

ON THE BURR TYPE IV DISTRIBUTION APPLIED TO LIFETIME SURVIVAL DATA

CLAIRE JOY G. MARIQUIT¹ AND BERNADETTE F. TUBO²



¹Department of Physical Sciences and Mathematics
Mindanao State University at Naawan
Naawan, Misamis Oriental, Philippines

²Department of Mathematics and Statistics
Mindanao State University - Iligan Institute of Technology
Iligan City, Philippines

¹clairejoy.mariquit@msunaawan.edu.ph ²bernadette.tubo@g.msuiit.edu.ph

Abstract

This study presents the Burr type IV ($Burr_{IV}$) distribution, which is one of the twelve continuous distributions within the Burr family. The paper thoroughly evaluates its properties and applications in survival analysis. Key distributional characteristics such as survival, hazard, quantile, cumulative distribution, and probability density functions are derived. Density plots are simulated to illustrate common patterns, including J-shaped, U-shaped, and reversed J-shaped forms, frequently observed in engineering and reliability contexts. The applicability of the $Burr_{IV}$ distribution is demonstrated through three real-life datasets, each exhibiting distinct density shapes. The findings affirm the flexibility and robustness of the $Burr_{IV}$ distribution in modeling a variety of data shapes, underscoring its potential for broader applications in survival analysis and related fields.

Keywords: Burr family, survival analysis, maximum likelihood estimation, probability density function

1. INTRODUCTION

Survival analysis centers on analyzing life data in which the time until the event of interest occurs. The Burr distribution, introduced by Burr in 1942 [5], is mostly a unimodal family of distributions that is used to analyze various kinds of life data with unknown underlying distribution due to its adaptability and flexibility.

Some of the widely studied distributions under this family are Burr type III or $Burr_{III}$ and Burr type XII or $Burr_{XII}$ distributions. Only few researches have been conducted to study in-depth other types of this family, like $Burr_V$, $Burr_{VII}$ and $Burr_X$ distributions, mostly on classical and Bayesian approaches in estimating unknown parameters of the distributions. $Burr_I$ and $Burr_{II}$ distributions are known as the famous uniform and generalized logistic distributions, respectively. However, remaining distributions have not received any considerable research as of this writing; such is the case with $Burr_{IV}$ distribution.

The $Burr_{III}$ distribution is extensively known and applied in various applications in the area of finance and engineering. It has at least two unknown parameters which include shape, scale and location parameters.

Let $F_{\bullet}(x)$ indicate the cumulative distribution function (*cdf*) of a specific Burr type (\bullet). The *cdfs* of $Burr_{III}$ and $Burr_{IV}$ are respectively,

$$F_{III}(x) = (1 + x^{-c})^{-k}, \quad x \geq 0 \quad \text{and}$$

$$F_{IV}(x) = \left[1 + \left(\frac{c-x}{x} \right)^{\frac{1}{c}} \right]^{-k}, \quad 0 \leq x \leq c,$$

where c and k are shape parameters.

Looking at $Burr_{III}$ and $Burr_{IV}$ distribution functions, clearly $F_{IV}(x)$ can be obtained by replacing x in $F_{III}(x)$ with $x^* = \left(\frac{c-x}{x}\right)^{-c^{-2}}$, that is, $F_{IV}(x) = F_{III}(x^*)$. This implies that the $Burr_{IV}$ distribution with two shape parameters c and k , to some degree, can be treated as an extension of $Burr_{III}$ distribution.

Most of the approaches used to estimate the parameters of Burr distributions include Maximum Likelihood Estimation (MLE) and Bayesian Estimation. Ramos et al. [22] reiterated that MLE is widely used due to its good properties, under regular conditions. The MLEs have consistency, asymptotic unbiasedness, asymptotic efficiency and asymptotic normality.

In the latter years, Feroze et al. [7] conducted research on the MLE of $Burr_{III}$'s two shape parameters with samples having left censored values. The methodologies used were the same as what they have done on $Burr_V$, although a real life data analysis has been done to see its applicability to real world situations. The performance results and conclusions made to $Burr_{III}$ are identical to $Burr_V$'s.

There are already many distributions - extended or generalized existing in the literature used by many researchers to explain the behavior of a survival data. For example, the classical Weibull distribution and $Burr_{III}$ distribution are already popular tools for describing the behavior of a life data. However, for some parameter values, these distributions cannot capture more properly the behavior of the data more particularly when the trend or behavior of the data follows either a "J"-shape, "U"-shape or "Reversed J"-shape. These shapes are commonly observed in engineering and reliability scenarios. Since $Burr_{IV}$ distribution is an extension of $Burr_{III}$, it can be used as an alternative to these more famous lifetime distributions.

The remainder of this paper is structured to provide a comprehensive understanding of the $Burr_{IV}$ distribution. Section 2 introduces the Burr type family, laying the groundwork for further discussion. In Section 3, we delve into the distributional properties of $Burr_{IV}$, where we derive key functions such as the *cdf*, probability density function (*pdf*), survival function, hazard function, and quantile functions. The analysis continues in Section 4, where we assess the behavior of the shape parameters of $Burr_{IV}$ through simulation studies. Following this, Section 5 presents applications of the $Burr_{IV}$ distribution to three real-life data sets that exhibit characteristics similar to the density shapes identified in the previous section. Finally, Section 6 concludes the paper with remarks summarizing the findings and implications of this study.

2. THE BURR TYPE FAMILY OF DISTRIBUTIONS

In 1942, Irving W. Burr proposed a family of cumulative distribution functions characterized by three positive real parameters: c, k and r . Since Burr's initial proposal, there has been a significant influx of research focusing on the adaptability and flexibility of Burr distributions in the contexts of reliability and survival analysis. These characteristics make them particularly attractive for modeling datasets where the underlying distribution is unknown.

2.1. $Burr_I$ and $Burr_{II}$ distributions

$Burr_I$ and $Burr_{II}$ distributions, famously known as uniform and generalized logistic distributions respectively, are explored by books [4] on statistical inference, [9] on statistical distributions, and [15] on continuous univariate distributions.

It was emphasized by Gorny and Cramer [12] that one major application of the uniform distribution in lifetime analysis and models of ordered data is given by the fact that (ordered) life time data from a particular distribution can often be written as a quantile transformation of the corresponding data from a (standard) uniform distribution. Also, uniform distributions have been used as a model for various life testing setups.

For the Burr_{II}, Hossain and Balakrishnan [14] stipulated that the general logistic distribution can be used in robustness studies of some classical procedures based on normality or skewed distributions if Type II censored. Moreover, Balakrishnan and Leung [3] affirmed several application of this distribution that includes the following: importance as a growth curve, usage in probit analysis, dosage response studies, and bioassays, and as replacement to the normal distribution.

2.2. Burr_{III} distribution

Considered to be one of the most famous Burr distribution, Burr_{III} is widely used as an alternative to many lifetime distributions including Weibull and Burr_{XII} distributions, as stressed by Feroze et al. [7]. Gamchi et al. [10] also remarked that this distribution can accommodate different hazard lifetime data and can properly approximate many well-known distributions such as Weibull, gamma, and log-normal.

Panahi [19] as well pointed out the effective utilization of this distribution in analyzing materials engineering data such as fracture toughness data in place of the others. The Burr_{III} distribution is used extensively in various fields such as survival and reliability analysis, environmental studies, economics, meteorology and water resources, forestry among others as remarked by Ul-Haq et al. [18]. This distribution is also applied to wages, income and wealth datasets.

2.3. Burr_X distribution

For Burr_X distribution, also known as the Generalized Rayleigh distribution, Kundu and Raqab [16] described that this two-parameter distribution can be used quite effectively in modeling strength data and also modeling general lifetime data. It has increasing importance in several areas of applications such as lifetime tests, health, agriculture, biology, and other sciences as discussed by Fernandez et al. [6]. One specific application was mentioned by Rabie and Li [21] where a mathematical model was obtained for the expected lifetime of AIDS patients by using this distribution and fitting the data observed of the infected persons for Burr_X distribution.

2.4. Burr_{XII} distribution

A wide study on real-life application of Burr_{XII} have been conducted due to its capacity to assume various shapes that often permits a good fit when used to describe biological, clinical or other experimental data. It has also been applied in areas of quality control, reliability studies, duration and failure time modeling as identified by Panahi and Asadi [20].

Feroze et al. [7] also mentioned that since Burr_{XII} distribution under specific assumptions, can be shown to follow an extreme value distribution, the distribution has been used to model extreme events such as flood frequencies, wind speeds, rainfalls and river discharge volumes. This distribution also, as explained by Ghitany and Al-Awadhi [11], includes the exponential, Weibull, and log-logistic distributions for particular limiting values of the parameters. and covers the curve shape characteristics of normal, log-normal, gamma, logistic, and several Pearson-type distributions.

However, eventhough there are studies on Burr_V and Burr_{VII} distributions both being good alternatives to many lifetime distributions as mentioned by Feroze and Aslam [8], the rest of the distributions from this family have not received significant analysis using any real-life data or scenarios.

3. SOME DISTRIBUTIONAL PROPERTIES OF THE Burr_{IV} DISTRIBUTION

In this section, the research will mirror the methods employed by Hakim et al. [13] in their work on finding some distributional properties of the Burr_{XII} distribution where it also has two shape parameters.

The probability density function for the general Burr_{IV} family distribution is given by

$$f_{IV}(x) = kx^{-2} \left(\frac{c-x}{x}\right)^{\frac{1}{c}-1} \left(\frac{c-x}{x} + 1\right)^{-k+1}, \quad 0 < x < c, \quad (1)$$

where $k > 0$ and $c > 0$ are shape parameters.

And the cumulative distribution function of the distribution is

$$F_{IV}(x) = \left[1 + \left(\frac{c-x}{x}\right)^{\frac{1}{c}}\right]^{-k}, \quad 0 \leq x \leq c. \quad (2)$$

The Burr_{IV} distribution can be viewed as an extension of the Burr_{III} distribution. By examining the *cdfs* of both distributions, clearly $F_{IV}(x)$ can be derived by substituting x in $F_{III}(x)$ with $x^* = \left(\frac{c-x}{x}\right)^{-c^{-2}}$, that is, $F_{IV}(x) = F_{III}(x^*)$.

The survival function $S_{IV}(x)$ gives the probability that the random variable X exceeds the specified time x . The survival function $S_{IV}(x) = P(X > X)$ can be ascertained from the probability density function by integrating over the probability density function from time x to infinity or just one minus the corresponding *cdf*,

$$S_{IV}(x) = P(X > x) = \int_x^{\infty} f(u) \, du = 1 - F_{IV}(x).$$

The hazard function $h_{IV}(x)$, is equal to the limit, as Δt approaches zero, of a survival probability statement, divided by Δx , where Δx denotes a small period of time,

$$h_{IV}(x) = \lim_{\Delta x \rightarrow 0} \frac{P(x \leq X < x + \Delta x | X \geq x)}{\Delta x}.$$

It gives instantaneous potential to fail at time x per unit of time, given survival up to time x . It is also known as failure rate or force of mortality. The hazard function can also be calculated by taking the ratio of the *pdf* and the survival function,

$$h(t) = \frac{f_{IV}(x)}{S_{IV}(x)}.$$

The quantile function can be computed using the inverse transformation method where, $X = F_{IV}(x) - u$ and u is the random variable with distribution $U(0,1)$. A summarized comparison of distributional properties of both distributions and a derivation from Burr_{III} to Burr_{IV} using their *cdfs* are detailed in Table 1.

Table 1: Comparison of Burr_{III} and Burr_{IV} Distributional Properties

Function	Burr Type III where $x > 0, c > 0, k > 0$	Burr Type IV where $0 \leq x \leq c, k > 0$
$F(x)$	$F_{III}(x) = (1 + x^{-c})^{-k}$	$F_{IV}(x) = \left[1 + \left(\frac{c-x}{x} \right)^{\frac{1}{c}} \right]^{-k}$
$f(x)$	$f_{III}(x) = ckx^{-c-1} (1 + x^{-c})^{-k-1}$	$f_{IV}(x) = kx^{-2} \left(\frac{c-x}{x} \right)^{\frac{1}{c}-1} \left(\frac{c-x}{x} + 1 \right)^{-k+1}$
$S(x)$	$S_{III}(x) = 1 - (1 + x^{-c})^{-k}$	$S_{IV}(x) = 1 - \left[1 + \left(\frac{c-x}{x} \right)^{\frac{1}{c}} \right]^{-k}$
$h(x)$	$h_{III}(x) = \frac{ckx^{-c-1}(1+x^{-c})^{-k-1}}{1-(1+x^{-c})^{-k}}$	$h_{IV}(x) = \frac{kx^{-2} \left(\frac{c-x}{x} \right)^{\frac{1}{c}-1} \left[\frac{c-x}{x} + 1 \right]^{-k+1}}{1 - \left[1 + \left(\frac{c-x}{x} \right)^{\frac{1}{c}} \right]^{-k}}$
$q(u)$	$q_{III}(u) = \left(u^{-1/k} - 1 \right)^{1/c}$ where $u \sim U(0,1)$	$q_{IV}(u) = c \left[\left(u^{-1/k} - 1 \right)^c - 1 \right]^{-1}$ where $u \sim U(0,1)$

4. SIMULATION RESULT ON THE SHAPE PARAMETERS OF THE BURR_{IV} DISTRIBUTION

The simulation study is conducted using R software to analyze the behavior of the parameters of the Burr_{IV} distribution. This involves identifying appropriate parameter values that produce various density function shapes observed in the distribution.

Graphs of the cumulative distribution function $F_{IV}(x)$ and the survival function $S_{IV}(x)$ of the Burr_{IV} random variable X for various values of shape parameters c and k are shown in Figure 1 and Figure 2.

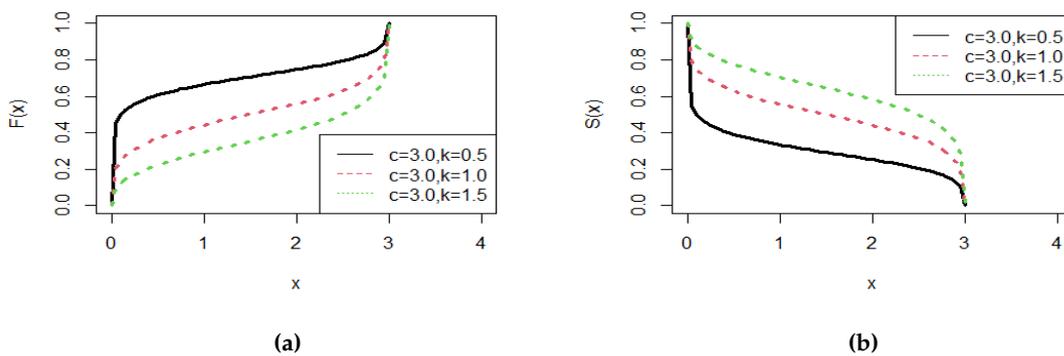


Figure 1: Plots of CDF (a) and Survival Function (b) of the Burr_{IV} distribution for a fixed value of c and various values of k

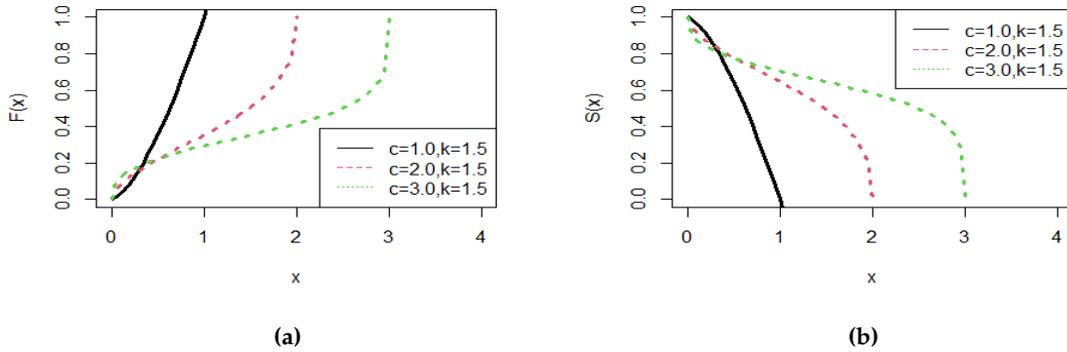


Figure 2: Plots of CDF (a) and Survival Function (b) of the Burr_{IV} distribution for a fixed value of k and various values of c

The probability density function of Burr_{IV}, $f_{IV}(x)$, represents the failure rate of the item at time x , or the instantaneous probability that item fails at time x . Graphs of the $f_{IV}(x)$ for various values of the parameters c and k are shown in Figure 3.

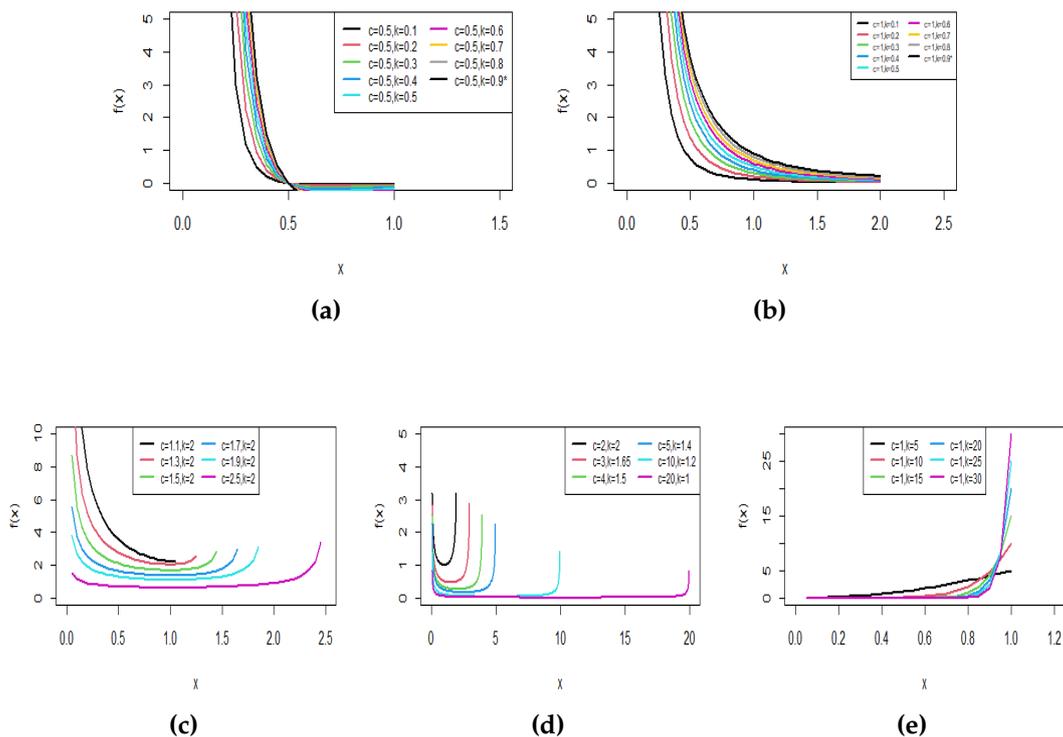


Figure 3: PDF plots for various values of c and k : (a)-(b) "Reversed J"-shaped, (c)-(d) "U"-shaped, and (e) "J"-shaped.

The density reflects a "Reversed J"-shape when parameter values are $c = 1$ and $0.1 \leq k \leq 0.9$ and when $0.5 < c \leq 1$ and $0.1 \leq k \leq 0.9$ (see Figure 3(a-b)). Figure 3(c-d) shows that for $c \geq 2$ and $1 < k \leq 2$, the "U"-shaped density function can be observed. The figures also indicate that the Burr_{IV} distribution when in "U"-shaped shows that its peak values are finite and the right peak ends at less than c since $0 < x < c$. Moreover, when $c \geq 2$ and $k > 2$, the density shape

shows the "J"-shaped trend (see Figure 3(e)). A summary table on various values of c and k with their corresponding density shape is given in Table 2. Therefore, different values of the shape parameters generate various density shapes or trend patterns for the Burr_{IV} distribution.

Table 2: Summary on various values of c and k with respective density shape details

Parameters		Density Plot Details
c	k	
$c \geq 2$	$1 < k \leq 2$	As c from 2 increases, to achieve U-shape, k from 2 decreases to 1
$c \geq 2$	$k > 2$	J-shaped
$c = 1$	$k > 5$	J-shaped
$0 < c < 0.1$	$0 < k < 1$	Rapidly flattens in $y=0$
$c = 1$	$0.1 \leq k \leq 0.9$	Reversed J-shaped
$0.5 < c < 1$	$0.1 \leq k \leq 0.9$	Reversed J-shaped

Graphs of the hazard rate function h depicts a bathtub-shaped for $c > 1$ in Figure 4(a) or increasing for $0 < c \leq 1$ in Figure 4(b).

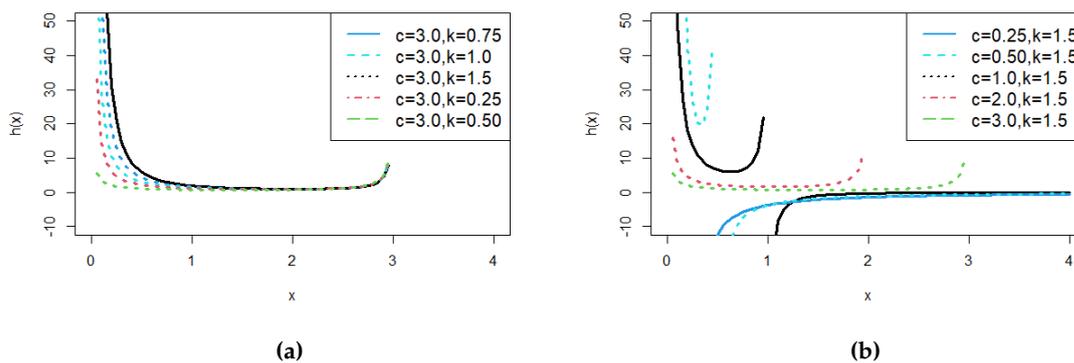


Figure 4: Plots of Hazard Function of the Burr_{IV} distribution. (a): fixed value of c and various values of k . (b): fixed value of k and various values of c

Since Burr_{IV} distribution has either a bathtub-shaped or an increasing hazard rate function, Burr_{IV} distribution provides flexibility for analyzing and modelling different types of survival and reliability data. The distribution can be used to model the survival time with increasing hazard rate, for example, is the conditional probability of a serious engine problem in the next month, given no problem so far, will increase with the life of the car. In addition, it can also be used to model the survival time whose hazard rate function is bathtub-shaped, for example, the age of death of infants and elderly wherein the hazard rate is expected to be decreasing at first and then overtime to increase in the end, reflecting higher hazard of infants and elderly.

5. APPLICATION TO LIFETIME SURVIVAL DATA

This section explores the suitability of the distribution for analyzing real-life scenarios. Actual lifetime survival data are fitted to the model using specific parameter values corresponding to different distributional shapes.

Three data sets with histogram shapes similar to Burr_{IV}'s density forms are analyzed and applied to show the Burr_{IV}'s potential in real-world circumstances when compared to other known distributions such as the Weibull and Burr_{III} distributions.

5.1. The J-shaped

We consider the data consisting of the failure times of 20 mechanical components from [17]. The data is right skewed and has a large kurtosis and heavy tail characteristic. Figure 5 shows the histogram of the data and superimposed "J" density shape.

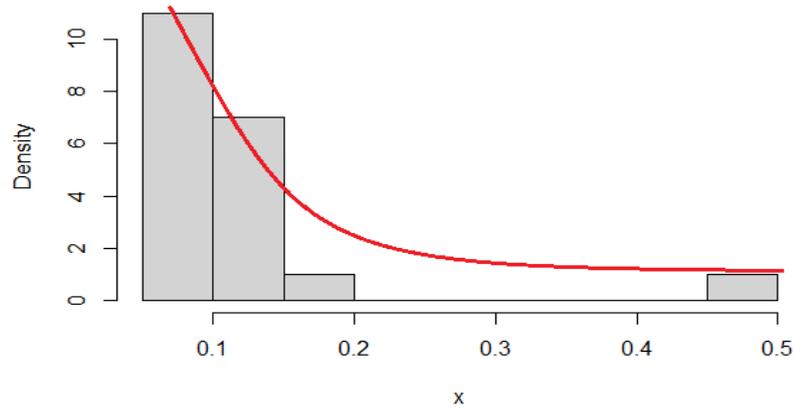


Figure 5: Histogram of Mecha Failure dataset

In using the *fitdistrplus* package in deriving the maximum likelihood estimates of the parameters of the candidate distributions, we have the following results as presented in Table 3. The goodness of fit statistics AIC, BIC and Loglik with adequacy values of KS, W^* and A^* for all the models are given in Table 4 for the first data set. It can be observed that the Burr_{IV} model has smaller values of AIC and BIC, and larger log likelihood compared to Weibull and the Burr_{III} distributions. So, the Burr_{IV} model could be chosen as the best fit for the first data set. However, KS, W^* , and A^* show that the Weibull distribution suits the data in a better manner.

Table 3: MLEs for the first data set

Distribution	\hat{c}	\hat{k}	$\hat{\tau}$	$\hat{\beta}$
Weibull	-	-	25.96762	1.642101
Burr _{III}	16.6364398	0.02695301		
Burr _{IV}	0.4945081	0.6554798		

Table 4: The goodness of fit tests and adequacy values for the first data set

Model	AIC	BIC	Loglik
Weibull	-48.84565	-46.85418	26.42282
Burr _{III}	-13.11638	-11.12491	8.558189
Burr _{IV}	-101.6659	-99.67447	52.83297
Model	KS	W^*	A^*
Weibull	0.2641364	0.4364214	2.537661
Burr _{III}	0.5103311	1.3816353	6.481232
Burr _{IV}	0.620643	2.657189	13.05256

5.2. The U-shaped

This second data ([1]) consists of the times to failure of 50 devices put on life test at time 0. The histogram of the data and superimposed "U" density shape is depicted at Figure 6.

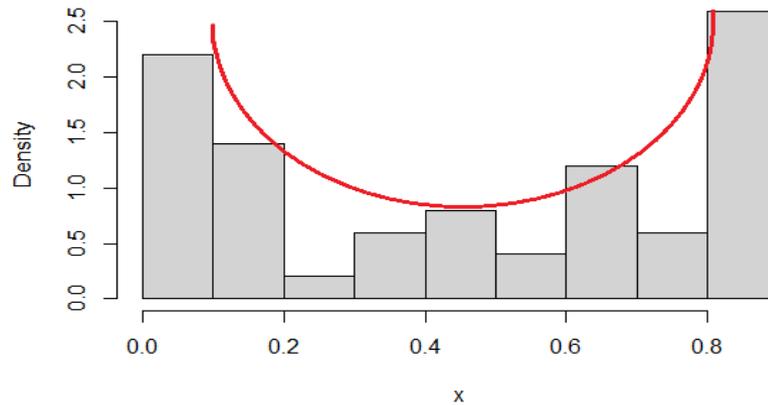


Figure 6: Histogram of Device Failure dataset

Mirroring procedures done in the first data set, it can be seen in Table 5 the maximum likelihood estimates of the parameters of the candidate distributions. The goodness of fit statistics AIC, BIC and Loglik with adequacy values of KS, W^* and A^* for all the models are given in Table 6 for the second data set. Still, the Burr_{IV} model has smaller values of AIC and BIC, and larger log likelihood compared to the other two distributions. However, KS, W^* , and A^* show that the Burr_{IV} distribution is not the most adequate distribution for explaining the data.

Table 5: MLEs for the second data set

Distribution	\hat{c}	\hat{k}	$\hat{\tau}$	$\hat{\beta}$
Weibull	-	-	2.137561	0.949043
Burr _{III}	86.2733934	0.00759508		
Burr _{IV}	0.8713916	0.7202177		

Table 6: The goodness of fit tests and adequacy values for the second data set

Model	AIC	BIC	Loglik
Weibull	25.48662	29.31066	-10.7433
Burr _{III}	-6.341407	-2.517361	5.170703
Burr _{IV}	-29.60651	-25.78246	16.80325
Model	KS	W^*	A^*
Weibull	0.1928	0.5295633	3.48898
Burr _{III}	0.1787837	0.4078772	2.491738
Burr _{IV}	0.2531941	0.9729153	9.793962

5.3. The Reversed J-shaped

The third data by Bader and Priest [2] represents 35 strength data for single carbon fibers (and impregnated 1000-carbon fiber tows). The histogram of measurements and the superimposed "Reversed J" density shape are shown at Figure 7.

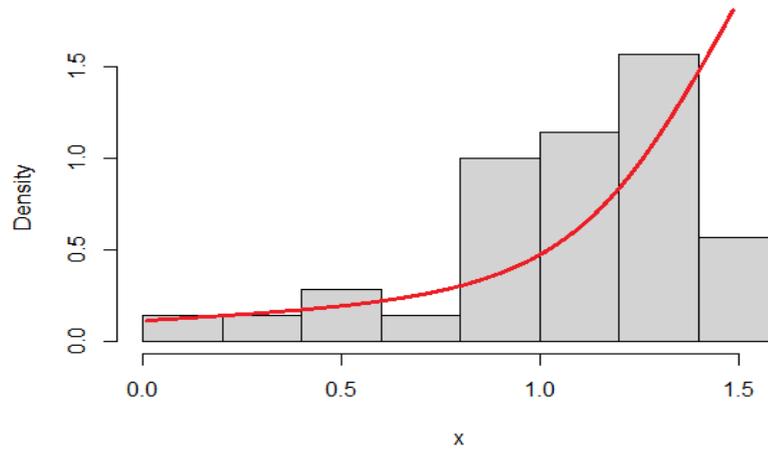


Figure 7: Histogram of Carbon Strength dataset

Table 7 presents the maximum likelihood estimates of the parameters obtained using the same procedure applied to the two previous data. The goodness of fit statistics for all the models are given in Table 8 for the third data set. Still, it has been observed that the Burr_{IV} model has the least values of AIC and BIC, and largest log likelihood among all candidate distributions. However, KS, W*, and A* show that the Weibull distribution suits the data in a better manner with Burr_{IV} model as next best. So, the Burr_{IV} model could be chosen as the best fit for the third data set.

Table 7: MLEs for the third data set

Distribution	\hat{c}	\hat{k}	$\hat{\tau}$	$\hat{\beta}$
Weibull	-	-	0.596878	3.55212
Burr _{III}	4.4716631	0.8079735		
Burr _{IV}	1.478	2.216528		

Table 8: The goodness of fit tests and adequacy values for the third data set

Model	AIC	BIC	Loglik
Weibull	32.19539	35.30609	-14.0977
Burr _{III}	47.56203	50.67272	-21.781
Burr _{IV}	-6.216309	-3.105613	5.108155
Model	KS	W*	A*
Weibull	0.1565853	0.2020518	1.374336
Burr _{III}	0.2827077	0.7875395	3.932118
Burr _{IV}	0.2001658	0.3083705	2.207361

6. CONCLUDING REMARKS

This study examined the Burr_{IV} distribution, an extension of the well-known Burr_{III} distribution, and derived key statistical distribution properties, including survival, hazard, and quantile functions. Various density function shapes “J-shaped, U-shaped, and reversed J-shaped” were identified for specific parameter values, highlighting the distribution’s flexibility in modelling lifetime survival data.

To assess its applicability, Burr_{IV} was fitted to three real datasets and compared to the Weibull and Burr_{III} distributions using goodness of fit tests. The results showed that Burr_{IV} consistently provided the best fit but was not always the most adequate model for specific datasets. While it outperformed alternatives for J-shaped and U-shaped densities, the Weibull distribution was superior for mechanical failure data, and Burr_{III} better suited device failure data. These findings demonstrate the potential of Burr_{IV} for modeling diverse real-lifetime survival scenarios.

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