MULTI- REFERENCE SKIP- LOT SAMPLING OF TYPE 3 (MR-SkSP-3)

Meby Joseph Manoj [1], Azarudheen S [2]

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[1,2] Department of Statistics and Data Science
CHRIST (Deemed to be University)
Bengaluru, India
mebyjosephmanoj0328@gmail.com, sazaru1990@gmail.com

Abstract

In the current industrial sector, the rate of defective products present in the lots has been decreasing and most of the products keeps up a good history of quality throughout the production also. Skip-lot sampling plans are the suitable acceptance sampling plan for the situations where the series of products shows a stable and excellent quality. The skip-lot sampling plans are still widely used because of its reduced sampling cost and efforts, because the plan only needs to inspect a fraction of the lots submitted after a continues series of lots with excellent quality. This approach makes the skip-lot plan more cost-effective than the other sampling plans, thus making it an economically important plan. The current study incorporated a modification on the skip-lot sampling of type 3 and designated it as multi- reference skip lot sampling of type 3. The proposed plan has the provision of having multiple reference plans in normal and skipping inspection of a skip-lot sampling plan, unlike the traditional skip-lot plans which has the same reference plan in all phases. The performance measures of the proposed plan are derived using the power series approach. A designing methodology to determine the optimal parameters for the plan using the unity value approach is also described with the help of a numerical illustration. Behaviour of the operating characteristic curves for varying set of parameters are also analysed for the plan. Comparison of the proposed plan is done between the conventional plans using performance measure values and graphical representations. This analysis shows that the new plan is able to effectively optimize the preferences of producer and consumer simultaneously, where the traditional plans fail to. The analysis is supported with the help of graphical representations and tabulated values.

Keywords: Quality control, acceptance sampling plan, skip-lot sampling plan, operating characteristic curve, producer’s risk, consumer’s risk, unity value approach.

1. Introduction

As we navigate through the radical technological era, and with the help of online platforms to purchase products, the options for the customers to choose from different brands is wide. Then quality becomes the important factor which customers choose to select the product. This led companies to put maximum effort in improving the quality and reach the peak level. Statistical Quality Control (SQC) provides statistical techniques to effectively monitor, maintain and improve the product quality. Acceptance sampling is one of the key techniques used for inspecting the final lot of products and it classifies them as confirming and non-confirming based on the sample inspection. In manufacturing industries, this lot inspection is used to evaluate the quality of the outgoing products; a government agency can inspect the food safety by inspecting the lots and ensuring the components are within the legal limits; and in an international trade, the purchased goods are accepted or rejected based on the lot inspection done on the samples. Since the binary decision about the products is taken based on the random samples, there exists some risk for committing errors. The type-1 error, also known as producer's risk which is the probability of rejecting a good lot and type-2 error, also known as consumer's risk which is the probability of accepting a bad lot. The optimal quality levels for the sampling plans are found out using these producer's and consumer's risk and are referred as acceptable quality level (AQL) and limiting quality level (LQL).

The acceptance sampling plans are generally classified into variable sampling plans and attribute sampling plans. Variable sampling plans is used to inspect the measurable quality parameters where the attribute sampling plans is applied to inspect the non-measurable quality parameters. Many attribute sampling plans have been developed as per the needs in the industry. Single sampling plan (SSP), double sampling plan (DSP), multiple sampling plan are the conventional attribute sampling plans used. Many modifications have been made to the traditional sampling plans according to the need for more reliable and less costly plans. More detailed studies about the Accepting sampling plans have been conducted by [12]. The first skip-lot sampling plan (SkSP-1), introduced by [6], is one of the acceptance sampling techniques. This plan, where individual lots are being inspected, was derived from the continuous sampling plan of type 1 (CSP-1) where the inspection happens for each unit. The SkSP-1 allows to skip some lots from the inspection if the preceding series of lots are reflecting a good quality with minimum defectives. This approach provides significant advantage in terms of reducing inspection cost and time. However, the lack of an inspection plan was a drawback for the primary plan. Then the skip-lot sampling plan was modified by [10], and introduced skip-lot sampling plan of type 2 (SkSP-2), by incorporating reference plans such as SSP or DSP. This introduction of the reference plans assisted to consider the preference of producer and consumer in the inspection and allowed flexibility in the approach to sample each lot depending on situations. SkSP-2 plan is a widely accepted sampling plan and has been adapted with many modifications and applications in current studies also. An attribute SkSP-2 plan with SSP as the reference plan with zero acceptance number was developed by [15]. At a later stage, [17] presented the SkSP-2 with the DSP as the reference plan to aim for less inspection cost by reducing the sample size. An optimal method to determine the plan parameters and to design the attribute SkSP-2 was done by [1]. The SkSP-2 with DSP as the reference plan under the binomial distribution was designed and investigated by [5]. Later [2] proposed the SkSP-2 with Intervened Poisson distribution (IPD) in the SSP as the reference plan which can be used when there are interventions in the process of production. The concept of resampling was assimilated to the two level SkSP-2 by [7] and the performance measures where derived using Markov chain formulation.

When the principles of CSP-2 was incorporated with that of skip-lot sampling plans, [13] developed a new plan as SkSP-3 with SSP as the reference plan. Later [16] studied the SkSP-3 with SSP as the reference plan with a characteristic of zero acceptance number and used the Markov chain approach to derive performance measures and. A SkSP-3 plan using the SSP as the reference plan was proposed, along with cost models for its economic design by [4]. Thereafter [3] developed the SkSP-3 with the IPD as the reference plan and designed the corresponding methodology also. The concept of resampling technique was introduced into skip-lot sampling plans by [8] and developed SkSP-3-R by deriving the performance measures using the Markov chain method. In a subsequent study, [14] introduced tightened single-level SkSP plan by tightening the inspection with stern limitations. They studied the performance measures of this TSkSP-3 using the Markov chain method and compared them with that of conventional plans.

The method of skip-lot inspection plan is to conduct the inspection in two phases, namely normal inspection and skipping inspection. The lots will be continuously checked during the normal inspection and after attaining a level of quality, skipping inspection is started. During the skipping inspection, the examination will be done only to fraction of lots while others are skipped. The skipping inspection continues until a defective is found, which then will start again from normal inspection. In all of the conventional skip-lot plans, both the normal and skipping inspection is done using same reference plan. This results in certain disadvantages for either the producer or the customer. When a skip-lot plan uses SSP as the reference plan in both inspections, some lots with acceptable level of quality is getting rejected due to the low acceptance rate for low proportion defectives. If the skip-lot plan uses DSP as the reference plan in both inspections, some of the lots with rejectable quality level will be accepted due to the high acceptance probability for high proportion defectives. The former plan favours the customer more in the inspection and latter one favours the producer more. This article proposes a new approach where the skip-lot sampling plan of type 3 can be conducted where the normal inspection and skipping inspection is having multiple and different reference plans and is termed as Multi Reference Skip-lot Sampling plan of type 3 (MR-SkSP-3). Here, the usage of two different reference plan for the two phaes of inspection will optimize the preference between the producer and consumer. Also, the proposed plan minimises the inspection cost and maintain high quality than the conventional plans. The article is followed as: Section 2 gives an introduction to the basic sampling plans which are required for the study. Section 3 consists of the designing and operating procedure for the new MR-SkSP-3 plan. Section 4 and section 5 includes the determination of optimal parameters for the proposed plan and a numerical example for the parameter determination, respectively. OC curve analysis and the influence of parameters of the proposed plan are studied in the Section 6. Also, the proposed plan is compared with that of conventional plans using the OC curves and tables of performance measures in the Section 7. In section 8, the key remarks and insights are provided in the conclusion.

2. OPERATING PROCEDURE OF THE SAMPLING PLANS.

2.1. SINGLE SAMPLING PLAN

The single sampling plan (SSP) which is considered the basic plan of acceptance sampling plan for attributes will sentence the products by examining a single sample from the lot and disposition will be decided based on the information from the sample. The acceptance or rejection of the lot will be based on whether the number of defects found in the sample of size n, is greater than the acceptance number c or not. The SSP under Poisson distribution was designed by [9] and the plan parameters were also derived. Afterward, [12] gave much more insight into the approach of using the unity value method for finding the sampling plans on Poisson distribution.
The probability of acceptance for the single sampling plan under the Poisson distribution is as follows;

$$P_a(p) = \sum_{x=0}^{c} \frac{e^{-\lambda}\lambda^x}{x!} \tag{1}$$

2.2. DOUBLE SAMPLING PLAN

The double sampling plan (DSP) is the modification of the SSP when the decision about the lot, whether to accept or reject it, is made from two different samples instead of a single sample. In DSP, a second sample is inspected when the first sample of size n_1 is not accepted because the defectives were more than the acceptance number c_1, but does not exceed the rejection number. In order to accept the lot after examining the second sample of size n_2, the number of defectives of both samples should be not greater than the c_2, which is the cumulative acceptance number for the whole. The tables for selecting DSP according to the MIL-STD-105D were developed by [11]. The use of DSP instead of the SSP will reduce the average sample number (ASN) of the plan which will keep the inspection cost and effort at the minimum.
The probability of acceptance for the double sampling plan under the Poisson distribution is as follows;

$$P_a(p) = \sum_{x=0}^{c_1} \frac{e^{-\lambda_1}\lambda_1^x}{x!} + \sum_{x=c_1+1}^{c_2} \sum_{y=0}^{c_2-x} \frac{e^{-\lambda_1}\lambda_1^x e^{-\lambda_2}\lambda_2^y}{x!\,y!} \tag{2}$$

2.3. SKIP-LOT SAMPLING PLAN -3

The Skip-lot Sampling plan -3 (SkSP-3) is one among the cumulative sampling plans and can be applied for a continuing stream of production from a trusted distributor. The plan is based on the principles of the continuous sampling plan (CSP-2) which is applied for the continuous inspection of products. Using the reference plan mentioned, the lots will be applied with 100% inspection until i succeeding lots are determined to be defect-free. After that, the skipping inspection is performed over the reference plan on a small fraction f of the total lots. When a lot is rejected during the skipping inspection, the proceeding k consecutive lots are inspected. If all the k lots are accepted, the skipping inspection will be continued, otherwise the inspection will start again from normal inspection.
The probability of acceptance for the Skip-lot Sampling plan -3 is as follows;

$$P_a(p) = \frac{fP + (1-f)P^i(2-P^i)}{f + (1-f)P^i(2-P^i)} \tag{3}$$

3. DESIGNING OF MULTI-REFERENCE SKIP-LOT SAMPLING PLAN OF TYPE 3 (MR-SKSP-3)

The SkSP-3 is a modification when the principles of CSP-2 is introduced to the skip-lot sampling procedure. It is performed in two inspection phases, the normal and skipping inspection as like all other skip-lot plans. The SkSP-3 plan conducts one more inspection when there is a rejection in the skipping inspection.

The MR-SkSP-3 is introduced here with the provision of having two different reference plans in the two phases of the plan. The MR-SkSP-3 plan has the single sampling plan as the reference plan in the normal inspection and double sampling plan as the reference plan in the skipping inspection. The inspection procedure of the proposed plan is as given in Figure 1.

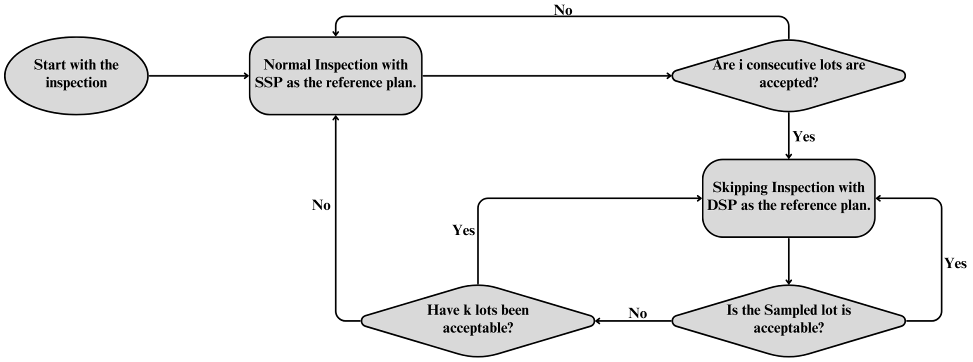

Figure 1: *Operating procedure for the MR-SkSP-3 plan*

The proposed MR-SkSP-3 plan will be having the parameters such as sample size n_1 for single sampling and n_1 and n_2 for double sampling, acceptance number c_1 for single sampling and c_1 and c_2 for double sampling, clearance number i for normal inspection, fraction of lots inspected f and clearance number k for skipping inspection.

A plan is assessed based on the performance measures and can be derived in various methods. The measures for the SkSP-2 using the power series approach were deribed by [10]. He derived the operating characteristic (OC) function for the plan by finding the number of expected lots in the both phases of inspection, normal and skipping inspection. Similarly for the MR-SkSP-3 plan, the expected number of lots during the normal inspection is:

$$U = \frac{1 - (P_{\text{SSP}})^i}{(P_{\text{SSP}})^i(1 - P_{\text{SSP}})} \tag{4}$$

Also in normal inspection, the expected number of failure sequence before having i consecutive accepted lots is:

$$G = \frac{1 - (P_{\text{SSP}})^i}{(P_{\text{SSP}})^i} \tag{5}$$

Likewise, during the skipping inspection, the expected number of lots before the first rejection occurs is:

$$V = \frac{2 - P_{\text{DSP}}^k}{f(1 - P_{\text{DSP}})(1 - P_{\text{DSP}}^k)} \tag{6}$$

where the P_{SSP} is the probability of acceptance of single sampling plan (SSP) and P_{DSP} is that of double sampling plan (DSP).

From these, the probability of rejection, P_r, of the MR-SkSP-3 plan is defined as,

$$P_r = \frac{\text{expected number of lots rejected}}{\text{expected number of lots submitted}} = \frac{G+1}{U+V} \tag{7}$$

Then, the probability of the acceptance, P_a, is given by,

$$P_a = 1 - P_r = 1 - \frac{G+1}{U+V} \tag{8}$$

There are many combinations of plan parameters for designing a sampling plan according to the specified requirements. The choice of selecting the combination of parameters for the plan depends upon the selection of the average sample size. During the inspection, the decision about the lot whether to accept or reject is made from this average number of sample units per lot. The average sample number (ASN) of the MR-SkSP-3 plan is given by:

$$ASN = \frac{ASN_{\text{SSP}} \cdot U + f \cdot ASN_{\text{DSP}} \cdot V}{U+V} \tag{9}$$

4. OPTIMAL PARAMETER SELECTION FOR MR-SKSP-3 PLAN.

For an effectively designed sampling plan to get best outputs, it should simultaneously fulfil the interest of both customer and producer. The consumer wants to avoid the acceptance of lots with bad quality, while the producer wants to minimise the rejection of lots with acceptable quality. The both can be done by minimising the type-1 and type-2 errors, also known as, producer's risk (α) and consumer's risk (β). In order to achieve this, the parameters are selected while the OC curve pass through the points, $(p_1, 1-\alpha)$ & (p_2, β). Here the level p_1 is the AQL associated with the producer's risk and p_2 is the LQL associated with the consumer's risk. In an OC curve the point $(p_1, 1-\alpha)$ will safeguard the interests of the producer and point (p_2, β) ensures the customer's interest. Hence the optimal parameters for the sampling plan can be determined when the OC curves pass through these points, so that the following inequalities are satisfied for the specified strength.

$$P_a(p_1) \geq 1 - \alpha \tag{10}$$

$$P_a(p_2) \leq \beta \tag{11}$$

Under the Poisson probability model for the expected defective per lot, the parameters can be obtained using the unity value approach. The np_1 & np_2 values which will satisfy the equations (10) and (11) are known as the unity values. The unity values are computed for different combinations of the parameters *f, i,* c_1 *&* c_2 , and is tabulated in Table 1 along with their probability values. It also contains the operating ratio, *(R=* np_2/np_1*)* , for the values ($\alpha = 0.05$ & $\beta = 0.10$). Using the operating ratio, we can differentiate the sampling plans. The selection of optimal parameters is presented detailed with a numerical illustration in next section.

Table 1: *Unity values & operating ratio for the MR-SkSP-3.*

f	i	k	c_1	c_2	P_a							$R = n_{p2}/n_{p1}$ (α=0.05, β=0.10)
					0.99	0.95	0.75	0.50	0.25	0.10	0.05	
0.25	5	2	1	5	1.3197	1.5264	1.8387	2.1246	2.7460	3.8805	4.7251	2.5423
0.33					1.2823	1.4877	1.7955	2.0837	2.7338	3.8806	4.7251	2.6085
0.50					1.2398	1.4353	1.7362	2.0300	2.7204	3.8796	4.7250	2.7029
0.67					1.2033	1.3988	1.6954	1.9929	2.7131	3.8790	4.7250	2.7732
0.75					1.1910	1.3842	1.6790	1.9781	2.7106	3.8787	4.7249	2.8021
0.25	4	2	1	5	1.3824	1.6163	1.9761	2.2886	2.8457	3.8875	4.7259	2.4053
	6				1.2668	1.4537	1.7331	2.0065	2.7079	3.8784	4.7249	2.6680
	8				1.1866	1.3398	1.5777	1.8477	2.6935	3.8772	4.7248	2.8938
	10				1.1096	1.2563	1.4660	1.7545	2.6924	3.8893	4.7261	3.0958
	12				1.0598	1.1884	1.3812	1.7069	2.6923	3.8893	4.7261	3.2726
0.25	5	1	1	5	1.3733	1.6060	1.9143	2.1861	2.7604	3.8802	4.7250	2.4161
		3			1.2758	1.4841	1.8006	2.0962	2.7413	3.8807	4.7251	2.6149
		5			1.2336	1.4362	1.7600	2.0706	2.7382	3.8808	4.7251	2.7021
0.25	5	2	0	6	1.1354	1.2561	1.4179	1.5376	1.7204	2.3081	2.9943	1.8375
			1		1.5169	1.7244	2.0101	2.2566	2.7765	3.8797	4.7250	2.2499
			2		1.8373	2.1653	2.6306	3.0727	3.9578	5.3155	6.2826	2.4549
			3		2.2293	2.6616	3.3637	4.0130	5.1439	6.6755	7.7447	2.5081
			4		2.6585	3.2981	4.2236	5.0235	6.3120	7.9894	9.1479	2.4224
0.25	5	2	0	1	0.2652	0.3958	0.6210	0.8689	1.4076	2.3009	2.9942	5.8131
				2	0.4277	0.5603	0.7560	0.9554	1.4191	2.3011	2.9943	4.1071
				3	0.6103	0.7295	0.9110	1.0754	1.4461	2.3016	2.9943	3.1552
				4	0.7858	0.9034	1.0757	1.2188	1.5034	2.3029	2.9943	2.5490
				5	0.9485	1.0794	1.2456	1.3748	1.5967	2.3041	2.9943	2.1346

5. NUMERICAL ILLUSTRATION

Consider the implementation of MR-SkSP-3 in an inspection process associated with producer's risk (α) at 0.05 and consumer's risk (β) at 0.10. The selection of optimal parameters for the inspection procedure for specified sets of strengths say $p_1 = 0.005$ & $p_2 = 0.02$, can be done using the following steps.

1. The given thresholds set for the inspection are $p_1 = 0.005$ & $p_2 = 0.02$. From this, the operating ratio, R, is found as,

$$R = \frac{p_2}{p_1} = \frac{0.02}{0.005} = 4 \tag{12}$$

2. The unity values and corresponding operating ratios for the plan are given in Table 1. From this, the R-value closest to that of calculated from *(12)* and is found as $R = 4.1071$.

3. The optimal parameter values for the inspection plan are selected from the table corresponding to this R- value. It is found as; $f = 0.25$, $i = 5$, $c_1 = 0$ & $c_2 = 2$.

4. From the table, the unity values for the specified strength of ($\alpha = 0.05$, $\beta = 0.10$), and corresponding to the obtained R-value is found out as $np_1 = 0.5603$ and $np_2 = 2.3011$.

5. From the unity values, the sample size for the inspection is calculated as largest of np_1/p_1 or np_2/p_2. Thus,

$$n = \frac{np_1}{p_1} \text{ or } \frac{np_2}{p_2} = \frac{0.5603}{0.005} \text{ or } \frac{2.3011}{0.02} = 112.06 \text{ or } 115.055 \tag{13}$$

From these values, the largest is chosen, thus the required sample size for the inspection is found out as 115 products. For the MR-SkSP-3 plan, both SSP and DSP is done using same sample size and it is assumed that, for DSP, $n_1 = n_2$. Accordingly, the optimal parameters for the MR-SkSP-3 plan with the given specifications are;

$$f = 0.25,\ i = 5,\ c_1 = 0,\ c_2 = 2\ \&\ n_1 = n_2 = 115.$$

6. OC CURVE ANALYSIS OF THE MR-SKSP-3

The OC function, which is one of the important performance measures of a sampling plan, is a function of the defective proportion, p and other parameters of the plan like c_1, c_2, n, f, i & k. The acceptance probability, P_a of a sampling plan can be found from equation (8), for each value of p. In general, the OC function is considered as a function of proportion defective. The plot of these P_a values along with their p values is the OC curve of the sampling plan. These curves evaluate the efficiency of the sampling plan on how well it will distinguish between acceptable and rejectable lots, reflecting it's discriminating power. Thus, it is important to study the influence of each parameter on the OC curves. Figures (2) to (7) plots several sets of OC curves of the MR-SkSP-3 plan for varying values of c_1, c_2, n, f, i & k.

Figure 2 plots the OC curves for the MR-SkSP-3 plan with fixed c_2, n, f, i & k values and varying values of acceptance number, c_1.

It can be seen that, when the value of c_1 is at minimum, the producer faces difficulty, as the probability for accepting the lots at lower p values are low. But, as the c_1 increases the producer protection improves, due higher acceptance for the high p values. Thus, at low c_1 values, the consumer protection is safe guarded, as the lot with low p values will also get rejected. Figure 3 shows the set of OC curves for the proposed plan by keeping the c_1, n, f, i & k and changing the value of c_2. While increasing the c_2 values, a slight increase in the probability of acceptance is observed only in the low proportion defective case. Beyond this low p levels, the OC curves remain unchanged and overlaps in most cases for all values of c_2. Therefore, changing the values of c_2 has only a minimal impact on the OC curve.

Figure 4 plots the OC curves of varying value for sample size, n and all other parameters kept constant. It can be seen that as n increases, the curves shift to the origin, indicating a lower acceptance probability. As the sample size is increasing, the producer is troubled with the risk of applying 100% inspection more often, leading to higher costs and longer inspection times. The consumer is protected with high values of n, by reducing the probability of accepting defective lots and the producer is safeguarded with smaller values of n for inspection, increasing acceptance probability and reducing inspection effort. Figure 5 shows the OC curves of the plan with varying values of clearance number, i. As it can be seen, when the value of i increases, the probability of acceptance decreases. This indicates that, the consumer is more safeguarded from the higher i values, and producer is protected from low values of i by increasing acceptance rates.

Figure 6 and 7 presents the OC curves by varying values of k and f, respectively, while keeping other parameters constant. A slight increase is observed in the acceptance probability as the values of the parameters k & f are increased. However, the curves of these plots remain nearly identical and overlapping each other, indicating that these parameters have a negligible effect on the acceptance probability. This implies that, adjusting the values of k & f does not significantly affect the overall performance of the OC function of the sampling plan.

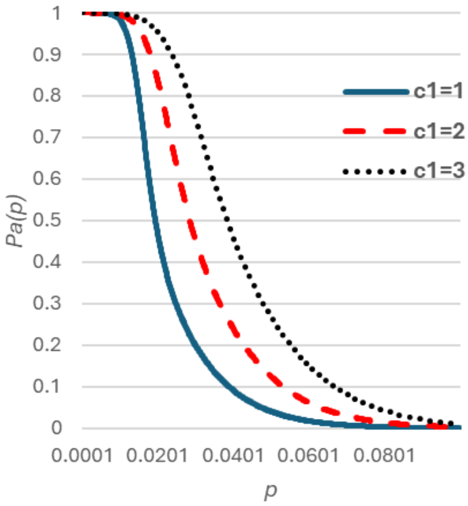

Figure 2: *OC curve with varying* c_1

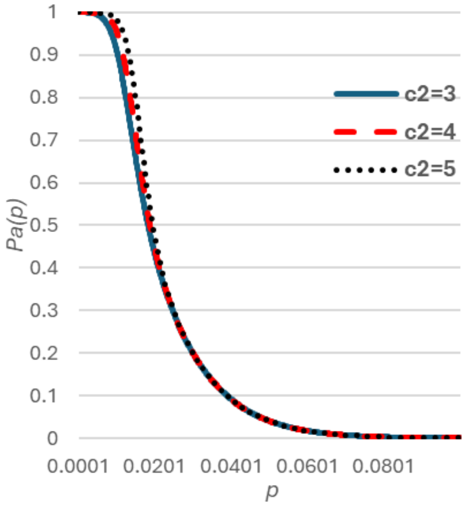

Figure 3: *OC curve with varying* c_2

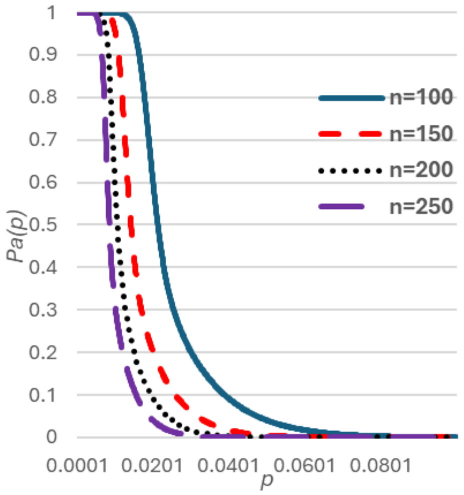

Figure 4: *OC curve with varying n*

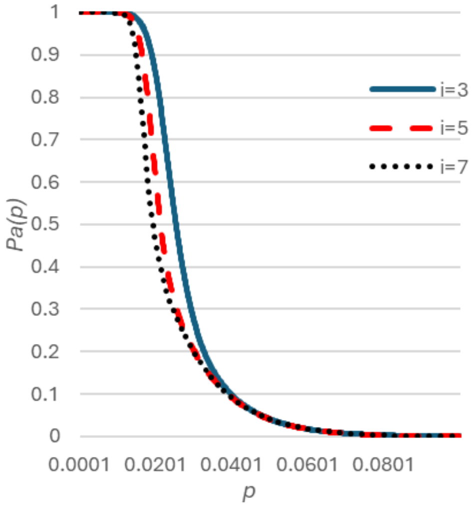

Figure 5: *OC curve with varying i*

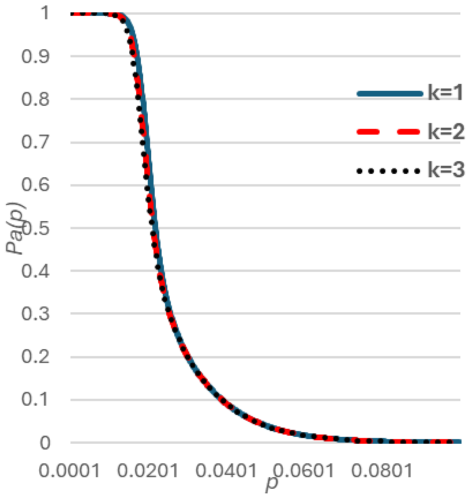

Figure 6: *OC curve with varying k*

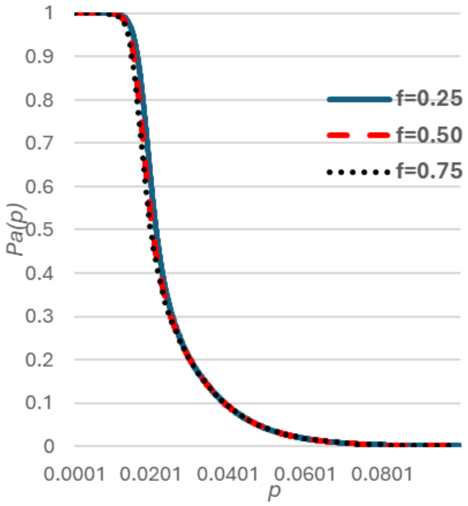

Figure 7: *OC curve with varying f*

7. COMPARATIVE ANALYSIS OF MR-SKSP-3 WITH CONVENTIONAL SKSP-3 PLANS

The comparative analysis of the MR-SkSP-3 plan is done with that of the conventional skip-lot sampling plans of type 3, which use the same reference plan in both normal and skipping phases. This analysis portrays the advantages of the proposed plan of having multiple reference plans in different phases, over the conventional plans. The MR-SkSP-3 plan of $(n = 100, c_1 = 2, c_2 = 5, i = 5, f = 0.25, k = 2)$ is compared with the SkSP-3 plan with SSP as the reference plan (SkSP-SSP) of parameters $(n = 100, c_1 = 1, i = 5, f = 0.25, k = 2)$ and with the SkSP-3 plan with DSP as the reference plan (SkSP-DSP) of parameters $(n_1 = n_2 = 100, c_1 = 2, c_2 = 5, i = 5, f = 0.25, k = 2)$. The analysis is done by comparing the performance measures like OC, ASN, and ATI. Figures (8) and (9) plot the OC curves and ASN curves of the aforementioned plans, respectively. From figure 8, it is observed that the OC curve of the proposed MR-SkSP-3 plan lies in between that of the conventional plans. From this, it is clear that the proposed plan exhibits a combined behavior of the conventional plans. When the proportion defective is low, i.e., when the sampled lots have very few defectives, the MR-SkSP-3 plan behaves like SkSP-DSP. Thus, it has a high acceptance rate for the lots because the plan allows lenient inspection and decreased sampling frequency. Similarly, when the proportion defective is higher, i.e., when the sampled lots have a greater number of defectives, the proposed plan mirrors the behavior of SkSP-SSP. This results in stricter inspection strategies, thus leading to a lower acceptance rate for the lots. Hence, using the proposed MR-SkSP-3 plan provides greater flexibility and adaptivity than the conventional plans in managing inspection effectiveness and strictness.

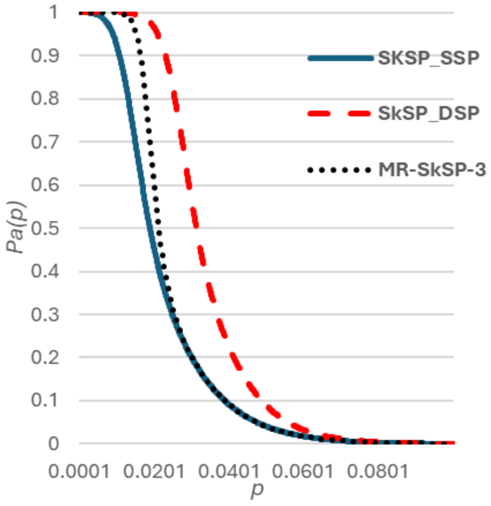

Figure 8: *Comparison of OC curves*

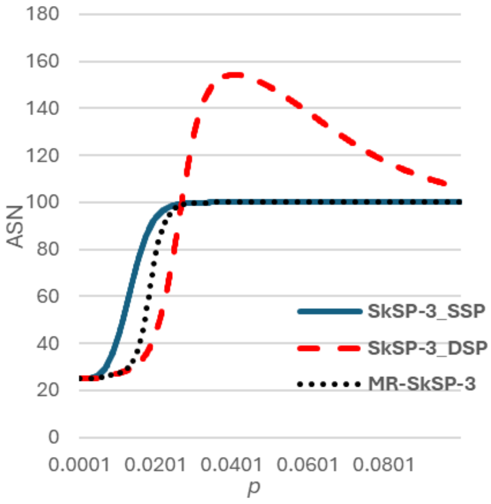

Figure 9: *Comparison of ASN curves*

The Figure 9 depicts the ASN curves of the plans mentioned above. Here also, it is seen that, the proposed MR-SkSP-3 plan is having a combined behaviour of both the conventional plans. When the proportion defective is low, the plan only needs fewer samples to inspect, just like the SkSP-DSP. This helps to reduce the inspection cost and resources during the inspection when the proportion defective is low. In the same situation, the SkSP-SSP is having a higher number of ASN compared to the others. But, when the proportion defective is more, the proposed plan is behaving similar to the SkSP-SSP. This shows that when there are more defectives in the sampled lots, the proposed plan will undergo stringent analysis which require large sample for inspection to control the high defective rate efficiently.

The tabulated values of the measures *OC, ASN, & ATI* for the aforementioned plans are given in tables 2 and 3. From table 2, it can be observed that the same pattern of combined behaviour among the plans, which was also seen in the graphs, is present here. Consider the case where the proportion defective is low, say $p = 0.01$. Under the conditions of SkSP-SSP ($P_a = 0.9251$), the producer's risk (α) for rejecting a good lot is 7.49%. For the same proportion defective, the plans SkSP-DSP ($P_a = 0.9999$) and MR-SkSP-3 ($P_a = 0.9995$) have reduced the producer's risk to 0.01% and 0.05% respectively. When the fraction defective is low, the proposed plan aligns with that of SkSP-DSP. Now, consider the case where the proportion defective is high, say $p = 0.05$. Under the conditions of SkSP-DSP ($P_a = 0.0886$), the consumer's risk (β) for accepting a lot with high defective rate is 8.86%. For the same defective proportion case, the plans SkSP-SSP ($P_a = 0.0404$) and MR-SkSP-3 ($P_a = 0.0404$) show a lower consumer's risk of 4.04% for both. Here, when the fraction defective is high, proposed MR-SkSP-3 plan aligns with that of SkSP-SSP. From this it is clear that when the proportion defective is low, the MR-SkSP-3 plan keeps the producer's risk minimal, avoiding the rejection of acceptable lots, similar to SkSP-DSP. Conversely, when the proportion defective is high, the MR-SkSP-3 plan minimizes the consumer's risk by avoiding the acceptance of rejectable lots, similar to SkSP-SSP. This illustration shows that, while the conventional plans safeguard either the producer or the consumer, the proposed plan protects both parties simultaneously by integrating the properties of the two different reference plans used in normal and skipping inspections. A similar pattern of behaviour is observed in both ASN and ATI.

Table 2: *Measures tabulated against p for SkSP-SSP, SkSP-DSP, MR-SkSP-3*

p	P_a			ASN			ATI		
	SkSP-SSP	SkSP-DSP	MR-SkSP-3	SkSP-SSP	SkSP-DSP	MR-SkSP-3	SkSP-SSP	SkSP-DSP	MR-SkSP-3
0.01	0.9251	0.9999	0.9995	41.6785	27.038	27.1015	0.0093	0.0100	0.0100
0.012	0.8529	0.9994	0.9962	53.5150	28.200	28.6769	0.0102	0.0120	0.0120
0.014	0.7548	0.9979	0.9798	66.7783	29.882	32.3325	0.0106	0.0140	0.0137
0.016	0.6454	0.9939	0.9210	78.7536	32.536	41.3978	0.0103	0.0159	0.0147
0.018	0.5416	0.9846	0.7846	87.6428	36.932	58.3723	0.0097	0.0177	0.0141
0.02	0.4530	0.9652	0.6016	93.3030	44.176	77.3348	0.0091	0.0193	0.0120
0.025	0.2974	0.8267	0.3148	98.7902	81.087	97.3861	0.0074	0.0207	0.0079
0.03	0.2009	0.5656	0.2024	99.8086	128.087	99.7249	0.0060	0.0170	0.0061
0.035	0.1362	0.3542	0.1363	99.9719	149.725	99.9685	0.0048	0.0124	0.0048
0.04	0.0916	0.2254	0.0916	99.9961	154.187	99.9962	0.0037	0.0090	0.0037
0.045	0.0611	0.1427	0.0611	99.9995	152.881	99.9995	0.0027	0.0064	0.0027
0.05	0.0404	0.0886	0.0404	99.9999	149.126	99.9999	0.0020	0.0044	0.0020
0.06	0.0174	0.0323	0.0174	100.0000	138.371	100.0000	0.0010	0.0019	0.0010
0.08	0.0030	0.0041	0.0030	100.0000	117.748	100.0000	0.0002	0.0003	0.0002
0.10	0.0005	0.0006	0.0005	100.0000	106.431	100.0000	0.0000	0.0001	0.0000

Table 3 shows us the performance measure values for the proposed plan along with that of conventional plans, with varying parameter values of the plan. It is seen that, when the values of parameters f, k & i are increasing, the OC and ATI measures are decreasing accordingly. While the ASN measure is increasing in the same instance. This shows that, while increasing the parameter values, the sampling plan is getting more lenient in inspection. Also, the SkSP-SSP is undergoing noticeable change while changing the parameters, while the MR-SkPS-3 plan and SkSP-DSP are only having a slight change in their values. This shows the compatibility of the proposed plan under different conditions.

8. CONCLUSION

The paper presents a multi-reference skip lot sampling plan of type-3, MR-SkSP-3, with the provision of having SSP as the reference plan in normal inspection and DSP as the reference plan in skipping inspection. Further, the inspection procedure of the proposed plan, analysis of plan using power measures and comparison of the plan with other conventional plans are also discussed in the paper. Tables are provided for determination of optimal parameters of plan, and these parameters are derived using the unity value approach. Instructive numerical illustration is also provided for the determination of optimal parameters for MR-SkSP-3. It has been shown that

Table 3: *Measures tabulated against parameter values for SkSP-SSP, SkSP-DSP, MR-SkSP-3*

f	k	i	P_a			ASN			ATI		
			SkSP-SSP	SkSP-DSP	MR-SkSP-3	SkSP-SSP	SkSP-DSP	MR-SkSP-3	SkSP-SSP	SkSP-DSP	MR-SkSP-3
0.25	2	2	0.964025	0.999888	0.999800	29.683313	27.010637	27.018049	0.009640	0.009999	0.009998
		5	0.925085	0.999883	0.999498	41.678496	27.038594	27.101481	0.009251	0.009999	0.009995
		7	0.888416	0.999880	0.999074	52.974359	27.057938	27.218641	0.008884	0.009999	0.009991
0.25	2	5	0.925085	0.999883	0.999498	41.678496	27.038594	27.101481	0.009251	0.009999	0.009995
0.33			0.907007	0.999844	0.999331	51.736290	36.044646	36.117367	0.009070	0.009998	0.009993
0.50			0.877428	0.999766	0.998997	68.192375	54.046544	54.122306	0.008774	0.009998	0.009990
0.67			0.854248	0.999689	0.998665	81.088545	72.034846	72.091557	0.008542	0.009997	0.009987
0.25	1	5	0.946191	0.999940	0.999743	36.979633	27.016154	27.048330	0.009462	0.009999	0.009997
	2		0.925085	0.999883	0.999498	41.678496	27.038594	27.101481	0.009251	0.009999	0.009995
	2		0.914213	0.999829	0.999264	44.099047	27.060065	27.152293	0.009142	0.009998	0.009993

the proposed plan effectively optimizes the balance for the producer and consumer preferences, outperforming conventional plans. While the traditional SkSP-SSP safeguards consumer when the proportion defective is high, it ignored the producer when the proportion defective is less. Also, the traditional SkSP-DSP safeguards the producer when the defective rate is low, but it bypassed the consumer during the high proportion defective cases. The proposed plan was able to overcome this issue by demonstrating exceptional adaptability and efficiency across different proportion levels and safeguards the producer and consumer simultaneously. Skip-lot sampling plan got widely accepted over the time because of its low inspection cost and time while keeping the inspection quality on top. Thus, the proposed study has a significant role by offering a suitable adaption for the situations where the traditional SkSPs fall back. The approach of implementing multiple reference plan allows the optimization of the conventional plans, this can be extended to the other skip-lot plans also.

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