OPTIMIZING FUZZY DETERIORATING INVENTORY MODEL WITH TIME DEPENDENT DEMAND AND PARTIAL BACKLOGGING UNDER RESALABLE RETURNS

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Abstract

This paper investigates with the development of a fuzzy inventory model with timevarying demand, deterioration and backlogging with resalable returns. The returns rate, demand and backlogging parameters are taken as trapezoidal fuzzy numbers. Numerical example is given to validate the proposed mathematical model which has been developed for determining the optimal cycle time and optimal total inventory cost and profit. Sensitivity analysis is also carried out to explore the effect of changes in the optimal solution with respect to change in various parameters. The aim of this paper is to develop inventory policies that minimize the total cost so that to get the maximum total profit in both crisp and fuzzy modeling, and comparison of crisp and fuzzy models. Our study focuses on de-fuzzifying the total cost using the signed distance method and comparing it with the crisp model. This inventory model incorporates fuzzy demand and constant holding cost per unit item under the reasonable return policy while considering the impact of deterioration as a linearly increasing function of time. The retailer allows its unsatisfactory costumers to return their products. We assume that the return product will be sold at the same price. Customers are allowed to return the product during any phase of the length of the replenishment cycle. The retailer does not return the full amount to its customers for the returned goods. He offers 80% of the initial amount of the product. The number of returns are assumed to be proportional to demand. The demand is dependent on time. Partial backlogging is an important issue in the inventory theory which is related how to deal with the unfulfilled demand that occurs due to the shortage of stock. In real practice some customers prefers to wait for backorder during the shortage time and some turns to buy from other sellers. The waiting time period for next replenishment determine whether the backlogging would be accepted or not. During the shortage period the longer the waiting time is, the smaller is the backlogging rate would be. So backlogging rate is a variable that depends on the waiting time for next replenishment.

Keywords: Optimization, Deteriorating Inventory, Returns Rate, Triangular Fuzzy Number, and Signed Distance Method.

1. Introduction and leterature survey

The inventory management system is an important aspect for any business firm is to get more profit . An inventory management depends on different parameters such as demand, deterioration, holding cost, shortages, backlogs, inflation and trade credit, etc. In inventory modeling uncertainty often arises due to the concept of fuzziness, which is considered the closest approximation to reality. Return policies offered by the seller attracts the customers. The return policy allows consumers who are unsatisfied with their purchase to return the product and get a refund either in terms of money or some gift vouchers as stated by the company or seller in its policies. In recent years, some researchers have focused on incorporating time-dependent rates, partial backlogging , return policy and deterioration in their models. For instance, De and Rawat [1] developed an a Fuzzy Inventory Model without Shortages using Triangular Fuzzy Number. Saha and Chakrabarti [2] developed fuzzy EOQ model for time-dependent deteriorating items and time-dependent demand with shortages. Jaggi et al. [3] Fuzzy Inventory Model for Deteriorating Items with Time Varying Demand and Shortages. Mahata [4] developed an EPQ-based inventory model for exponentially deteriorating items under retailer partial trade credit policy in supply chain. Saha and Chakrabarti [5] developed fuzzy EOQ model for timedependent deteriorating items and time-dependent demand with shortages, while Dutta and Kumar [6] introduced a partial backlogging inventory model for deteriorating items with time-varying demand and holding cost. Further, Kumar and Rajput [7] developed a Fuzzy Inventory Model for Deteriorating Items with Time Dependent Demand and Partial Backlogging. Sahoo et al. [8] presented a Fuzzy inventory model with exponential demand and time-varying deterioration. Jaggi et al. [9] focused on an inventory model with Inventory decisions for imperfect quality deteriorating items with exponential declining demand under trade credit and partially backlogged shortages. Malik and Garg [10] presented an Improved Fuzzy Inventory Model Under Two Warehouses. Boina et al. [11] presented an inventory model for Two-Storage Fuzzy Inventory Model with Time Dependent Demand and Holding Cost under Acceptable Delay in Payment . Kuppulakshmi et al. [12] introduced Fuzzy Inventory Model for Imperfect Items with Price Discount and Penalty Maintenance Cost.

Recent research has focused on incorporating environmental factors and technology investments into inventory models with deterioration. Yadav and Kumar [13] proposed a model that considers selling price, time-sensitive demand, and carbon emissions under green technology investment. Yadav et al. [14] introduced a two-warehouse model using an interval approach, including preservation technology to manage uncertainty. In another study, Yadav et al. [15] addressed deterioration during storage within a two-warehouse setup, aiming to reduce total costs. They further extended this by including reliability and carbon emission constraints with time-based demand [16]. Similarly, Mahata and Debnath [17] analyzed a price-sensitive inventory system with preservation investment to handle deterioration. Later, they explored the impact of green technology and flexible production on economic production models with carbon emissions [18], showing that cleaner and more flexible systems help reduce environmental costs. Debnath et al. [19] developed an EOQ model under uncertainty using a generalized intuitionistic fuzzy Laplace transform. Their approach helps handle imprecise data more effectively in inventory systems.

2. Methods

2.1. Definitions and Preliminaries

1. **Fuzzy set:** A fuzzy set allows for partial membership in which an element can belong to a set to a certain degree or degree of membership, which is represented by a value between 0 and 1. Mathematically, A fuzzy set defined on a universe of discourse $X = \{x_1, x_2, x_3, \ldots, x_n\}$ is given by $A = \{(x, \mu_A(x)) : x \in X\}$, where $\mu_A(x)$ is the membership function of A given by $\mu_A(x) : X \to [0, 1]$.
2. **Fuzzy Number:** A fuzzy number is a fuzzy set on the real line $\mathbb{R}$, if its membership function $\mu_A(x)$ has the following properties:
   (a) $\mu_A(x)$ is upper semi-continuous.
   (b) There exist some real numbers a_2 and a_3, $a_1 \leq a_2 \leq a_3 \leq a_4$ such that $\mu_A(x)$ is increasing on $[a_1, a_2]$, decreasing on $[a_3, a_4]$ and $\mu_A(x) = 1$ for each x on $[a_2, a_3]$.

(c) $\mu_A(x) = 0$ outside the interval $[a_1, a_4]$.

3. **Triangular Fuzzy Number:** A triangular fuzzy number is specified by the triplet (a_1, a_2, a_3) and defined by its membership function $\mu_A(x) : X \to [0, 1]$ as follows:

$$\mu_A(x) = \begin{cases} 0, & x \leq a_1 \\ \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 0, & x \geq a_3 \end{cases}$$

2.2. Notations and assumptions

The following notations are used for proposed fuzzy inventory model:

Table 1: *Notations used in the proposed fuzzy inventory model*

Symbol	Description
$\eta(t)$	Deterioration rate
$\gamma(t)$	Time-dependent demand
T	Length of one cycle
$Q_1(t)$	Inventory level at time $t \in [0, t_1]$
$Q_2(t)$	Inventory level at time $t \in [t_1, T]$
A	Ordering cost
c	Buying price of an item
c_d	Deterioration cost per item
c_p	Selling price of an item
c_o	Lost sale cost per item
c_s	Shortage cost per item
α	Deterioration coefficient parameter
μ, b	Demand coefficient parameters
h_c	Holding cost coefficient
$R(t)$	Return rate
σ	Return rate parameter
p	Backlogging parameter
TC^o	Total cost per unit time in crisp model
TC^*	Total cost per unit time in fuzzy model
TP^o	Total profit per unit time in crisp model
TP^*	Total profit per unit time in fuzzy model
$\tilde{\mu}, \tilde{b}$	Fuzzy demand parameters
$\tilde{\sigma}$	Fuzzy return rate parameter
$\tilde{p}$	Fuzzy backlogging parameter

The following assumptions have been adopted while developing the fuzzy inventory model:

1. Single item is used in the developed fuzzy inventory model.
2. Lead time is zero.
3. The demand rate is defined as:

$$\gamma(t) = be^{\mu t}$$

4. The inventory deteriorates as a linear function of time and given by: $\eta(t) = \alpha t$
5. It is assumed that customers return in proportion to demand, so return rate is given by:

$$R(t) = \sigma\,\gamma(t), \quad \text{where } 0 \le \sigma < 1$$

6. Shortages are allowed and partial backlogging is at constant rate p, where $0 \le p < 1$.
7. The holding cost per item is constant.
8. Time horizon is finite.

3. Mathematical Formulation of Model

At the start of the cycle, an initial lot consisting of Q_0 units is received. The retailer allows its customers to return the items. The inventory level changes from time $t = 0$ to $t = t_1$ due to time-dependent demand, returned items, and deterioration. From time $t = t_1$ to $t = T$, the inventory level changes due to returned items and partial backlogging.

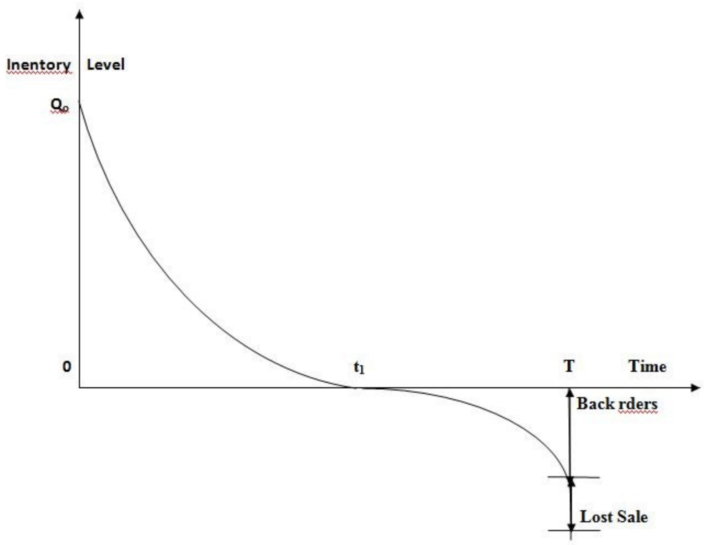

Figure 1: *Inventory depletion with backorders and lost sales.*

The inventory level $Q(t)$ at any time t in the interval $[0, T]$ is represented by the following differential equations:

$$\frac{dQ_1}{dt} = -\gamma(t) + \sigma\,\gamma(t) - \eta(t)\,Q_1(t) \quad \text{for } 0 \le t \le t_1 \tag{1}$$

$$\frac{dQ_2}{dt} = -p\,\gamma(t) + \sigma\,\gamma(t) \quad \text{for } t_1 \le t \le T \tag{2}$$

with boundary conditions: $Q_1(0) = Q_0$, $Q_1(t_1) = 0$, $Q_2(T) = 0$. The solutions of differential equations 1 and 2 with boundary conditions are as follows:

$$Q_1(t) = Q_0\left(1 - 0.5\,\alpha t^2 + 0.125\,\alpha^2 t^4\right) - (1-\sigma)b\left(t - \frac{\alpha t^3}{3} - \frac{\alpha^2 t^5}{60} - 0.5\,\mu t^2 + \mu\alpha t^4/8 + \mu t^3/6\right)$$

$$Q_2(t) = b(\sigma - p)\,\mu^{-1}\left(e^{\mu t} - e^{\mu T}\right)$$

The total cost of the system is the sum of all associated inventory costs calculated as above. Therefore, the total cost per unit time (or total average cost per cycle) is given by:

1. **Ordering cost:**

$$OC = A$$

2. **Purchase cost:**

$$PC = cQ_0$$

3. **Deterioration Cost:**

$$DC = c_d \left[Q_0 - \int_0^{t_1} be^{\mu t}\, dt + \int_0^{t_1} \sigma be^{\mu t}\, dt + \int_{t_1}^{T} (\sigma - p) be^{\mu t}\, dt \right]$$

$$= c_d \left[Q_0 - b(1-\sigma)\mu^{-1}(e^{\mu t_1} - 1) - b(\sigma - p)\mu^{-1}(e^{\mu T} - e^{\mu t_1}) \right]$$

4. **Holding Cost:**

$$HC = h_c \left[\int_0^{t_1} Q_1(t)\, dt + \int_{t_1}^{T} Q_2(t)\, dt \right]$$

$$= h_c \Big[Q_0 \left(t_1 - 0.17\alpha t_1^3 \right) - (1-\sigma) b \left(0.5 t_1^2 - 0.08\alpha t_1^4 - 0.16\mu t_1^3 \right) + b(\sigma - p)\mu^{-2}(e^{\mu T} - e^{\mu t_1}) - b(\sigma - p)\mu^{-1}(T - t_1)e^{\mu T} \Big]$$

5. **Shortage Cost:**

$$SC = c_s \int_{t_2}^{T} Q_2(t) dt = -c_s b(\sigma - p)\mu^{-2} \left(e^{\mu T} - e^{\mu t_1} \right) - b(\sigma - p)\mu^{-1}(T - t_1)e^{\mu T}$$

6. **Lost Sale Cost:**

$$LSC = c_o \int_{t_2}^{T} (1-p) D(t)\, dt = c_o b(1-p)\mu^{-1}(e^{\mu T} - e^{\mu t_1})$$

So, the total cost per unit time of inventory system is:

$$\begin{aligned} TC &= \frac{1}{T}[OC + PC + DC + HC + SC + LSC] \\ &= \frac{1}{T}\Big[A + c_Q Q_0 + c_d[Q_0 - b(1-\sigma)\mu^{-1}(e^{\mu t_1} - 1) - b(\sigma - p)\mu^{-1}(e^{\mu t_1} - e^{\mu T})] \\ &+ h_c[Q_0(t_1 - 0.17\alpha t_1^3) - (1-\sigma)b(0.5t_1^2 - 0.08\alpha t_1^4 - 0.16\ \mu t_1^3) + b(\sigma - p)\mu^{-2}(e^{\mu T} - e^{\mu t_1}) \\ &- b(\sigma - p)\mu^{-1}(T - t_1)e^{\mu T}] - c_s[b(\sigma - p)\mu^{-2}(e^{\mu T} - e^{\mu t_1}) - b(\sigma - p)\mu^{-1}(T - t_1)e^{\mu T}] \\ &+ c_b(1-p)\mu^{-1}(e^{\mu T} - e^{\mu t_1}) \Big] \end{aligned} \tag{3}$$

Sales revenue collected over the cycle (SR):

$$SR = c_s Q_0 + 0.2c_o \int_0^T R(t) dt = (Q_0 + 0.2\sigma) c_o b \mu^{-1}(e^{\mu T} - 1)$$

The total profit per unit time is calculated as follows:

$$TP = \frac{1}{T}[SR - OC - PC - HC - DC - SC]$$

$$= \frac{1}{T}(Q_0 + 0.2\sigma)c_o b\mu^{-1}(e^{\mu T} - 1) - \frac{1}{T}\Big[A + c_Q Q_0 + c_d[Q_0 - b(1-\sigma)\mu^{-1}(e^{\mu t_1} - 1)$$
$$- b(\sigma - p)\mu^{-1}(e^{\mu t_1} - e^{\mu T})] + h_c[Q_0(t_1 - 0.17\alpha t_1^3) - (1-\sigma)b(0.5t_1^2 - 0.08\alpha t_1^4 - 0.16\ \mu t_1^3)$$
$$+ b(\sigma - p)\mu^{-2}(e^{\mu T} - e^{\mu t_1}) - b(\sigma - p)\mu^{-1}(T - t_1)e^{\mu T}]$$
$$- c_s[b(\sigma - p)\mu^{-2}(e^{\mu T} - e^{\mu t_1}) - b(\sigma - p)\mu^{-1}(T - t_1)e^{\mu T}]$$
$$+ c_b(1 - p)\mu^{-1}(e^{\mu T} - e^{\mu t_1})\Big]$$

Fuzzy inventory model Suppose we examine an inventory model in a fuzzy setting where the uncertainty in parameters is present. Let's assume that the parameters p, b, σ and p can vary within certain boundaries.
Let $\tilde{b} = (b_1, b_2, b_3)$, $\tilde{\mu} = (\mu_1, \mu_2, \mu_3)$, $\tilde{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ and $\tilde{p} = (p_1, p_2, p_3)$ are triangular fuzzy numbers.

The total cost per unit time in fuzzy model is:

$$\tilde{TC} = \frac{1}{T}[OC + PC + DC + \tilde{HC} + \tilde{SC} + LSC] \tag{4}$$

Using signed distance method (SDM) the total cost per unit time is:

$$\tilde{TC} = \frac{1}{4}(\text{TC}_1 + 2\text{TC}_2 + \text{TC}_3) \tag{5}$$

where

$$\text{TC}_1 = \frac{1}{T}[OC + PC + DC_1 + HC_1 + SC_1]$$

$$\text{TC}_2 = \frac{1}{T}[OC + PC + DC_2 + HC_2 + SC_2]$$

$$\text{TC}_3 = \frac{1}{T}[OC + PC + DC_3 + HC_3 + SC_3]$$

$$\text{TC}_1 = \frac{1}{T}\Big[A + c_Q Q_0 + c_d[Q_0 - b_1(1-\sigma_1)\mu_1^{-1}(e^{\mu_1 t_1} - 1) - b_1(\sigma_1 - p_1)\mu_1^{-1}(e^{\mu_1 t_1}$$
$$- e^{\mu_1 T})] + h_c[Q_0(t_1 - 0.121\alpha_1 t_1^3) - (1-\sigma_1)b_1(0.5t_1^2 - 0.083\alpha_1 t_1^4 - 0.17\ \mu_1 t_1^3 + 0.025\mu_1\alpha_1 t_1^5)$$
$$+ b_1(\sigma_1 - p_1)\mu_1^{-2}(e^{\mu_1 T} - e^{\mu_1 t_1} - b_1(\sigma_1 - p_1)\mu_1^{-1}(T - t_1)e^{\mu_1 T}] - c_s[b_1(\sigma_1 - p_1)\mu_1^{-2}(e^{\mu_1 T} - e^{\mu_1 t_1})$$
$$- b_1(\sigma_1 - p_1)\mu_1^{-1}(T - t_1)e^{\mu_1 T}] + c_b(1 - p_1)\mu_1^{-1}(e^{\mu_1 T} - e^{\mu_1 t_1})\Big]$$

$$\text{TC}_2 = \frac{1}{T}\Big[A + c_Q Q_0 + c_d[Q_0 - b_2(1-\sigma_2)\mu_2^{-1}(e^{\mu_2 t_1} - 1) - b_2(\sigma_2 - p_2)\mu_2^{-1}(e^{\mu_2 t_1} - e^{\mu_2 T})]$$
$$+ h_c[Q_0(t_1 - 0.121\alpha_2 t_1^3) - (1-\sigma_2)b_2(0.5t_1^2 - 0.083\alpha_2 t_1^4 - 0.17\ \mu_2 t_1^3 + 0.025\mu_2\alpha_2 t_1^5)$$
$$+ b_2(\sigma_2 - p_2)\mu_2^{-2}(e^{\mu_2 T} - e^{\mu_2 t_1}) - b_2(\sigma_2 - p_2)\mu_2^{-1}(T - t_1)e^{\mu_2 T}]$$
$$- c_s[b_2(\sigma_2 - p_2)\mu_2^{-2}(e^{\mu_2 T} - e^{\mu_2 t_1}) - b_2(\sigma_2 - p_2)\mu_2^{-1}(T - t_1)e^{\mu_2 T}]$$
$$+ c_b b_2(1 - p_2)\mu_2^{-1}(e^{\mu_2 T} - e^{\mu_2 t_1})\Big]$$

$$\mathrm{TC}_3 = \frac{1}{T}\Big[A + c_Q Q_0 + c_d[Q_0 - b_3(1-\sigma_3)\mu_3^{-1}(e^{\mu_3 t_1} - 1) - b_3(\sigma_3 - p_3)\mu_3^{-1}(e^{\mu_3 t_1} - e^{\mu_3 T})] + h_c[Q_0(t_1 - 0.121\alpha_3 t_1^3) - (1-\sigma_3)b_3(0.5t_1^2 - 0.083\alpha_3 t_1^4 - 0.17\,\mu_3 t_1^3 + 0.025\mu_3\alpha_3 t_1^5) + b_3(\sigma_3 - p_3)\mu_3^{-2}(e^{\mu_3 T} - e^{\mu_3 t_1}) - b_3(\sigma_3 - p_3)\mu_3^{-1}(T - t_1)e^{\mu_3 T}] - c_s[b_3(\sigma_3 - p_3)\mu_3^{-2}(e^{\mu_3 T} - e^{\mu_3 t_1}) - b_3(\sigma_3 - p_3)\mu_3^{-1}(T - t_1)e^{\mu_3 T}] + c_b b_3(1 - p_3)\mu_3^{-1}(e^{\mu_3 T} - e^{\mu_3 t_1})\Big]$$

4. Optimal Solution Procedure

The primary aim of the model is to find the values of t_1 and T, which can minimize the total cost per unit time TC^* and $\tilde{TC}^0$ to get the maximum total profit throughout a complete cycle over time T. The conditions required to minimize the total cost per unit time during the cycle time T are:

$$\frac{\partial(\mathrm{TC})}{\partial t_1} = 0 \tag{6}$$

$$\frac{\partial(\mathrm{TC})}{\partial T} = 0 \tag{7}$$

$$\frac{\partial(\tilde{\mathrm{TC}})}{\partial t_1} = 0 \tag{8}$$

$$\frac{\partial(\tilde{\mathrm{TC}})}{\partial T} = 0 \tag{9}$$

The condition for sufficiency is that TC and $\tilde{TC}$ are convex functions over the cycle time T. The optimal values t_1° and T° of t_1 and T respectively are obtained by solving equations 6 and 7. To obtain the optimal value $\tilde{TC}^\circ$, we substitute the optimal values t_1° and T° in equation 3. Similarly, the optimal values t_1^* and T^* of t_1 and T, respectively, in the fuzzy model is obtained by solving equations 8 and 9 respectively. Finally, the fuzzy optimal value TC^* is calculated by substituting the optimal values t_1^* and T^* of t_1 and T in equation 5. The condition for sufficiency is that TC is a convex function over the cycle time T.

$$\frac{\partial^2 TC}{\partial T^2} > 0 \quad for\ all \quad T > 0$$

5. Results and discussion

To illustrate the results of **proposed inventory** model developed above, a numerical example has been solved. Data have been assumed randomly from the literature in appropriate units to obtain optimal values of t_1, T, TC°, TC^*, TP° and TP^* and to perform the sensitive analysis.

Example: A numerical example has been presented to illustrate the proposed inventory model. The main objective is to determine the optimal values of t_1, T which minimizes total cost per unit time and maximize the total profit. The values of different parameters are as follows:

$$A = 100,\ Q_0 = 1000,\ a_2 = 0.005,\ a_1 = 0.003,\ a_3 = 0.007,\ c_d = 0.005,\ h_c = 0.5,\ c = 10,$$
$$u_1 = 0.10,\ u_2 = 0.20,\ u_3 = 0.30,\ b_1 = 26,\ b_2 = 35,\ b_3 = 44,\ o_1 = 0.1,\ o_2 = 0.2,\ o_3 = 0.3,$$
$$s = 0.5,\ p_1 = 0.5,\ p_2 = 0.6,\ p_3 = 0.7,\ c_s = 0.5,\ c_b = 17,\ c_0 = 0.5$$

MATLAB is used to obtain the optimal values of t_1, T, TC and TP. To demonstrate the convexity of total cost per unit of inventory system, graphical representations of the convexity of TC° and TC^* with respect to decision variables are presented in Figure 2 and Figure 3, respectively.

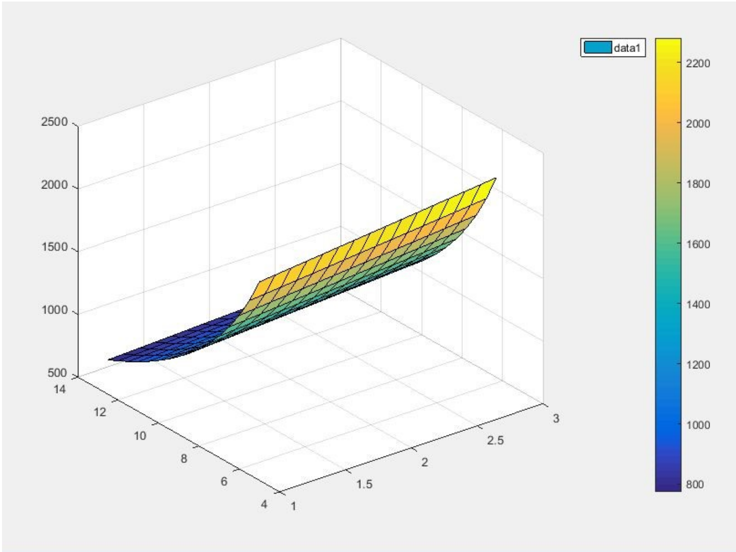

Figure 2: *Convexity of total cost in crisp model.*

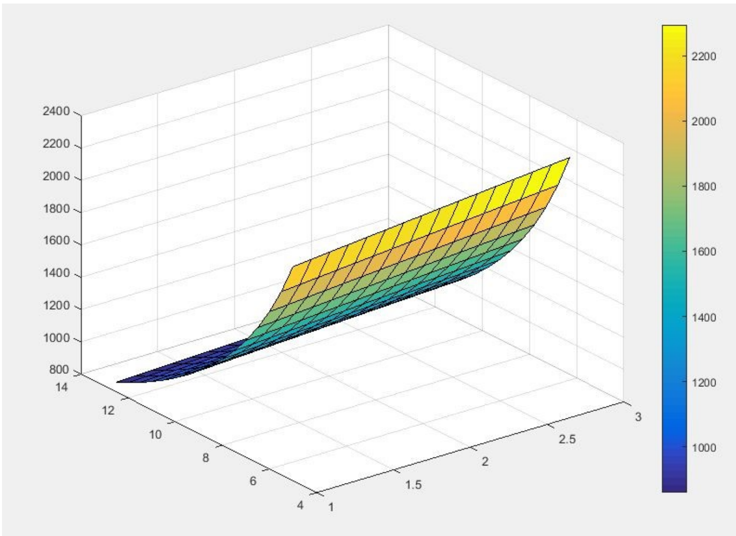

Figure 3: *Convexity of total cost in fuzzy model.*

6. Sensitivity Analysis

The study conducted a sensitivity analysis by modifying the values of critical parameters to determine their impact on optimal values. Each key parameter was varied individually while others are remained constant to obtain the optimal values. The resultant changes in optimal values t_1°, T°, TC°, TC^*, TP° and TP^* are recorded in following tables:

1. **When b changes in crisp model:**

b	t_1°	T°	TC°	TP°
15	1.90	7.36	1583.70	749.30
20	1.84	7.27	1625.10	744.01
25	1.80	7.20	1663.70	735.56
30	1.75	7.12	1703.50	729.27
35	1.70	7.05	1741.70	721.85

2. **When b changes in fuzzy model:**

b	t_1^*	T^*	b_1	b_2	b_3	TC*	TP*
15	1.90	7.35	6	15	24	1586.50	752.27
20	1.85	7.27	11	20	29	1629.40	744.62
25	1.80	7.19	16	25	34	1670.50	736.10
30	1.76	7.12	21	30	39	1710.10	726.15
35	1.71	7.05	26	35	44	1749.50	716.83

3. **When μ changes in crisp model:**

μ	t_1°	T°	TC°	TP°
0.10	1.71	7.20	1676.40	709.64
0.15	1.77	7.21	1662.30	726.15
0.20	1.80	7.20	1663.70	735.56
0.25	1.82	7.18	1680.80	753.37
0.30	1.83	7.16	1671.20	744.01

4. **When μ changes in fuzzy model:**

μ	t_1^*	T^*	μ_1	μ_2	μ_3	TC*	TP*
0.10	1.70	7.20	0.05	0.10	0.15	1679.00	705.51
0.15	1.77	7.21	0.10	0.15	0.20	1662.10	724.67
0.20	1.81	7.21	0.15	0.20	0.24	1663.20	734.07
0.25	1.82	7.18	0.20	0.25	0.30	1670.80	742.25
0.30	1.83	7.16	0.25	0.30	0.35	1680.50	751.31

5. **When σ changes in crisp model:**

σ	t_1°	T°	TC°	TP°
0.10	1.74	7.10	1712.10	701.05
0.15	1.77	7.15	1687.80	718.13
0.20	1.80	7.20	1663.70	735.56
0.25	1.83	7.24	1641.20	754.64
0.30	1.85	7.29	1616.50	773.16

6. **When σ changes in fuzzy model:**

σ	t_1^*	T^*	σ_1	σ_2	σ_3	TC*	TP*
0.10	1.76	7.09	0.01	0.10	0.25	1717.90	709.83
0.15	1.78	7.16	0.05	0.15	0.35	1684.3	733.63
0.20	1.82	7.21	0.10	0.20	0.40	1660.10	750.29
0.25	1.84	7.26	0.15	0.25	0.45	1686.50	768.16
0.30	1.87	7.31	0.20	0.30	0.50	1655.60	785.98

7. **When p changes in crisp model:**

p	t_1°	T°	TC°	TP°
0.4	1.90	7.38	1572.30	770.09
0.5	1.85	7.30	1616.10	751.15
0.6	1.80	7.20	1663.70	735.56
0.7	1.75	7.10	1710.90	720.99
0.8	1.70	7.02	1754.50	704.34

8. **When p changes in fuzzy model:**

p	t_1^*	T^*	p_1	p_2	p_3	TC*	TP*
0.4	1.91	7.38	0.3	0.4	0.5	1542.60	804.49
0.5	1.85	7.29	0.4	0.5	0.6	1620.50	754.09
0.6	1.82	7.21	0.5	0.6	0.7	1660.10	750.29
0.7	1.75	7.10	0.6	0.7	0.8	1715.30	720.52
0.8	1.71	7.01	0.7	0.8	0.9	1761.80	704.09

The following key observations can be drawn Based on the above tables:

1. As the demand parameters μ increases, optimal value of total cost per unit time TC° and TC^* fluctuates while corresponding profit decreases. As the demand parameters b increases, optimal value of total cost per unit time TC° and TC^* increases, the corresponding profit decreases.

2. As the parameters p increases, there is an increase in the optimal value of total cost per unit time TC° and TC^* and there is a decrease in the corresponding profit.

3. As the parameters σ increase, the optimal value of total cost per unit time TC° and TC^* decreases and there is an increase in the corresponding profit.

4. The optimal value of total cost per unit time in the fuzzy model is less than total cost per unit time in the crisp model and the profit is more in fuzzy model compared to crisp model.

7. Conclusion

This work developed an inventory framework for deteriorating items that incorporates time‘varying demand, a constant per‘unit holding charge, and partial backlogging in the presence of resalable returns. A detailed numerical illustration and accompanying sensitivity study were carried out for both crisp and fuzzy formulations. The investigation reveals that unit total cost and the associated profit react strongly to shifts in the selling price parameter, and the demand variability parameter, while showing only moderate responsiveness to changes in the demand scale and the salvage value parameter. Moreover, the fuzzy version consistently achieves a lower unit cost and, consequently, a higher profit when compared with its crisp counterpart. Future research could broaden the model by incorporating shortages, trade‘credit arrangements, or the time value of money to capture a wider range of real‘world managerial conditions.

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