RELIABILITY ANALYSIS AND PERFORMANCE OPTIMIZATION OF A 2-OUT-OF-4 REDUNDANT SYSTEM USING RPGT AND METAHEURISTIC ALGORITHMS

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Abstract

In this paper focuses on the reliability analysis and performance optimization of a 2-out-of-4 redundant system, modeled using the Regenerative Point Graphical Technique (RPGT). The study aims to evaluate system behavior under various failure and repair rate conditions and to determine optimal parameters that enhance system reliability and availability. The system consists of four units A, B, C, and D arranged in a 2-out-of-4 configuration, meaning the system continues to function as long as at least two units are operational. A state-space model is developed, where each state corresponds to a distinct combination of operational, failed, and repair conditions of these units. The transitions between these states are governed by exponentially distributed failure and repair processes, and are represented using a directed graph structure. Transition probabilities are derived from these time-dependent exponential distributions, allowing precise evaluation of system behavior. To enhance the system's performance, three metaheuristic optimization algorithms—Cuckoo Search Algorithm (CSA), Particle Swarm Optimization (PSO), and Genetic Algorithm (GA)—are employed. These algorithms are applied to optimize key system parameters: failure rates (λ) and repair rates (μ). The objective is to maximize the Mean Time to System Failure (MTSF) and steady-state availability, and to minimize the expected number of inspections required by the repair personnel. The comparative analysis of optimization results reveals the relative effectiveness of each algorithm. PSO consistently provides the highest values for MTSF and availability, indicating stronger performance in identifying optimal solutions. CSA also performs well, showing close results to PSO. GA, while effective, yields comparatively lower reliability indices. The outcomes demonstrate how advanced optimization techniques can be successfully integrated with RPGT-based modeling to develop a detailed and practical understanding of complex redundant systems. These insights support better maintenance planning, enabling organizations to improve reliability and reduce downtime in industrial, communication, and critical control systems. This methodology can be extended to other system configurations and industries, making it a valuable tool in the field of reliability engineering and operational research

Keywords: Reliability analysis, Cuckoo Search Algorithm (CSA), Particle Swarm Optimization (PSO), Genetic Algorithm (GA), Maintenance strategies, System performance optimization

I. Introduction

High reliability and continuous operation are essential in industrial, communication, and power systems. Redundancy, especially the 2-out-of-4 configuration, enhances reliability by allowing system functionality with at least two working units. Using Regenerative Point Graphical Technique (RPGT), we can model these systems through state-space diagrams that reflect various working and failed states, incorporating exponential failure and repair rates to compute metrics like MTSF and availability. However, identifying optimal rate combinations analytically is challenging. Metaheuristic algorithms like CSA, PSO, and GA effectively explore these parameters to maximize MTSF, improve availability, and minimize inspection needs. This paper presents an RPGT-based model of a 2-out-of-4 system with priority repair, derives state transitions, and applies CSA, PSO, and GA to optimize system performance. Results compare each method's effectiveness, offering guidance for cost-effective maintenance strategies in complex systems. Singla et al. [1] analyzed a four-unit system where full performance requires all units active, reduced capacity occurs with three, and failure happens with two or more down. Each unit has unique failure/repair costs, and a single server handles repairs. In [2], they studied a system with two active and one backup unit maintained by an external, potentially failing server. In [3], they examined a three-unit parallel system with varying failure/repair rates, using RPGT and deep learning (Adam, SGD) for optimization. In [4], they explored how reliability and warranty policies affect productivity and investment in automated industries. In [5], they used the Markov process and Laplace transforms to assess how preventive maintenance influences a three-unit system’s failure time, profit, and efficiency through sensitivity analysis. In [6], they optimized a repairable 2-out-of-4 system with evolutionary algorithms for redundancy and maintenance scheduling. In [7], they modeled a poly-tube system using Markov chains and PSO, identifying key failure/repair impacts on performance. In [8], they analyzed a two-subsystem series model with preventive maintenance using Chapman-Kolmogorov equations and GA for optimizing availability and MTTF. In [9], they applied the TLBO algorithm to constrained/unconstrained problems, showing its versatility. Dang [10] reviewed metaheuristics (PSO, GA, ACO) for improving software fault prediction and future research. Zhou [11] evaluated metaheuristics in cloud load balancing, finding PSO best for make-span and performance. Khorshidi [12] optimized reliability and cost in multi-state k-out-of-n systems, showing GA outperforms ICA, though ICA is faster.

II. Notations and Assumptions

The following notations and assumptions are used to discuss the system:

I. Notations

◯ / ⬭ / □	: Full working state / Degraded state/ Failed state
β_i	: Constant RR for $i = 1, 2, 3, 4$
A, B/ C, D	: Full working state/ Cold stand by

II. Assumptions

The two units having the arrangement of series manner, hence both play important role for finding reliability metrics.

- The system is working with 100 % efficiency with two stand by.
- Upon Failure, The system will shift to stand by.

- The malfunction and recovery rate are considered exponential distributed.

III. System Transition Diagram and Model Description

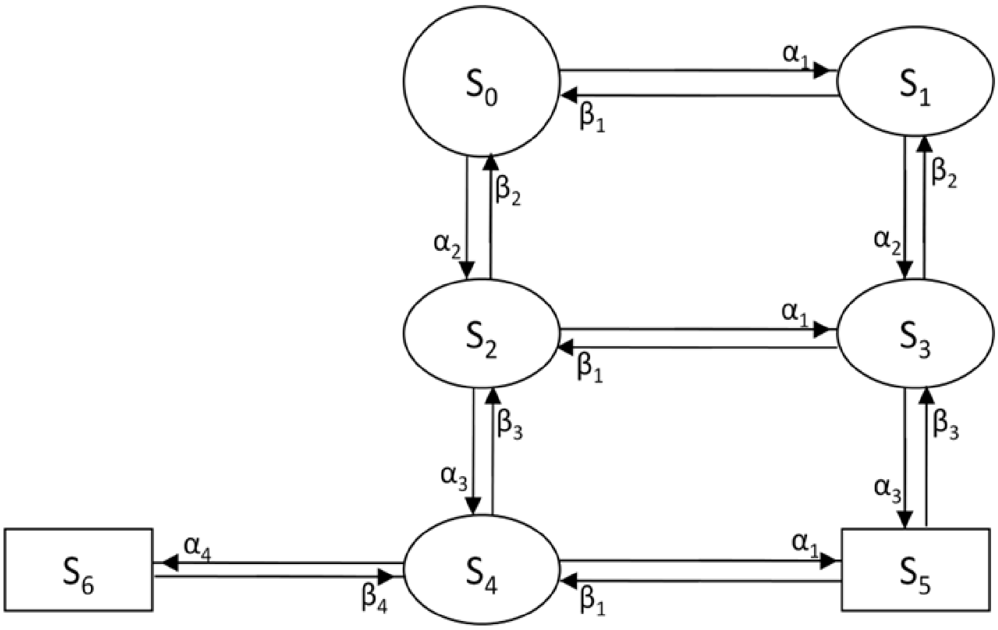

Figure 1: *Transition diagram*

The different possible transition for four units is represented in Figure 1.
S_0 = Units A and B are working, C and D are in standby, and the system is working.
S_1 = Units B and C are working, Unit D is in standby, Unit A is under repair, and the and the system is working.
S_2 = A and C are working, D is standby, and B is under repair.
S_3 = C and D are working, and B and A are under repair.
S_4 = A and D are working, and B and C are under repair.
S_5 = A, B, and C are under repair, and D unit is good.
S_6= B, C, and D are under repair, and the A unit is good

IV. Mathematical Modelling

I. Transition probabilities $q_{i,j}(t)$

Table 1: *Transition Probabilities*

"$q_{i,j}(t)$"	$p_{i,j}$ = "$q^*_{i,j}(0)$"
$q_{0,1}(t) = \alpha_1 e^{-(\alpha_1+\alpha_2)t}$	$p_{0,1} = \alpha_1/(\alpha_1+\alpha_2)$
$q_{0,2}(t) = \alpha_2 e^{-(\alpha_1+\alpha_2)t}$	$p_{0,2} = \alpha_2/(\alpha_1+\alpha_2)$
$q_{1,0}(t) = \beta_1 e^{-(\beta_1+\alpha_2)t}$	$p_{1,0} = \beta_1/(\beta_1+\alpha_2)$
$q_{1,4}(t) = \alpha_2 e^{-(\beta_1+\alpha_2)t}$	$p_{1,4} = \alpha_2/(\beta_1+\alpha_2)$
$q_{2,0}(t) = \beta_2 e^{-(\beta_2+\alpha_1+\alpha_3)t}$	$p_{2,0} = \beta_2/(\beta_2+\alpha_1+\alpha_3)$
$q_{2,3}(t) = \alpha_1 e^{-(\beta_1+\alpha_1+\alpha_3)t}$	$p_{2,3} = \alpha_1/(\beta_1+\alpha_1+\alpha_3)$
$q_{2,4}(t) = \alpha_3 e^{-(\beta_1+\alpha_1+\alpha_3)t}$	$p_{2,4} = \alpha_3/(\beta_1+\alpha_1+\alpha_3)$
$q_{3,1}(t) = \beta_2 e^{-(\beta_2+\beta_1+\alpha_3)t}$	$p_{3,1} = \beta_2/(\beta_2+\beta_1+\alpha_3)$
$q_{3,2}(t) = \beta_1 e^{-(\beta_2+\beta_1+\alpha_3)t}$	$p_{3,2} = \beta_1/(\beta_2+\beta_1+\alpha_3)$
$q_{3,5}(t) = \alpha_3 e^{-(\beta_2+\beta_1+\alpha_3)t}$	$p_{3,5} = \alpha_3/(\beta_2+\beta_1+\alpha_3)$

Table 1 shows the transition probability for moving from state 0 to state 6, whereas Table 2 represent the mean sojourn times after finding the reliability for various states.

Table 2: *Mean Sojourn Times*

$R_i(t)$	$\mu_i = R_i^*(0)$
$R_0(t) = e^{-(\alpha_1+\alpha_2)t}$	$\mu_0 = 1/(\alpha_1 + \alpha_2)$
$R_1(t) = e^{-(\beta_1+\alpha_2)t}$	$\mu_1 = 1/(\beta_1 + \alpha_2)$
$R_2(t) = e^{-(\beta_2+\alpha_1+\alpha_3)t}$	$\mu 2 = 1/(\beta_2 + \alpha_1 + \alpha_3)$
$R_3(t) = e^{-(\beta_2+\beta_1+\alpha_3)t}$	$\mu_3 = 1/(\beta_2 + \beta_1 + \alpha_3)$
$R_4(t) = e^{-(\beta_3+\alpha_1+\alpha_4)t}$	$\mu_4 = 1/(\beta_3 + \alpha_1 + \alpha_4)$
$R_5(t) = e^{-(\beta_3+\beta_1)t}$	$\mu_5 = 1/(\beta_3 + \beta_1)$
$R_6(t) = e^{-(\beta_4)t}$	$\mu_6 = 1/\beta_4$

V. Measurement of parameter

The various transition probabilities of working system with 0 as initial state and base state $\xi = 4$ are explained below.
$V_{0,0} = 1$(Verified)
$V_{4,4} = 1$(Verified)
$V_{4,6} = (4,6,) = p_{4,6}$

I. Mean time to system failure (MTSF) (T_0)

The un-failed states to which the system transits before visiting any failed state are: $0 \leq j \leq = 4$, and taking $\xi' = 0'$, then

$$\text{MTSF } (T_0) = = \left[\Sigma_{i,sr}\left\{\frac{\left\{pr\left(\xi^{\overset{sr(sff)}{-}} i\right)\right\}\mu i}{\Pi_{m_1\neq\xi}\left\{1\text{-}V_{\overline{m_1 m_1}}\right\}}\right\}\right] / \left[1\text{-}\Sigma_{sr}\left\{\frac{\left\{pr\left(\xi^{\overset{sr(sff)}{-}} \xi\right)\right\}}{\Pi_{m_2\neq\xi}\left\{1\text{-}V_{\overline{m_2 m_2}}\right\}}\right\}\right]$$

$$T_0 = \alpha_1(\beta_1 + \alpha_4 + 2\alpha_3 + \alpha_2)(\beta_1 + \alpha_4 + \alpha_2 + \alpha_3)(\beta_1 + \alpha_1 + 2\alpha_2 + \alpha_4) / (\alpha_1 + \alpha_4 + \alpha_2 + \alpha_3)(\beta_1 + \alpha_2 + 3\alpha_3 + \alpha_4 + \alpha_1)^3$$

II. Availability of System (A_0)

The states are where system is available are $0 \leq j \leq 4$ taking base state $\xi' = 4$ total fraction of time for which system is available

$$A_0 = \left[\Sigma_{j,sr}\left\{\frac{\{pr(\xi^{sr\rightarrow} j)\}fj,\mu j}{\Pi_{m_1\neq\xi}\left\{1\text{-}V_{\overline{m_1 m_1}}\right\}}\right\}\right] / \left[\Sigma_{i,s_r}\left\{\frac{\{pr(\xi^{sr\rightarrow} i)\}\mu_i^1}{\Pi_{m_2\neq\xi}\left\{1\text{-}V_{\overline{m_2 m_2}}\right\}}\right\}\right]$$

$$A_0 = [1/(\alpha_1 + \alpha_4 + \alpha_3 + \alpha_2) + \alpha_1(\beta_1 + 2\alpha_2 + \alpha_1 + \alpha_4 + \alpha_3)/ (\alpha_1 + 4\alpha_3 + \alpha_2 + \alpha_4)(\beta_1 + \alpha_1 + 3\alpha_3 + \alpha_2)(\beta_1 + \alpha_4 + \alpha_3 + \alpha_2) (\beta_1 + \alpha_2 + 2\alpha_4 + \alpha_1)]/1/(\alpha_1 + \alpha_3 + 2\alpha_4 + \alpha_2) + \{1/(\alpha_3/\beta_3) + (\alpha_4/\beta_4)\}$$

III. Expected number of inspections by the repair man(V_0)

The states where the repairman's visit is fresh are S_1, S_2, taking $\xi = 0$,the number of repairmen's visits.

$$V_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr\to}j)\}}{\Pi_{k_1\neq\xi}\left\{1\text{-}V_{\overline{k_1k_1}}\right\}}\right\}\right] / \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr\to}i)\}\mu_i^1}{\Pi_{k_2\neq\xi}\left\{1\text{-}V_{\overline{k_2k_2}}\right\}}\right\}\right]$$

$$V_0 = \alpha_1(\beta_1 + \alpha_2 + \alpha_1 + 3\alpha_4 + \alpha_3)/(\alpha_1 + \alpha_4 + \alpha_2 + \alpha_3)(\beta_1 + 2\alpha_4 + 2\alpha_4 + \alpha_3)$$
$$(\beta_1 + \alpha_4 + \alpha_2 + \alpha_3)(\beta_1 + \alpha_3 + 3\alpha_4 + \alpha_2)(\beta_1 + 2\alpha_4 + 2\alpha_4 + \alpha_1) +$$
$$[(\alpha_1 + \alpha_4 + \alpha_3)]/(\alpha_1 + \alpha_4 + \alpha_2 + \alpha_3)$$

VI. Methodology for optimization the parameters

Dataset: The dataset includes ten records, each with four normalized features. Workload (W) indicates equipment usage (0–100), Failure Rate (λ) reflects fault likelihood (0–100), and Sensor Index (S) captures unit health based on sensor data—higher values imply more wear. The target variable, Maintenance Priority (p), ranges from 0 to 75 and indicates how urgently maintenance is needed. The goal is to train supervised models to predict p from W, λ, and S, enabling proactive maintenance decisions.

Table 3: *Parameter*

Parameter	W	λ	S	P
Range	0-100	0-100	0-100	0-75

The Optimization of RPGT-Based Reliability Metrics Using CSA, PSO, and GA is explored below.

I. Mean Time to System Failure (T_0)

The Cuckoo Search Algorithm (CSA), PSO, GA was used to optimize the failure (α) and repair (β) rates. The optimized values and the resulting T_0 are shown below.

Table 4: *Optimized Parameters and MTSF Value (T_0)*

Algorithm	α_1	α_2	α_3	α_4	β_1	T_0
CSA	1.2413	2.7539	0.9261	1.5427	3.1186	0.2158
PSO	1.1852	2.6891	0.9987	1.4723	3.0432	0.2197
GA	1.3794	2.8476	0.8742	1.6389	3.1954	0.2135

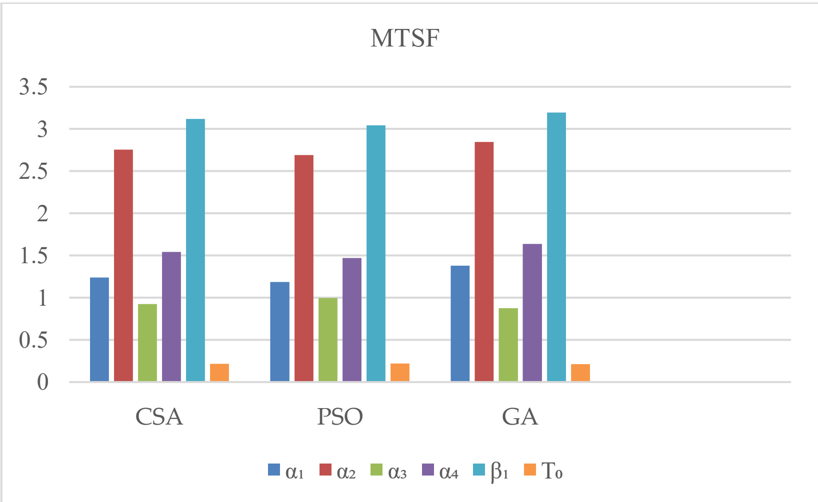

Figure 2: *Optimized Parameters and MTSF Value (T_0)*

II. Availability Optimization

CSA, PSO, and GA were applied to optimize a nonlinear availability function using failure and repair rates (α_1–α_4, β_1) to maximize system availability (A_0).

Table 5: *Optimized Parameters and Availability Value (T_0)*

Algorithm	α_1	α_2	α_3	α_4	β_1	A_0
CSA	1.2413	2.7539	0.9261	1.5427	3.1186	0.085429
PSO	1.3786	2.9874	0.9142	1.6349	3.0562	0.087113
GA	1.1842	2.6645	0.8963	1.5217	3.1325	0.084307

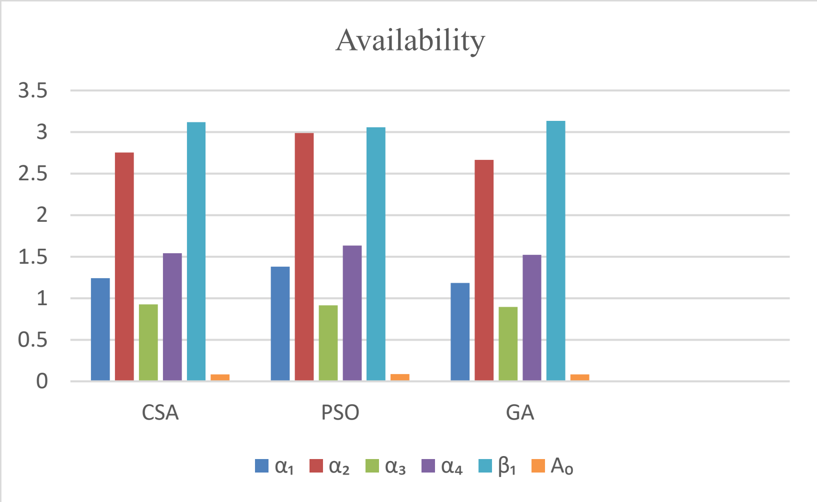

Figure 3: *Optimized Parameters and Availability Value (A_0)*

PSO achieved the highest system availability, followed closely by CSA, while GA was less effective, showing PSO's superior optimization ability.

III. Expected Number of Inspections (V_0) using CSA

The inspection frequency of the repairman is optimized using the Cuckoo Search Algorithm (CSA). Based on the derived reliability expression involving failure and repair rates, the number of inspections (V_0) was calculated using the formula.

Table 6: *Optimized Parameters and Expected fractional number of inspection Value (V_0)*

Algorithm	α_1	α_2	α_3	α_4	β_1	V_0
CSA	1.2413	2.7539	0.9261	1.5427	3.1186	0.184
PSO	1.3089	2.8467	0.8975	1.6134	3.0921	0.179
GA	1.1932	2.7011	0.9493	1.5018	3.1457	0.193

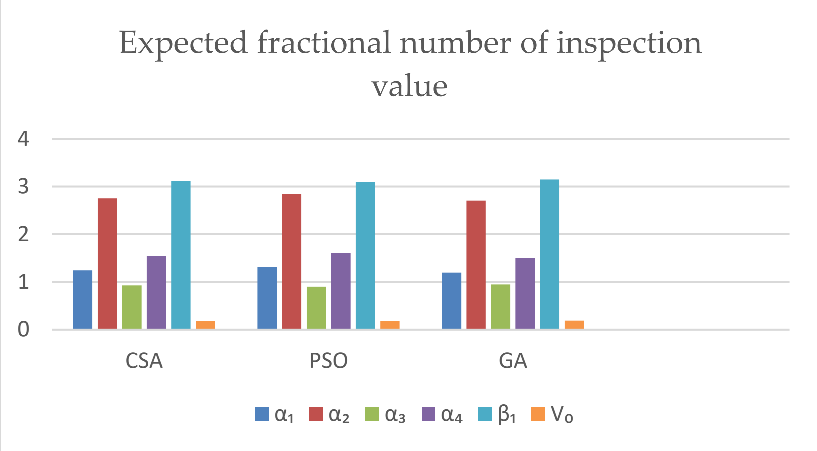

Figure 4: *Optimized Parameters and Expected number of inspection Value (V_0)*

PSO gave the lowest V_0, indicating the fewest expected inspections and efficient repairs, followed by CSA with balanced scheduling, while GA required the most frequent inspections.

VII. Conclusion

In this study, a 2-out-of-4 redundant repairable system was modeled and analyzed using the RPGT, with the primary goal of optimizing key reliability performance indices: T_0 (MTSF), A_0 (Availability), and V_0 (Expected Inspections). PSO consistently delivered the best overall performance, providing highest MTSF, maximum availability, and least inspection frequency. SA (Cuckoo Search Algorithm) was a close second, offering balanced results and strong optimization capability.GA (Genetic Algorithm), though effective, was less optimal in this context, with relatively lower reliability values and higher inspection requirements. Therefore, PSO is concluded to be the most effective optimization technique for the presented system model, followed by CSA, with GA being the least effective under the same conditions.

Conflicts of interests: The authors declare that there are no conflicts of interest.

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