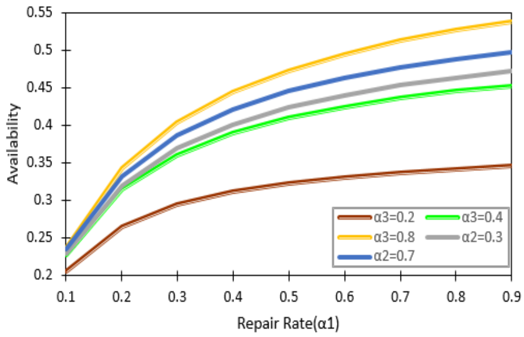
Figure 3: *Profit vs Replacement rate (γ)*

4.3. Impact of Repair Rates

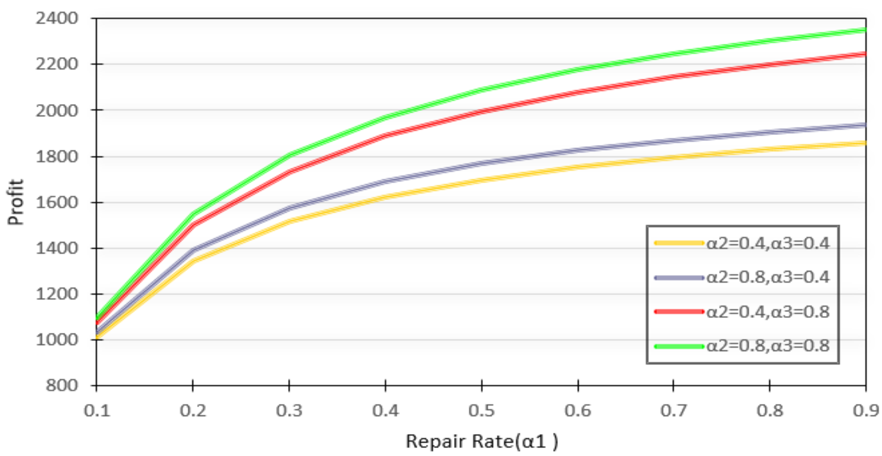
Numerical experimentation has been done to check the consequences of change in various repair rates of both units on system. From figure 4, the following observations can be made: With the increase in repair rate α_1, all the three factors MTSF, availability and profit of system shifts upwards. However, the increase is different in different measures. Figures 4a and 4b defines the change in MTSF and availability with respect to α_2 and α_3. It is visible that MTSF does not change much with increase in value of α_2 while availability has considerable increment. Figures 4a and 4b also establish that there is great increase in MTSF and availability with increasing values of α_3. Also, the increase is much more in availability as compared to that of MTSF. From figure 4c, it can be in view that there is increase in profitability of system in case of increment of both α_2 as well as α_3, but greater in later case.



(a) *MTSF vs α_2 and α_3*



(b) *Availability vs α_2 and α_3*



(c) *Effect of α_2 and α_3 on Profit*

Figure 4: *Impact of Repair Rates*

5. Conclusion

This study is presented to gauge the behavior of various repair and failure rates on the reliability metrics of a system consisting of two different parallel units. Numerical results signify that increasing the failure rates reduces these metrics to a remarkable extent. Additionally, an increase in the repair rate of all units will enhance profit. Graphical representations suggest that the failure rate should be controlled to maximize system profit, especially the failure rate of the unit with only one mode of malfunction, as it most significantly influences the reliability characteristics. However, a continuous increase in the replacement rate will not significantly boost profit, as it will also raise the system's costs, adversely affecting profitability. Therefore, replacement should also be managed carefully.

References

[1] Chander, S., and Bansal, R. K. (2005). Profit analysis of single-unit reliability models with repair at different failure modes. Proc. INCRESE IIT Kharagpur, India, 577-587.

[2] Malik, S. C., and Pawar, D. (2010). Reliability and economic measures of a system with inspection for on-line repair and no repair activity in abnormal weather. Bulletin of Pure and Applied Sciences, 29(2), 355-368.

[3] Malhotra, R., and Taneja, G. (2013). Reliability and availability analysis of a single unit system with varying demand. Mathematical Journal of Interdisciplinary Sciences, 2(1), 77-88.

[4] Kumar, A., and Saini, M. (2016). Analysis of a single-unit system with weibull failure and repair densities subject to server failure. Malaysian Journal of Science, 35(1), 15-22.

[5] Danjuma, M. U., Yusuf, B., and Yusuf, I. (2022). Reliability, availability, maintainability, and dependability analysis of cold standby series-parallel system. Journal of computational and cognitive engineering, 1(4), 193-200.

[6] Malhotra, R., and Taneja, G. (2015). Comparative study between a single unit system and a two-unit cold standby system with varying demand. Springerplus, 4, 1-17.

[7] Sultan, K. S., and Moshref, M. E. (2021). Stochastic analysis of a priority standby system under preventive maintenance. Applied Sciences, 11(9), 3861.

[8] Ahmad, S., Sharma, G., and Sharma, U. (2018). Modelling and profit evaluation of a repairable system working with one operative unit and three cold standby units. International Journal of Mathematics Trends and Technology-IJMTT, 64.

[9] Yusuf, I., and Umar, S. M. (2019). Reliability Modelling and Assessment of Multi Standby Hybrid System. Reliability: Theory & Applications, 14(3), 39-45.

[10] Hu, L., Huang, W., Wang, G., and Tian, R. (2018). Redundancy Optimization of an Uncertain ParallelSeries System with Warm Standby Elements. Complexity, 2018(1), 3154360.

[11] Kolledath, S., Kumar, K., and Pippal, S. (2018). Survey on queueing models with standbys support. Yugoslav Journal of Operations Research, 28(1), 3-20.

[12] El-Ghamry, E., Muse, A. H., Aldallal, R., and Mohamed, M. S. (2022). Availability and Reliability Analysis of akOutofn Warm Standby System with CommonCause Failure and Fuzzy Failure and Repair Rates. Mathematical Problems in Engineering, 2022(1), 3170665.

[13] Ghosh, J., Pawar, D., and Malik, S. C. (2022). Analysis of triple-unit system with operational priority. Reliability: Theory & Applications, 17(3 (69)), 202-210.

[14] Goel, L. R., and Gupta, P. (1984). Stochastic analysis of a two-unit parallel system with partial and catastrophic failures and preventive maintenance. Microelectronics Reliability, 24(5), 881-883.

[15] Ram, M., and Manglik, M. (2016). Reliability measures analysis of an industrial system under standby modes and catastrophic failure. International journal of operations research and information systems (IJORIS), 7(3), 36-56.

[16] Tyagi, V., Arora, R., Ram, M., and Yadav, O. P. (2019). 2-Out-of-3: F System analysis under catastrophic failure. Nonlinear Studies, 26(3).

[17] Li, H., Yuan, R., and Fu, J. (2019). A reliability modeling for multi-component systems considering random shocks and multi-state degradation. IEEE Access, 7, 168805-168814.

[18] Sirohi, A., Poonia, P. K., and Raghav, D. (2021). Reliability analysis of a complex repairable system in series configuration with switch and catastrophic failure using copula repair. J. Math. Comput. Sci., 11(2), 2403-2425.