REDUCING RENTAL COSTS IN TWO-STAGE HYBRID FSSP: BRANCH AND BOUND VS. HEURISTIC APPROACHES

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Abstract

This paper proposes a Branch and Bound (BB) based heuristic to solve a two-stage hybrid Flow Shop Scheduling Problem (HFSSP), incorporating practical constraints such as machine rental costs, transportation time, and job weightage. The scheduling environment includes multiple parallel machines at the first stage, where each job can be processed on any one machine, followed by a single rented machine at the second stage. To effectively model uncertainty in processing times, triangular fuzzy numbers are used. Additionally, transportation time between stages and job weightage are integrated into the model to reflect realistic industrial conditions. The proposed BB algorithm systematically explores the space of feasible job sequences, pruning non-promising branches to derive an optimal or near-optimal schedule under the given constraints. Its performance is benchmarked against two widely used heuristic approaches—NEH and GRASP-NEH—focusing on minimizing total elapsed time and rental cost of the second-stage machine. Experimental results indicate that the BB approach consistently outperforms the heuristic methods, particularly in reducing rental costs and providing better control over the utilization of the rented resource. While heuristic methods offer computational speed, they often compromise on cost efficiency and precision in handling fuzzy parameters. Overall, the study demonstrates the effectiveness of the BB method as a robust and efficient alternative for solving complex scheduling problems, especially in scenarios where minimizing rental costs is a critical objective. The integration of fuzzy processing times, transportation delays, and job priorities further enhances the practical relevance of the proposed approach.

Keywords: Rental Cost Optimization, Fuzzy Processing Time, Triangular Fuzzy Numbers, Hybrid Scheduling, Parallel Machines

I. Introduction

Efficient job scheduling in manufacturing and service environments remains a vital concern for industries aiming to optimize resource utilization and operational costs. Among various scheduling models, the Flow Shop Scheduling Problem (FSSP) is one of the most widely studied, where each job follows the same processing route across multiple machines. However, traditional FSSPs often fall short of capturing the complexity of modern production systems that involve multiple parallel machines, rented resources, transportation delays, and uncertainties in processing times. These realistic factors give rise to the Hybrid Flow Shop Scheduling Problem (HFSSP)—a more generalized and challenging variant of the classical flow shop problem.

In practical scenarios, certain stages in the production process may involve rented machines, where minimizing rental time becomes economically significant. Furthermore, uncertainty in processing durations, caused by fluctuating workloads or machine variability, is better represented using triangular fuzzy numbers rather than deterministic values. In addition, transportation time between stages and job-specific weightage further complicate the scheduling environment and must be accounted for to develop viable and cost-effective schedules. This study addresses a two-stage HFSSP where jobs are processed on parallel equipotential machines at the first stage and then on a single rented machine at the second stage. To solve this problem, we propose a Branch and Bound (BB)-based heuristic that systematically explores feasible sequences and eliminates suboptimal ones, balancing accuracy and computational efficiency. The BB algorithm explores feasible job sequences while eliminating inefficient options, aiming to minimize the rental cost of the second-stage machine. The performance of the proposed method is compared with that of heuristic algorithms, demonstrating the BB method's effectiveness in handling fuzzy parameters and real-world constraints. González-Neira et al. [1] have studied solving the Hybrid Flow Shop Scheduling Problem (HFSP), which is known to be NP-hard, using the simple yet effective GRASP meta-heuristic. Unlike complex hybrid algorithms, GRASP offers good performance across multiple optimization objectives without multi-objective modeling. Computational results show GRASP outperforms traditional dispatching rules and is practical for real-world industrial applications. The GRASP method is further combined with NEH and compared with NEH by Öztop [2]. Johnson [3] presents a decision rule for optimal scheduling of items on two machines to minimize total elapsed time, considering setup and work times. It also explores a restricted version of a three-machine problem. Lomnicki [4] combines Roy's graph-theoretical approach to job scheduling with the branch-and-bound method, providing an exact solution for the three-machine scheduling problem. The algorithm's functionality is demonstrated through numerical examples, and further research possibilities are suggested. Nawaz et al. [5] present a simple heuristic algorithm named NEH for flow-shop scheduling, prioritizing jobs with higher total process time. Kahraman et al. [6] propose an efficient genetic algorithm to minimize makespan in Hybrid Flow Shop (HFS) scheduling problems. Ignall et al. [7] present the branch-and-bound technique by Little et al. and Land and Doig applied to two flow-shop scheduling problems. Computational results for 2- and 3-machine problems show improved solutions for minimizing completion time and makespan, respectively, compared to other techniques. Tang et al. [8] propose a hybrid optimization algorithm combining Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) for solving the NP-Hard Flow Shop Scheduling Problem (FSSP). Experimental results show that this hybrid method outperforms existing approaches, providing a more efficient solution. Morsy et al. [9] explore the use of Pythagorean fuzzy sets to model processing time uncertainty in scheduling, aiming to minimize machine rental costs in complex manufacturing systems. It highlights the effectiveness of fuzzy modeling in improving decision-making under uncertainty. Alburaikan et al. [10] present an effective algorithm to solve the problem of rental cost under fuzzy environment. The problem under consideration is a 3-stage FSSP having a single machine at each stage. Singla et al. [11] consider job weightage and setup times while reducing idle time to reduce rent. Sathish et al. [12] in their study present an effective heuristic to reduce the rental cost for the problem with fuzzy processing times. Kaushik et al. [13] compare the result of BB with CDS and NEH for HFSSP, and the results show the effectiveness of BB over the heuristics.

In continuation of the existing research, the present study focuses on a two-stage hybrid flow shop scheduling problem that incorporates realistic industrial constraints such as parallel equipotential machines, a rented second-stage machine, transportation time, and job weightage. To effectively handle uncertainty in processing times, triangular fuzzy numbers are used. A Branch and Bound (BB)-based heuristic is proposed to minimize both total elapsed time and machine rental cost. The performance of the proposed method is evaluated against well-known heuristics such as NEH and GRASP-NEH, and the results clearly demonstrate the superiority of the BB approach in achieving more cost-effective and accurate scheduling outcomes under complex and uncertain conditions.

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II. Practical Situation

Several machines are used by auto manufacturers during the initial assembly phase, which includes welding, engine fitting, and component installation. The autos are shipped to an outside factory for painting when assembly is finished. The corporation rents the painting machine, and there's a transportation time from the assembly line to the painting facility for the cars. In order to keep transportation times under control and paint cars on schedule, the corporation must reduce the cost of renting the painting facility. Based on consumer demand, each automobile model has distinct priorities, and because processes can be unpredictable, there can be variations in the time required for each step.

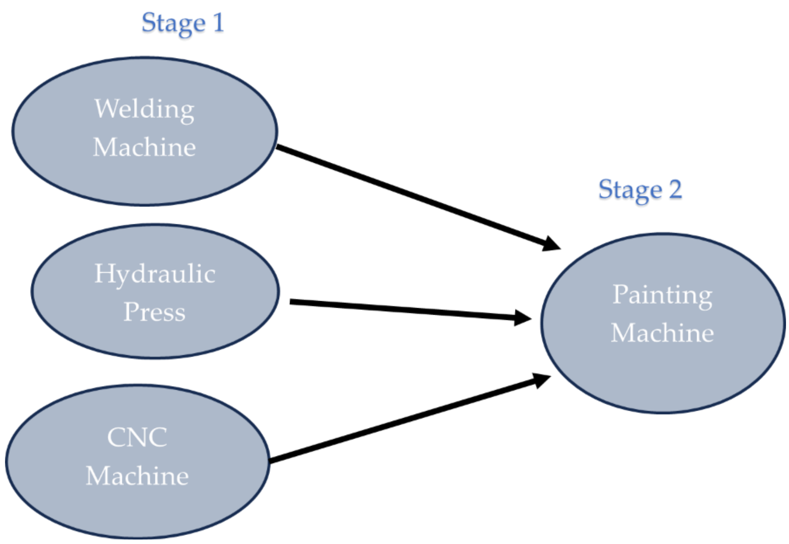

Figure 1: *Pictorial Representation of the problem in Automobile Manufacturing Unit*

III. Problem Formulation

Using equipotential machines in the first stage and a single machine in the second, this research aims to tackle a particular problem. For the second stage, rent is to be paid for the equipment. As such, the machine is rented when a task is assigned to it and returned as soon as all tasks are finished, in compliance with the rental conditions. It follows that for the duration that the equipment is being used, the rent must be paid. Job weightage and cost of transportation are also taken into account. Table 1 contains the mathematical representation of the problem.

Notations

f_{ij}=Operational Cost on j^{th} machine for i^{th} job {(i = 1, 2, ..., s); (j = 1, 2, ..., r)}

$(\kappa_{i1},\kappa_{i2},\kappa_{i3})$ = Processing time on machine M for job i

$(\zeta_{i1},\zeta_{i2},\zeta_{i3})$= Processing time on machine N for job i

t_i = transportation time of job i from machine M to machine N

η_j = Total available time of machine M_j

θ_i=Weight of job i

Table 1: *Mathematical Representation of Data*

Job/Machine	M						P.T. on M	Transportation time	P.T. on N	Job Weightage
	M_1	M_2	.	.	.	M_r				
1	f_{11}	f_{12}	.	.	.	f_{1r}	$(\kappa_{11},\kappa_{12},\kappa_{13})$	t_1	$(\zeta_{11},\zeta_{12},\zeta_{13})$	θ_1
2	f_{21}	f_{22}	.	.	.	f_{2r}	$(\kappa_{21},\kappa_{22},\kappa_{23})$	t_2	$(\zeta_{21},\zeta_{22},\zeta_{23})$	θ_2
3	f_{31}	f_{32}	.	.	.	f_{3r}	$(\kappa_{31},\kappa_{32},\kappa_{33})$	t_3	$(\zeta_{31},\zeta_{32},\zeta_{33})$	θ_3
.										
.										
.										
S	f_{s1}	f_{s2}	.	.	.	f_{sr}	$(\kappa_{s1},\kappa_{s2},\kappa_{s3})$	t_s	$(\zeta_{s1},\zeta_{s2},\zeta_{s3})$	θ_s
	η_1	η_2	.	.	.	η_k				

Algorithm Used:

Step 1: Defuzzify triangular fuzzy number using Yager's method.

For the fuzzy number $(\kappa_{i1}, \kappa_{i2}, \kappa_{i3})$ crisp value comes out to be $k_i = \frac{k_{i1}+4*k_{i2}+k_{i3}}{6}$

Step 2: Change the processing times of machines M and N according to the condition of transportation cost. i.e. the processing times of i^{th} job on machine M is to be $g_i = k_i + t_i$ and on machine N is to be $\boldsymbol{h_i} = \boldsymbol{\zeta}_i + t_i$.

Step 3 : In the next step if min{g_i,h_i}=g_i then change the processing time as $g_i' = \frac{g_i+\theta_i}{\theta_i}$, $h_i' = \frac{h_i}{\theta_i}$ and if min{g_i,h_i}=h_i then change the processing time as $h_i' = \frac{h_i+\theta_i}{\theta_i}$, $g_i' = \frac{g_i}{\theta_i}$

Step 4: Use Modi Method to optimize the utilization cost of the machines at first stage

Step 5: Use BB method to get the optimized schedule

Step 6: Prepare an in-out table for the schedule and get the total elapsed time

Step 7: Calculate the total utilization time of machine N

Step 8: Earliest time= Total elapsed time – utilization time of N

Step 9: Change he starting time of first job on machine N by earliest time and modify the schedule accordingly.

Step 10: Calculate the rent from the in-out table as: (Ending time of last job on machine N - starting time of first job on machine N) * rent

IV. Numerical Illustration

Here is a numerical example for the better understanding of the algorithm devised in this paper. This example consists of 4 equipotential machines at first stage and a single machine at second stage to complete 5 jobs. The processing times of jobs are considered in triangular fuzzy numbers. The scheduling problem outlined in Table 2 is solved by systematically applying the proposed algorithm.

Table 2: *Numerical Example*

Job/Machine	Machine M					Tr. time	P.T. on N	Job Weightage
	M1	M2	M3	M4	P.T. on M			
1	10	15	13	14	(10,14,19)	6	(2,5,7)	2
2	20	25	27	24	(16,24,26)	5	(4,5,9)	3
3	10	15	13	19	(12,19,21)	8	(3,7,8)	5
4	23	29	27	30	(20,24,30)	9	(1,3,7)	1
5	26	27	32	21	(24,26,29)	10	(5,8,9)	4
	15.9	12.6	21.7	16.82				

Table 3: *Problem with Crisp Processing Times*

Job/Machine	Machine M					Tr. time	P.T. on N	Job Weightage
	M1	M2	M3	M4	P.T. on M			
1	10	15	13	14	14.17	6	4.83	2
2	20	25	27	24	23	5	5.50	3
3	10	15	13	19	18.17	8	6.50	5
4	23	29	27	30	24.33	9	3.33	1
5	26	27	32	21	26.17	10	7.67	4
	15.9	12.6	21.7	16.82				

Table 4: *Transportation time Integration*

Job/Machine	Machine M					P.T. on N	Job Weightage
	M1	M2	M3	M4	P.T. on M		
1	10	15	13	14	20.17	10.83	2
2	20	25	27	24	28	10.50	3
3	10	15	13	19	26.17	14.50	5
4	23	29	27	30	33.33	12.33	1
5	26	27	32	21	36.17	17.67	4
	15.9	12.6	21.7	16.82			

Table 5: *Problem after the third step of Algorithm*

Job/Machine	Machine M					P.T. on N
	M1	M2	M3	M4	P.T. on M	
1	10	15	13	14	10.09	6.42
2	20	25	27	24	9.33	4.50
3	10	15	13	19	5.23	3.90
4	23	29	27	30	33.33	13.33
5	26	27	32	21	9.04	5.42
	15.9	12.6	21.7	16.82		

Table 6: *After Applying Modi Method*

Job/Machine	Machine M				P.T. on N
	M1	M2	M3	M4	
1	-	2.31	-	7.78	6.42
2	-	9.33	-	-	4.50
3	-	-	5.23	-	3.90
4	15.9	0.96	16.47	-	13.33
5	-	-	-	9.04	5.42

Table 3 presents the data after defuzzification of the triangular fuzzy processing times using Yager's method (Step 1). In Table 4, transportation times are incorporated into the processing times as outlined in Step 2. Table 5 reflects the adjustments made after applying the job weightage criterion (Step 3). The optimization of machine utilization cost for the parallel machines at the first stage, performed using the MODI method (Step 4), is displayed in Table 6. The data in Table 7 shows the process of BB method step by step and the final sequence of jobs after BB methos comes out to be 3-1-2-5-4. Table 8 represents the in time and out time of the jobs on machines.

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Table 7: *Bounds of BB method*

Jobs	g′	g′′	G=max(g′,g′′)
1	41.35	25.6	41.35
2	42.9	25.6	42.9
3	38.8	26.2	38.8
4	50.04	25.6	50.04
5	42.61	25.6	42.61
31	41.35	26.2	41.35
32	42.9	27.12	42.9
34	55.27	26.2	55.27
35	42.61	26.2	42.61
312	45.21	27.12	45.21
314	55.27	26.2	55.27
315	50.39	26.2	50.39
3124	55.27	27.12	55.27
3125	50.39	35.03	50.39

Table 8: *In-Out Table*

Job/Machine	Machine M				Tr. Time	P.T. on N
	M1	M2	M3	M4		
3	-	-	0-18.17	-	8	26.17-32.67
1	-	-	18.17-31.87	31.87-32.34	6	38.34-43.17
2	-	0-22	-	32.34-33.34	5	43.17-48.67
5	-	-	-	33.34-59.51	10	69.51-77.18
4	0-21.4	-	31.87-34.8	-	9	77.18-80.51

Utilization time of machine N = 27.83. Earliest time for machine N=80.51-27.83 = 52.68
New Schedule is given in Table 9.

Table 9: *Modified In-Out Table*

Job/Machine	Machine M				Tr. time	P.T. on N
	M1	M2	M3	M4		
3	-	-	0-18.17	-	8	52.68-59.18
1	-	-	18.17-31.87	31.87-32.34	6	59.18-64.01
2	-	0-22	-	32.34-33.34	5	64.01-69.51
5	-	-	-	33.34-59.51	10	69.51-77.18
4	0-21.4	-	31.87-34.8	-	9	77.18-80.51

Consider the rent of Machine N is to be Rs. 3/- per unit of time then according to previous schedule the rent becomes (80.51-26.17) * 3 =Rs. 163.02/- and according to new schedule it comes out to be (80.51-52.68) * 3= Rs. 83.49/-.

V. Solution using the NEH and GRASP-NEH method

I. Steps for NEH method & GRASP-NEH method

In this approach, the first four steps of the proposed algorithm—namely defuzzification of fuzzy processing times, adjustment of processing times to include transportation time, modification based on job weightage, and optimization of parallel machine allocation using the MODI method—are applied initially to preprocess the input data. The resulting crisp and adjusted job data is then used as input for the classical NEH heuristic to determine the job sequence.
Step 5 : Calculate the total processing times of all jobs on all machines as TT_i
Step 6: Arrange in descending order and consider the first job of the sequence with the maximum TT.

II. For NEH

Step 7: For i=2:S

Consider the job Ji and calculate the fitness value for all the possible positions of Ji in the sequence and insert Ji at the position where the fitness value is minimum.

III. For GRASP-NEH

Step 7: For k=1:x

a. Take J_k as first job
b. For each job from the set of unscheduled jobs calculate the cost (makespan) C_t
c. Find the minimum and maximum of C_t as C_{max} and C_{min}.
d. Create the RCL (restricted candidate list) of the jobs t satisfying the condition (here the threshold value γ is taken to be 0.1)
$$C_t - C_{min} \leq \gamma * (C_{max} - C_{min})$$
e. Select the random job from RCL to append in the schedule.
f. Repeat the steps from b. to e. until the complete schedule is prepared.
g. This scheduled is modified with the NEH algorithm.

Step 8: Find the best schedule among the x schedules.

IV. Solution using NEH

Table 10: *Total Processing Time*

Job/Machine	Machine M				P.T. on N	TT_i
	M1	M2	M3	M4		
1	-	2.31	-	7.78	6.42	16.51
2	-	9.33	-	-	4.50	13.83
3	-	-	5.23	-	3.90	9.13
4	15.9	0.96	16.47	-	13.33	46.66
5	-	-	-	9.04	5.42	14.46

Table 10 gives the TT_i for all the jobs. According to the descending order of TT the sequence of jobs comes out to be 4-1-5-2-3. Now NEH algorithm is applied on this sequence. The process for NEH is given in Table 11. The final sequence is 1-5-2-3-4. The in-out table is given in Table 12.

Table 11: *NEH Method*

Sequence	Fitness Value
4-1	53.08
1-4	46.66
1-4-5	52.08
1-5-4	46.66
5-1-4	46.66
1-5-4-2	51.16
1-5-2-4	46.66
1-2-5-4	46.66
2-1-5-4	47.21
1-5-2-4-3	50.56
1-5-2-3-4	46.66
1-5-3-2-4	46.66
1-3-5-2-4	46.66
3-1-5-2-4	46.66

Table 12: *In-Out Table for NEH algorithm*

Job/Machine	Machine M				P.T. on N
	M1	M2	M3	M4	
1	-	-	0-13.7	13.7-14.17	20.17-25
5	-	-	-	14.17-40.34	50.34-58.01
2	-	-	13.7-31.87	-	58.01-64.51
3	-	0-22	-	40.34-41.34	64.51-70.01
4	0-21.4	-	31.87-34.8	-	70.01-73.34

Utilization time of Machine N=53.17. Rental Cost = Rs. 159.51

V. Solution using GRASP-NEH

Jobs in descending order of TT_i are : 4-1-5-2-3
Consider the first job as job 4. The cost function is given as Table 13 and Table 14.

Table 13*: GRASP procedure*

Scheduled Sequence	Unscheduled Job	C_t
4	1	53.08
4	2	51.16
4	3	50.56
4	5	52.08

C_{min} = 38.56, C_{max} = 43.84. Now according to the condition

$C_t \leq C_{min} + \gamma * (C_{max} - C_{min})$ =50.812. So, job 3 lie in RCL.

Table 14: *GRASP procedure (Cont.)*

Scheduled Sequence	Unscheduled Job	C_t
4-3	1	56.98
4-3	2	55.06
4-3	5	55.98
4-3-2	1	61.48
4-3-2	5	60.48

So, the sequence after Grasp algorithm comes out to be 4-3-2-5-1. Now, the NEH heuristic is applied to this sequence (Table 15).

Table 15: *GRASP-NEH algorithm*

Sequence	Elapsed Time
4-3	50.56
3-4	46.66
3-4-2	51.16
3-2-4	46.66
2-3-4	46.66
3-2-4-5	52.08
3-2-5-4	46.66
3-5-2-4	46.66
5-3-2-4	46.66
3-2-5-4-1	53.08
3-2-5-1-4	46.66
3-2-1-5-4	47.21
3-1-2-5-4	46.66
1-3-2-5-4	46.66

So the final sequence comes out to be 3-2-5-1-4. The in-out table for the GRASP-NEH is given in Table 16.

Table 16: *In-Out table for GRASP-NEH*

Job/Machine	Machine M				Tr. time	P.T. on N
	M1	M2	M3	M4		
3	-	-	0-18.17	-	8	26.17-32.67
2	-	0-22	-	22-23	6	32.67-38.17
5	-	-	-	23-49.17	5	59.17-66.84
1	-	-	18.17-31.87	49.17-49.64	10	66.84-71.67
4	0-21.4	-	31.87-34.8	-	9	71.67-75

Utilization time for machine N is 48.83 units of time so according to rental policy the rent becomes Rs. 146.49.

VI. Comparison of Results

Data presented in Table 17 draws the comparison of the results obtained above using the heuristics. Figure 2 gives the graphical representation of the comparison. The BB method shows a marked advantage in minimizing rental cost, achieving the lowest values among all tested approaches. This aligns with the method's strength in systematically considering all possible job sequences under fuzzy and transportation-adjusted conditions. However, BB tends to generate longer total elapsed times, likely due to its cost-prioritized scheduling decisions. The NEH and GRASP-NEH heuristics, on the other hand, offer shorter processing times but at the expense of higher rental costs. GRASP-NEH, due to its randomized construction phase followed by a greedy improvement, performs slightly better than NEH in some instances but still does not outperform BB in terms of rental cost. This side-by-side comparison underscores a performance-cost trade-off: heuristic methods favor speed and efficiency, while the BB method offers optimal control over cost, particularly relevant when machine rental expenses dominate scheduling priorities.

Table 17: *Comparison of the results*

	Using present heuristic	Using GRASP-NEH[2]	Using NEH[5]
Rental Cost	83.49	146.49	159.51
Job Sequence	3-1-2-5-4	3-2-5-1-4	1-5-2-3-4
Total elapsed Time	80.51	75	73.34

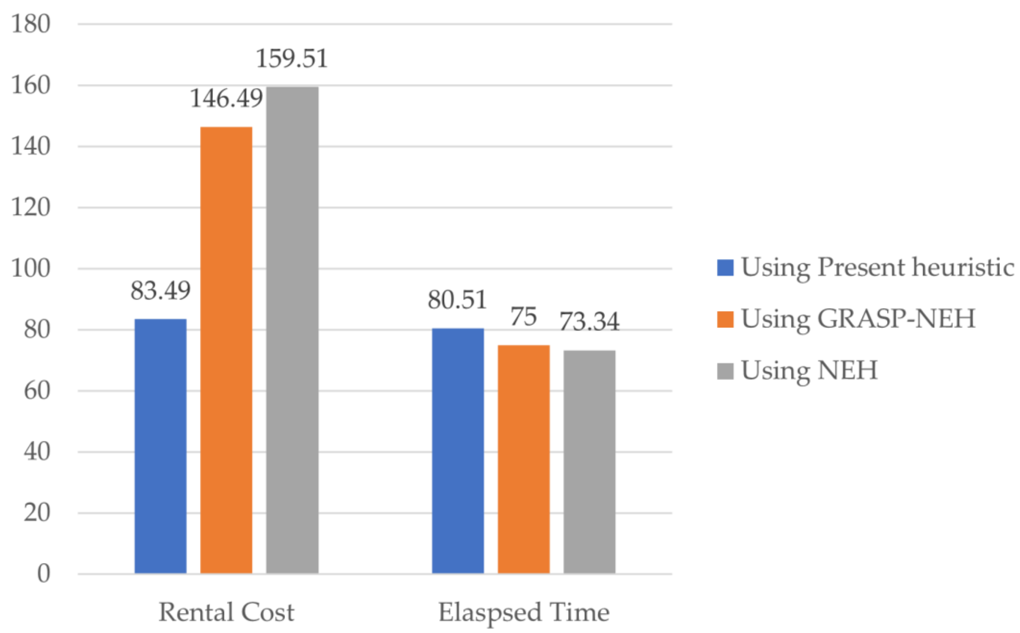

Figure 2: *Comparison of Results*

VII. Conclusion

This study presents a comparative analysis of the Branch and Bound (BB), NEH, and GRASP-NEH methods for solving a two-stage flow shop scheduling problem with transportation time, machine rental costs, and job weightage. While the BB approach does not yield the minimum total elapsed time, it significantly reduces the rental cost of the second-stage machine, which is a critical objective in the given problem setting. The results demonstrate the effectiveness of the BB method in optimizing cost-related constraints, particularly under fuzzy processing conditions, whereas heuristic methods offer faster but less cost-efficient solutions. Thus, BB proves to be a suitable technique in scenarios where minimizing rental expenses outweighs the need for shorter completion times.

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