A NON- MARKOVIAN RETRIAL QUEUE WITH NON PERSISTENT CUSTOMERS, GENERAL RETRIAL TIMES ALONG WITH REOCCURRING CUSTOMERS

S. Palaniammal[1], K. Kumar[2], R.Keerthana[3]

•

[1] Department of Mathematics, Sri Krishna Adithya College of Arts and Science,India.
[2] Department of Mathematics,Sri Krishna College of Engineering and Technology,India.
[3] V.S.B. Engineering College, India
principal@skacas.ac.in, kkumaar007@gmail.com, keerthirn1996@gmail.com

Abstract

A non-Markovian retrial queue with non persistent customers, general retrial times along with reoccurring customers is taken into concern in this place study. In this model, the retrial periods for reoccurring customers have an exponential distribution, while all the service periods and retrial periods for transitory customers are pretended to follow a general distribution. The PGF for the total amount of customers and the average amount of customers in the invisible waiting region is acquired by applying the supplementary variable method. We compute the waiting period delivery. Out of attention, special cases are conferred. Numerical outcomes are reveals.

Keywords: non-Markovian retrial queue, transitory and reoccurring customers, non persistent customers, general retrial times, supplementary variable technique.

1. Introduction

Whenever the customers recognise that the server is occupied, they are moved to an orbit, that is an invisible awaiting region. A lot of experts have studied the retrial queue in the recent past. There are referrals available for an additional review of the retrial queues [1, 2, 3, 4, 5]. Numerous queuing systems that have multiple attempts follow the traditional retrial strategy. A second discipline, known as permanent retrial policy, does, however, naturally exist in situations where the server requires searching for customers [6].

In the context of retrials, Farahmand [7] examined a system of permanent customers from two perspectives: traditional and permanent retrial policy. Since the literature on retrial queues with continuous repeated efforts has rapidly expanded [8, 9]. In contrast, Boxma and Cohen [10] studied an M/G/1 queue that has a fixed amount of permanent customers who rejoin the queue after getting their services. Gomez-Corral [11] went into great detail about a retrial queuing system that includes general retrial times and FCFS discipline.

Our primary goal is to extend the second instance in [7] by taking consideration of a more typical and widespread scenario: distinct service distribution for each kind of customers and normal retrial rates for customers who are transitory. The primary justification for examining this queuing system is that numerous depictions of computer and communication networks use its structure.

A primary customer can become dissatisfied and permanently leave the system without service if they discover that a server is busy. For an additional review of the non-persistent customers, Keerthana [15] presented the results regarding the probability generating function (PGF) for the number of customers in the orbit while discussing the retrial queue with working vacations, nonpersistent customers, and a waiting server.

In this article, we consider a non-Markovian retrial queue with non persistent customers, general retrial times along with reoccurring customers. This article is organised as follows. In section 2, the model is described. In section 3, performance measures are defined. In Section 4, a few unique scenarios are explored. In section 5, numerical findings are presented. In section 6, the conclusion is given.

2. Model Description

We discuss a non-Markovian retrial queue with non persistent customers, general retrial times and reoccurring customers, in which the arrival rate of the primary customers is λ and the arrival process is Poison. If a customer enters and discovers that the server is busy, he either becomes non-persistent and exits the system with a probability of (1-α) or enters an orbit with a probability of α. We assume there is no waiting area, thus if a transitory customer approaches and finds the server is occupied, they leave the service area and enter the orbit. Both of the customers who are at the top of the orbit are free to get nearer to the server at anytime after the service is finished. For transitory customers, the retrial time is distributed according to a distribution function and a general distribution $R_t(x)$; the pdf and LST are denoted by the expressions $r_t(x)$ and $R_t^*(x)$, respectively. For reoccuring customers, the retrial time is distributed exponentially with a parameter of ν. If there is a competition between transitory customers and orbit customers during each customer's service completion epoch. For transitory customers a general distribution with distribution functions governs service times $S_t(x)$,$s_t(x)$ and $S_t^*(x)$ as pdf and LST, respectively, and for reoccurring customers a general distribution with distribution functions governs service times, pdf, and LST of $S_r(x), s_r(x)$, and $S_r^*(x)$. It is assumed that service times, retrial periods, and inter-arrival times are unrelated to one another. Reoccurring customers always return to orbit after a service is completed, but transitory customers permanently exit the system. Let's apply the following sequence of random variables.

For the transitory customer at the top of the orbit, the remaining retial time is indicated by $R_t^\circ(t)$ if $M(t) = 0$ and $N(t) > K$.

For the transitory customer, the remaining service time is indicated by $S_t^\circ(t)$ if $M(t) = 1$ and $N(t) \geq K$.

For the reoccuring customer, the remaining service time is indicated by $S_r^\circ(t)$ if $M(t) = 2$ and $N(t) < K$.

$$M(t) = \begin{cases} 0 & \text{if the system is idle at time } t \\ 1 & \text{if the system is busy with transitory customers} \\ 2 & \text{if the system is busy with reoccuring customers} \end{cases}$$

so that the supplementary variables $R_t^\circ(t), S_t^\circ(t), S_r^\circ(t)$ and $R_s^\circ(t)$ are introduced to acquire the bivariate Markov Processes $(N(t), B(t)); x \geq 0$, where $B(t) = R_t^\circ(t)$, if $M(t) = 0; S_t^\circ(t)$, if $M(t) = 1; S_r^\circ(t)$, if $M(t) = 2$.

$$\begin{aligned} P_{0,K} &= \lim_{t\to\infty} P[M(t) = 0, N(t) = K] \\ P_{0,n} &= \lim_{t\to\infty} P[M(t) = 0, N(t) = n, x < R_t^\circ(t) \leq x + dx], n \geq K+1, x \geq 0 \\ P_{t,n} &= \lim_{t\to\infty} P[M(t) = 1, N(t) = n, x < S_t^\circ(t) \leq x + dx], n \geq K, x \geq 0 \\ P_{r,n} &= \lim_{t\to\infty} P[M(t) = 2, N(t) = n, x < S_r^\circ(t) \leq x + dx], n \geq K-1, x \geq 0 \end{aligned}$$

We have defined the limiting probabilities, which are listed above.

$$P_{0,n}^*(\theta) = \int_0^\infty e^{-\theta x} P_{0,n}(x)dx, \qquad P_{0,n}^*(0) = \int_0^\infty P_{0,n}(x)dx$$

$$P_0^*(z,\theta) = \sum_{n=K+1}^{\infty} P_{0,n}^*(\theta)z^n \qquad P_0(z,0) = \sum_{n=K+1}^{\infty} P_{0,n}(0)z^n$$

$$P_1^*(z,\theta) = \sum_{n=K}^{\infty} P_{1,n}^*(\theta) z^n \qquad P_1(z,0) = \sum_{n=K}^{\infty} P_{1,n}(0) z^n$$
$$P_2^*(z,\theta) = \sum_{n=K-1}^{\infty} P_{2,n}^*(\theta) z^n \qquad P_2(z,0) = \sum_{n=K-1}^{\infty} P_{2,n}(0) z^n$$

We have defined the LST and PGF, which are listed above.
In steady state, the following differential difference equations were used to depict the system:

$$(\lambda+\nu)P_{0,K} = P_{1,K}(0) + P_{2,K-1}(0) \tag{1}$$
$$-\frac{d}{dx}P_{0,n}(x) = -(\lambda+\nu)P_{0,n} + P_{1,n}(0)r_t(x) + P_{2,n-1}(0)r_t(x); n \geq K+1 \tag{2}$$
$$-\frac{d}{dx}P_{1,K}(x) = -\lambda\alpha P_{1,K}(x) + P_{0,K+1}(0)s_t(x) + \lambda P_{0,K}s_t(x) \tag{3}$$
$$\begin{aligned}-\frac{d}{dx}P_{1,n}(x) &= -\lambda\alpha P_{1,n}(x) + \lambda\alpha P_{1,n-1}(x) + P_{0,n+1}(0)s_t(x) \\ &\quad +\lambda s_t(x)\int_0^\infty P_{0,n}(x)dx; n \geq K+1\end{aligned} \tag{4}$$
$$-\frac{d}{dx}P_{2,K-1}(x) = -\lambda\alpha P_{2,K-1}(x) + \nu P_{0,K}s_r(x) \tag{5}$$
$$-\frac{d}{dx}P_{2,n}(x) = -\lambda\alpha P_{2,n}(x) + \lambda\alpha P_{2,n-1}(x) + \nu s_r(x)\int_0^\infty P_{0,n+1}dx; n \geq K \tag{6}$$

Taking the LST on both sides from (2) to (6) yields

$$\theta P_{0,n}^*(\theta) - P_{0,n}(0) = (\lambda+\nu)P_{0,n}^*(\theta) - P_{1,n}(0)R_t^*(\theta) - P_{2,n-1}(0)R_t^*(\theta), n \geq K+1 \tag{7}$$
$$\theta P_{1,K}^*(\theta) - P_{1,K}(0) = \lambda\alpha P_{1,K}^*(\theta) - P_{0,K+1}(0)S_t^*(\theta) - \lambda P_{0,K}S_t^*(\theta) \tag{8}$$
$$\begin{aligned}\theta P_{1,n}^*(\theta) - P_{1,n}(0) &= \lambda\alpha P_{1,n}^*(\theta) - \lambda\alpha P_{1,n-1}(\theta) - P_{0,n+1}(0)S_t^*(\theta) \\ &\quad -\lambda\int_0^\infty P_{0,n}(x)dxS_t^*(\theta), \quad n \geq K+1\end{aligned} \tag{9}$$
$$\theta P_{2,K-1}^*(\theta) - P_{2,k-1}(0) = \lambda\alpha P_{2,k-1}^*(\theta) - \nu P_{0,k}S_r^*(\theta) \tag{10}$$
$$\theta P_{2,n}^*(\theta) - P_{2,n}(0) = \lambda\alpha P_{2,n}^*(\theta) - \lambda P_{2,n-1}^*(\theta) - \nu\int_0^\infty P_{0,n+1}(x)dxS_r^*(\theta), \quad n \geq K \tag{11}$$

(8) x $\sum_{n=K+1}^{\infty} z^n$gives

$$\begin{aligned}\theta\sum_{n=K+1}^{\infty}P_{0,n}^*(\theta)z^n - \sum_{n=K+1}^{\infty}P_{0,n}(0)z^n &= (\lambda+\nu)\sum_{n=K+1}^{\infty}P_{0,n}^*(\theta)z^n - \sum_{n=K+1}^{\infty}P_{1,n}(0)z^nR_t^*(\theta) \\ &\quad - \sum_{n=K+1}^{\infty}P_{2,n-1}(0)z^nR_t^*(\theta)\end{aligned}$$

$$\begin{aligned} &= P_0(z,0) - R_t^*(\theta)\{P_1(z,0) + zP_2(z,0)\} + R_t^*(\theta)z^K(\lambda+\nu)P_{0,K} \\ P_0^*(z,\theta)[\theta-(\lambda+\nu)] &= P_0(z,0) - R_t^*(\theta)\{z^K(\lambda+\nu)P_{0,K} - P_1(z,0) + zP_2(z,0)\}\end{aligned} \tag{12}$$

Placing $\theta = \lambda+\nu$ in (12), results

$$P_0(z,0) = R_t^*(\lambda+\nu)[P_1(z,0) + zP_2(z,0) - (\lambda+\nu)P_{0,k}z^K] \tag{13}$$

Placing $\theta = 0$ in (12), results

$$P_0^*(z,0) = \frac{[1-R_t^*(\lambda+\nu)]}{\lambda+\nu}[P_1(z,0) + zP_2(z,0) - (\lambda+\nu)P_{0,k}z^K] \tag{14}$$

$\Big[(9) \text{ x } \sum_{n=K+1}^{\infty} z^n\Big]$ gives,

$$\theta \sum_{n=K+1}^{\infty} P^*_{1,n}(\theta) z^n - \sum_{n=K+1}^{\infty} P_{1,n}(0) z^n = \lambda\alpha \sum_{n=K+1}^{\infty} P^*_{1,n}(\theta) z^n - \lambda\alpha \sum_{n=K+1}^{\infty} P^*_{1,n-1}(\theta) z^{n-1+1}$$
$$- S^*_t(\theta) \sum_{n=K+1}^{\infty} P_{0,n+1}(0) z^{n+1-1} - \lambda \sum_{n=K+1}^{\infty} P^*_{0,n}(0) z^n S^*_t(\theta)$$

Adding with $(8) \times z^K$, we get

$$P^*_1(z,\theta)[\theta - (\lambda\alpha - \lambda\alpha z)] = P_1(z,0) - \frac{P_0(z,0) S^*_t(\theta)}{z} - \lambda P^*_0(z,0) S^*_t(\theta) - \lambda P^*_{0,K} z^K S^*_t(\theta) \tag{15}$$

Placing $\theta = \lambda\alpha - \lambda\alpha z$ in (15), results

$$P_1(z,0) = \frac{S^*_t(\lambda\alpha - \lambda\alpha z)}{z} \Big[P_0(z,0) + \lambda z P^*_0(z,0) + \lambda P_{0,k} z^{K+1}\Big] \tag{16}$$

Placing $\theta = 0$ in (15), results

$$P^*_1(z,0) = \frac{1 - [S^*_t(\lambda\alpha - \lambda\alpha z)]}{z(\lambda\alpha - \lambda\alpha z)} \Big[P_0(z,0) + \lambda z P^*_0(z,0) + \lambda P_{0,k} z^{K+1}\Big] \tag{17}$$

(Sub.) (14) in (16), results

$$\begin{aligned} P_1(z,0) &= \Big[\frac{\lambda z[1 - R^*_t(\lambda+\nu)][P_1(z,0) + z P_2(z,0) - (\lambda+\nu) P_{0,K} z^K]}{\lambda+\nu} \\ &\quad + P_0(z,0) + \lambda P_{0,K} z^{K+1}\Big] \Big[\frac{S^*_t(\lambda\alpha - \lambda\alpha z)}{z}\Big] \end{aligned} \tag{18}$$

$\Big[(11) \text{ x } \sum_{n=K}^{\infty} z^n\Big]$ gives

$$\theta \sum_{n=K}^{\infty} P^*_{2,n}(\theta) z^n - \sum_{n=K}^{\infty} P_{2,n}(0) z^n = \lambda\alpha \sum_{n=K}^{\infty} P^*_{2,n}(\theta) z^n - \lambda\alpha \sum_{n=K}^{\infty} P^*_{2,n-1}(\theta) z^{n-1+1}$$
$$- \nu \sum_{n=K}^{\infty} P^*_{0,n+1}(0) z^{n+1-1} S^*_r(\theta)$$

Adding with $(10) \times z^{K-1}$, we get

$$P^*_2(z,\theta)[\theta - (\lambda\alpha - \lambda\alpha z)] = P_2(z,0) - \frac{\nu}{z} S^*_r(\theta)[P^*_0(z,0) + P_{0,K} z^K] \tag{19}$$

Placing $\theta = \lambda - \lambda z$ in (19), results

$$\begin{aligned} P_2(z,0) &= \Bigg[\Big[\frac{1 - S^*_t(\lambda\alpha - \lambda\alpha z)}{\lambda+\nu}\Big] \Big[P_1(z,0) + z P_2(z,0) - (\lambda+\nu) P_{0,k} z^K\Big] + P_{0,k} z^K\Bigg] \\ &\quad \times \Bigg[\frac{\nu S^*_r(\lambda\alpha - \lambda\alpha z)}{z}\Bigg] \end{aligned} \tag{20}$$

Placing $\theta = 0$ in (19), results

$$-(\lambda\alpha - \lambda\alpha z) P^*_2(z,0) = P_2(z,0) - \frac{\nu}{z}[P^*_0(z,0) + P_{0,K} z^K] \tag{21}$$

Rearrage the equation (13),(16) and (20) in the system of equations, results

$$
\begin{aligned}
(\lambda+\nu)P_{0,K}z^K R_t^*(\lambda+\nu) &= -P_0(z,0)+P_1(z,0)R_t^*(\lambda+\nu)+zP_2(z,0)R_t^*(\lambda+\nu) \\
\lambda z P_{0,K}z^K R_t^*(\lambda+\nu)(\lambda+\nu)S_t^*(\lambda\alpha-\lambda\alpha z) &= -P_0(z,0)(\lambda+\nu)S_t^*(\lambda\alpha-\lambda\alpha z)+P_1(z,0)[z(\lambda+\nu) \\
&\quad -\lambda z(1-R_t^*(\lambda+\nu))]-zP_2(z,0)S_t^*(\lambda\alpha-\lambda\alpha z) \\
&\quad \times\lambda z(1-R_t^*(\lambda+\nu)) \\
\nu(\lambda+\nu)R_t^*(\lambda+\nu)P_{0,K}z^K S_r^*(\lambda\alpha-\lambda\alpha z) &= -\nu P_1(z,0)S_r^*(\lambda\alpha-\lambda\alpha z)(1-R_t^*(\lambda+\nu))+P_2(z,0) \\
&\quad \times[z(\lambda+\nu)-\nu z S_r^*(\lambda\alpha-\lambda\alpha z)(1-R_t^*(\lambda+\nu))
\end{aligned}
$$

By solving the above equations using determinent method, results

$$
\begin{aligned}
P_0(z,0) &= \frac{1}{D_1(z)}\Big\{(\lambda+\nu)R_t^*(\lambda+\nu)P_{0,K}z^{K+1}[(\lambda+\nu)-\lambda S_t^*(\lambda\alpha-\lambda\alpha z) \\
&\quad -\nu S_r^*(\lambda\alpha-\lambda\alpha z)]\Big\} \qquad (22) \\
P_1(z,0) &= \frac{1}{D_1(z)}\Big\{(\lambda+\nu)R_t^*(\lambda+\nu)P_{0,K}z^K S_t^*(\lambda\alpha-\lambda\alpha z)[\lambda(1-z) \\
&\quad -\nu[S_r^*(\lambda\alpha-\lambda\alpha z)-1]]\Big\} \qquad (23) \\
P_2(z,0) &= \frac{1}{D_1(z)}\Big\{(\lambda+\nu)R_t^*(\lambda+\nu)\nu S_r^*(\lambda\alpha-\lambda\alpha z)[S_t^*(\lambda\alpha-\lambda\alpha z)-z]P_{0,K}z^{K-1}\Big\} \qquad (24)
\end{aligned}
$$

Substituting (23), (24) in (14)

$$
P_0^*(z,0)=\frac{1}{D_1(z)}\Big\{[1-R_t^*(\lambda+\nu)][\lambda(1-S_t^*(\lambda\alpha-\lambda\alpha z)+\nu(1-S_r^*(\lambda\alpha-\lambda\alpha z))]P_{0,K}z^{K+1}\Big\} \qquad (25)
$$

Substituting (22), (25) in (17)

$$
\begin{aligned}
P_1^*(z,0) &= \frac{1}{(\lambda\alpha-\lambda\alpha z)D_1(z)}\Big\{[1-S_t^*(\lambda\alpha-\lambda\alpha z)]R_t^*(\lambda+\nu)[\lambda(1-z) \\
&\quad +\nu(1-S_r^*(\lambda\alpha-\lambda\alpha z))](\lambda+\nu)P_{0,K}z^K\Big\} \qquad (26)
\end{aligned}
$$

Substituting (24), (25) in (21)

$$
\begin{aligned}
P_2^*(z,0) &= \frac{1}{(\lambda\alpha-\lambda\alpha z)D_1(z)}\Big\{(\lambda+\nu)R_t^*(\lambda+\nu)[S_t^*(\lambda\alpha-\lambda\alpha z)-z][1-S_r^*(\lambda\alpha-\lambda\alpha z)] \\
&\quad \times\nu P_{0,K}z^{K-1}\Big\} \qquad (27)
\end{aligned}
$$

where,

$$
\begin{aligned}
D_1(z) &= [1-R_t^*(\lambda+\nu)][\lambda S_t^*(\lambda\alpha-\lambda\alpha z)+\nu S_r^*(\lambda\alpha-\lambda\alpha z)]z-(\lambda+\nu)[z-R_t^*(\lambda+\nu) \\
&\quad \times S_t^*(\lambda\alpha-\lambda\alpha z)] \qquad (28)
\end{aligned}
$$

We define $P(z)=P_0^*(z,0)+P_1^*(z,0)+P_2^*(z,0)+P_{0,K}z^K$

$$
P(z)=\frac{(\lambda+\nu)R_t^*(\lambda+\nu)P_{0,K}z^{K-1}}{\lambda\alpha D_1(z)}\Big\{\lambda\alpha(1-z)z+\nu S_t^*(\lambda\alpha-\lambda\alpha z)(1-S_r^*(\lambda\alpha-\lambda\alpha z))\Big\} \qquad (29)
$$

As the PGF for the number of customers in the orbit is represented in above relation in that the defined term $D_1(z)$ is given in (28). To determine that $P_(0,K)$ arises in (30), apply the normalising condition $P(1)=1$ and utilising the L'Hospitals rule in the result of (29).

$$
P_{0,K}=\frac{(\lambda+\nu)R_t^*(\lambda+\nu)[1-\lambda\alpha E(S_t)]-\lambda\alpha(1-R_t^*(\lambda+\nu))[\lambda E(S_t)+\nu E(S_r)]}{(\lambda+\nu)\alpha R_t^*(\lambda+\nu)[1+\nu\alpha E(S_r)]} \qquad (30)
$$

3. Performance Measures

The following parameters are considered, L_s - average orbit size during peak hours and, L_{sw} - average customers waiting time at peak hours in the orbit. Then

$$\begin{aligned} L_s &= \frac{d}{dz}P(z)\Big|_{z=1} \\ &= \Big[\frac{N_1'(z)}{D_1'(z)} + \frac{N_2''(z)}{-2\lambda\alpha D_1'(z)} + \frac{N_3''(z)}{-2\lambda\alpha D_1'(z)}\Big]\Big|_{z=1} \times P_{0,K} \end{aligned}$$

where

$$\begin{aligned} N_1(z) &= [1-R_t^*(\lambda+\nu)][\lambda(1-S_t^*(\lambda\alpha-\lambda\alpha z)+\nu(1-S_r^*(\lambda\alpha-\lambda\alpha z))]z^{K+1} \\ D_1(z) &= [1-R_t^*(\lambda+\nu)][\lambda S_t^*(\lambda\alpha-\lambda\alpha z)+\nu S_r^*(\lambda\alpha-\lambda\alpha z)]z-(\lambda+\nu)[z-R_t^*(\lambda+\nu) \\ &\quad \times S_t^*(\lambda\alpha-\lambda\alpha z)] \\ N_2(z) &= [1-S_t^*(\lambda-\lambda\alpha z)]R_t^*(\lambda+\nu)[\lambda(1-z)+\nu(1-S_r^*(\lambda\alpha-\lambda\alpha z))](\lambda+\nu)z^K \\ N_3(z) &= (\lambda+\nu)R_t^*(\lambda+\nu)[S_t^*(\lambda\alpha-\lambda\alpha z)-z][1-S_r^*(\lambda\alpha-\lambda\alpha z)]\nu z^{K-1} \end{aligned}$$

At $z=1$ L_s turns

$$L_s = \Big[\frac{N_1'(1)}{D_1'(1)} + \frac{N_2''(1)}{-2\lambda\alpha D_1'(1)} + \frac{N_3''(1)}{-2\lambda\alpha D_1'(1)}\Big] \times P_{0,K}$$

By Little's formula, $L_{sw} = \dfrac{L_s}{\lambda}$
where,

$$\begin{aligned} N_1'(1) &= \lambda\alpha[1-R_t^*(\lambda+\nu)][\lambda E(S_t)+\nu E(S_r)] \\ N_2''(1) &= 2\lambda^2\alpha(\lambda+\nu)R_t^*(\lambda+\nu)E(S_t)(1+\nu\alpha E(S_r)) \\ N_3''(1) &= -2\lambda\nu\alpha(\lambda+\nu)R_t^*(\lambda+\nu)E(S_r)(\lambda\alpha E(S_t)-1) \\ D_1'(1) &= \lambda\alpha[1-R_t^*(\lambda+\nu)][\lambda E(S_t)+\nu E(S_r)]-(\lambda+\nu)R_t^*(\lambda+\nu)[1-\lambda\alpha E(S_t)] \end{aligned}$$

4. Special cases

(a) The current model coincides with M/G/1 queue with recurrent customers model for taking the values $\nu \to \infty, \alpha = 1$, and $R_t^*(\lambda+\nu) = 1$
(b) The current model concides with an M/G/1 retrial queue model for taking the value $K = 0$, and $\alpha = 1$.

5. Numerical results

Table 1 and Fig. 1 and represent the variation between the arrival rate of customers versus the average orbit size during peak hours in the presence of various service times. Also, to attain the above mentioned graph and table, the following values are assumed. The value of a retrial time for reoccurring customers is 2.1, the value of service time for transitory customers is 3.1, and the value of service time for reoccurring customers is 4.2 and $\alpha = 1.2$. From this we observed that the increasing value of the arrival rate of customers and service time of customers, the mean orbit size will be reduced; it shows that the derived model is stable. Table 2 and Fig. 2 represent the variation between the arrival rate of customers versus the average customers waiting time at peak hours in the presence of various service times. Also, to attain the abovementioned graph and table, the following values are assumed. The value of a retrial time for reoccurring customers is 2, the value of service time for transitory customers is 3.5, and the value of service time for reoccurring customers is 4 and $\alpha = 1.5$. From this we observed that the increasing value of the arrival rate of customers and service time of customers, the mean customers waiting time at peak hours in the orbit will be reduced; it shows that the derived model is stable. Table 3 and

Table 1: *λ versus μ*

λ	$\mu = 2.2$	$\mu = 2.4$	$\mu = 2.6$
1.0	0.2991	0.3198	0.3374
1.2	0.2688	0.2956	0.3184
1.4	0.2306	0.2643	0.2928
1.6	0.1847	0.2258	0.2605
1.8	0.1309	0.1801	0.2216
2.0	0.0694	0.1272	0.1761

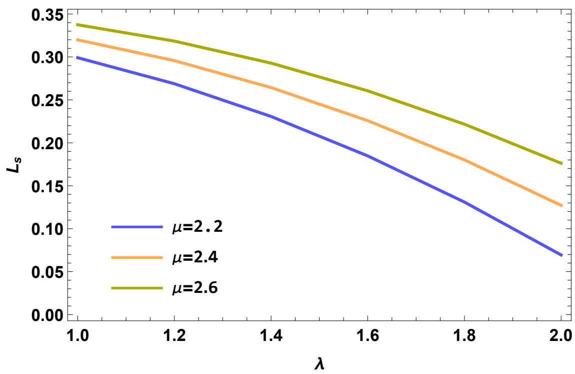

Figure 1: *λ versus μ*

Table 2: *λ versus μ*

λ	$\mu = 3.2$	$\mu = 4.2$	$\mu = 5.2$
1.0	0.3492	0.3741	0.3928
1.2	0.2809	0.3077	0.3277
1.4	0.2285	0.2571	0.2785
1.6	0.1861	0.2164	0.2392
1.8	0.1502	0.1824	0.2066
2.0	0.1190	0.1530	0.1785

Fig. 3 represent the variation between the arrival rate of customers versus the average orbit size during peak hours in the presence of various retrial times. Also, to attain the abovementioned graph and table, the following values are assumed. The value of a retrial time for transitory customers is 3.7, the value of service time for transitory customers is 5.5, and the value of service time for reoccurring customers is 7.5 and $\alpha = 1.2$. From this we observed that the increasing value of the arrival rate of customers and service time of customers, the mean orbit size will be reduced; it shows that the derived model is stable. Table 4 and Fig. 4 represent the variation between the arrival rate of customers versus the average customers waiting time at peak hours in the presence of various retrial times. Also, to attain the abovementioned graph and table, the following values are assumed. The value of a retrial time for transitory customers is 4, the value of service time for transitory customers is 4.8, and the value of service time for reoccurring customers is 7.2 and $\alpha = 1.2$. From this we observed that the increasing value of the arrival rate of customers and service time of customers, the mean customers waiting time at peak hours in

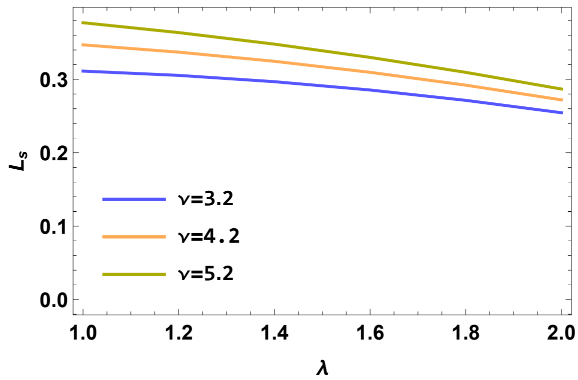

Figure 2: λ *versus* μ

the orbit will be reduced; it shows that the derived model is stable.

Table 3: λ *versus* ν

λ	$\nu = 3$	$\nu = 3.5$	$\nu = 4$
1.0	0.3112	0.3470	0.3771
1.2	0.3054	0.3370	0.3637
1.4	0.2968	0.3245	0.3479
1.6	0.2854	0.3095	0.3298
1.8	0.2713	0.2920	0.3095
2.0	0.2545	0.2720	0.2867

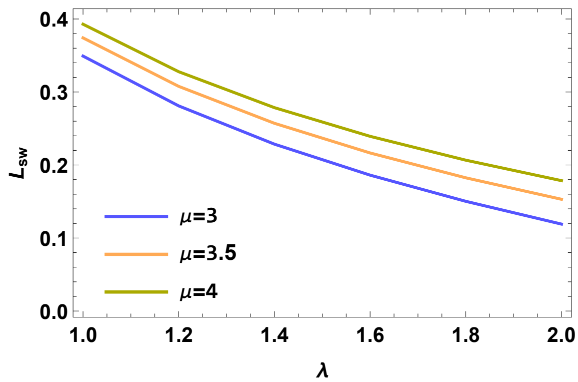

Figure 3: λ *versus* ν

Table 4: λ *versus* ν

λ	$\nu = 2.1$	$\nu = 3.1$	$\nu = 4.1$
1.0	0.2903	0.3349	0.3716
1.2	0.2446	0.2775	0.3045
1.4	0.2096	0.2343	0.2547
1.6	0.1814	0.2002	0.2157
1.8	0.1577	0.1720	0.1838
2.0	0.1370	0.1480	0.1570

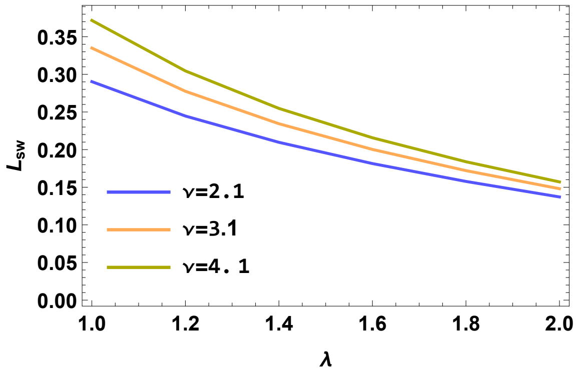

Figure 4: *λ versus ν*

6. Conclusion

A non-Markovian retrial queue with non persistent customers, general retrial times along with reoccurring customers is assessed in this work. We obtained the PGF for the total amount of customers and the average amount of customers in the invisible waiting region. The average waiting time is also determined. We discussed the performance measures and deliberated some special cases, and numerical outcomes are displayed.

References

[1] Yang T. and Templeton, J. G. C. (1987). A Survey on Retrial Queue. *Queue. Syst.*, 2:201–233.
[2] Falin, G. I. (1990). A Survey on Retrial Queues. *Queueing Systems Theory and Applications*, 7:127–168.
[3] Falin, G. I. and Templeton, J.G. C. Retrial queues, CRC Press, 1997.
[4] Artalejo, J. R. (1999). Accessible bibliography on retrial queue. *Math. Comput. Modell.*, 30:1–6.
[5] Artalejo, J. R.(1999). A classified bibliography of research on retrial queues: Progress in 1990-1999. Top, 7(2): 187–211.
[6] Neuts, M. F. and Ramalhoto, M. F. (1984). A Service model in which the server is required to search for customers. *Journal of applied probability*, 21:157–166.
[7] Farahmand, K.(1990). Single line queue with repeated demands. *Queueing Systems*, 6:223–228.
[8] Farahmand, K.(1996). Single line queue with recurrent repeated demands. *Queueing Systems*, 22:425–435.
[9] Martin, M. and Artalejo, J. R. (1995). Analysis of an M/G/1 queue with two types of impatient units *Advances in Applied Probability*, 27(3): 840–861.
[10] Boxma, O. J., and Cohen, J. W. (1991). The M/G/1 queue with permenent customers. *IEEE Journal on selected areas in communications*, 9(2):179–184.
[11] Gomez-Corral A. (1999). Stochastic analysis of a single server retrial queue with general retrial times. *Naval research logistics*, 46:561–581.
[12] Medhi J. Stochastic Models in Queueing Theory, Second Edition, 2003.
[13] Yang, T., Posner, M.J. M., Templeton, J.G. C. and Li H. (1994) An approximation method for the M/G/1 retrial queue with general retrial times. *European journal of operational research*, 76:552–562.
[14] Choi, B. D., Park, K. K., and Pearce, C.E. M. (1993). An M/M/1 retrial queue with control policy and general retrial times. *Queueing systems*, 14:275–292.
[15] Keerthana, R. (2025). An M/G/1 Retrial queue with working vacation, non persistent customers and a waiting server. *Reliability: Theory and Applications*, 20(1 (82)):1089–1099.
[16] Murugan, S.P. B. and Keerthana R. (2023). An M/G/1 Feedback retrial queue with working vacation and a waiting server. *Journal of computational analysis and applications*, 61.