A COMPARATIVE ANALYSIS OF RESPONSE SURFACE METHODOLOGY IN LINEAR PROGRAMMING

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Abstract

A linear mathematical programming approach has been employed to tackle the problem of land allocation in this comparative study. One design of experiment technique used in creating new processes and improving their efficiency is response surface approach. Different search methods have been employed in this study to identify the best distribution of agricultural land. The results of all formulated mathematical programming problem are obtained by using R software and different R functions like mexchalgorithm() and qconalgorithm (), argumants to these problems are b1, b2, b3 were developed. Furthermore, the Branch and Bound method has been utilized to provide the proper integer solution when the problem's solution turns out to be non-integer.

Keywords: Response surface methodology, Linear programming, branch and bound methods, simplex methods.

1. Introduction

Optimization techniques play an important role to explore different management options, to solve diverse decision problems including risk assessment in agriculture systems, watershed development plan, and annual agriculture land cultivation plan and in forest planning. Innovative production techniques are required to keep up with the expanding demand of the world's population. Farmers in the agricultural industry struggle with how to allocate fertilizer and fields, staff to fields, and the ideal amount of plants to have on their farm. Allocation issues often arise in other academic disciplines, including salesperson to area, operator to machinery, and product to product mix. In fact, there are resource allocation issues in every subject of study, where the utilization of resources must be optimized to fulfill the organization's goals experiments in the field are frequently used in agriculture research to understand the numerous factors that affect production and output quality. The most important part of agricultural field trials that may be studied with the use of mathematical programming is planning. To study issues in agriculture, a unique area of mathematical in which an experimental design not only enables one to analyze the experimental data and build empirical models to obtain the most accurate representation of the particular situation, but the present study also shows how experimental design are used to solve linear programming problems. The process by which an experiment is prepared so that the required data is collected & analyzed using statistical techniques to reach a valid & objective conclusion is known as experimental design Montgomery [1].

Response surface methodology is widely used in design of experiments to determine the value of the independent variables that maximizes (or minimizes) the response. Response surface methodology is one of the design of experiment (DOE) methods used in developing new processes and optimizing their performance. RSM is applicable when several input variables potentially influence some quality characteristics or performance measures. Onukogu [2] states that response surface methodology can be seen as a bridge linking experimental design with the subject of our constrained optimization. Hansen and Hurwitz [3] presented the classical non response theory for eliciting responses from a subsample of the non- respondents. Sarvarian & Soleimani [4] used optimization techniques for cropping pattern with limited resources. Lone *et.al* [5] discussed linear programming approach for optimal cropping pattern. Koufie [6] discussed optimum combination of food crop for enterprises. Lone *et.al* [7] used Branch and Bound Method for multi-objective problem. Montgomery a valid conclusion is drawn when the data is collected and analysis using statistical methods about an experimental design. Response surface methodology (RSM) was introduced by Box and Wilson [8]. Onukogu states that response surface methodology can be seen as a bridge linking experimental design with the subject of our constrained optimization. Mayers and Montgomery [9] state that response surface is a mathematical and statistical technique useful for optimizing the stochastic problems. The capability of locating the local optimizer of RSM problem by using more recent line search techniques in just one move has been mentioned in Onukogu and Esele [10]. A detailed information about the optimal experimental design can be found in Pazaman [11], Atkinson and Donev [12]. The optimal solutions of linear programming ploblems was solved through different methods as simplex method and as a polynomial time interior point algorithm given by Karmarkar [13], a modification of karmarkar's interior point algorithm given by Vanderbei *et al.* [14]. Ekezie and Nzenwa [15] developed a minimum exchange algorithm, Osita and Mary [16] developed a quick Convergent Inflow algorithm. In this comparative study different methods like minimum exchange algorithm, a quick Convergent Inflow algorithm, Simplex Method, Interior Point Method and Branch and Bound method has been used to determine optimal allocation of formulated programming problem in agriculture.

2. Response surface methodology

One design of experiment (DOE) technique used in creating new processes and improving their efficiency is response surface approach. RSM is a group of mathematical and statistical methods used in problem modeling and analysis. Although it is more commonly known as the procedure for selecting and fitting a suitable response surface model based on experimental data. The primary objective of RSM is to maximise or minimise the response variable, which is affected by a number of independent (predictor) factors. RSM is particularly useful when numerous input variables may have an impact on certain quality traits or performance metrics. For example, the growth of the plant is affected by a certain amount of water $x_1(say)$ and sunlight $x_2(say)$. With the variations in the amount of water and sunlight, plant can grow with any combination of treatments. When there are continuous variations in treatment combinations RSM is useful for developing and optimizing the response variable. In particular, the response variable (y) denotes the growth of plant is a function of x_1 and x_2 , and is expressed as fallows

$$y = f(x_1 + x_2) + e \tag{1}$$

Where, e represents experimental error which is normally distributed with mean zero and variance σ^2. x_1 and x_2 are independent variables where the response variable(y) depends on them. if expected response is denoted by $E(y) = f(x_1 + x_2) = \eta$,then the surface denoted by $f(x_1 + x_2) = \eta$ is called response surface. The optimization based on response surface is known as experimental optimization. RSM can be viewed from the three major stand points (Cornell, 1990).

- If the system response is fairly well known, the RSM technique is utilised to identify the best value of the response.
- RSM approaches are employed to at least get a better understanding of the entire response system in the event that determining the best (optimum) value is outside the scope of the experiment's resources.
- In place of the complex analysis that takes hours to complete, a comparable response surface in a condensed version can be produced in a few runs.

In general, suppose a set of predictor variables is denoted by $x_1, x_2, \ldots, x_n$ and the unknown response function $f(X)$ is represented by a regression function $Y(X)$ with additive error, thus we write

$$Y(X) = f(x_1, x_2, \ldots, x_n) + e \tag{2}$$

Every significant work on the design of experiments used to address unconstrained optimization problems now includes RSM as a mainstay. The issue of minimizing or maximizing a function in the absence of any constraints is addressed by unconstrained optimization. RSM is a bridge linking the subject of experimental design with constrained optimization The problem is to find the local optimizer $X^* = (x_1^*, x_2^*, \ldots, x_n^*)'$ such that

$$F(X^*) \begin{cases} \leq f(X) \ if \ f(X) \text{ is minimized} \\ \geq f(X) \ if \ f(X) \text{ is maximized} \end{cases} \tag{3}$$

2.1 Line search techniques

Line search techniques are used to solve problems in linear programming, which improves performance. The line search approach finds a descent direction along which the objective function will be reduced and then computes a step size that determines how far should move along the direction. It also requires the best decision on: The starting point of the sequence, the regression model, and the design of experiments. Therefore, it is important to consider how these factors interact to affect performance and the most effective method for determining each. Additionally, modern line search methods are employed to find the RSM problem's local optimizer.
The line search equation is:

$$X_{i,1} = \overline{X} - p_i d_i \tag{4}$$

Where, $X_i = (X_{i1}, \ldots, X_{in})'$ is the starting point.
$d_i = T_i \, g_i$ is the direction of search and
p_i is the step size that determines how far X should move along the direction.
T_i is matrix of transformation
g_i is gradient vector at j^{th} step.
The basic iterative steps of a line search procedure are:

- Perform an experiment to determine the direction of search d_i . Move to the point

$$X_{i+1} = \overline{X} - p_i d_i \tag{5}$$

- Stop if local optimizer $X_1 = X^*$, otherwise set 1=0 and return to step (a).

This paper uses the notion of optimum experimental design to solve a constrained optimization issue involving linear programming approach in agriculture using the line search technique. The search's goal is to identify the response surface's optimum with the fewest possible iterations and the least amount of computing required during each iteration. In this comparative study different methods have been used to determine the optimal allocation.

3. Linear programming

Linear programming (LP) is a mathematical programming technique applicable for the solution of problems in which the objective function and the constraints appear as linear functions of the decision variables. LP is a mathematical technique useful for allocation of 'scarce' or 'limited' resources to several competing activities on the basis of a given criterion of optimality.

The general LPP with n decision variables and m constraints is stated in the following form:

$$\begin{aligned} &\min/\max\ Z = C_1X_1 + \ldots + C_nX_n \\ &\textit{subject to} \\ &\quad a_{11}X_1 + \ldots + a_{1n}X_n\ (\le, =, \ge)\ b_1 \\ &\quad a_{21}X_1 + \ldots + a_{2n}X_n\ (\le, =, \ge)\ b_2 \\ &\quad \ldots \qquad\qquad \ldots \\ &\quad a_{m1}X_1 + \ldots + a_{mn}X_n\ (\le, =, \ge)\ b_m \\ &\quad X_1\ \ldots, X_n \ge 0 \end{aligned} \tag{6}$$

In matrix notation, it can be expressed as

$$\begin{aligned} &\min/\max\ Z = C^T X \\ &\text{subject to} \\ &\qquad AX\ (\le, =, \ge)\ B \\ &\qquad X \ge 0 \end{aligned} \tag{7}$$

Where, $C^T = (C_1, \ldots, C_n); \mathbf{B}^T = (b_1, \ldots, b_m), X^T = (X_1, \ldots, X_n)$ are row vectors and $A = (a_{ij})$ is an $m \times n$ matrix.

4. Similarities between linear programming problem (LPP) and RSM

The following are some similarities between LPP and RSM:

- In the LPP, the decision variables can only have non-negative values, whereas the independent variables in the RSM can have any value. Both are regarded as quantitative.
- Within a finite dimensional area that fulfills the response function, the independent variables in an RSM model are free to take any value. Similar to the feasible zone in LPP, the experimenter region in RSM acts as a constraint and serves the same purpose.
- With the usage of RSM, the experimental region is continuous, negating the necessity to use slack surplus variables or other techniques to solve LPP.

4.1 Minimum Exchange algorithm & Quick Convergent Inflow algorithm

The sequence of steps involved in minimum exchange algorithm & a quick Convergent Inflow algorithm can be found in Ekezie and Nzenwa [16] & Osita and Mary [16].

4.2 Simplex method

Most real – life problems when formulated as an LP model have more than two variables and are too large to be interpreting with the help of graphical solution method. We therefore need a more efficient method known as Simplex method to suggest an optimal solution for such problems. The simplex method examines the extreme points in a systematic manner, repeating the same set of steps of the algorithm until an optimal solution is found. It is for this reason that it is also called the iterative method.

4.3 Interior point method

In simplex-based methods for solving LPPs the computational time and efforts increase exponentially with the increase in the dimensions of the problem. Such methods therefore become slow and computationally expensive for large problems. On the other hand, presented interior point algorithm is a polynomial time algorithm, this new approach finds the optimum solution by starting at a trial solution and shooting through the interior of the feasible solution space. Although the advantage of this method is that it is used for solving large LPPs. The given problem is solved using R programme package based on modification of interior point algorithm. This algorithm makes it easier to conceptualize and leads to computational simplicity. Graphically the solution of a two-dimensional linear programming problem is also shown at the sequence of points calculated by the interior point method.

4.4 Branch and bound method

A well-structured, systematic search of the space of all feasible solutions to constrained optimization problems with a finite number of feasible answers is what the Branch and Bound approach entails. Integer programming issues including the scheduling problem, the traveling salesman problem, the plant location problem, the assignment problem, etc. have been partially solved using branch and bound algorithms. This method is helpful for LP issues when there are numerous viable solutions, making it economically unfeasible to enumerate all of them. If a sub problem's answer does not result in an ideal integer solution, a new sub problem is chosen for branching until an optimal solution is reached at the point where no more sub problems may be formed.

5. Numerical Example of Linear programming problem

A farmer has more than 4 hectares farm on which he plants two crops: corn and rice. For each hectare of rice planted, his expense is 3 thousand units and for each hectare of corn planted, his expense is Rs 2 thousand units. Now, each hectare of corn requires one hundred cubic feet of space for storage and yields a profit of two thousand units; each hectare of rice requires two hundred cubic feet space for storage and yields a profit of Rs 1 thousand units. But the storage space required for rice is twice than corn. The farmer is expected to get more than 6 thousand units of profit using his past experience. Also, the expense on the corn is twice than that of rice with farmer is expect to have more than 6 thousand units of money. Now, the farmer is in such a situation he wants to know how many hectares of each crop should be planted in order to minimize his expense. What will be his minimum expense if he fallows this strategy?

Let x_1 and x_2 be the area required in hectares for planting rice and corn plants respectively. The mathematical formulation of the LPP is given below:

$$\begin{aligned}
&\min \ f(x) = 3x_1 + 2x_2 \\
&\textit{subject to} \\
&\qquad 2x_1 + x_2 \geq 6 \\
&\qquad x_1 + x_2 \geq 4 \\
&\qquad x_1 + 2x_2 \geq 6 \\
&\qquad x_1, x_2 \geq 0
\end{aligned} \tag{8}$$

The solution of the above linear programming problem is obtained using R software and the programmers were developed in R for this particular problem.

5.1. Minimum Exchange algorithm

```
>mexchalgorithm=function(b1,b2,b3)
{
>obj.coef<-matrix(c(3,2),nrow=1,ncol=2)
>const1<-matrix(c(2,1),nrow=1)
> const2<-matrix(c(1,1),nrow=1)
>const3<-matrix(c(1,2),nrow=1)
>b1<-6
>b2<-4
>b3<-6
>E1=matrix(c(3,2,3,0,0,2,3,4),nrow=4,ncol=2)
>average1<-matrix(c(mean(E1[,1]),mean(E1[,2])))
>Mo<-t(E1)%*%E1
>Mo<-(t(E1)%*%E1)/2
>y<-matrix(c(9,10,15,8),ncol=1)
>b<-((solve(Mo)%*%t(E1)))%*%y
>d1<-b[1,]/(sqrt(sum(b[1,]^2,b[2,]^2)))
>d2<-b[2,]/(sqrt(sum(b[1,]^2,b[2,]^2)))
>d<-matrix(c(d1,d2),ncol=1)
>p1<-((const1%*%average1)-b1)/(const1%*%d)
>p2<-((const2%*%average1)-b2)/(const2%*%d)
>p3<-((const3%*%average1)-b3)/(const3%*%d)
>po<-min(p1,p2,p3)
>x1<-average1-(d%*%po)
>obv1<-(obj.coef%*%x1)
>reducedE1<-E1[-3,]
>E2<-insertRow(reducedE1,2,x1)
>average2<-matrix(c(mean(E2[,1]),mean(E2[,2])))
>M1<-(t(E2)%*%E2)/2
>x2<-average2-(d%*%(min(p2,p3)))
>obv2<-(obj.coef%*%x2)
>reducedE2<-E2[-3,]
>E3<-insertRow(reducedE2,3,x2)
>average3<-matrix(c(mean(E3[,1]),mean(E3[,2])))
>M2<-(t(E3)%*%E3)/2
>x3<-average3-(d%*%p3)
>obv3<-(obj.coef%*%x3)
>optsol<-x3
```

```
>list(E1=E1,average1=average1,Mo=Mo,y=y,b=b,po=po,x1=x1,obv1=obv1,reducedE1=reducedE1,E2=E2
,average2=average2,M1=M1,x2=x2,obv2=obv2,reducedE2=reducedE2,E3=E3,average3=average3,M2=
M2,obv3=obv3,optsol=optsol)
}
>mexchalgorithm(6,4,6)
```

5.2. A quick Convergent Inflow Algorithm

```
>qconalgorithm=function(b1,b2,b3)
{
>obj.coef<-matrix(c(3,2),nrow=1,ncol=2)
>const1<-matrix(c(2,1),nrow=1)
>const2<-matrix(c(1,1),nrow=1)
>const3<-matrix(c(1,2),nrow=1)
>b1<-b1
>b2<-b2
>b3<-b3
>E1=matrix(c(1,4,4,1),nrow=2,ncol=2)
>average1<-matrix(c(mean(E1[,1]),mean(E1[,2])))
>Mo<-E1%*%E1
>g<-matrix(c(3,2),nrow=2,ncol=1)
>zo<-Mo%*%g
>do<-(solve(Mo))%*%zo
>do1<-do%*%(1/(sqrt(sum(do[1,]^2,do[2,]^2))))
>po1<-((const1%*%average1)-b1)/(const1%*%do1)
>po2<-((const2%*%average1)-b2)/(const2%*%do1)
>po3<-((const3%*%average1)-b3)/(const3%*%do1)
>po<-min(po1,po2,po3)
>x1<-t(average1-(do1%*%po))
>obv1<-(obj.coef%*%t(x1))
>E2<-insertRow(E1,3,x1)
>average2<-matrix(c(mean(E2[,1]),mean(E2[,2])))
>M1<-t(E2)%*%E2
>z1<-M1%*%g
>d1<-(solve(M1))%*%z1
>do2<-d1%*%(1/sqrt(sum(d1[1,]^2,d1[2,]^2)))
>po11<-((const1%*%average2)-b1)/(const1%*%do2)
>po12<-((const2%*%average2)-b2)/(const2%*%do2)
>po13<-((const3%*%average2)-b3)/(const3%*%do2)
>popt1<-min(po11,po12,po13)
>x2<-average2-(do2%*%popt1)
>obv2<-(obj.coef%*%x2)
>optimality_check<-(x2-t(x1))
>optval<-x2
>optsol<-obv2
>list(E1=E1,average1=average1,Mo=Mo,g=g,zo=zo,do=do,do1=do1,po=po,x1=x1,obv1=obv1,E2=E2,ave
rage2=average2,M1=M1,z1=z1,d1=d1,x2=x2,obv2=obv2,
optimality_check=optimality_check,optval=optval,optsol=optsol)
}
>qconalgorithm(6,4,6)
```

6. Simplex method

```
>simplex.lp<-
lp(objective.in=c(3,2),const.mat=matrix(c(2,1,1,1,1,2),nrow=3),const.rhs=c(6,4,6),const.dir=c(">=",">=",">
="))
>simplex.lp
Success: the objective function is 10
>simplex.lp$solution
[1] 2 2
```

6.1. Interior point method

```
>Obj.coef<- c(3,2)
> const1<- c(2,1)
> const2<- c(1,1)
> const3<- c(1,2)
>interior_point(-1,c=Obj.coef, M=rbind(const1,const2,const3), bM = c(6,4,6))
$`The optimum value of Z is`
[1] 10.00001
$`The optimal solution X`
[,1]
[1,] 2.000002
[2,] 2.000001
$`Number of iterations`
[1] 10
>solve2dlp(-1,c=Obj.coef, M=rbind(const1,const2,const3), bM = c(6,4,6))
```

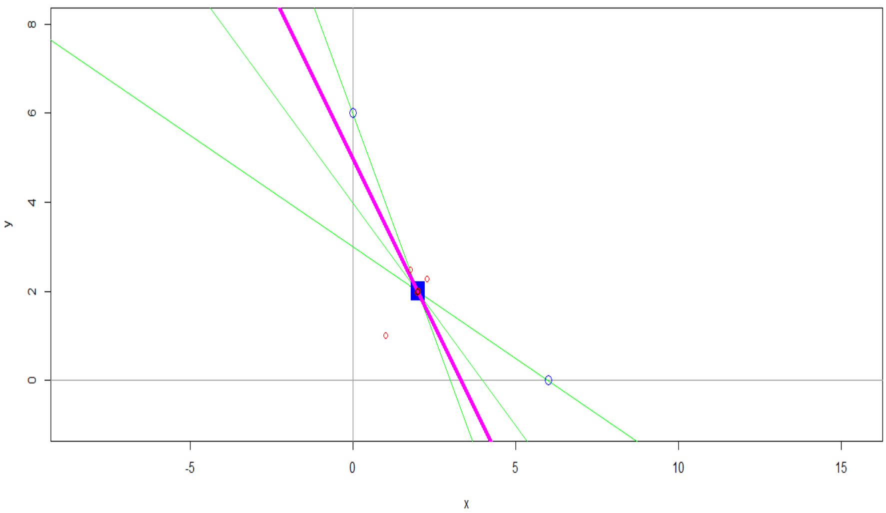

Fig. 1: *Graphical solution to the linear programming problem*

The graphical solution to the linear programming problem in shown in fig. 1. Lines corresponding to the constraints are coloured in green, the corner points in blue and the optimal point in magenta (with the objective function in blue). The sequence of interior points is drawn in red.

6.2. Branch and bound method

```
>integ.lp<-lp(objective.in=c(3,2),const.mat=matrix(c(2,1,1,1,1,2),nrow=3),const.rhs=c(6,4,6),const.dir=c(">=",">=",">="),int.vec=c(1,2))
>integ.lp
Success: the objective function is 10
>integ.lp$solution
[1] 2 2
```

Table 1: *Summary statistics for the minimization problem*

Method used	Optimal solution	Optimal value	Iterations
Minimum Exchange algorithm	1.406417 1.890737	8.000725	3
Quick Convergent Inflow algorithm	1.9375 2.1250	10.0625	2
Simplex method	2 2	10	2
Interior point method	2.000002 2.000001	10.00001	10
Branch and bound method	2 2	10	0

7. Result & Discussion

The summary provides information on the performance of the algorithms used to solve the above minimization linear programming problem. It provides us information about the optimal solution, optimal value and number of iterations. It can be easily seen from the table 1, that the minimum value of the objective function is provided by using the Minimum Exchange algorithm method to solve the LPP with three iterations. Even though the simplex method, Interior point method and Branch and bound method provides us the objective function value same but more than obtained from Minimum Exchange algorithm. Simplex method takes two iterations to reach the optimal value but Interior point method takes 10 iterations. As it has been already mentioned that simplex method become slow and computationally expensive for large problems as compared to interior point algorithm which finds the optimum solution by starting at a trial solution and makes it easier to conceptualize and leads to computational simplicity for solving large LPPs. Further, Quick Convergent Inflow algorithm takes only two iterations provides the optimum value more than the rest of the methods.

Now one of the important facts about these methods is that, for practical purposes if the solution is non-integer then the LPP is solved using Branch and Bound method instead of rounding the non-integer sample sizes to the nearest integral values. However, in some situation for small samples the rounded off allocation may become infeasible and non-optimal. Thus, the advantage of Branch and Bound method over rest of all methods, as it provides us an integer optimal allocation.

8. Conclusion

In conclusion, the study demonstrates that the use of the line search algorithm provides a more effective and convenient approach for solving linear programming problems, particularly when the objective is to determine the minimum value. Compared to other traditional algorithms, the line search method shows superior performance in terms of both accuracy and computational efficiency.

Furthermore, the findings suggest that the experimental design procedure employed in this work is relatively straightforward to implement and consistently yields lower minimum values than alternative techniques. This simplicity, coupled with its effectiveness, makes it a promising strategy for future applications in optimization and statistical computing.

Additionally, the study has led to the development and implementation of customized R functions such as mexchalgorithm() and qconalgorithm(), which are tailored to handle specific problem scenarios. These functions utilize arguments labeled as b1, b2, and b3, which represent key parameters in the optimization process. The integration of these user-defined functions enhances the flexibility and applicability of the proposed methods, paving the way for further exploration in both theoretical and practical domains of linear programming.

References

[1] Montgomery, D.C. 1976. Design and Analysis of Experiments. John Wiley and Sons, New York.

[2] Onukogu I.B.1997. Foundations of optimal exploration of response surface. Ephrata, Press Nsukka, Nigeria.

[3] Hansen, M.H. and Hurwitz, W.N. 1946. The problem of non response in sample surveys. Journal of American Statistical Association40: 517-529.

[4] Sarvarian, J., & Soleimani, P. (2022). Optimization of cropping pattern, taking into account limited water resources, cultivated area and biodiversity in the Mehran Plain. Water and Irrigation Management, 11(4), 725-737.

[5] Mushtaq, A. Lone, Mir. S. A., Maqbool. S. and Khan. I.(2014). Modeling Crop Pattern System using Linear Programming. Golden Research Thoughts,3(12): 1-4

[6] Koufie, A. (2020). Optimum combination of food crop farm enterprise among smallholder farmers in the Assin north district of Ghana (Doctoral dissertation, University of Cape Coast).

[7] Mushtaq, A. Lone, Mir. S. A., Maqbool. S. and Bhat. M. A. (2015). An integer solution using Branch and Bound Method in Multi-objective stratified sampling design. International Journal of Advanced Scientific and Technological Research,4(5): 172-181.

[8] Box, G.E.P. and Wilson, K.B. 1951. On the experimental attainment of optimum conditions. Journal of the Royal Statistical Society13 :1-45.

[9] Myers, R.H. and Montgomery, D.C. 1995. Response Surface Methodology. John Wiley & Sons, New York.

[10] Onukogu, I.B. and Esele, I.I.1987. Response Surface exploration with spline functions: biased regression. Statistica XLVIT,n,2 : 288-298.

[11] Pazman, A. 1986. Foundation of optimum Experimental Design, D. Riedel Publishing Company.

[12] Atkinson, A.C. and Donev, A.N. 1992. Optimum experimental design.Oxford Science Publication.

[13] Karmarkar, N. 1984. A new polynomial - time algorithm for linear programming. Combinatorica4 : 373-394.

[14] Vanderbei, R.J., Meketon, M.S. and Freedman, B.A.1986. Modificationof Karmarkars's linear oprogramming algorithm. Algorithmica1 : 395-407.

[15] Ekezie, D.D. and Nzenwa, U. 2013. Using experimental design procedure in solving linear programming problem. International Journal of Current Research5(5) : 1093-1100.

[16] Osita, O. and Mary, I. 2013. A quick convergent inflow algorithm for solving linear programming problems. Journal of Statistics Applications and Probability2(2) : 31-42.