CENSORED COUNT DATA REGRESSION MODELS: NUMBER OF CAESAREAN SECTION DELIVERIES USING INTEGRATED NESTED LAPLACE APPROXIMATION

[1]Srinu Setti*, [2]B.Muniswamy

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[1*]Research Scholar, Department of Statistics, Andhra University, Visakhapatnam-530003, Andhra Pradesh, India

[2]Professor, Department of Statistics, Andhra University, Visakhapatnam-530003, Andhra Pradesh, India

*Corresponding author: [1]srinujesusj@gmail.com

[2]munistats@gmail.com

Abstract

Count data depicts the frequency of an occurrence within a specific time frame. Consider the frequency of caesarean operations that women undergo during their lives. Almost every academic field, including management, economics, medicine, and industrial organizations, relies on count data. Count data is extensively utilized across various fields, including marketing, public health, and biomedical science. This study aims to estimate the posterior means, variances, or quantiles of the NCSD for women aged 15 to 49 in Andhra Pradesh, India, using the RCPRM and RCNBRM methodologies. The RCPRM and RCNBRM are used to determine the optimal fit. The secondary dataset NFHS-5 is used for the study. This research utilizes INLA to model the NCSD. Subsequently, it analyzed delivery patterns among pregnant women. Information criteria DIC and WAIC are utilized to compare for the best fit. The DIC values 4608.60 and 4593.90 of RCNBRM are less than the DIC values 4864.74 and 4860.68 of RCPRM. Thus, from the results, it is inferred that RCNBRM is the best fit for NCSD and that Breech Presentation, the present age of the respondent, High Blood Pressure, Child is Twin, Prolonged Labour, Education Level, and Heart Disease are significant determinants of the NCSD. Therefore, government policymakers need to consider these variables while making healthcare policies for women aged 15 to 49 years who are of childbearing age.

Keywords: Number of Caesarean Section Deliveries, RCPRM, RCNBRM, INLA, DIC, WAIC

I. Introduction

The fundamentals of the Censored Poison (CP) are defined by Cameron and Trivedi [1] and Winkelmann [2]. It is presumed that the censoring technique operates independently of the count variable [3]. Recently, Poisson Regression Model (PRM) has been adapted to handle censored count data. Censorship is typically linked to survival analysis; however, count data can also be subject to censorship, with right-censoring being the most prevalent kind. This transpires when it is established that the actual count surpasses the stated figure. The PRM is modified for the censored situation with a predetermined censoring threshold. In censored scenarios, it is necessary to modify the probability function and log-likelihood functions to reflect alterations in the response distribution, in accordance with the econometric principle of censoring, which stipulates that a single cut-point determines the censoring for all impacted cases. Numerous survival models demonstrate a continuous temporal response. A discrete response survival model is identified. The answer may be interpreted as counts rather than defined in temporal terms. Traditionally, these models have been depicted as piecewise models; however, they could represent utilizing a count model that accounts for censored observations. CP and Censored Negative Binomial (CNB) models are applicable for the analysis of this count response data. The models are applicable to any type of count response, regardless of their use in survival contexts. The only distinction is that filtering is allowed as a function within the framework of survival analysis [4].

II. Methods

I. Variables selected for the study

The variables are as follows:

NCSD	=	"Number of caesarean section deliveries"
BP	=	"Breech presentation"
HD	=	"Currently has heart disease"
HBP	=	"High blood pressure"
PL	=	"Prolonged labour"
CT	=	"Child is twin"
Age	=	"Current Age"
EL	=	"Education level"
TR	=	"Type of place of residence"

Where

$y =$ "Number of caesarean section deliveries"
$x_1 =$ "Breech presentation"
$x_2 =$ "Currently has heart disease"
$x_3 =$ "High blood pressure"
$x_4 =$ "Prolonged labour"
$x_5 =$ "Child is twin"
$x_6 =$ "Current Age"
$x_7 =$ "Educational level"

Hence we have the mathematical model as:

$$\begin{aligned} &Number\ of\ caesarean\ section\ deliveries \\ &= \beta_0 + \beta_1(Breech\ presentation) \\ &+ \beta_2(Currently\ has\ heart\ disease) + \beta_3(High\ blood\ pressure) \\ &+ \beta_4(Prolonged\ labour) + \beta_5(Child\ is\ twin) + \beta_6(Current\ Age) \\ &+ \beta_7(Educational\ level) \end{aligned} \tag{1}$$

The above model can be written in Statistical terms as:

$$\begin{aligned} NCSD = \beta_0 + \beta_1(BP) + \beta_2(HD) + \beta_3(HBP) + \beta_4(PL) + \beta_5(CT) + \beta_6(Age) \\ + \beta_7(EL) + U \end{aligned} \tag{2}$$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7 + U \tag{3}$$

Where y is the predictand variable, $x_1, x_2, x_3, x_4, x_5, x_6, x_7$ are predictor variables, $\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7$ are parameters and U is disturbance term.

Then the structure of a count model will be

$$\log(\mu) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7 \tag{4}$$

In GLM theory, the link function serves to linearize the relationship between the linear predictor, $x'\beta$, and the fitted value, , μ or $\hat{y}$ or estimated mean, $E(y)$. Consequently, μ is delineated in relation to the inverse relationship.

$$\mu = e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7} \tag{5}$$

$$\log(\mu_i) = \sum_{i=1}^{n} \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_j X_{ji} \tag{6}$$

II. Right Censoring

Right censoring, which limits observed counts from the right, is the primary method utilized for count data. In summary, it is established that the value of Y is at least equal to a predetermined threshold c for specific instances where the response variable Y surpasses c. This scenario can be applied in various circumstances, such as when a study seeks to investigate the correlation between high income and heavy smoking. An individual may be categorized as a heavy smoker in such a study if they consume ten or more cigarettes daily. The daily cigarette consumption, as the response variable, can be quantified in discrete values of 0, 1, 2,... or 10 or extra, encompassing persons who smoke 11 or extra cigarettes. The endogenous variable is thus censored at c=10. The exogenous variable, income, is accurately represented for the entire sample of censored individuals who smoke greater than 10 cigarettes everyday [4]. A significant body of research on censored count data has been published. Terza, J.V. [5] conducted an analysis of censored data concerning the frequency of shopping at a certain place over a defined point of time. In the experiment, the observation frequencies were categorized as zero, one, two, or more. Thus, individuals who made four or more purchases would still be classified as having made three or more purchases. Caudill, S. B. and Mixon, F. G. [6] examined the censoring situation regarding their endogenous variable, the number of children in a household, within the context of fertility [7].

According to Joseph, M. Hilbe [4], the comprehension of appropriate censoring is as follows: A right-censored cut at c indicates that the highest observable value of the response is c, with all values surpassing c recorded as c; thus, this value in the data effectively represents "greater than or equal to c." If c = 1, then any response values exceeding one are modified to equal 1. All values have been included in the model analysis.

Values are obscured due to their placement within the data set or as a consequence of external influences, including loss to follow-up or delayed entry. Censoring can be defined within the contexts of econometrics or survival analysis. Censorship, in this context, resembles truncation; however, the data beyond the cut-point(s) are modified to the nearest value included within the model. If the cut-off is set at 3, counts of 0, 1, and 2 are not removed as in truncation but are instead modified to 3. Censored data sets are identifiable by multiple values at each cut-point.

As previously indicated right censoring at c is the most prevalent form of censoring and is therefore the focus of this study. Many values of y^* remain inadequately observed due to their classification as greater than or equal to c, resulting in indeterminate true values. The observed outcome y_i can be expressed as follows, incorporating the pertinent censoring scheme:

In a standard Poisson setting, all y^* are observed, whereas in a CP, the true y_i^* is only observed lower a censoring position c [2].

Then

$$y_i = \begin{cases} y_i^*, & if\ y_i^* < c_i \\ c_i, & if\ y_i^* \geq c_i \end{cases} \tag{7}$$

Winkelmann, R. [2] writes that the censoring point c_i may differ for each observation [6]. If c is constant, a model with a constant point exists [5].

If y_i is censored, it is understood that.

$$P(y_i \geq c_i) = \sum_{j=c_i}^{\infty} P(y_i = j) = \sum_{j=c_i}^{\infty} f(j) = 1 - \sum_{j=0}^{c_i-1} f(j) = 1 - F(c_i - 1) \tag{8}$$

An indicator variable d_i is defined as follows.

$$d_i = \begin{cases} 1, & if\ y_i^* \geq c_i \\ 0, & otherwise \end{cases} \tag{9}$$

The likelihood function is explained as:

$$\mathcal{L} = \prod_{i=1}^{n} [f(y_i)]^{1-d_i}\, P[(y_i \geq c_i)]^{d_i} \tag{10}$$

Therefore, the log-likelihood function is expressed as follows:

$$\ell = \sum_{i=1}^{n} (1 - d_i) \log f(y_i) + \sum_{i=1}^{n} d_i \log[P(y_i \geq c_i)] \tag{11}$$

Then, from the equation

$$\ell = \sum_{i=1}^{n} (1 - d_i) \log f(y_i) + \sum_{i=1}^{n} d_i \log[1 - F(c_i - 1)] \tag{12}$$

where the pmf and cdf of y_i^* are denoted by $f(.)$ and $F(.)$, respectively. For further information, see [8] and [9].

The conditional mean is given by

$$E(y_i, \mu_i) = c_i - \sum_{j=0}^{c_i-1} f(j)(c_i - j) \tag{13}$$

III. Dispersion Index Test

In order to check over-dispersion quantitatively, dispersion index is computes as in equation

$$d = \frac{Var(Y)}{E(Y)} \tag{14}$$

where d is the dispersion index. Given count variable Y, the variable is over-dispersed, if $d > 1$. If the d is greater than 1, that indicates over-dispersion of the data. In such cases Negative Binomial may be more appropriate model.

IV. Right-Censored Poisson Regression Model (RCPRM)

The outcome variable, NCSD, is right-censored at one, indicating a single C-section delivery, which is used to conform to the framework. The NCSD is initially applied in RCPRM and subsequently applied in RCNBRM for the data to be over-dispersed. The INLA is employed for parameter estimation in the RCPRM and RCNBRM. The INLA methodology guarantees computing efficiency.

Winkelmann [2] and Cameron and Trivedi [1] provide an overview of the fundamentals of the censored Poisson model. Here, it is assumed that the count variable has no bearing on the censoring process [3]. As stated by Rafal Raciborski [7], examine the Poisson random variable's probability function in equation (15).

$$f(y_i, \mu_i) = \frac{e^{-\mu_i}\ \mu_i^{y_i}}{y_i!}\ , y_i = 0,1,2, \dots\dots\dots;\ \mu_i > 0. \tag{15}$$

Where $\mu_i = e^{x_i'\beta}$ is the log link function, β is a vector of unknown parameters, x_i is a vector of exogenous variables.

The likelihood for the RCPRM can, therefore, be found using the likelihood function equation (16) as follows:

$$\mathcal{L} = \prod_{i=1}^{n} \left[\frac{e^{-\mu}\mu^{y}}{y!}\right]^{1-d_i} \left[1 - \sum_{j=0}^{c_i-1} \frac{e^{-\mu}\mu^{j}}{j!}\right]^{d_i} \tag{16}$$

The log-likelihood function of RCPRM shown in equation (18) is

$$\ell = \sum_{i=1}^{n} \left[(1-d_i)\log\left[\frac{e^{-\mu}\mu^{y}}{y!}\right] + d_i \log\left[1 - \sum_{j=0}^{c_i-1} \frac{e^{-\mu}\mu^{j}}{j!}\right]\right] \tag{17}$$

$$\ell = \sum_{i=1}^{n}(1-d_i)\log\left[\frac{e^{-\mu}\mu^{y}}{y!}\right] + \sum_{i=1}^{n} d_i \log\left[1 - \sum_{j=0}^{c_i-1} \frac{e^{-\mu}\mu^{j}}{j!}\right] \tag{18}$$

The conditional mean is given by equation (13)

$$E(y_i, \mu_i) = C_i - \sum_{j=0}^{c_i - 1} f(j)(C_i - j)$$

Here

$$f(j) = \frac{e^{-\mu}\mu^{j}}{j!} \tag{19}$$

$$E(y_i, \mu_i) = c_i - c_i \sum_{j=0}^{c_i - 1} \frac{e^{-\mu_i}{\mu_i}^{j}}{j!} + j \sum_{j=0}^{c_i - 1} \frac{e^{-\mu_i}{\mu_i}^{j}}{j!} \tag{20}$$

$$E(y_i, \mu_i) = \mu \tag{21}$$

$$E(y_i^2, \mu_i) = \mu^2 + \mu \tag{22}$$

$$V(y_i, \mu_i) = E(y_i^2) - [E(y_i)]^2 \tag{23}$$

$$V(y_i, \mu_i) = \mu \tag{24}$$

Hence, the mean and variance of RCPRM are μ and μ, respectively, which shows that the RCPRM converges to PRM.

V. Right-Censored Negative Binomial Regression Model (RCNBRM)

The pmf for NBRM is explained as

$$p\left(y_i; \frac{1}{\alpha}, \mu_i\right) = \frac{\Gamma\left(y_i + \frac{1}{\alpha}\right)}{\Gamma\left(\frac{1}{\alpha}\right)\Gamma(y_i + 1)} \left(\frac{1}{1 + \alpha\,\mu_i}\right)^{\frac{1}{\alpha}} \left(\frac{\alpha\mu_i}{1 + \alpha\,\mu_i}\right)^{y_i} \tag{25}$$

Where $\mu_i = e^{x_i' \beta}$ is the log link function, β is a vector of unknown parameters and x_i is a vector of independent variables. Equation (10) censored likelihood function yields the likelihood function of the appropriate CNB model as follows:

$$\mathcal{L} = \prod_{i=1}^{n} \left[\frac{\Gamma\left(y_i + \frac{1}{\alpha}\right)}{\Gamma\left(\frac{1}{\alpha}\right)\Gamma(y_i + 1)} \left(\frac{1}{1 + \alpha\,\mu_i}\right)^{\frac{1}{\alpha}} \left(\frac{\alpha\mu_i}{1 + \alpha\,\mu_i}\right)^{y_i}\right]^{1-d_i} \times \left[1 - \Gamma\left(\frac{1}{\alpha}\right)\left(\frac{1}{1 + \alpha\,\mu_i}\right)^{\frac{1}{\alpha}} \sum_{j=0}^{a} \frac{\Gamma\left(j + \left(\frac{1}{\alpha}\right)\right)}{\Gamma(j + 1)} \left(\frac{\alpha\mu_i}{1 + \alpha\,\mu_i}\right)^{j}\right]^{d_i} \tag{26}$$

The initial term on the right represents the likelihood of contribution from non-censored observations, whereas the subsequent term accounts for the contribution from censored observations [2].

The log likelihood function for the sample can be expressed as:

$$\ell = \sum_{i=1}^{n}(1-d_i)\log\left[\frac{\Gamma\left(y_i+\frac{1}{\alpha}\right)}{\Gamma\left(\frac{1}{\alpha}\right)\Gamma(y_i+1)}\left(\frac{1}{1+\alpha\,\mu_i}\right)^{\frac{1}{\alpha}}\left(\frac{\alpha\mu_i}{1+\alpha\,\mu_i}\right)^{y_i}\right] + \sum_{i=1}^{n} d_i\log\left[1-\Gamma\left(\frac{1}{\alpha}\right)\left(\frac{1}{1+\alpha\,\mu_i}\right)^{\frac{1}{\alpha}}\sum_{j=0}^{\alpha}\frac{\Gamma\left(j+\left(\frac{1}{\alpha}\right)\right)}{\Gamma(j+1)}\left(\frac{\alpha\mu_i}{1+\alpha\,\mu_i}\right)^{j}\right] \tag{27}$$

Refer to [5], [6], [7] and [10] for further details on these models.
The conditional mean is expressed as

$$E(y_i;\mu_{i,}\alpha) = c_i - \sum_{j=0}^{c_i-1} f(j)(c_i-j) \tag{28}$$

Where

$$f(j) = \frac{\Gamma\left(j+\frac{1}{\alpha}\right)}{\Gamma\left(\frac{1}{\alpha}\right)\Gamma(j+1)}\left(\frac{1}{1+\alpha\,\mu_i}\right)^{\frac{1}{\alpha}}\left(\frac{\alpha\mu_i}{1+\alpha\,\mu_i}\right)^{j} \tag{29}$$

$$E(y_i;\mu_i,\alpha) = c_i - c_i\sum_{j=0}^{c_i-1}\frac{\Gamma\left(j+\frac{1}{\alpha}\right)}{\Gamma\left(\frac{1}{\alpha}\right)\Gamma(j+1)}\left(\frac{1}{1+\alpha\,\mu_i}\right)^{\frac{1}{\alpha}}\left(\frac{\alpha\mu_i}{1+\alpha\,\mu_i}\right)^{j} + j\sum_{j=0}^{c_i-1}\frac{\Gamma\left(j+\frac{1}{\alpha}\right)}{\Gamma\left(\frac{1}{\alpha}\right)\Gamma(j+1)}\left(\frac{1}{1+\alpha\,\mu_i}\right)^{\frac{1}{\alpha}}\left(\frac{\alpha\mu_i}{1+\alpha\,\mu_i}\right)^{j} \tag{30}$$

$$E(y_i;\mu_i,\alpha) = \mu \tag{31}$$

$$E(y_i^2;\ \mu_i,\alpha) = \mu^2+\mu \tag{32}$$

$$V(y_i;\mu_i,\alpha) = E(y_i^2) - [E(y_i)]^2 \tag{33}$$

$$V(y_i;\mu_i,\alpha) = \mu \tag{34}$$

The negative binomial distribution is characterized by its dependence on the mean (μ) and the dispersion parameter (α). If α approaches zero, the distribution converges to the Poisson distribution. The PRM represents the model equation for NBRM. Using a linear combination of predictors, the logarithm of the outcome is forecasted. If α approaches 0, the CNB distribution converges to the CP distribution. As α approach 0, the RCNB distribution converges to the RCP distribution [10].

VI. Model Selection

Marginal Log-likelihood (MLIK)

According to P. McCullagh and J.A. Nelder FRS [11], one method for removing undesired nuisance parameters is to use the marginal likelihood for a suitable subset of the data vector. This strategy does not always work well, but when it does, it is definitely preferable to select a big chunk of the original data as feasible to minimize information loss. The marginal likelihood estimates remain consistent regardless of whether the postulated models are valid. Note that the marginal log-likelihood is also known as the restricted log likelihood [12].

Deviance Information Criterion (DIC)

According to Hilbe and Greene [13], the Wald, likelihood ratio, and Score or Lagrange Multiplier tests were designed to assess the specification of the conditional mean function by testing parameter constraints. Non-nested (and nested) models are frequently compared using 'information criteria' statistics, which are roughly equivalent to adjusted R^2s in the field of maximum likelihood estimation. The Akaike information criterion (AIC) is a widely used statistic [11] and [14]

AIC is likely the most widely used information criterion in the frequentist statistical framework. Spiegelhalter et al.'s [15] deviance information criterion (DIC) is widely regarded as a key element in Bayesian model selection during the previous two decades. DIC is viewed as a Bayesian variant of AIC. Alike AIC, it weighs model adequacy against complexity, is deals with well theoretical replicated data, and predicts observed data. Nevertheless, unalike AIC, DIC takes prior knowledge into consideration. DIC is built using the posterior mean of the log-likelihood or deviance and has various advantageous properties [16]

As a result, DIC can be seen as a Bayesian counterpart of AIC, with a similar rationale but a broader range of applications. Additionally, it works with any model, requires very little extra analysis or Monte Carlo sampling, and functions rather well [15]. DIC is defined as

$$DIC = -2(d - P_{dic}) \tag{35}$$

Where

$$d = \frac{1}{S}\sum_{s=1}^{S} \log[P(y|\theta_s)] \tag{36}$$

$$P_{dic} = \max\,[\log[P(y|\theta)] - d \tag{37}$$

Where $max[\,]$ is the maximum function over all posterior samples, and S is the number of posterior samples [17].

The model exhibiting the lowest DIC value is deemed the superior fit when evaluating models. If two models have similar DIC values, neither is favoured over the other [4].

Watanabe-Akaike Information Criterion (WAIC)

A new statistical learning theory using algebraic geometry methods has recently emerged. The resolution theorem in algebraic geometry can be applied in singular learning theory to transform a log-likelihood function into a standardized form, notwithstanding the presence of singularities. Consequently, the asymptotic characteristics of the posterior distribution are comprehended, allowing for the extension of AIC and BIC concepts to unique statistical models. The real log canonical threshold [18] establishes the asymptotic Bayesian marginal likelihood, whereas the information criterion [19, 20, 21] assesses the average Bayes generalization error [20].

WAIC is defined as:

$$WAIC = -2(lpd - P_{waic}) \tag{38}$$

Where

$$lpd = \sum_{i=1}^{n} \log\left[\frac{1}{S}\sum_{s=1}^{S} p(y|\theta_s)\right] \tag{39}$$

$$P_{waic} = \sum_{i=1}^{n} var\,[\log\,[p(y|\,\theta)]] \tag{40}$$

Where S is the number of posterior samples, n is the number of data points, $var[\,]$ is the variance function over the posterior samples, lpd is the log predictive density of the data, and P_{waic} is the effective number of parameters.

III. Results

I. Dispersion Index Test

The dispersion index test is in the following table

Table 1: *Dispersion Index Test*

Data	Variance	Mean	Dispersion
y	0.9583	0.8507	1.1265

The Table 1 explains the dispersion index test. The variance, 0.9583 is greater than the mean 0.8507 of the response variable, which could mean that a different model, like Negative Binomial may be more appropriate. In such situations, the Negative Binomial model might be a better fit.

II. Application - RCPRM using INLA

The posterior means, variances, or quantiles are estimated. The mean or mode of the resultant posterior distribution of a parameter is referred to as the parameter of interest [4].

The model NCSD is applied in RCPRM by using INLA. The findings are presented below:

Table 2: *Summary of fixed effects - RCPRM*

Fixed effects:	Mean	Standard deviation	0.025 quantile	0.5 quantile	0.975 quantile	Mode	KLD
(Intercept)	-1.781	0.225	-2.222	-1.781	-1.340	-1.781	0
BPYes	-0.286	0.099	-0.480	-0.286	-0.092	-0.286	0
BPDon't know	0.409	0.161	0.093	0.409	0.725	0.409	0
HDYes	-1.098	0.708	-2.486	-1.098	0.291	-1.098	0
HDDon't know	0.000	31.623	-61.980	0.000	61.980	0.000	0
HBPYes	0.458	0.458	0.29	0.458	0.627	0.458	0
HBPDon't know	-0.064	0.206	-0.467	-0.064	0.339	-0.064	0
PLYes	0.382	0.089	0.208	0.382	0.557	0.382	0
PLDon't know	-1.081	0.263	-1.596	-1.081	-0.567	-1.081	0
CT1st of multiple	0.048	0.295	-0.530	0.048	0.625	0.048	0
CT2nd of multiple	0.091	0.290	-0.478	0.091	0.660	0.091	0
CT3rd of multiple	-68.765	14.083	-96.366	-68.765	-41.164	-68.765	0
CT4th of multiple	0.000	31.623	-61.980	0.000	61.980	0.000	0
CT5th of multiple	0.000	31.623	-61.980	0.000	61.980	0.000	0
Age	0.006	0.007	-0.007	0.006	0.019	0.006	0
ELPrimary	0.062	0.140	-0.212	0.062	0.335	0.062	0
ELSecondary	0.471	0.105	0.265	0.471	0.677	0.471	0
ELHigher	0.737	0.113	0.517	0.737	0.958	0.737	0

Table 2 displays the values of RCPRM. The parameters of interest of the posterior distribution of the covariates BP, Don't know, HBP, Yes, PL, Yes, EL secondary, and EL higher more than 0.300, whereas HD, Don't know, CT4th of multiple, and CT5th of multiple is 0.000.

III. Application - RCNBRM using INLA

The model NCSD is fitted in RCNBRM using INLA. The posterior means, variances, or quantiles are estimated. The values are given below:

Table 3: *The values of posterior means, variances, or quantiles - RCNBRM*

Fixed effects:	Mean	Standard deviation	0.025 quantile	0.5 quantile	0.975 quantile	Mode	KLD
(Intercept)	-1.740	0.226	-2.183	-1.740	-1.298	-1.740	0
BPYes	-0.313	0.100	-0.508	-0.313	-0.117	-0.313	0
BPDon't know	0.481	0.176	0.124	0.485	0.817	0.485	0

HDYes	-1.044	0.717	-2.449	-1.044	0.364	-1.044	0
HDDon't know	0.000	31.623	-62.009	0.000	62.009	0.000	0
HBPYes	0.133	0.142	-0.101	0.112	0.433	0.068	0
HBPDon't know	-0.017	0.216	-0.448	-0.015	0.401	-0.015	0
PLYes	0.032	0.148	-0.211	0.009	0.349	-0.036	0
PLDon't know	-0.776	0.342	-1.582	-0.747	-0.189	-0.702	0
CT1st of multiple	0.119	0.296	-0.462	0.119	0.700	0.119	0
CT2nd of multiple	0.100	0.291	-0.471	0.100	0.671	0.100	0
CT3rd of multiple	-45.426	33.774	-102.331	-46.917	15.160	-45.578	0
CT4th of multiple	0.000	31.623	-62.009	0.000	62.009	0.000	0
CT5th of multiple	0.000	31.623	-62.009	0.000	62.009	0.000	0
Age	0.021	0.008	0.004	0.022	0.035	0.023	0
ELPrimary	0.068	0.140	-0.206	0.068	0.342	0.068	0
ELSecondary	0.444	0.107	0.234	0.444	0.653	0.444	0
ELHigher	0.678	0.115	0.452	0.678	0.903	0.678	0

The parameters of means, variances, or quantiles are computed. Table 3 shows the computed values of the parameters of means, variances, or quantiles of RCNBRM. The parameters of mean or mode of the covariates BP, Do not know, EL secondary, and EL higher are greater than 0.300, whereas HD, Do not know, CT4th of multiple, and CT5th of multiple are 0.000.

Table 4: *Comparing models*

Model	Model selection criteria		
	MLIK	DIC	WAIC
RCPRM	-2290.03	4864.74	4860.68
RCNBRM	-2291.39	4608.60	4593.90

Table 4 clearly shows that the MLIK of RCNBRM, -2291.39, is greater than that of RCPRM, -2290.03. The DIC and WAIC values of RCNBRM, 4608.60 and 4593.90, are smaller than those of RCPRM, 4864.74 and 4860.68, respectively. Thus, RCNBRM seems to be the best fit for the NCSD.

IV. Discussion

I. Findings

This article succinctly outlines the INLA algorithm for estimating the marginal posterior parameters of interest and hyperparameters in Bayesian spatial and spatiotemporal models. The parameters of interest, the mean or mode of RCPRM, are as follows. The mean or mode of the intercept is -1.781, BP Yes & BP Do not know is -0.286 & 0.409, HD Yes & HD Do not know is -1.098 & 0.000, HBP Yes & HBP Do not know is 0.458 & -0.064, PL Yes & PL Do not know is 0.382 & -1.081, CT1st multiple, CT2nd multiple, CT3rd multiple, CT4th multiple & CT5th multiple is 0.048, 0.091, -68.765, 0.000 & 0.000, age is 0.006, EL Primary, EL Secondary, EL Higher is 0.062, 0.471 & 0.737.

The parameters of interest, mean, and mode of RCNBRM are as follows: Mean or mode of the intercept is -1.740, BP Yes & BP Do not know is -0.313 & 0.481, HD Yes & HD Do not know is -1.044 & 0.000, HBP Yes & HBP Do not know is 0.133 & -0.017, PL Yes & PL Do not know is 0.032 & -0.776, CT1st multiple, CT2nd multiple, CT3rd multiple, CT4th multiple & CT5th multiple is 0.119, 0.100, -45.426, 0.000 & 0.000, age is 0.021, EL Primary, EL Secondary & Higher is 0.068, 0.444 & 0.678. The right-censored NCSD dataset used for RCPRM and RCNBRM demonstrates the INLA estimate outcomes.

II. Conclusion

INLA offers significant computational advantages over alternative methods for addressing problems involving random and fixed effects across specific regions and time in spatiotemporal analysis. This research article presents the computation of the INLA fixed effects of additive models. The RCNBRM is the superior model in comparison to the RCPRM. The NCSD is modelled using Bayesian spatial-temporal methods in INLA specification. The NCSD is comparable to other regression models. The RCNBRM is identified as the most effective, concluding that factors such as BP, HD, HBP, PL, CT, Age, and EL significantly influence the NCSD. Therefore the government policy makers need to consider these variables while making the health care polices for the women of age from 15 to 49 years old, who are in the age of child bearing.

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Declaration of conflicting interests

The authors declared no conflicts of interest

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