A RANKING FUNCTION APPROACH BASED ON CENTROID FOR GENERALIZED HEXAGONAL FUZZY NUMBERS TO SOLVE FUZZY MULTI-OBJECTIVE TRANSPORTATION PROBLEMS

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Abstract

In a real-world scenario, the decision maker is unsure about the precise value of transportation parameters due to uncontrolled factors such as fuel prices, weather conditions, product availability, and requirements. Therefore,o fuzzy numbers are used to handle these uncertainties. This Paper aims to find an efficient solution to fuzzy multi-objective transportation problems (FMOTP) where all parameters (objective functions, availabilities, requirements) are given in the form of generalized hexagonal fuzzy numbers. In the Proposed approach, FMOTP is transformed into Fuzzy Single-Objective Transportation Problems (FSOTP) using the geometric mean technique. A new ranking function based on the centroid and in-center point method for hexagonal fuzzy numbers defuzzifies the FSOTP into a Crisp Single-Objective Transportation Problem (CSOTP). The optimal solution for the transformed CSOTP is obtained using the proposed approach based on the zero-entry cell method. Using the optimal solution of CSOTP the fuzzy efficient solution of FMOTP is obtained. Additionally, two numerical problems are solved to elaborate the Proposed approach, and results are compared with other existing methods. A comparison and analysis of the results show that the proposed approach provides a more optimized solution for FMOTP. This approach is easily applicable to real-life transportation problems in which decision-makers are unsure about the exact value of parameters.

Keywords: Ranking function, Centroid, Incentre Point, Generalized Hexagonal fuzzy number, Fuzzy Transportation Problem.

1. Introduction

The transportation problem (TP) is a significant optimization problem in logistics and supply chain management. Its objective is to find the most cost-efficient method of transporting items with multiple origins to numerous destinations while ensuring that supply and demand restrictions are fulfilled. Classical transportation problems operate on the assumption that all variables, including prices, suppliers, and demands, are accurately known. Nevertheless, in practical situations, these characteristics often exhibit uncertainty or imprecision as a result of various causes such as market volatility, estimating inaccuracies, and insufficient information. Fuzzy set theory has been extensively used to address uncertainties in issues related to transportation.

Fuzzy transportation problems (FTPs) use fuzzy numbers to describe unknown parameters, enabling a more accurate representation of real-world scenarios. An improved method in this field involves the utilization of hexagonal fuzzy numbers, which provide a more adaptable and inclusive representation of uncertainty compared with ordinary triangular or trapezoidal fuzzy numbers.

F.L. Hitchcock [1] first formulated the Transportation Problems (TP) in the form of LPP in 1941. An LPP form was used to represent the TP, which a simplex algorithm can solve. The Stepping Stone approach offers an alternative approach for obtaining the information from the Simplex Method developed by Charnes and Cooper [2]. Lotfi A. Zadeh [3] was given the concept of fuzziness in 1965. Zimmermann H.J [4]. was the first to use an appropriate membership function to solve an LP problem with multiple objectives. Avishek et.al.[5] presented a novel approach for ranking and defuzzification, which involves converting a fuzzy hexagonal number into a real number to determine its importance in addressing decision-making issues. Sudha and Revathi [6] introduced a novel ranking approach for hexagonal fuzzy numbers (HFNs) and used it to solve fuzzy LPP. Leela-Apiradee et. al. [7] established the concept of hexagonal fuzzy number cardinality, which is a strategy used to categorize hexagonal fuzzy numbers. They also offered a ranking function for Hexagonal fuzzy numbers, particularly with their probable mean values. Thirupathi et al. [8] proposed a novel ranking method based on the fuzzy hexagonal number. This method utilizes the centroid formula of triangles and rectangles, taking into account the distance from the origin to the centroid. S.Adilakshmi et al. [14] proposed a ranking function based on the centroid and circumcentre methods for HFNs which are used to find the fuzzy critical path to the fuzzy network.

Several methods exist for establishing an initial basic feasible solution for a balanced transportation problem [9-13]. Various transportation models are available in various forms, as individuals seek to use the most effective resources to maximize profit and minimize costs. In the corporate sector, several sorts of transportation difficulties are encountered. These challenges include finding the most cost-effective method of transferring products. The aim is to reduce the total cost of transportation. [15,16]. Ashour [17] introduced the cost-minimization fuzzy transportation problems using HFNs that take supply and demand into account were presented by. according to the generalized trapezoidal fuzzy number ranking. Z.A.M.S. Juman et. al. [18] presented the two methods based on two different ranking functions for pentagonal fuzzy numbers to solve interval-valued transportation problems. Kamini et.al. [19] proposed a zero-point technique and genetic algorithm to solve MOTP with hexagonal fuzzy numbers. Sahayasudha A. et. al. [20] presented an alpha-cut operation for symmetric HFNs to solve transportation problems. M. et al. [21] presented the DEA approach to solving FMOTP. Sharma M.K. et al. [22] proposed a novel approach to solve MOTP using the Fermatean fuzzy technique. Akram, M. et.al. [23] presented a method using data envelope analysis, for decision-making units with multiple inputs and outputs ranked according to their relative efficiency P. Indira and M. Jayalakshmi [24] proposed an approach to find the minimum transportation cost of triangular TP using the alpha cut, and row-column minima method. Sharma, R., and Tyagi, S. L [25] proposed an algorithm to solve multi-objective transportation problems with generalized trapezoidal fuzzy numbers based on the proposed ranking function.

This paper presents an approach to solving fuzzy multi-objective transportation problems (FMOTP) with generalized hexagonal fuzzy numbers based on the ranking function for hexagonal fuzzy numbers by separating hexagons into three quadrilaterals. The proposed ranking functions are based on centroid and in-centre methods. The geometric mean is used to convert FMOTP into fuzzy single-objective transportation problems(FSOTP). The zero-entry cell approach is used to get the fuzzy efficient solution of FMOTP.

2. ABBREVIATIONS

1. Linear Programming Problem - LPP

2. Transportation Problem - TP

3. Fuzzy Transportation Problem - FTP

4. Hexagonal Fuzzy Number - HFN

5. Generalized Hexagonal Fuzzy Number - GHFN

6. Multi-Objective Transportation Problem - MOTP

7. Fuzzy Multi-Objective Transportation Problem - FMOTP

8. Decision Maker- DM

9. Fuzzy Transportation Cost - FTC

10. Minimum Transportation Cost - MTC

11. Single objective transportation problem - SOTP

12. Crisp Single Objective Transportation Problem - CSOTP

3. Basic Definitions

1. Fuzzy Number: A fuzzy set $\tilde{A}$ is said to be fuzzy number if its membership function $\tilde{A} : \mathbb{R} \rightarrow \lceil 0, 1 \rceil$ has satisfy the following conditions:
$\tilde{A}(\lambda y_1 + (1 - \lambda)y_2) \geq \min\{A(\tilde{y_1})\}, A(\tilde{y_2}))$
there exist a y $\in \mathbb{R}$ such that $A(\tilde{y}) = 1$
$\tilde{A}$ is piece-wise continuous

2. Hexagonal Fuzzy Numbers : A fuzzy set $\tilde{A} = (\alpha^1, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6; w)$ where $\alpha^1 \leq \alpha^2 \leq \alpha^3 \leq \alpha^4 \leq \alpha^5 \leq \alpha^6$ and $0 < w \leq 1$ with membership function defined as:

$$\mu_{A(\tilde{y})} = \begin{cases} \frac{w}{2} \left\{ \frac{y-\alpha^1}{\alpha^2-\alpha^1} \right\} & \text{if } \alpha^1 \leq y \leq \alpha^2 \\ \frac{w}{2} + \frac{w}{2} \left\{ \frac{y-\alpha^2}{\alpha^3-\alpha^2} \right\} & \text{if } \alpha^2 \leq y \leq \alpha^3 \\ w & \text{if } \alpha^3 \leq y \leq \alpha^4 \\ w + \frac{w}{2} \left\{ \frac{y-\alpha^4}{\alpha^5-\alpha^4} \right\} & \text{if } \alpha^4 \leq y \leq \alpha^5 \\ \frac{w}{2} \left\{ \frac{\alpha^6-y}{\alpha^6-\alpha^5} \right\} & \text{if } \alpha^5 \leq y \leq \alpha^6 \\ 0 & \text{Otherwise} \end{cases}$$

is called generalized hexagonal fuzzy numbers. If w = 1, then it is called a normal hexagonal fuzzy number.

3. Properties of Hexagonal Fuzzy Numbers (HFNs): let $\tilde{A} = (\alpha^1, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6; w_1)$ and $\tilde{B} = (\beta^1, \beta^2, \beta^3, \beta^4, \beta^5, \beta^6; w_2)$ be any two HFNs. then
$\tilde{A} + \tilde{B} = (\alpha^1 + \beta^1, \alpha^2 + \beta^2, \alpha^3 + \beta^3, \alpha^4 + \beta^4, \alpha^5 + \beta^5, \alpha^6 + \beta^6; min(w_1, w_2))$
$\tilde{A} - \tilde{B} = (\alpha^1 - \beta^6, \alpha^2 - \beta^5, \alpha^3 - \beta^4, \alpha^4 - \beta^3, \alpha^5 - \beta^2, \alpha^6 - \beta^1; min(w_1, w_2))$
$\sigma \tilde{A} = (\sigma\alpha^1, \sigma\alpha^2, \sigma\alpha^3, \sigma\alpha^4, \sigma\alpha^5, \sigma\alpha^6)$, where σ is any constant.

4. Proposed Ranking Function

The ranking function is used to compare the fuzzy numbers. Assuming that the natural order is preserved, the ranking function $\mathfrak{R}: T(\mathbb{R}) \rightarrow \mathbb{R}$ defined on the set of real numbers maps each fuzzy number into a real number where $T(\mathbb{R})$ is the set of fuzzy numbers.
A new ranking function is proposed for generalized hexagonal fuzzy numbers based on the centroid and incentre points. Let $\tilde{A} = (\alpha^1, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6; w)$ be any GHFN represented by hexagonal AGIJHF as given in Fig. 1. In this ranking function, a hexagonal AGIJHF is divided into three quadrilaterals, namely ABCG, GCDH, and HDEF.
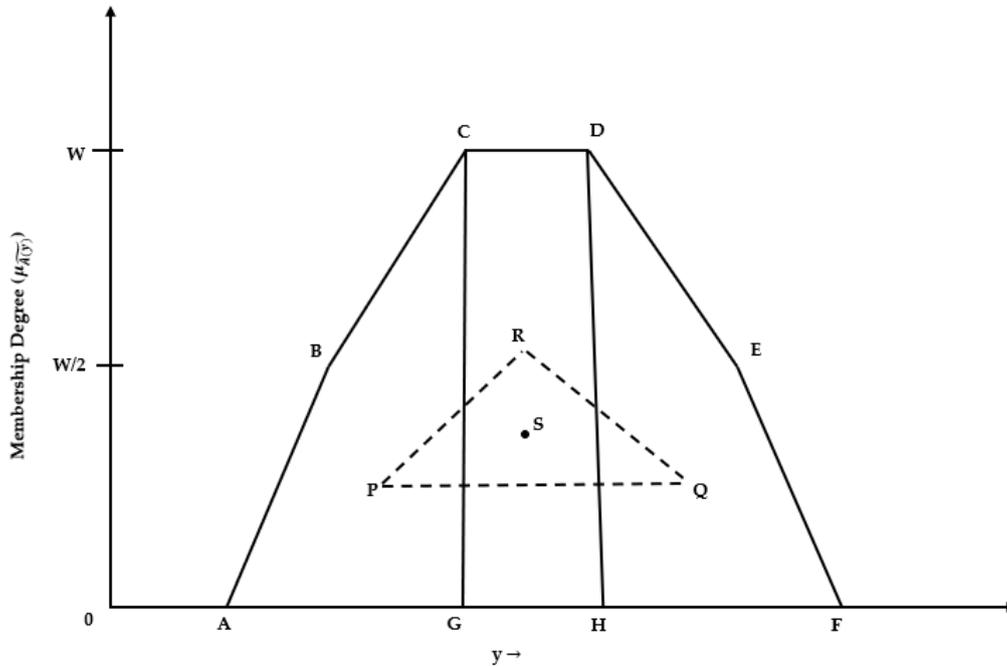
Figure 1: *Representation of hexagonal fuzzy number*

The vertices of ABGC is $A(\alpha_1, 0)$, $B(\alpha_2, \frac{w}{2})$, $C(\alpha_3, w)$, $G(\alpha_3, 0)$.
The vertices of GCDH is $G(\alpha_3, 0)$, $C(\alpha_3, w)$, $D(\alpha_4, w)$, $H(\alpha_4, 0)$.
The vertices of HDEF is $H(\alpha_4, 0)$, $D(\alpha_4, w)$, $E(\alpha_5, \frac{w}{2})$, $F(\alpha_6, 0)$.
Now, let P, Q, and R give the ABCG, HDEF, and GCDH centroids. The coordinates of P, Q, and R
is $(\frac{\alpha_1+\alpha_2+2\alpha_3}{4}, \frac{3w}{8})$, $(\frac{2\alpha_4+\alpha_5+\alpha_6}{4}, \frac{3w}{8})$, and $(\frac{\alpha_3+\alpha_4}{2}, \frac{w}{2})$ respectively .
Now, adjoining the centroids P, Q, and R, a triangle PQR is formed as given in Fig.2. Let S be the
incentre point of the triangle PQR.
The coordinates of S are $(I_x, I_Y) = (\frac{cx_1+bx_2+ax_3}{a+b+c}, \frac{cy_1+by_2+ay_3}{a+b+c})$

where a = $\frac{(2\alpha_4+\alpha_5+\alpha_6-2\alpha_3-\alpha_2-\alpha_1)}{4}$, b = $\frac{(4(2\alpha_4-\alpha_1-\alpha_2)^2+w^2)^{\frac{1}{2}}}{8}$, and c = $\frac{(4(2\alpha_3-\alpha_5-\alpha_6)^2+w^2)^{\frac{1}{2}}}{8}$.
Here in the incentre point (I_x, I_Y), the x-coordinate value I_x provides the ranking function such
that $I_x \in (\alpha_1, \alpha_6)$
Hence ranking of $\tilde{A}$ is given by $\mathfrak{R}(\tilde{A}) = I_x = (\frac{cx_1+bx_2+ax_3}{a+b+c})$

5.   Properties of Ranking functions

let $\tilde{A} = (\alpha^1, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6; w_1)$ and $\tilde{B} = (\beta^1, \beta^2, \beta^3, \beta^4, \beta^5, \beta^6; w_2)$ be any two HFNs. Then the
properties of the ranking function are given as:
$\tilde{A} \leq \tilde{B} \Longleftrightarrow \mathfrak{R}(\tilde{A}) \leq \mathfrak{R}(\tilde{B})$
$\tilde{A} \equiv \tilde{B} \Longleftrightarrow \mathfrak{R}(\tilde{A}) \equiv \mathfrak{R}(\tilde{B})$
$\tilde{A} \geq \tilde{B} \Longleftrightarrow \mathfrak{R}(\tilde{A}) \geq \mathfrak{R}(\tilde{B})$

6.   Mathematical model for FMOTP with Hexagonal fuzzy numbers

The FMOTP with k objectives in mathematical form is given as:
$(\tilde{T}_k)$ Min $\tilde{Z}_k(y) = \Sigma_{i=1}^{m}\Sigma_{j=1}^{n}\tilde{a}_{ij}^{(k)}y_{ij}$ for k = 1,2,.....
Subject to
$\Sigma_{i=1}^{m}y_{ij} = \tilde{d}_j$: for fixed j=1,2,.......n
$\Sigma_{j=1}^{n}y_{ij} = \tilde{s}_i$: for fixed i=1,2,.......,m

$y_{ij} \geq 0$
Where,

$\tilde{s}_i$ = the product's fuzzy availability at the i-th source

$\tilde{d}_j$ = the product's fuzzy requirement at the j-th destinations

$\tilde{a}_{ij}^{(k)}$ = the fuzzy cost for transporting one unit of the given product from i-th source to j-th destination of k-th objective

y_{ij} = The transported quantity of product from i-th source to j-th destination.

here $\tilde{a}_{ij} = (a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4, a_{ij}^5, a_{ij}^6)$, $\tilde{s}_i = (s_i^1, s_i^2, s_i^3, s_i^4, s_i^5, s_i^6)$, $\tilde{d}_j = (d_j^1, d_j^2, d_j^3, d_j^4, d_j^5, d_j^6)$ are the normal hexagonal fuzzy numbers.

If k=1 then FMOTP is called fuzzy single-objective transportation problem.

7. Efficient Solution

A feasible solution $Y^0 = \{y_{ij}^0, i = 1, 2, ..., m, j = 1, 2, ..., n\}$ is called an efficient solution to the problem (T) is there does not exist any feasible solution Y of MOTP such that $Z_1(Y) \leq Z_1(Y^0)$ and $Z_2(Y) \leq Z_2(Y^0)$.

8. Our Proposed Approach

The proposed algorithm obtains the Compromise efficient fuzzy solution of FMOTP in which some parameters are hexagonal fuzzy numbers. The proposed approach steps are as follows:

Step I: First, the FMOTP $(\tilde{T}_k)$ is represented in tabular form as given in Table 1.

Table 1: *Tabular representation of FMOTP*

Destination → source ↓	A_1	A_2	......	A_n	Availability $\{s_i\}$
B_1	$\tilde{a}_{11}^{(1)}$ $\tilde{a}_{11}^{(2)}$: $\tilde{a}_{11}^{(k)}$	$\tilde{a}_{12}^{(1)}$ $\tilde{a}_{12}^{(2)}$: $\tilde{a}_{12}^{(k)}$	....	$\tilde{a}_{1n}^{(1)}$ $\tilde{a}_{1n}^{(2)}$: $\tilde{a}_{1n}^{(k)}$	$\tilde{s}_1$
B_2	$\tilde{a}_{21}^{(1)}$ $\tilde{a}_{21}^{(2)}$: $\tilde{a}_{21}^{(k)}$	$\tilde{a}_{22}^{(1)}$ $\tilde{a}_{22}^{(2)}$: $\tilde{a}_{22}^{(k)}$	....	$\tilde{a}_{2n}^{(1)}$ $\tilde{a}_{2n}^{(2)}$: $\tilde{a}_{2n}^{(k)}$	$\tilde{s}_2$
:	:	:	:	:	:
B_m	$\tilde{a}_{m1}^{(1)}$ $\tilde{a}_{m1}^{(2)}$: $\tilde{a}_{m1}^{(k)}$	$\tilde{a}_{m2}^{(1)}$ $\tilde{a}_{m2}^{(2)}$: $\tilde{a}_{m2}^{(k)}$	....	$\tilde{a}_{mn}^{(1)}$ $\tilde{a}_{mn}^{(2)}$: $\tilde{a}_{mn}^{(k)}$	$\tilde{s}_m$
Requirement (d_j)	$\tilde{d}_1$	$\tilde{d}_2$	......	$\tilde{d}_n$	

Where $\tilde{a}_{ij}^{(k)}$, $\tilde{s}_i$, and $\tilde{d}_j$ $\forall 1 \leq i \leq m, 1 \leq j \leq n$ are represented by hexagonal fuzzy numbers and written in the form $\tilde{a}_{ij}^{(k)} = (\alpha_{ij}^{1(k)}, \alpha_{ij}^{2(k)}, \alpha_{ij}^{3(k)}, \alpha_{ij}^{4(k)}, \alpha_{ij}^{5(k)}, \alpha_{ij}^{6(k)})$, $\tilde{s}_i = (s_i^1, s_i^2, s_i^3, s_i^4, s_i^5, s_i^6)$, and $\tilde{d}_j = (d_j^1, d_j^2, d_j^3, d_j^4, d_j^5, d_j^6)$.

Step II: The FMOTP is converted into FSOTP using the geometric mean. The FSOTP $\tilde{T}$ is given in the form:

$(\tilde{T})$ Min $\tilde{Z}(y) = \Sigma_{i=1}^{m}\Sigma_{j=1}^{n}\tilde{b}_{ij}y_{ij}$ for k = 1,2,.....

Subject to

$\Sigma_{i=1}^{m}y_{ij} = \tilde{d}_j$: for fixed j=1,2,......,n

$\Sigma_{j=1}^{n}y_{ij} = \tilde{s}_i$: for fixed i=1,2,......,m

$y_{ij} \geq 0$

The fuzzy single-objective transportation problem (FSOTP) in tabular form is then represented in Table 2.

Table 2: *The converted FSOTP using geometric mean*

Destination → source ↓	A_1	A_2	......	A_n	Availability $\{s_i\}$
B_1	$\tilde{b}_{11}$	$\tilde{b}_{12}$	......	$\tilde{b}_{1n}$	$\tilde{s}_1$
B_2	$\tilde{b}_{21}$	$\tilde{b}_{22}$	......	$\tilde{b}_{2n}$	$\tilde{s}_2$
:	:	:	......	:	
B_m	$\tilde{b}_{m1}$	$\tilde{b}_{m2}$	......	$\tilde{b}_{mn}$	$\tilde{s}_m$
Requirement $(\tilde{d}_j)$	$\tilde{d}_1$	$\tilde{d}_2$	......	$\tilde{d}_n$	

Where, $\forall i, j; 1 \leq i \leq m, 1 \leq j \leq n$ $\tilde{b}_{ij}$ is a hexagonal fuzzy number and written in the form, $\tilde{b}_{ij} = (\beta_{ij}^1, \beta_{ij}^2, \beta_{ij}^3, \beta_{ij}^4, \beta_{ij}^5, \beta_{ij}^6)$.

Here β_{ij}^1 is geometric mean of $(\alpha_{ij}^{1(1)}, \alpha_{ij}^{1(2)}, ..., \alpha_{ij}^{1(k)})$ so

$\beta_{ij}^1 = (\prod(\alpha_{ij}^{1(1)}, \alpha_{ij}^{1(2)}, ..., \alpha_{ij}^{1(k)}))^{\frac{1}{k}}$

Similarly β_{ij}^2 is geometric mean of $(\alpha_{ij}^{2(1)}, \alpha_{ij}^{2(2)}, ..., \alpha_{ij}^{2(k)})$ so

$\beta_{ij}^2 = (\prod(\alpha_{ij}^{2(1)}, \alpha_{ij}^{2(2)}, ..., \alpha_{ij}^{2(k)}))^{\frac{1}{k}}$,

β_{ij}^3 is geometric mean of $(\alpha_{ij}^{3(1)}, \alpha_{ij}^{3(2)}, ..., \alpha_{ij}^{3(k)})$ so

$\beta_{ij}^3 = (\prod(\alpha_{ij}^{3(1)}, \alpha_{ij}^{3(2)}, ..., \alpha_{ij}^{3(k)}))^{\frac{1}{k}}$,

β_{ij}^4 is geometric mean of $(\alpha_{ij}^{4(1)}, \alpha_{ij}^{4(2)}, ..., \alpha_{ij}^{4(k)})$ so

$\beta_{ij}^4 = (\prod(\alpha_{ij}^{4(1)}, \alpha_{ij}^{4(2)}, ..., \alpha_{ij}^{4(k)}))^{\frac{1}{k}}$,

β_{ij}^5 is geometric mean of $(\alpha_{ij}^{5(1)}, \alpha_{ij}^{5(2)}, ..., \alpha_{ij}^{5(k)})$ so

$\beta_{ij}^5 = (\prod(\alpha_{ij}^{5(1)}, \alpha_{ij}^{5(2)}, ..., \alpha_{ij}^{5(k)}))^{\frac{1}{k}}$,

β_{ij}^6 is geometric mean of $(\alpha_{ij}^{6(1)}, \alpha_{ij}^{6(2)}, ..., \alpha_{ij}^{6(k)})$ so

$\beta_{ij}^6 = (\prod(\alpha_{ij}^{6(1)}, \alpha_{ij}^{6(2)}, ..., \alpha_{ij}^{6(k)}))^{\frac{1}{k}}$.

Step III: FSOTP $\tilde{T}$ is converted into CSOTP (T) using the proposed ranking function in section 4. The Converted CSOTP is given in Table 3.

Table 3: *The transformed CSOTP using ranking Function*

Destination → source ↓	A_1	A_2	......	A_n	Availability $\{s_i\}$
B_1	b_{11}	b_{12}	......	b_{1n}	s_1
B_2	b_{21}	b_{22}	......	b_{2n}	s_2
:	:	:	......	:	
B_m	b_{m1}	b_{m2}	......	b_{mn}	s_m
Requirement (d_j)	d_1	d_2	......	d_n	

Here $b_{ij} = \Re(\tilde{b}_{ij}) \forall i, j; 1 \leq i \leq m, 1 \leq j \leq n, s_i = \Re(\tilde{s}_i)\forall i; 1 \leq i \leq m$ and $d_j = \Re(\tilde{d}_j)\forall j; 1 \leq j \leq n$

Step IV: First check whether or not the CSOTP (T) is balanced. if it is balanced TP go to the next step. if it is not balanced make the transportation problem balanced.

To make the transportation problem balanced there are two cases:

(a) If availabilities exceed the requirements then add a dummy column to satisfy the excess quantity of availabilities. The transportation cost of elements in the dummy column is assumed to be zero.

(b) If requirements exceed the availabilities then add a dummy row to satisfy the excess quantity of requirements. The transportation cost of elements in the dummy row is assumed to be zero.

Step V: In this step, the balanced TP matrix is reduced to at least one zero-entry cell matrix. to reduce the matrix steps are the following:

(i) The lowest cost of each row of balanced TP is subtracted from each row.

(ii) The lowest cost of each column of balanced TP is subtracted from each column.

Ensure that every row and column of balanced TP has one zero-entry cell.

Step VI: Put the zero-entry cell in the square bracket and count the no. of zero-entry cells in the corresponding row and column of the selected zero-entry cells.

Allocate the maximum possible allocation to that cell with the minimum number of zeros in that corresponding row and column. If there is a tie situation, the following rule is applicable:

(a) then, a maximum allocation will be allocated to the cell with the lowest objective function value.

(b) In the case of another tie, the maximum allocation will be allocated to the zero-entry cell whose objective function values sum to the most in the corresponding row/column.

(c) If there is also a tie, then select the chosen zero-entry cells in (b) for which the maximum possible allocation is allocated.

Ignore that row/column where the requirement/availability is satisfied.

Step VII: Repeat the process until all requirements/availability are not satisfied.

The flowchart of the proposed algorithm is given in Figure 1.

To elaborate the proposed approach, two numerical examples are considered.

Example 1: A FMOTP with two objective functions is considered as given in Table 4.

Table 4: *Tabular representation of FMOTP*

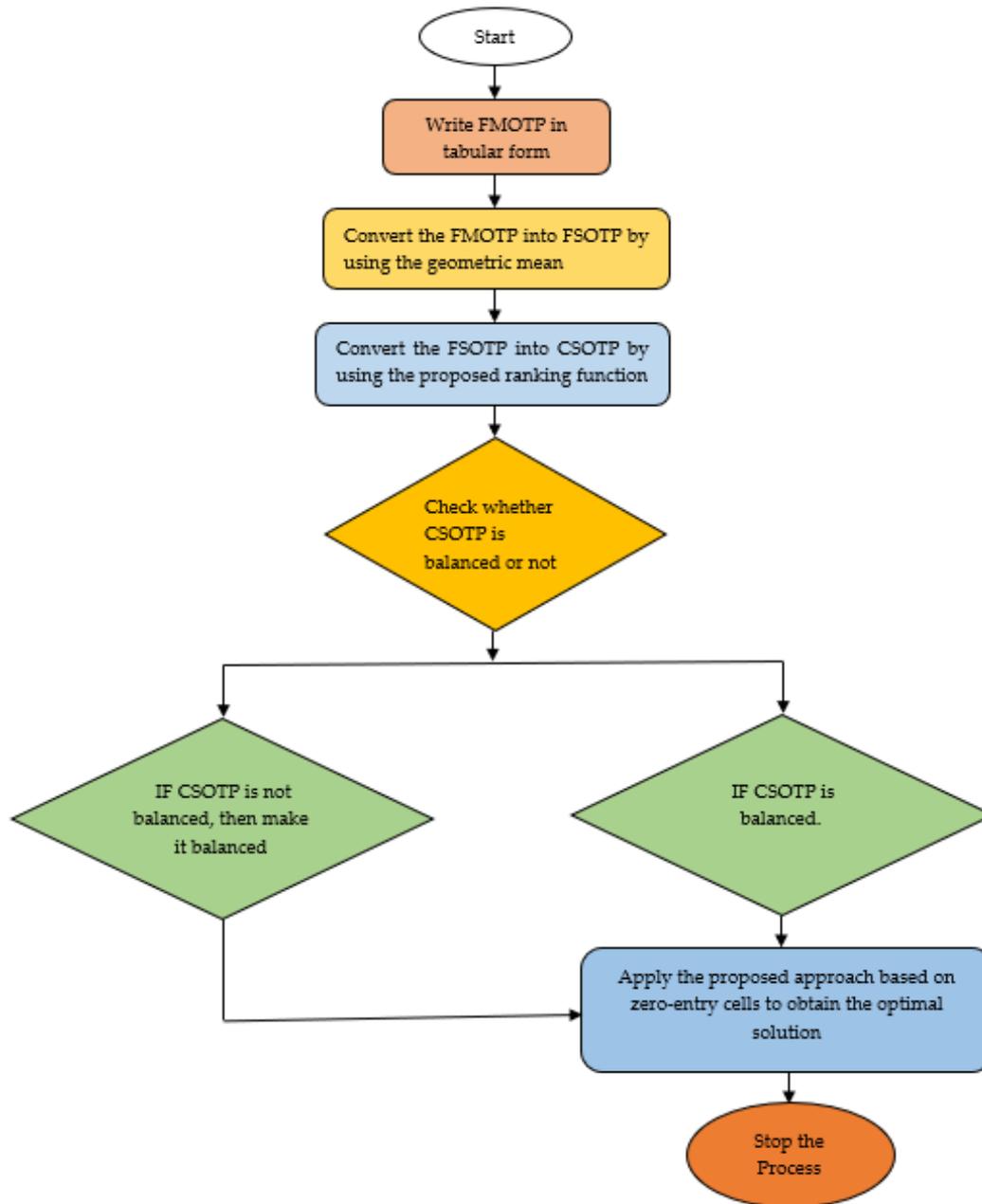| Destination $\rightarrow$ source $\downarrow$	A_1	A_2	A_3	A_4	Availability $\{s_i\}$
B_1	$\tilde{a}_{11}^{(1)}$ $\tilde{a}_{11}^{(2)}$	$\tilde{a}_{12}^{(1)}$ $\tilde{a}_{12}^{(2)}$	$\tilde{a}_{13}^{1}$ $\tilde{a}_{13}^{(2)}$	$\tilde{a}_{14}^{(1)}$ $\tilde{a}_{14}^{(2)}$	$\tilde{s}_1$
B_2	$\tilde{a}_{21}^{(1)}$ $\tilde{a}_{21}^{(2)}$	$\tilde{a}_{22}^{(1)}$ $\tilde{a}_{22}^{(2)}$	$\tilde{a}_{23}^{(1)}$ $\tilde{a}_{23}^{(2)}$	$\tilde{a}_{24}^{(1)}$ $\tilde{a}_{24}^{(2)}$	$\tilde{s}_2$
B_3	$\tilde{a}_{31}^{(1)}$ $\tilde{a}_{31}^{(2)}$	$\tilde{a}_{32}^{(1)}$ $\tilde{a}_{32}^{(2)}$	$\tilde{a}_{33}^{(1)}$ $\tilde{a}_{33}^{(2)}$	$\tilde{a}_{34}^{(1)}$ $\tilde{a}_{34}^{(2)}$	$\tilde{s}_3$
B_4	$\tilde{a}_{41}^{(1)}$ $\tilde{a}_{41}^{(2)}$	$\tilde{a}_{42}^{(1)}$ $\tilde{a}_{42}^{(2)}$	$\tilde{a}_{43}^{(1)}$ $\tilde{a}_{43}^{(2)}$	$\tilde{a}_{44}^{(1)}$ $\tilde{a}_{44}^{(2)}$	$\tilde{s}_4$
Requirement (d_j)	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{d}_3$	$\tilde{d}_4$	

Figure 2: *Flowchart of the proposed approach*

Coefficient matrix for a first objective function

$$C_1 = \begin{bmatrix} \tilde{a}_{11}^{(1)} & \tilde{a}_{12}^{(1)} & \tilde{a}_{13}^{(1)} & \tilde{a}_{14}^{(1)} \\ \tilde{a}_{21}^{(1)} & \tilde{a}_{22}^{(1)} & \tilde{a}_{23}^{(1)} & \tilde{a}_{24}^{(1)} \\ \tilde{a}_{31}^{(1)} & \tilde{a}_{32}^{(1)} & \tilde{a}_{33}^{(1)} & \tilde{a}_{34}^{(1)} \\ \tilde{a}_{41}^{(1)} & \tilde{a}_{42}^{(1)} & \tilde{a}_{43}^{(1)} & \tilde{a}_{44}^{(1)} \end{bmatrix}$$

$$= \begin{bmatrix} (2,3,5,7,8,10) & (3,5,7,9,10,12) & (3,7,11,14,15,16) & (7,9,10,12,15,20) \\ (4,6,7,10,12,15) & (3,6,8,12,14,15) & (8,10,11,13,14,16) & (6,8,9,11,13,18) \\ (7,8,12,13,15,19) & (9,10,11,13,17,19) & (2,4,5,8,12,16) & (5,8,9,12,14,20) \\ (3,4,7,9,13,17) & (6,9,11,14,18,22) & (2,5,6,8,11,15) & (7,11,13,16,17,21) \end{bmatrix}$$

Coefficient matrix for the second objective function

$$
C_2 = \begin{bmatrix}
\tilde{a}_{11}^{(2)} & \tilde{a}_{12}^{(2)} & \tilde{a}_{13}^{(2)} & \tilde{a}_{14}^{(2)} \\
\tilde{a}_{21}^{(2)} & \tilde{a}_{22}^{(2)} & \tilde{a}_{23}^{(2)} & \tilde{a}_{24}^{(2)} \\
\tilde{a}_{31}^{(2)} & \tilde{a}_{32}^{(2)} & \tilde{a}_{33}^{(2)} & \tilde{a}_{34}^{(2)} \\
\tilde{a}_{41}^{(2)} & \tilde{a}_{42}^{(2)} & \tilde{a}_{43}^{(2)} & \tilde{a}_{44}^{(2)}
\end{bmatrix}
$$

$$
= \begin{bmatrix}
(5,9,12,13,15,19) & (4,7,10,12,14,16) & (3,5,7,10,12,14) & (2,3,4,6,7,10) \\
(6,7,9,10,13,14) & (8,10,11,13,14,16) & (9,10,12,15,18,22) & (2,4,5,8,11,17) \\
(8,10,12,15,17,20) & (3,5,8,10,11,15) & (6,7,9,13,15,16) & (5,6,8,11,12,16) \\
(6,9,11,14,15,19) & (5,8,9,13,17,22) & (8,11,12,16,18,23) & (2,5,7,10,14,17)
\end{bmatrix}
$$

Availabilities:
$s_1 = (4,6,7,10,12,14)$, $s_2 = (5,7,10,13,16,20)$, $s_3 = (6,8,10,12,14,15)$, $s_4 = (7,9,12,14,15,18)$.
Requirements:
$d_1 = (6,7,9,12,14,18)$, $d_2 = (3,5,8,9,11,15)$, $d_3 = (9,11,12,15,17,18)$, $d_4 = (4,7,10,13,15,16)$
Step I: In this step, the FMOTP is transformed into FSOTP using the geometric mean. The converted FSOTP is given in Table 5.

Table 5: *The converted FSOTP using geometric mean*

Destination → source ↓	A_1	A_2	A_3	A_4	Availability $\{s_i\}$
B_1	$\tilde{b}_{11}$	$\tilde{b}_{12}$	$\tilde{b}_{13}$	$\tilde{b}_{14}$	$\tilde{s}_1$
B_2	$\tilde{b}_{21}$	$\tilde{b}_{22}$	$\tilde{b}_{23}$	$\tilde{b}_{24}$	$\tilde{s}_2$
B_3	$\tilde{b}_{31}$	$\tilde{b}_{32}$	$\tilde{b}_{33}$	$\tilde{b}_{34}$	$\tilde{s}_3$
B_4	$\tilde{b}_{41}$	$\tilde{b}_{42}$	$\tilde{b}_{43}$	$\tilde{b}_{44}$	$\tilde{s}_4$
Requirement $(\tilde{d}_j)$	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{d}_3$	$\tilde{d}_4$	

Here $\tilde{a}_{11}^{(1)} = (2,3,5,7,8,10)$ and $\tilde{a}_{11}^{(2)} = (5,9,12,13,15,19)$

For FSOTP, the objective function values
$\tilde{b}_{11} = (\sqrt{2 \times 5}, \sqrt{3 \times 9}, \sqrt{5 \times 12}, \sqrt{7 \times 13}, \sqrt{8 \times 13}, \sqrt{10 \times 19}) = (3.16, 5.2, 7.75, 9.54, 10.95, 13.78)$
$\tilde{b}_{12} = (3.46, 5.92, 8.37, 10.39, 11.83, 13.86)$, $\quad \tilde{b}_{13} = (3, 5.92, 8.77, 11.83, 13.42, 14.97)$,
$\tilde{b}_{14} = (3.74, 5.2, 6.32, 8.49, 10.25, 14.14)$, $\quad \tilde{b}_{21} = (4.9, 6.48, 7.94, 10, 12.49, 14.49)$,
$\tilde{b}_{22} = (4.9, 7.75, 9.38, 12.49, 14, 15.49)$, $\quad \tilde{b}_{23} = (8.49, 10, 11.49, 13.96, 15.87, 18.76)$,
$\tilde{b}_{24} = (3.46, 5.66, 6.71, 9.38, 11.96, 17.49)$, $\quad \tilde{b}_{31} = (7.48, 8.94, 12, 13.96, 15.97, 19.49)$,
$\tilde{b}_{32} = (5.2, 7.07, 9.38, 11.4, 13.67, 16.88)$, $\quad \tilde{b}_{33} = (3.46, 5.29, 6.71, 10.20, 13.42, 16)$,
$\tilde{b}_{34} = (5, 6.93, 8.49, 11.49, 12.96, 17.89)$, $\quad \tilde{b}_{41} = (4.24, 6, 8.77, 11.22, 13.96, 17.97)$,
$\tilde{b}_{42} = (5.48, 8.49, 9.95, 13.49, 17.49, 22)$, $\quad \tilde{b}_{43} = (4, 7.42, 8.49, 11.31, 14.07, 18.57)$,
$\tilde{b}_{44} = (3.74, 7.42, 9.54, 12.65, 15.43, 18.89)$
Availabilities:
$s_1 = (4,6,7,10,12,14)$, $s_2 = (5,7,10,13,16,20)$, $s_3 = (6,8,10,12,14,15)$, $s_4 = (7,9,12,14,15,18)$.
Requirements:
$d_1 = (6,7,9,12,14,18)$, $d_2 = (3,5,8,9,11,15)$, $d_3 = (9,11,12,15,17,18)$, $d_4 = (4,7,10,13,15,16)$

Now FMOTP is converted into CSOTP using the proposed ranking function.
Here $\tilde{b}_{11} = (\beta_{11}^1, \beta_{11}^2, \beta_{11}^3, \beta_{11}^4, \beta_{11}^5, \beta_{11}^6) = (3.16, 5.2, 7.75, 9.54, 10.95, 13.78)$
$\Re(\tilde{b}_{11}) = I_x = \left(\frac{cx_1 + bx_2 + ax_3}{a+b+c}\right)$
where a $= \frac{(2\beta_{11}^4 + \beta_{11}^5 + \beta_{11}^6 - 2\beta_{11}^3 - \beta_{11}^2 - \beta_{11}^1)}{4} = 4.99$, b $= \frac{(4(2\beta_{11}^4 - \beta_{11}^1 - \beta_{11}^2)^2 + w^2)^{\frac{1}{2}}}{8} = 2.68$,
and c $= \frac{(4(2\beta_{11}^3 - \beta_{11}^5 - \beta_{11}^6)^2 + w^2)^{\frac{1}{2}}}{8} = 2.32$
$x_1 = \frac{\beta_1 + \beta_2 + 2\beta_3}{4} = 5.92$, $x_2 = \frac{2\beta_4 + \beta_5 + \beta_6}{4} = 10.95$, and $x_3 = \frac{\beta_3 + \beta_4}{2} = 8.64$

$\Re(\tilde{b}_{11}) = \frac{2.32 \times 5.92 + 2.68 \times 10.95 + 4.99 \times 8.64}{4.99 + 2.68 + 2.32} = 8.64$

Similarly, all the fuzzy values $\tilde{a}_{ij}^{(k)}$ for $1 \leq j \leq 4, 1 \leq i \leq 4$ and $1 \leq k \leq 2$, availabilities $\tilde{s}_i$ for $1 \leq i \leq 4$, and requirements $\tilde{d}_j$ for $1 \leq j \leq 4$ can be converted in crisp values by using ranking function.

The CSOTP is given in Table 6.

Table 6: *The transformed Crisp SOTP using ranking function*

Destination → source ↓	A_1	A_2	A_3	A_4	Availability $\{s_i\}$
B_1	8.64	9.38	10.3	7.41	8.5
B_2	8.97	10.94	12.73	8.05	11.5
B_3	12.98	10.39	8.45	9.99	11
B_4	10	11.72	9.9	11.09	13
Requirement $(\tilde{d}_j)$	10.5	8.5	13.5	11.5	

Here $\Sigma_{i=1}^{m} s_i = \Sigma_{j=1}^{n} d_j = 44$. So it is a balanced transportation problem.

Step II: We now reduce the Balanced TP to a zero-entry cell matrix. The reduced zero-entry cell matrix is given in Table 7.

Table 7: *The CSOTP with zero-entry cells*

Destination → source ↓	A_1	A_2	A_3	A_4	Availability $\{s_i\}$
B_1	8.64	9.38	10.3	[7.41]	8.5
B_2	8.97	10.94	12.73	[8.05]	11.5
B_3	12.98	10.39	[8.45]	9.99	11
B_4	[10]	[11.72]	[9.9]	11.09	13
Requirement $(\tilde{d}_j)$	10.5	8.5	13.5	11.5	

In each row and column of the zero-entry cells, count the number of cells with zero entries.

The no. of zero-entry cells for the selected cell at b_{14} position = 2
The no. of zero-entry cells for the selected cell at b_{24} position = 2
The no. of zero-entry cells for the selected cell at b_{32} position = 2
The no. of zero-entry cells for the selected cell at b_{41} position = 3
The no. of zero-entry cells for the selected cell at b_{42} position = 3
The no. of zero-entry cells for the selected cell at b_{43} position = 3
Here cells at b_{14}, b_{24}, b_{32} positions has the same number of zero-entry cells. So choose that cell which has minimum objective function value.

Cell at b_{14} position has the minimum objective value (7.41). so we allocate the maximum possible allocation, min(8.5,11.5) = 8.5 to the cell at b_{14} position as shown in Table 8.

Table 8: *The CSOTP with the first allocation*

Destination → source ↓	A_1	A_2	A_3	A_4	Availability $\{s_i\}$
B_1	8.64	9.38	10.3	7.41 (8.5)	8.5
B_2	8.97	10.94	12.73	8.05	11.5
B_3	12.98	10.39	8.45	9.99	11
B_4	10	11.72	9.9	11.09	13
Requirement $(\tilde{d}_j)$	10.5	8.5	13.5	11.5	

Step III: Repeat the process to satisfy all the availabilities and requirements and to make the possible allocation in the remaining rows and columns, hence the 2nd, 3rd, 4th, 5th, 6th, and 7th

allocations as $3,11,8.5,2,8.5,2.5$ at cells $b_{14}, b_{24}, b_{33}, b_{41}, b_{42}, b_{43}$ respectively. the final optimum allocation is given in Table 9.

Table 9: *final Optimum allocation table for Example 1*

Destination → source ↓	A_1	A_2	A_3	A_4	Availability $\{s_i\}$
B_1	8.64	9.38	10.3	7.41(8.5)	8.5
B_2	8.97(8.5)	10.94	12.73	8.05(3)	11.5
B_3	12.98	10.39	8.45(11)	9.99	11
B_4	10(2)	11.72(8.5)	9.9(2.5)	11.09	13
Requirement $\tilde{(d_j)}$	10.5	8.5	13.5	11.5	

Example 2: A fuzzy transportation problem with all the fuzzy parameters as hexagonal fuzzy numbers is given in Table 10.

Table 10: *Tabular representation of FSOTP*

Destination → source ↓	A_1	A_2	A_3	A_4	Availability $\{s_i\}$
B_1	$\tilde{b}_{11}$	$\tilde{b}_{12}$	$\tilde{b}_{13}$	$\tilde{b}_{14}$	$\tilde{s}_1$
B_2	$\tilde{b}_{21}$	$\tilde{b}_{22}$	$\tilde{b}_{23}$	$\tilde{b}_{24}$	$\tilde{s}_2$
B_3	$\tilde{b}_{31}$	$\tilde{b}_{32}$	$\tilde{b}_{33}$	$\tilde{b}_{34}$	$\tilde{s}_3$
Requirement $\tilde{(d_j)}$	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{d}_3$	$\tilde{d}_4$	

where $\tilde{b}_{11} = (1,2,3,4,5,6)$, $\tilde{b}_{12} = (1,3,5,6,8,10)$, $\tilde{b}_{13} = (9,11,13,16,18,20)$, $\tilde{b}_{14} =(2,4,6,8,10,12)$
$\tilde{b}_{21} = (1,3,5,7,8,10)$, $\tilde{b}_{22} = (0,2,4,6,8,10)$ $\tilde{b}_{23} = (2,3,4,5,6,7)$, $\tilde{b}_{24} =(3,6,9,10,13,16)$
$\tilde{b}_{31} = (3,5,7,8,10,12)$, $\tilde{b}_{32} = (4,8,12,16,20,24)$ $\tilde{b}_{33} = (3,4,5,6,7,8)$, $\tilde{b}_{34} =(6,8,10,11,13,15)$
Availabilities:
$s_1 = ((1,3,5,6,8,10)$, $s_2 = (2,4,6,7,9,11)$, $s_3 = (3,6,9,17,20,23)$
Requirements:
$d_1 = (5,7,9,10,12,14)$, $d_2 = (1,3,5,6,8,10)$, $d_3 = (1,2,3,4,5,6)$, $d_4 = (2,4,6,7,9,11)$ The Fuzzy SOTP is transformed into Crisp SOTP using the proposed ranking Function in Table 11.

Table 11: *Tabular representation of Crisp SOTP*

Destination → source ↓	A_1	A_2	A_3	A_4	Availability $\{s_i\}$
B_1	3.5	5.5	14.5	7	5.5
B_2	6	5	4.5	9.5	6.5
B_3	7.5	14	5.5	10.5	13
Requirement $\tilde{(d_j)}$	9.5	5.5	3.5	6.5	

Apply the proposed approach the final optimum allocation is obtained.

Table 12: *Optimum allocation table for Example 2*

Destination → source ↓	A_1	A_2	A_3	A_4	Availability $\{s_i\}$
B_1	3.5(5.5)	5.5	14.5	7	5.5
B_2	6	5(5.5)	4.5(1)	9.5	6.5
B_3	7.5(4)	14	5.5(2.5)	10.5(6.5)	13
Requirement $\tilde{(d_j)}$	9.5	5.5	3.5	6.5	

9. RESULTS AND DISCUSSION

This section analyzes the results obtained from two illustrative examples using the proposed approach. The discussion is structured into two parts: (i) Result Analysis and (ii) Physical Interpretation.

9.1. Result analysis

This section presents the analysis of the results obtained using the proposed approach on two illustrative examples. The final allocation details, fuzzy and crisp compromise efficient solutions, are presented in Tables 13 and 14. The comparative results with existing methods are shown in Table 15.

Table 13: *Final Solution table for Example 1*

Obtained Allocations	$y_{11} = 0, y_{12} = 0, y_{13} = 0, y_{14} = 8.5$
	$y_{21} = 8.5, y_{22} = 0, y_{23} = 0, y_{24} = 3,$
	$y_{31} = 0, y_{32} = 0, y_{33} = 11, y_{34} = 0,$
	$y_{41} = 2, y_{42} = 8.5, y_{43} = 2.5, y_{44} = 0.$
Fuzzy Compromise efficient solution of MOTP	$[(195.5, 292.5, 349, 465, 607, 786),$
	$(214.5, 287.5, 353, 481.5, 587.5, 713.5)]$
Crisp Compromise efficient solution of MOTP	$(407, 417.25)$
Nature of Crisp Compromise Crisp Solution	Non-Degenerate

Table 14: *Final Solution table for Example 2*

Obtained Allocations	$y_{11} = 5.5, y_{12} = 0, y_{13} = 0, y_{14} = 0$
	$y_{21} = 0, y_{22} = 5.5, y_{23} = 1, y_{24} = 0,$
	$y_{31} = 4, y_{32} = 0, y_{33} = 2.5, y_{34} = 6.5,$
Fuzzy Compromise efficient solution of MOTP	$(66, 107, 148, 178.5, 219.5, 260.5)$
Crisp Compromise efficient solution of MOTP	(163.25)
Nature of Crisp Compromise Crisp Solution	Non-Degenerate

The final solution tables for both examples show that the nature of the solution is nondegenerate and optimal.

Table 15: *The result comparison table*

	Method Name	Optimal Solution
Example 1	Sharma M.K. et. al. [19]	(1442.06, 1888.81)
	Proposed Method	(407, 417.25)
Example 2	Sahayasudha, A. et. al. [20]	662
	Proposed Method	163.25

For Example 1, the proposed method yielded a crisp compromise solution that is significantly lower than the values reported by Sharma M.K. et al. [19]. As shown in Table 15, the solution values obtained using the proposed approach yield a 71.78% reduction in the first objective and a 77.90% reduction in the second objective compared to the existing method. This considerable decrease in objective values indicates a more cost-effective and efficient allocation under the proposed model.
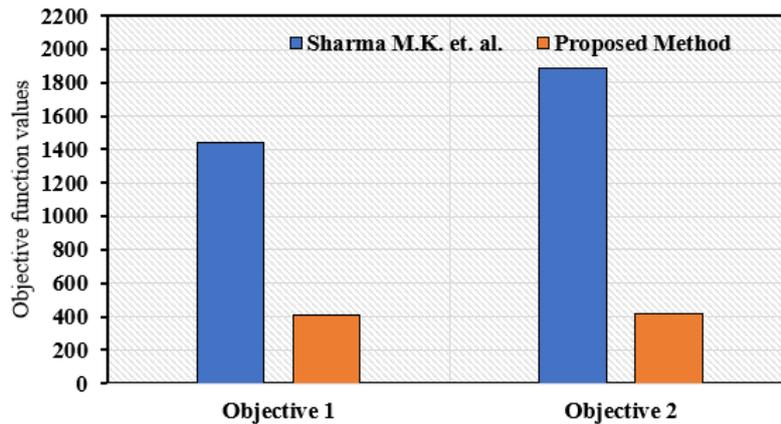
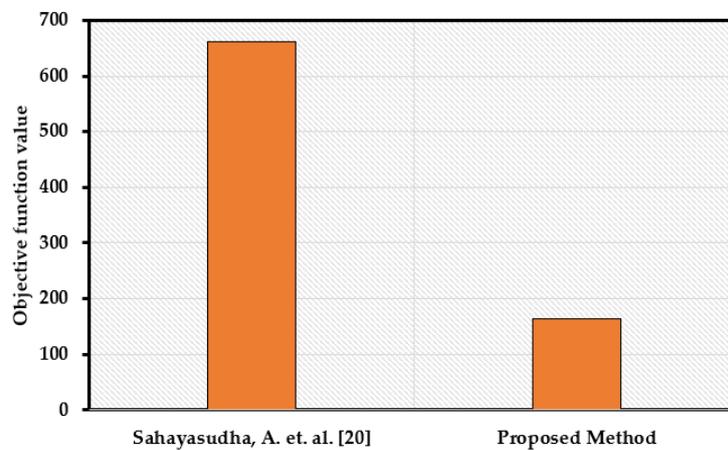**Figure 3:** *Result comparison for Example 1*

Figure 4: *Result comparison for Example 2*

Similarly, in Example 2, the solution obtained through the proposed method demonstrates a significant improvement compared to the method presented by Sahayasudha et al. [20]. Specifically, there is a 75.34% reduction in the overall objective value, as observed in Table 15. This further validates the strength of the proposed method in yielding optimized results even in more complex scenarios. A graphical representation of the observations is shown in Figures 3 and 4., clearly illustrating the improved performance of the proposed method across both examples. The observed reduction in objective values across the examples indicates that the proposed method provides consistent and efficient solutions, demonstrating its practical effectiveness in addressing fuzzy multi-objective transportation problems

9.2. Physical interpretation of the results

The solution obtained, as presented in Table 13 and Table 14 for example 1 and example 2 respectively, is physically interpreted as follows:

9.2.1 Interpretation of the results for Example 1

(I) For the first objective function:

Using the proposed approach, the minimum fuzzy transportation cost (FTC) is [(195.5,292.5,349,465,607, 786)]. It has the following physical interpretation:

(i) In the decision maker's estimation, the minimum transportation cost (MTC) will be greater than Rs. 195.5 and less than Rs. 786 units.

(ii) The decision-maker is 100% satisfied overall with the statement that transportation costs will be 349-465. It is easily seen that the optimal value of the first objective function is 407, which lies between 349 and 465.

(iii) The following values of the remaining minimum transportation cost can be used to determine the decision-maker's overall level of satisfaction:

If y is the MTC, then the overall decision-maker satisfaction level for y = $\mu_{\tilde{A}_1}(y) \times 100$

where

$$\mu_{\tilde{A_1}(y)} = \begin{cases} \frac{1}{2}\left\{\frac{y-195.5}{97}\right\} & \text{if } 195.5 \leq y \leq 292.5 \\ \frac{1}{2} + \frac{1}{2}\left\{\frac{y-292.5}{56.5}\right\} & \text{if } 292.5 \leq y \leq 349 \\ 1 & \text{if } 349 \leq y \leq 465 \\ 1 + \frac{1}{2}\left\{\frac{y-465}{142}\right\} & \text{if } 465 \leq y \leq 607 \\ \frac{1}{2}\left\{\frac{786-y}{179}\right\} & \text{if } 607 \leq y \leq 786 \\ 0 & \text{Otherwise} \end{cases}$$

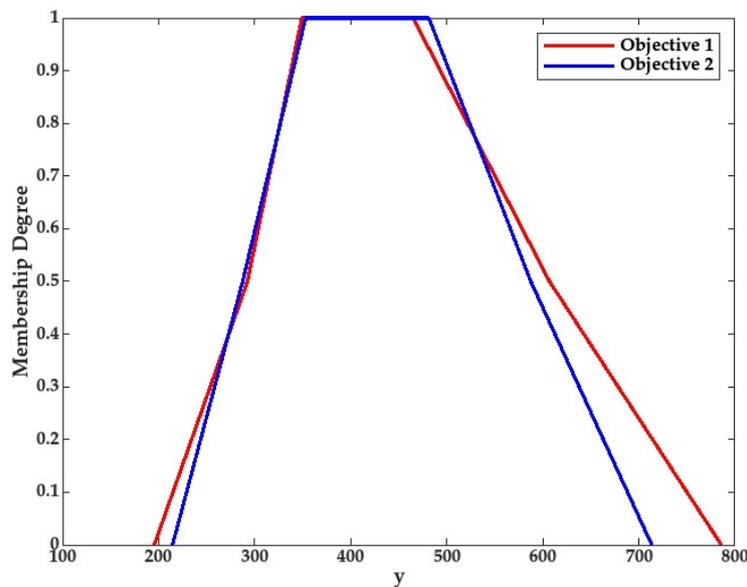Fig. 5 and Fig.6. shows the fuzzy optimal solution for example 1 and example 2.

Figure 5: *Fuzzy objective values graph for Example 1*

(II) For Second Objective Function:

Using the proposed approach, the minimum FTP is [(214.5,287.5,353,481.5,587.5,713.5)]. It has the following physical interpretation:

(i) The MTC, in the decision-maker's estimation, will be greater than Rs. 214.5 and less than Rs. 713.5 units.

(ii) The decision-maker is 100% satisfied overall with the statement that transportation costs will be 353.5-481.5. It is easily seen that the optimal value of the first objective function is 417.5, which lies between 353 and 481.5

(iii) The following values of the remaining minimum transportation cost can be used to determine the decision-maker's overall level of satisfaction:

If y is the MTC, then the overall level of satisfaction of the decision maker for y = $\mu_{\tilde{A}_2}(y) \times 100$. Here $\mu_{\tilde{A}_2}(y)$ is given as :

$$\mu_{A_2\tilde{(y)}} = \begin{cases} \frac{1}{2}\left\{\frac{y-214.5}{73}\right\} & \text{if } 214.5 \leq y \leq 287.5 \\ \frac{1}{2} + \frac{1}{2}\left\{\frac{y-287.5}{65.5}\right\} & \text{if } 287.5 \leq y \leq 353 \\ 1 & \text{if } 353 \leq y \leq 481.5 \\ 1 + \frac{1}{2}\left\{\frac{y-481.5}{106}\right\} & \text{if } 481.5 \leq y \leq 587.5 \\ \frac{1}{2}\left\{\frac{713.5-y}{126}\right\} & \text{if } 587.5 \leq y \leq 713.5 \\ 0 & \text{Otherwise} \end{cases}$$

used to determine the decision-maker's overall level of satisfaction.

9.2.2 Interpretation of the results for Example 2

Using the proposed algorithm, the minimum FTP is [(66,107,148,178.5,219.5,260.5)] It has the following physical interpretation:

(i) In the decision-maker's estimation, the MTC will be greater than Rs. 66 and less than Rs. 260.5 units.

(ii) The decision-maker is 100% satisfied overall with the statement that transportation costs will be 148-178.5. It is easily seen that the optimal value of the first objective function is 163.25, which lies between 148 and 178.5.
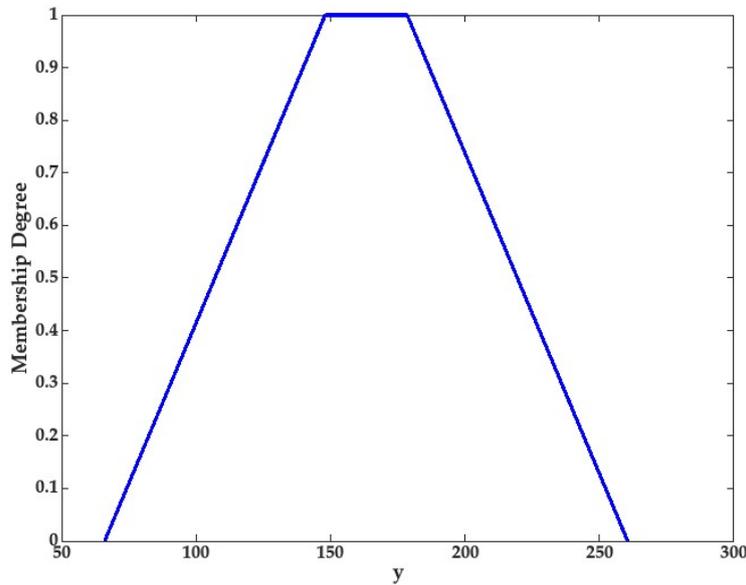

Figure 6: *Fuzzy objective values graph for Example 2*

(iii) The following values of the remaining minimum transportation cost can be used to determine the decision-maker's overall level of satisfaction:

If y is the MTC, then the overall level of satisfaction of the decision maker for y = $\mu_{\tilde{A}}(y) \times 100$
Where,

$$\mu_{A\tilde{(y)}} = \begin{cases} \frac{1}{2}\left\{\frac{y-66}{41}\right\} & \text{if } 66 \leq y \leq 107 \\ \frac{1}{2} + \frac{1}{2}\left\{\frac{y-107}{41}\right\} & \text{if } 107 \leq y \leq 148 \\ 1 & \text{if } 148 \leq y \leq 178.5 \\ 1 + \frac{1}{2}\left\{\frac{y-178.5}{41}\right\} & \text{if } 178.5 \leq y \leq 219.5 \\ \frac{1}{2}\left\{\frac{260.5-y}{41}\right\} & \text{if } 219.5 \leq y \leq 260.5 \\ 0 & \text{Otherwise} \end{cases}$$

10. Conclusion

A new approach is presented to solve the fuzzy multi-objective transportation problem in which all the transportation parameters are represented by generalized hexagonal fuzzy numbers. The geometric mean technique is applied to transform the FMOTP into a fuzzy single-objective transportation problem. The approach is based on a new ranking function and zero-entry cell technique. The ranking function is based on the centroid and incentre points defuzzify the FMOTP. The defuzzified value for each parameter lies within the range of the hexagonal fuzzy number. Then zero-entry cell technique is applied to the crisp single-objective transportation problem to get the optimal solution . Two numerical examples are solved to validate our proposed approach. The analysis and comparison of results with some other existing methods from the literature show that the proposed approach provides more optimized and better results than some other existing methods. The approach is simple to understand and easy to use in real-life situations in which the decision-makers are not sure about the precise value of transportation parameters and these parameters are given by HFNs. It can be used for many optimization problems, like as traveling salesman problems and assignment problems.

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